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# Stability of compactification in Einstein-Yang-Mills theories after inflation

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We study multidimensional Einstein-Yang-Mills cosmologies. The stability of the compactifying solutions in the radiation-dominated period that followed the inflationary expansion of the external dimensions is examined. It is shown that for suitable values of the multidimensional cosmological constant the compactifying solution is the true ground state of the system.

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## I. INTRODUCTION

In multidimensional field theories, commonly referred to as generalized Kaluza-Klein (GKK) theories, the extra dimensions serve the purpose of unifying different four-dimensional fields. The metric and gauge fields can, for instance, be seen in four dimensions, as different manifestations of one single multidimensional object, namely, the metric [1–3]. Alternatively, a gauge field in a multidimensional spacetime can lead in four dimensions to a gauge field and a Higgs field with a self-interacting symmetry-breaking potential [4–9].

Of course, agreement between GKK theories with our everyday experience requires that the extra or internal dimensions must have a very small characteristic size. It has been proposed by Cremmer, Scherk, and Luciani [10] that the smallness of the extra dimensions could be the result of a spontaneous symmetry-breaking phenomenon in the multidimensional theory. These authors found that in a multidimensional Einstein-Yang-Mills (EYM) theory the symmetry between all the dimensions could be spontaneously broken by the existence of solutions corresponding to a factorization of the spacetime in

$$E^{4+d}=M^4\times I^d\ , \tag{1.1}$$

$M^4$ being the four-dimensional Minkowski spacetime and $I^d$ a $d$-dimensional compact space with a size of the order of the Planck length, $L_{\rm pl}=\sqrt{16\pi k}\approx 10^{-33}$ cm, being $k$ the four-dimensional gravitational constant [2,7–10].

However, for a given compactifying solution to correspond to the ground state of the theory it must be stable both with respect to classical and quantum fluctuations. It has been shown that some of the compactifying solutions in EYM systems are stable against symmetric [7–9] and general [11–13] small classical fluctuations.

In a cosmological setting the multidimensional spacetime is considered to have, in large scales, the form

$$E^{4+d}=\mathbb{R}\times G^{\rm ext}/H^{\rm ext}\times G^{\rm int}/H^{\rm int} \tag{1.2}$$

admitting local coordinates $\hat{x}^{\hat{\mu}}=(t,x^i,\xi^m)$, where $\hat{\mu}=0,1,\ldots,3+d$; $i=1,2,3$; $m=4,\ldots,d+3$, $\mathbb{R}$ denoting a timelike direction and $G^{\rm ext}/H^{\rm ext}$ ($G^{\rm int}/H^{\rm int}$) the space of external (internal) spatial dimensions realized as a coset space of the external (internal) isometry group $G^{\rm ext}$ ($G^{\rm int}$). In this approach spontaneous compactification occurs if, as a result of the cosmological evolution, the scale factor $a(t)$ of the external space increases up to its observed macroscopic value while the scale factor $b(t)$ of the internal space is kept static or slowly varying and very small (see, e.g., [9,14–17]). The classical stability at zero temperature of some solutions of this type in EYM theories has been proved in Refs. [9,17]. In Ref. [16] these solutions were found to survive semiclassically a period of inflationary expansion of the external dimensions for the different models of inflation: old, new, extended, and chaotic.

We shall assume that temperature can be introduced in a multidimensional EYM cosmological model by considering gauge fields with nonvanishing external space components of the strength tensor. These components are assumed to be associated, after inflation, with the radiation that dominates the energy density of the Universe and fixes its temperature. This approach is complementary to the phenomenological treatment developed in Refs. [18] on which one generalizes to $d$ dimensions the usual four-dimensional thermodynamical arguments.

This paper is organized as follows. In Sec. II the dynamical equations of multidimensional EYM cosmologies are derived and some of their properties are studied. In Sec. III we show that the effective potential for the dilaton field depends crucially on the temperature. At zero temperature the solution corresponding to compactified internal dimensions is (for suitable values of the cosmological constant of the multidimensional theory [9]) classically stable but semiclassically unstable; however, this solution for a nonvanishing temperature becomes the true ground state of the dilaton field. In Sec. IV the stability of compactification along the directions of the internal gauge-field components is discussed. Section V contains our conclusions.

## II. CLASSICAL DYNAMICAL EQUATIONS

In this section we shall obtain the equations describing the dynamics of a multidimensional EYM cosmological model. An important point to emphasize is that since we are interested in the post inflationary period, for which the three-dimensional external Universe is radiation dominated, we shall not set to zero, as done in Refs. [9,16,17], the external-space components $\hat{A}_i$ of the gauge field.

We shall consider that the large-scale dynamics of the post inflationary Universe is dominated by the bosonic sector of the multidimensional theory. Though this may not seem a very good assumption we believe that the inclusion of fermions will not lead to any qualitative change in the analysis of the stability of the vacuum.

Following the original Kaluza-Klein idea [1–3] we would expect the bosonic sector of the multidimensional theory to be that of pure gravity with dynamics described by the action

$$S_{\rm gr}[\hat{g}_{\hat{\mu}\hat{\nu}}]=\frac{1}{16\pi\hat{k}}\int_{E^{4+d}}d\hat{x}\sqrt{-\hat{g}}\,(\hat{R}-2\hat{\Lambda})\ , \tag{2.1}$$

where $\hat{g}=\det(\hat{g}_{\hat{\mu}\hat{\nu}}),\hat{R}$ is the scalar curvature, $\hat{k}$ and $\hat{\Lambda}$ are, respectively, the gravitational and cosmological constants in $D=4+d$ dimensions. Assuming that the spacetime has the factorized form (1.2) and that the space of internal dimensions $G^{\rm int}/H^{\rm int}$ is a compact space with a very small size, then it follows that the theory (2.1) leads to an effective four-dimensional Einstein-Yang-Mills theory with a gauge group $K(K\subset G^{\rm int})$ and a multiplet of scalar fields [1–3]. However, this theory has serious drawbacks as it lacks stable compactifying solutions and from it one cannot obtain the correct chiralities and masses for the fermions in the reduced theory. One must therefore consider either alternative theories of gravity or theories with a more complicated bosonic sector including, in addition to the metric, gauge and other bosonic fields in the multidimensional spacetime. These are the GKK theories referred to above. Of course, this procedure raises doubts on the motivation of the GKK theories as consistency with observations requires that the $D$-dimensional theory is almost as complicated as the four-dimensional one. It can be argued, however, that GKK theories and in particular the multidimensional EYM theory, correspond to the bosonic sector (or at least part of it) of a superstring theory. Furthermore, since the existence of extra dimensions is not in contradiction with observations, one should anyway study this possibility, aiming to understand, for instance, the reason the dimensionality of macroscopic spacetime is four and not any other number.

