A MESH FREE NUMERICAL METHOD FOR ACOUSTIC WAVE PROPAGATION PROBLEMS IN PLANAR DOMAINS WITH CORNERS AND CRACKS

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The numerical solution of acoustic wave propagation problems in bounded 2D domains with corners or cracks is addressed. In particular, a modification of the Method of Fundamental Solutions (MFS) will be developed and applied to Boundary Value Problems (BVP) for the Helmholtz equation

$$\begin{cases} \Delta u + k^2 u = 0 & \text{in} \quad \Omega \\ \mathcal{B}u = 0 & \text{on} \quad \Gamma. \end{cases} \tag{1}$$

Here Ω is the (Lipschitz) domain with boundary Γ, $k > 0$ is the frequency of the problem, $\mathcal{B}$ is a boundary operator specifying the boundary conditions for the BVP and u is the unknown solution. Dirichlet, Neumann and mixed Dirichlet-Neumann boundary conditions for resonance and non-resonance frequencies will be simulated. In the resonance case the solution u corresponds to an eigenmode for the Laplace operator in Ω. In the non-resonance case direct scattering problems from plane and spherical incident waves will be simulated.

The classical MFS, e.g. [1], is a mesh-free and integration-free boundary collocation method, where the unknown solution u is approximated by superposition of fundamental solutions from the corresponding PDE, with source points (singularities) located in the exterior of the domain Ω. As it is well known, the solution of problem (1) exhibits a singular behavior at the corners' or cracks' tips. Therefore, from an interpolation point of view, no linear combination of the analytic fundamental solutions can guarantee an accurate approximation for the solution u in Ω.

This issue is solved by enriching the MFS approximation basis with appropriate corner-adapted singular particular solution from the PDE. More precisely, a *span* of eigenfunctions for the wedge sectors defined by the corners or cracks is included in the MFS formulation. Such functions can be easily derived by solving the Helmholtz equation in polar coordinates and they have been already used in the Method of Particular Solutions for solving eigenproblems in polygonal domains, e.g. [2]. Analogous approach was used in [3] for the solution of BVPs for the Laplace equation in planar domains with cracks.

Numerical examples will be presented in order to illustrate the accuracy of the new method.

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