As mentioned previously we shall consider multidimensional EYM theories. In these theories the basic difficulties of pure Einstein theories can be naturally circumvented [7–17]. Let the gauge group $\hat{K}$ of the $D$-dimensional theory be a simple compact Lie group. The action is given by

$$S[\hat{g}_{\hat{\mu}\hat{\nu}},\hat{A}_{\hat{\mu}},\hat{\chi}]=S_{\rm gr}[\hat{g}_{\hat{\mu}\hat{\nu}}]+S_{\rm gf}[\hat{A}_{\hat{\mu}},\hat{g}_{\hat{\mu}\hat{\nu}}]+S_{\rm inf}[\hat{\chi},\hat{g}_{\hat{\mu}\hat{\nu}}]\ , \tag{2.2}$$

where $S_{\rm gr}[\hat{g}_{\hat{\mu}\hat{\nu}}]$ is given by (2.1):

$$S_{\rm gf}[\hat{A}_{\hat{\mu}},\hat{g}_{\hat{\mu}\hat{\nu}}]=\frac{1}{8\hat{e}^2}\int_{E^{4+d}}d\hat{x}\sqrt{-\hat{g}}\,{\rm Tr}\hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}}\ , \tag{2.3a}$$

$$S_{\rm inf}[\hat{\chi},\hat{g}_{\hat{\mu}\hat{\nu}}]=-\int_{E^{4+d}}d\hat{x}\sqrt{-\hat{g}}\,[\tfrac{1}{2}(\partial_{\hat{\mu}}\hat{\chi})^2+\hat{U}(\hat{\chi})]\ , \tag{2.3b}$$

$\hat{F}_{\hat{\mu}\hat{\nu}}=\partial_{\hat{\mu}}\hat{A}_{\hat{\nu}}-\partial_{\hat{\nu}}\hat{A}_{\hat{\mu}}+[\hat{A}_{\hat{\mu}},\hat{A}_{\hat{\nu}}]$, $\hat{e}$ is the gauge coupling constant, $\hat{\chi}$ is the inflaton field responsible for the inflationary expansion of the external space, and $\hat{U}(\hat{\chi})$ is the potential for $\hat{\chi}$. It is assumed that the potential $\hat{U}(\hat{\chi})$ is bounded from below, has a global minimum and that without loss of generality $\hat{U}_{\rm min}=0$.

To study cosmological models associated with action (2.2) we must restrict ourselves to spatially homogeneous and (partially) isotropic field configurations, which means that these are symmetric under the action of the group $G^{\rm ext}\times G^{\rm int}$. To find these configurations we shall use the theory of symmetric fields [3–9,12,13,19]. Let us for definiteness consider the case with the gauge group $\hat{K}={\rm SO}(N),N\geq 3+d$ and

$$E^{4+d}=\mathbb{R}\times S^3\times S^d\ , \tag{2.4}$$

where $S^3$ ($S^d$) is the three- ($d$-)dimensional sphere. The group of spatial homogeneity and isotropy is, in this case,

$$G^{\rm HI}={\rm SO}(4)\times{\rm SO}(d+1)\ , \tag{2.5}$$

while the group of spatial isotropy is

$$H^I={\rm SO}(3)\times{\rm SO}(d)\ , \tag{2.6}$$

which is in agreement with the alternative realization of $E^{4+d}$ as

$$E^{4+d}=\mathbb{R}\times{\rm SO}(4)/{\rm SO}(3)\times{\rm SO}(d+1)/{\rm SO}(d)=\mathbb{R}\times[{\rm SO}(4)\times{\rm SO}(d+1)]/[{\rm SO}(3)\times{\rm SO}(d)]\ . \tag{2.7}$$

In the theory of symmetric fields a central role is played by the so-called Cartan one-form which, in the present case is defined as follows. Let $\sigma(y)\in{\rm SO}(4)\times{\rm SO}(d+1)$, $y\in[{\rm SO}(4)\times{\rm SO}(d+1)]/[{\rm SO}(3)\times{\rm SO}(d)]=S^3\times S^d$ be some choice of representatives in the cosets $y$, such that $[\sigma(y)]=y$, i.e., a (local) section in the principal bundle ${\rm SO}(4)\times{\rm SO}(d+1)\to S^3\times S^d$. The Cartan one-form on $S^3\times S^d$ is defined as the pullback of the canonical left-invariant form on the group ${\rm SO}(4)\times{\rm SO}(d+1)$ [20]:

$$\omega_{(y)}=\sigma^{-1}(y)d\sigma(y)\ . \tag{2.8}$$

The form $\omega$ takes values on ${\rm so}(4)\oplus{\rm so}(d+1)={\rm Lie}[{\rm SO}(4)\times{\rm SO}(d+1)]$, the Lie algebra of the group ${\rm SO}(4)\times{\rm SO}(d+1)$, and therefore can be decomposed as

$$\omega=\sum_{\alpha=1}^{d+3}\omega^{\alpha}T_{\alpha}+\sum_{1\le i<j\le 3}\omega^{ij}\frac{T_{ij}^{(4)}}{2}+\sum_{1\le m<n\le d}\tilde{\omega}^{mn}\frac{\tilde{T}_{mn}^{(d+1)}}{2}\,, \tag{2.9}$$

where $\{T_{ij}^{(4)},\ 1\le i<j\le 4,\ \tilde{T}_{mn}^{(d+1)},\ 1\le m<n\le d+1\}$, $(T_{ij}^{(4)})_{i'j'}=\delta_{ii'}\delta_{jj'}-\delta_{ij'}\delta_{i'j}$, $(\tilde{T}_{mn}^{(d+1)})_{m'n'}=\delta_{mm'}\delta_{nn'}-\delta_{mn'}\delta_{m'n}$ is a basis in the Lie algebra so(4)⊕so($d+1$) of $G^{HI}$ and

$$T_{\alpha}=\frac{T_{\alpha 4}^{(4)}}{2}\quad\text{for } \alpha=1,2,3\,,\qquad T_{\alpha}=\frac{\tilde{T}_{\alpha-3d+1}^{(d+1)}}{2}\quad\text{for } \alpha=4,\dots,d+3\,, \tag{2.10}$$

with the standard commutation relations.

The one-forms $\omega^{\alpha},\alpha=1,\dots,d+3$ in (2.9) form a local moving coframe in $S^3\times S^d$. In this coframe the components of a SO(4)×SO($d+1$)-invariant metric on $S^3\times S^d$ are independent of the local coordinates $(x^i,\xi^m)$. Moreover the most general form of a SO(4)×SO($d+1$)-invariant metric in $E^{4+d}$ reads

$$\hat{g}=-\tilde{N}^2(t)dt^2+\tilde{a}^2(t)\sum_{i=1}^{3}\omega^i\omega^i+b^2(t)\sum_{m=4}^{d+3}\omega^m\omega^m\,, \tag{2.11}$$

where $\tilde{a}(t)$, $b(t)$ and the lapse function $\tilde{N}(t)$ are arbitrary nonvanishing functions of time. Notice that in (2.11) $\sum_{i=1}^{3}\omega^i\omega^i$ and $\sum_{m=4}^{d+3}\omega^m\omega^m$ coincide with the standard metrics $d\Omega_3^2$ and $d\Omega_d^2$ in the three- and $d$-dimensional spheres, respectively. For later purposes, we perform the following conformal change of the variables that characterize the four-dimensional part of the metric:

$$\tilde{N}^2(t)=\left[\frac{b_0}{b(t)}\right]^d N^2(t)\,, \tag{2.12a}$$

$$\tilde{a}^2(t)=\left[\frac{b_0}{b(t)}\right]^d a^2(t)\,. \tag{2.12b}$$

The SO(4)×SO($d+1$)-invariant ansatz for the inflation field $\hat{\chi}$ reads

$$\hat{\chi}(t,x^i,\xi^m)=\hat{\chi}(t)\,. \tag{2.13}$$

To fix a sector of SO(4)×SO($d+1$)-symmetric gauge fields one must choose a homomorphism $\lambda$ of the isotropy group SO(3)×SO($d$) to the gauge group SO($N$) [4–9]:

$$\lambda\text{:SO}(3)\times\text{SO}(d)\to\text{SO}(N)\,. \tag{2.14}$$

Here we choose $\lambda$ to be the simplest embedding defined by the branching rule

$$\mathbf{N}\downarrow_{\lambda(\text{SO}(3)\times\text{SO}(d))}=(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{d})+(N-3-d)(\mathbf{1},\mathbf{1})\,. \tag{2.15}$$

Then the SO(4)×SO($d+1$)-symmetric ansatz for the gauge field is [4–9,19]:

$$\hat{A}(t)=\tfrac{1}{2}\sum_{p,q=1}^{N-3-d}B^{pq}(t)T_{3+d+p\,3+d+q}^{(N)}dt+\tfrac{1}{2}\sum_{1\le i<j\le 3}T_{ij}^{(N)}\omega^{ij}+\tfrac{1}{2}\sum_{4\le m<n\le 3+d}T_{mn}^{(N)}\tilde{\omega}^{m-3n-3}$$
$$+\sum_{i=1}^{3}\left[\tfrac{1}{4}f_0(t)\sum_{j,k=1}^{3}\varepsilon_{jik}T_{jk}^{(N)}+\tfrac{1}{2}\sum_{p=1}^{N-3-d}f_p(t)T_{i\,d+3+p}^{(N)}\right]\omega^i+\sum_{m=4}^{d+3}\left[\tfrac{1}{2}\sum_{q=1}^{N-3-d}g_q(t)T_{m\,d+3+q}^{(N)}\right]\omega^m\,, \tag{2.16}$$

where $f_p(t)$, $p=0,\dots,N-3-d$; $g_q(t)$, $q=1,\dots,N-3-d$; $B^{pq}(t)$, $1\le p<q\le N-3-d$ are arbitrary functions and $T_{pq}^{(N)}$, $1\le p<q\le N$ are the generators of the gauge group SO($N$).

By substituting (2.11)–(2.13) and (2.16) into action (2.2) we obtain a one-dimensional effective action for the functions of time $t$ that parametrize the symmetric field configurations:

$$S^{\text{eff}}=S^{\text{eff}}[a,\psi,f_0,\mathbf{f},\mathbf{g},\chi,N,\hat{B}]=16\pi^2\int_{t_1}^{t_2}dt\,Na^3\left\{-\frac{3}{8\pi k}\frac{1}{a^2}\left[\frac{\dot{a}}{N}\right]^2+\frac{3}{32\pi k}\frac{1}{a^2}+\frac{1}{2}\left[\frac{\dot{\psi}}{N}\right]^2+\frac{1}{2}\left[\frac{\dot{\chi}}{N}\right]^2\right.$$
$$+e^{d\beta\psi}\frac{3}{4e^2}\frac{1}{a^2}\left[\frac{1}{2}\left[\frac{\dot{f}_0}{N}\right]^2+\frac{1}{2}\left[\frac{\mathcal{D}_t\mathbf{f}}{N}\right]^2\right]$$
$$\left.+e^{-2\beta\psi}\frac{d}{4e^2}\frac{1}{b_0^2}\frac{1}{2}\left[\frac{\mathcal{D}_t\mathbf{g}}{N}\right]^2-W(\psi,a,f_0,\mathbf{f},\mathbf{g},\chi)\right\}\,, \tag{2.17}$$

where $k=\hat{k}/v_d b_0^d$, $e^2=\hat{e}^2/v_d b_0^d$, $\beta=\sqrt{16\pi k/d(d+2)}$, $v_d$ is the volume of $S^d$ for $b=1$, with $\psi=\beta^{-1}\ln(b/b_0)$ and $\chi=\sqrt{v_d b_0^d}\,\hat{\chi}$ denoting the dilaton and the inflaton fields respectively. In (2.17) the dots denote time derivative and $\mathcal{D}_t$ is the covariant derivative with respect to the SO($N-3-d$) gauge field $\hat{B}(t)$ in $\mathbb{R}$:

$$\mathcal{D}_t\mathbf{f}(t)=\frac{d}{dt}\mathbf{f}(t)+\hat{B}(t)\mathbf{f}(t)\,, \tag{2.18a}$$

$$\mathcal{D}_t\mathbf{g}(t)=\frac{d}{dt}\mathbf{g}(t)+\hat{B}(t)\mathbf{g}(t)\,, \tag{2.18b}$$

where $\mathbf{f}=\{f_p\}$, $\mathbf{g}=\{g_p\}$, $p=1,\ldots,N-3-d$, and $\hat{B}$ is the $(N-3-d)\times(N-3-d)$ antisymmetric matrix $\hat{B}=(B_{pq})$. The potential $W$ in (2.17) is given by

$$W=e^{-d\beta\psi}\left[-e^{-2\beta\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2}+e^{-4\beta\psi}\frac{1}{b_0^4}\frac{d(d-1)}{8e^2}V_2(\mathbf{g})+\frac{\Lambda}{8\pi k}+U(\chi)\right]$$
$$+e^{-2\beta\psi}\frac{1}{a^2b_0^2}\frac{3d}{4}\frac{1}{8e^2}(\mathbf{f}\cdot\mathbf{g})^2+e^{d\beta\psi}\frac{1}{a^4}\frac{3}{4e^2}V_1(f_0,\mathbf{f})\;, \tag{2.19}$$

where $\Lambda=v_d b_0^d\hat{\Lambda}$, $U(\chi)=v_d b_0^d\hat{U}(\hat{\chi}/\sqrt{v_d b_0^d}\,)$ and

$$V_1(f_0,\mathbf{f})=\tfrac{1}{8}[(f_0^2+\mathbf{f}^2-1)^2+4f_0^2\mathbf{f}^2]\;, \tag{2.20a}$$

$$V_2(\mathbf{g})=\tfrac{1}{8}(\mathbf{g}^2-1)^2\;. \tag{2.20b}$$

Notice that the effective Lagrangian in (2.17) does not depend on the time derivatives of $N$ and $\hat{B}$. This means that these variables play the role of Lagrange multipliers associated with local symmetries of (2.17). The lapse function $N$ is associated with the invariance of $S^{\text{eff}}$ with respect to arbitrary time reparametrizations while $\hat{B}$ is related with local $SO(N-d-3)$ invariance [19]. The equations of motion can be easily obtained by applying the variational principle to (2.17). These equations are identical to the equations that one obtains by substituting the *Ansätze* (2.11)–(2.13) and (2.16) directly in the multidimensional equations of motion meaning that the *Ansätze* are consistent. In the "gauge" $N=1$, $\hat{B}=0$ one finds the following equations.

(i) Friedmann equation:

$$\left[\frac{\dot a}{a}\right]^2=-\frac{1}{4a^2}+\frac{8\pi k}{3}\left[\frac{\dot\psi^2}{2}+\frac{\dot\chi^2}{2}+\frac{1}{8e^2}e^{d\beta\psi}\frac{6}{a^2}\left[\frac{\dot f_0^2}{2}+\frac{\dot{\mathbf{f}}^2}{2}\right]+\frac{d}{4e^2}\frac{1}{b_0^2}e^{-2\beta\psi}\frac{\dot{\mathbf{g}}^2}{2}+W\right]\;, \tag{2.21}$$

(ii) Klein-Gordon equation for the dilaton field:

$$\ddot\psi+3\frac{\dot a}{a}\dot\psi=-\frac{\partial W}{\partial\psi}+\beta\frac{3d}{4e^2}\frac{1}{a^2}e^{d\beta\psi}\left[\frac{\dot f_0^2}{2}+\frac{\dot{\mathbf{f}}^2}{2}\right]-\beta\frac{d}{2e^2}\frac{1}{b_0^2}e^{-2\beta\psi}\frac{\dot{\mathbf{g}}^2}{2}\;, \tag{2.22}$$

(iii) Klein-Gordon equation for the inflaton field:

$$\ddot\chi+3\frac{\dot a}{a}\dot\chi=-e^{-d\beta\psi}\frac{\partial U}{\partial\chi}\;, \tag{2.23}$$

(iv) Yang-Mills equations:

$$\ddot f_0+\frac{\dot a}{a}\dot f_0+d\beta\dot\psi\dot f_0=-\frac{1}{a^2}\frac{\partial V_1}{\partial f_0}\;, \tag{2.24a}$$

$$\ddot{\mathbf{f}}+\frac{\dot a}{a}\dot{\mathbf{f}}+d\beta\dot\psi\dot{\mathbf{f}}=-\frac{1}{a^2}\frac{\partial V_1}{\partial\mathbf{f}}-e^{-(d+2)\beta\psi}\frac{1}{4b_0^2}(\mathbf{f}\cdot\mathbf{g})\mathbf{g}\;, \tag{2.24b}$$

$$\ddot{\mathbf{g}}+3\frac{\dot a}{a}\dot{\mathbf{g}}-2\beta\dot\psi\dot{\mathbf{g}}=-\frac{1}{b_0^2}e^{-(d+2)\beta\psi}\frac{d-1}{2}\frac{\partial V_2}{\partial\mathbf{g}}-\frac{1}{a^2}\frac{3}{4}(\mathbf{g}\cdot\mathbf{f})\mathbf{f}\;, \tag{2.24c}$$

$$3b_0^2e^{(d+2)\beta\psi}(\dot f_p f_q-\dot f_q f_p)+da^2(\dot g_p g_q-\dot g_q g_p)=0\;. \tag{2.24d}$$

Aiming to examine the stability of compactification we now turn to the study of solutions of equations (2.21)–(2.24) corresponding to static configurations of the gauge and inflaton fields. The static configuration

$$\chi=\chi^v,\quad f_0=f_0^v\;,\qquad \mathbf{f}=\mathbf{f}^v,\ \text{and}\ \mathbf{g}=\mathbf{g}^v \tag{2.25}$$

is a solution of equations (2.23) and (2.24) if

$$\left.\frac{\partial U}{\partial\chi}\right|_{\chi=\chi^v}=0,\quad \left.\frac{\partial V_1}{\partial f_p}\right|_{f_0=f_0^v,\ \mathbf{f}=\mathbf{f}^v}=0\;,\qquad p=0,\ldots,N-3-d \tag{2.26a}$$

$$\left.\frac{\partial V_2}{\partial g_p}\right|_{\mathbf{g}=\mathbf{g}^v}=0,\quad p=1,\ldots,N-d-d\;,$$

and

$$(\mathbf{f}^v\mathbf{g}^v)=0\;. \tag{2.26b}$$

Substituting the static configuration (2.25) in (2.21) and (2.22) the equations for $a(t)$ and $\psi(t)$ become

$$\left[\frac{\dot a}{a}\right]^2=-\frac{1}{4a^2}+\frac{8\pi k}{3}\left[\frac{\dot\psi^2}{2}+\Omega(\psi,a)\right]\;, \tag{2.27}$$

$$\ddot\psi+3\frac{\dot a}{a}\dot\psi=-\frac{\partial\Omega}{\partial\psi}\;, \tag{2.28}$$

where

$$\Omega(\psi,a)=e^{-d\beta\psi}\left[-e^{-2\beta\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2}+e^{-4\beta\psi}\frac{1}{b_0^4}\frac{d(d-1)}{8e^2}v_2+\frac{\Lambda}{8\pi k}\right]+e^{d\beta\psi}\frac{1}{a^4}\frac{3}{4e^2}v_1\;, \tag{2.29}$$

with $v_1=V_1(f_0^v,\mathbf{f}^v)$, $v_2=V_2(\mathbf{g}^v)$ and we have assumed that $U(\chi^v)=0$.

During inflation the scale factor $a(t)$ of the external dimensions grows exponentially and therefore one can neglect the last term in the effective potential (2.29) for the dilaton. Let us briefly recall this situation, which was studied in detail in Refs. [9] and [16]. If $v_2=0$, as in the case of pure gravity, there are no stable compactifying solutions, i.e., solutions for which $b=b^v\approx\sqrt{16\pi k}$. If $v_2>0$ then the shape of the potential $\Omega_\infty(\psi)=\Omega(\psi,\infty)$ depends on the value of the cosmological constant $\hat{\Lambda}$ of the multidimensional theory [9]. Notice that although in our case the only extremum of $V_2(\mathbf{g})$ [see (2.20b)] with $v_2>0$ is the unstable local maximum $\mathbf{g}^v=0$, models with an absolute minimum of $V_2$ for which $v_2>0$ can be easily found by considering either nonregular embeddings $\lambda$ or internal spaces $G^{\rm int}/H^{\rm int}$ with a nonsimple isotropy group $H^{\rm int}$ (see discussion in Sec. IV below). For $\Lambda>c_2/16\pi k$ ($c_2=[(d+2)^2(d-1)/(d+4)]e^2/16v_2$) there are no compactifying solutions (see Fig. 1) and for

$$\frac{c_1}{16\pi k}<\Lambda<\frac{c_2}{16\pi k} \tag{2.30}$$

$[c_1=d(d-1)e^2/16v_2]$ a compactifying solution exists which is classically stable but semiclassically unstable (Fig. 2). Nevertheless, it has been shown [16] that this solution survives the inflationary period without tunneling to the true decompactified vacuum. This occurs as in the thin-wall approximation [21]; the tunneling rate of the inflaton field is much greater than the corresponding one to the dilaton field [16]. A value of $\Lambda<c_1/16\pi k$ leads to a negative value of the effective four-dimensional cosmological constant $\Lambda^{(4)}$—the value of $\Omega_\infty 8\pi k$ at the local minimum. Since the four-dimensional cosmological constant must satisfy the bound

$$|\Lambda^{(4)}|<10^{-120}\frac{1}{16\pi k}\ , \tag{2.31}$$

it then follows that the multidimensional cosmological constant $\hat{\Lambda}=\Lambda/v_d b_0^d$ has to be fine-tuned in such a way that

$$|\Lambda^{(4)}|=|v_d b_0^d\hat{\Lambda}-\frac{c_1}{16\pi k}|<10^{-120}\frac{1}{16\pi k}\ . \tag{2.32}$$

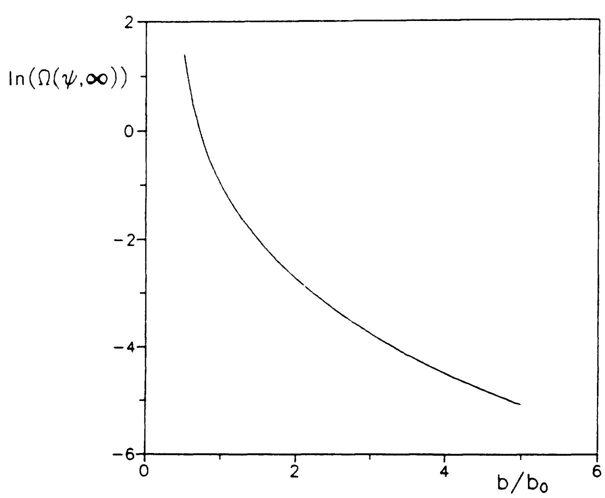


FIG. 1. Potential $\Omega_\infty(\psi)$ for $d=6$ and $\Lambda>c_2/16\pi k$ ($c_2=[(d+2)^2(d-1)/(d+4)]e^2/16v_2$).

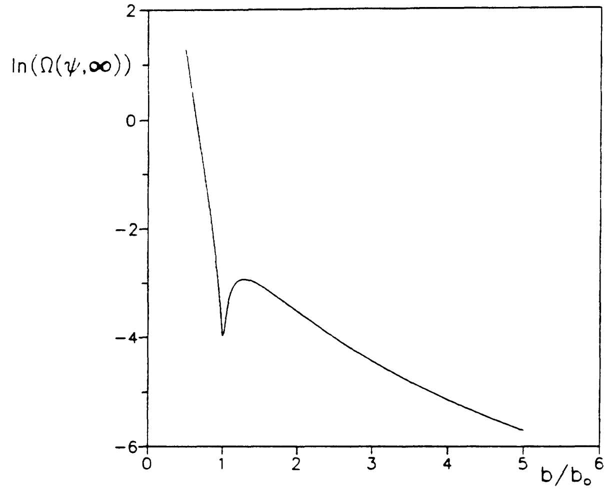


FIG. 2. Potential $\Omega_\infty(\psi)$ for $d=6$ and $c_1/16\pi k<\Lambda<c_2/16\pi k$ $[c_1=d(d-1)e^2/16v_2]$.

Finally let us comment on the possibility of achieving inflation driven by the dilaton without the need of introducing the inflaton. The exponential form of the potential (2.29) leads to a power-law behavior of the scale factor $a(t)=a_0t^p$ with $p<1$. In Ref. [22] it has been suggested that the inclusion of a phenomenological viscosity term would imply in a flow of energy from the dilaton to radiation improving considerably the power-law behavior such that one could easily obtain $p>1$. However, this flow of energy cannot in principle be obtained in a fundamental way by considering the interaction of the dilaton with radiation described by the external components of the gauge field [see action (2.17)]. Actually, this interaction leads to a interchange of energy between the two fields without a preferred direction [23]. We therefore think that to achieve a successful period of inflation one needs necessarily to consider an inflaton sector.

In the next section we shall use the analogue of Eq. (2.28) to study the dynamics of the dilaton field in the radiation-dominated period that follows inflation.

## III. STABILITY OF COMPACTIFICATION AFTER INFLATION

After the period of accelerated expansion of the external dimensions and subsequent reheating the Universe became radiation dominated. We shall model this situation by considering that the main contribution to the temperature comes from nonvanishing external-space components of the gauge field $\hat{F}_{\mu\nu}$ ($\mu,\nu=0,1,2,3$). From the left-hand side (LHS) of (2.19) and (2.21) we see that the contribution of the external components of the gauge field to the energy density is given by (we assume that $\mathbf{f}$ is orthogonal to $\mathbf{g}$)

$$\rho_{EGF}=\frac{1}{a^4}\frac{3}{4e^2}e^{d\beta\psi}\left[a^2\left[\frac{\dot{f}_0^2}{2}+\frac{\dot{\mathbf{f}}^2}{2}\right]+V_1(f_0,\mathbf{f})\right]\ . \tag{3.1}$$

Therefore after a period of inflationary expansion of the extra dimensions, during which $e^{\beta\psi}\approx 1$, the contribution to the energy density from the static solution $f_0=f_0^v$, $\mathbf{f}=\mathbf{f}^v$ of the previous section becomes negligibly small. For the Universe to be radiation dominated after inflation we assume that the vacuum energy density is transformed into thermal energy described by (3.1) via large nonstatic vacuum expectation values for $f_0$ and $\mathbf{f}$.

Let us now study the dynamics implied by the system of Eqs. (2.21)–(2.24) in the post-inflationary period. We assume that both the inflaton $\chi$ and the internal components of the gauge fields described by $\mathbf{g}$ are given by static configurations $\chi^v$, $\mathbf{g}^v$ corresponding to the extrema (minima) of the associated potentials [see the first and third equations in (2.26a)]. The stability of the compactifying solution along the $\mathbf{g}$ direction is studied in Sec. IV. For these configurations and choosing $\mathbf{f}=f\mathbf{u}$, where $\mathbf{u}$ is a fixed unit vector orthogonal to $\mathbf{g}^v$, we solve Eqs. (2.23), (2.24c), and (2.24d) exactly and obtain from (2.21), (2.22), (2.24a) and (2.24b) a simplified system of equations for $a$, $\psi$, $f_0$, and $f$. These equations can then be solved approximately as follows. We suppose that the kinetic energy of the dilaton is small

$$\frac{4\pi k}{3}\dot{\psi}^2 \ll \left[\frac{\dot{a}}{a}\right]^2 , \tag{3.2}$$

which implies that

$$\rho_{EGF}=\frac{c}{a^4}e^{d\beta\psi} , \tag{3.3a}$$

where $c$ is a constant approximately equal to the first integral of Eqs. (2.24a), (2.24b) being given by

$$c=\frac{3}{4e^2}\left[a^2\left[\frac{\dot{f}_0^2}{2}+\frac{\dot{\mathbf{f}}^2}{2}\right]+V_1(f_0,\mathbf{f})\right] . \tag{3.3b}$$

By introducing the temperature associated with the external components of the gauge field, $T=\delta/a$, where due to inflation $\delta$ is a large constant, $\delta \gtrsim 10^{30}$, we obtain

$$\rho_{EGF}=\sigma T^4 e^{d\beta\psi} , \tag{3.3}$$

with $\sigma=c/\delta^4$ being a constant of order one and after inflation $T\lesssim 10^{-5}/\sqrt{16\pi k}$.

Thus we get for the dilaton field the dynamical equation

$$\ddot{\psi}-3\frac{\dot{T}}{T}\dot{\psi}=-\frac{\partial\tilde{\Omega}}{\partial\psi} , \tag{3.4}$$

where

$$\tilde{\Omega}(\psi,T)=e^{-d\beta\psi}\left[\frac{2e^2}{d(d-1)}\frac{1}{v_2(16\pi k)^2}(e^{-2\beta\psi}-1)^2+\frac{\Lambda^{(4)}}{8\pi k}\right]+\sigma T^4 e^{d\beta\psi} , \tag{3.5}$$

and assuming that $v_2>0$ we have set $b_0^2=4\pi k d(d-1)v_2/e^2$. For this choice of $b_0$ the point $\psi=0$, i.e., $b=b_0$, is very close to the minimum of (3.5) corresponding to a solution of spontaneous compactification. We see then that the influence of the scale factor of the external dimensions in the dynamics of the dilaton is twofold. On one hand it changes the effective potential for the dilaton by turning it temperature ($T\sim 1/a$) dependent (see difference between Figs. 2 and 3). On the other hand, being dominated by the radiation energy density, the RHS of Eq. (2.21) describes an expanding Friedman universe with $\dot{a}>0$. This means that the second term in the LHS of Eq. (3.4) is a viscositylike term which makes the stable extrema (two in our case) of the potential $\tilde{\Omega}(\psi,T)$ asymptotically stable. Unlike the zero-temperature case [$\tilde{\Omega}(\psi,0)$] the potential (3.5) has now an infinite potential barrier for large $\psi$. After inflation one has that $T\lesssim 10^{-5}/\sqrt{16\pi k}$ and $|\Lambda^{(4)}|\ll 16\pi k T^4$. In this case the potential (3.5) is of the double-well type and it has two minima $\psi_-$ and $\psi_+$ (Fig. 3). The first one is very close to zero,

$$\psi_-\cong-\frac{\sqrt{d(d+2)}d^2(d-1)v_2}{16e^2}(16\pi k)^{3/2}\sigma T^4 , \tag{3.6a}$$

and, accordingly,

$$\tilde{\Omega}_-=\tilde{\Omega}(\psi_-,T)\cong\sigma T^4 . \tag{3.6b}$$

For $\psi_+$ we have

$$\psi_+=\frac{1}{d}\left[\frac{d(d+2)}{16\pi k}\right]^{1/2}\ln\left[\frac{1}{16\pi k T^2}\left[\frac{2e^2}{d(d-1)\sigma}\right]^{1/2}\right] \tag{3.6c}$$

and

$$\tilde{\Omega}_+=\tilde{\Omega}(\psi_+,T)\cong\frac{T^2}{16\pi k}\left[\frac{2e^2\sigma}{d(d-1)}\right]^{1/2}\left[1+\frac{1}{v_2}\right] . \tag{3.6d}$$

There since $\tilde{\Omega}_-\ll\tilde{\Omega}_+$, $\psi=\psi_-$ is for $16\pi k|\Lambda^{(4)}|\ll(16\pi k)^2T^4\ll 1$ the true ground state. On the other hand, this corresponds to the situation in which the internal dimensions are compactified, that is

$$b_-=\left[\frac{4\pi k d(d-1)v_2}{e^2}\right]^{1/2}\exp\left[\left[\frac{16\pi k}{d(d-1)}\right]^{1/2}\psi_-\right]\cong\left[\frac{4\pi k d(d-1)v_2}{e^2}\right]^{1/2} . \tag{3.7}$$

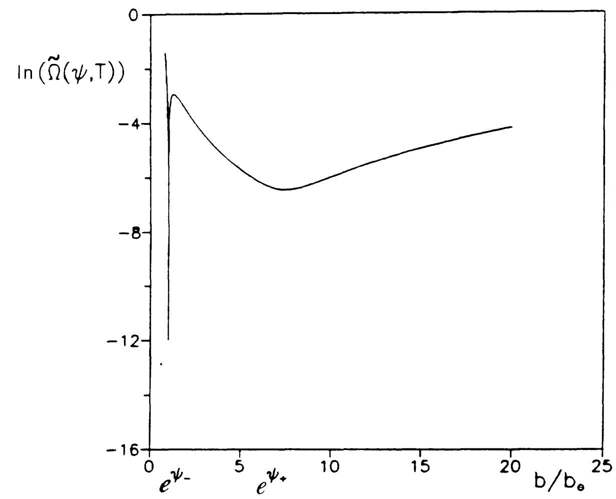


FIG. 3. Potential $\tilde{\Omega}(\psi,T)$ for $d=6$ and $T=10^{-3}/\sqrt{16\pi k}$.

The false vacuum $\psi=\psi_+$ corresponds to the situation in which the internal dimensions are decompactified in the limit $T\rightarrow 0$:

$$b_+=\left[\frac{4\pi k d(d-1)v_2}{e^2}\right]^{1/2}\exp\left[\left[\frac{16\pi k}{d(d-1)}\right]^{1/2}\psi_+\right]\rightarrow\infty\ . \tag{3.8}$$

Thus, we see that for $0<16\pi k\Lambda^{(4)}\ll(16\pi k)^2T^4\ll 1$ (with the corresponding $\hat{\Lambda}$) the decompactified internal dimensions are semiclassically unstable. In this situation, even if after inflation the internal dimensions were decompactified, the multidimensional Universe would tend to tunnel to the solution for which the internal dimensions were compactified. This result concerning the metastability of $\psi_+$ is much stronger than the one related with the semiclassical stability of the compactifying solutions.

By using the thin-wall approximation [21], we find that if after inflation the internal dimensions were decompactified, the lifetime of the $\psi_+$ state per unit of volume is given by

$$\tau=Ce^B \tag{3.9}$$

where $B=27\pi^2S^4/2\varepsilon^3$, and in our case $S\approx 10^{-1}(16\pi k)^{-3/2}$, the difference of energies between the two minima $\varepsilon\approx T^2(16\pi k)^{-1}$ which gives $B\approx 10^{28}$. Thus even assuming that the preexponential factor $C$ is of the order of one in Planck units the $\psi_+$ state would take much longer than the present age of the Universe to tunnel to the compactified state $\psi_-$, which is, of course, incompatible with our everyday experience. This conclusion holds even considering spharelonlike configurations as the probability of over the barrier transitions is proportional to $\exp(-E_{\rm sph}/T)$ [24] and for $d=6$ we get that $E_{\rm sph}\approx 10^{-1}/\sqrt{16\pi k}$.

We emphasize that the temperature in expressions (3.4)–(3.8) has a time dependence implied by the Friedmann equation (2.21) which according to our previous considerations can be rewritten as

$$\left[\frac{\dot{T}}{T}\right]^2=\frac{8\pi k}{3}\left[\frac{\dot{\psi}^2}{2}+\tilde{\Omega}(\psi,T)\right]\ . \tag{3.10}$$

This implies that the solutions (3.6a) and (3.6c) are only approximate solutions of Eq. (3.4). An argument in favor of considering $\psi_-$ in (3.6a) as a good approximation comes from the fact that the frequency $\omega$ of small oscillations around $\psi_-$ is much larger than $|\dot{T}/T|$:

$$\omega=\frac{e}{d\sqrt{(d-1)(d+2)v_2}}\frac{1}{\sqrt{\pi k}}\gg\left|\frac{\dot{T}}{T}\right|\ .$$

However, in general and especially for values of $\psi$ far from $\psi_-$, the potential (3.5) should be considered indicative only of the qualitative behavior of $\psi$. A more rigorous approach consists in considering the coupled equations (3.4) and (3.9) and even (2.21), (2.22), (2.24a), and (2.24b). To illustrate this let us show the stability of the compactifying solutions we have obtained after inflation by studying the coupled equations (3.4) and (3.10). For convenience [25–27], we consider Raychaudhuri's equation, which is obtained by differentiating both terms in (3.10) and using (3.4). The Friedmann equation (3.10) then plays the role of a constraint for the four-dimensional dynamical system in the variables $(\psi,\dot{\psi},T,\dot{T})$. We obtain therefore a closed system of differential equations for the variables $(\psi,\dot{\psi},H=-\dot{T}/T)$. For simplicity of analysis let us introduce the dimensionless variables $(x,y,z)$ and the dimensionless time $\eta$:

$$\begin{aligned} x&=\mu_1^{-1}\psi\ ,\\ \eta&=\mu_2 t\ ,\\ y&=x'=\mu_1^{-1}\mu_2^{-1}\dot{\psi}\ ,\\ z&=\mu_2^{-1}H\ , \end{aligned} \tag{3.11}$$

where the prime denotes a derivative with respect to $\eta$ and

$$\begin{aligned} \mu_1&=\left[\frac{3}{4\pi k}\right]^{1/2},\\ \mu_2&=\left[\frac{e^2}{2\pi k d^2(d-1)(d+2)v_2}\right]^{1/2}. \end{aligned} \tag{3.11a}$$

In these variables we obtain the dynamical system

$$\begin{aligned} x'&=y\ ,\\ y'&=-3yz-\frac{d\alpha}{2}[z^2-v(x)-y^2]-\frac{dv(x)}{dx}\ ,\\ z'&=-2z^2-y^2+4v(x)\ , \end{aligned} \tag{3.12a}$$

and the constraint

$$z^2-2v(x)-y^2=\frac{8\pi k}{3}\mu_2^{-2}\sigma T^4e^{d\alpha x}\ , \tag{3.12b}$$

where $\alpha=\sqrt{12/d(d+2)}$ and

$$v(x)=\frac{1}{8\alpha^2}e^{-d\alpha x}(e^{-2\alpha x}-1)^2 \ . \quad (3.13)$$

The constraint (3.12b) means just that the physical region $A_{\rm ph}$ is given by the points satisfying [25,26]

$$A_{\rm ph}=\{(x,y,z):z^2-2v(x)-y^2\geq 0\} \ . \quad (3.14)$$

The region $A_{\rm ph}$ is the union of two sets, with $z\geq 0$ and $z\leq 0$, respectively, intersecting only at the critical point $P=(0,0,0)$. As we are interested in an expanding universe we shall consider only the set $A^{\dagger}_{\rm ph}$ with $z\geq 0$ defined by

$$A^{\dagger}_{\rm ph}=\{(x,y,z)\in A_{\rm ph}:z\geq 0\} \ . \quad (3.15)$$

The boundary of the region (3.14), (3.15) is an invariant manifold of the system (3.12a) describing the vanishing temperature case studied in Ref. [9]. The critical point $P=(0,0,0)$ corresponds to spontaneous compactification of the internal dimensions as $x=0$ means that $b=b_0=\sqrt{4\pi k d(d-1)v_2/e^2}$.

The asymptotic stability of $P$ in the physical region can be shown by finding an appropriate Lyapunov function. Notice that close to $P$ the physical region $A^{\dagger}_{\rm ph}$ corresponds to the cone

$$z^2-x^2-y^2+O(x^3)\geq 0 \ , \quad z\geq 0 \ . \quad (3.16)$$

Consider now the following region $U$ in $A^{\dagger}_{\rm ph}/\{P\}$:

$$U=\left\{(x,y,z)\in A^{\dagger}_{\rm ph}/\{P\}:|x|\leq\frac{1}{2\alpha}\ln\left[\frac{d+4}{d}\right],\ z\leq\frac{1}{2\alpha(d+4)}\left[\frac{d}{d+4}\right]^{d/4}\right\} ; \quad (3.17)$$

it is then clear that in $U\cup P$ the simple function $F=z$ plays the role of a Lyapunov function: $F(0,0,0)=0$, $F$ is positive in $U$ and from the third equation in (3.12a) we see that $dF/d\eta<0$ in $U$. Physically, the condition $dz/d\eta<0$ in $U$ implies that the solutions of system (3.12) are compelled to approach $P$.

## IV. STABILITY OF COMPACTIFICATION ALONG THE INTERNAL GAUGE FIELD DIRECTIONS

As we have seen above for a stable compactifying solution to exist the vacuum condensate of the internal gauge field components must be nontrivial, i.e., $v_2>0$. In Sec. II we have obtained, by using the simplest embedding (2.15) of SO($d$) in the group SO($N$), an effective potential $V_2$ (for the functions $\mathbf{g}$ that parametrize the internal gauge field) with two different types of extrema. The first $\mathbf{g}^{v}:\|\mathbf{g}^{v}\|=1$ corresponds to a trivial configuration $\hat{A}_m$ (pure gauge with $v_2=0$). In this case the potential for $\psi$ does not have stable compactifying extrema. For the second, $\mathbf{g}^{v}=0$, one has $v_2=V_2(\mathbf{g}^{v})>0$ which means that for suitable values of the multidimensional cosmological constant [see (2.30)] there are compactifying solutions which are stable against fluctuations of the inflaton field $\chi$. However, these solutions are unstable against fluctuations of $\mathbf{g}$ as can be seen from the fact that $\mathbf{g}^{v}=0$ is a local maximum of $V_2(\mathbf{g})$ in (2.20b). Fortunately for nontrivial embeddings of SO($d$) in the gauge group (second reference in [8]) or for internal spaces $G^{\rm int}/H^{\rm int}$ with nonsemisimple isotropy groups [7,9] there are models for which the absolute minimum $v_2$ of $V_2$ is positive. As is well known [28] for the absolute minimum $v_2$ to be positive it is necessary and sufficient that the homomorphism $\lambda^{(2)}$ of the isotropy group $H^{\rm int}$ of the internal space to the gauge group cannot be continued to a homomorphism of the group $G^{\rm int}$.

Willing to keep the form (2.4) for the spacetime (the internal space being a sphere) we shall use the method of nontrivial embeddings. The analysis of internal spaces with nonsemisimple isotropy groups (such as the complex projective spaces $CP^n$) can be straightforwardly performed. Following the third reference in [5] we consider an embedding

$$\lambda^{(2)}[\mathrm{SO}(d)]\subset \mathrm{SO}(N) \quad (4.1)$$

defined by the branching rule

$$\mathbf{N}\downarrow_{\lambda^{(2)}[\mathrm{SO}(d)]}=\nu\mathbf{d}+(N-\nu d)\mathbf{1} \ . \quad (4.2)$$

In Sec. II we have considered the case $\nu=1$. From (4.2) we conclude that for

$$\nu>N-\nu d \quad (4.3)$$

the homomorphism $\lambda^{(2)}$ cannot be extended to a homomorphism of SO($d+1$) which implies that the symmetric gauge field associated with (4.2) has a potential $V_2$ with a positive absolute minimum. Therefore we shall further assume that

$$\mu=N-\nu d<\nu \ . \quad (4.4)$$

The explicit form of $\tilde{\lambda}^{(2)}$, the embedding of so($d$) in so($N$) corresponding to (4.2), reads

$$\tilde{\lambda}^{(2)}(\tilde{T}^{(d+1)}_{mn})=\sum_{s=1}^{\nu}T^{(N)}_{3+(s-1)d+m\ 3+(s-1)d+n} \ . \quad (4.5)$$

The Ansatz for the SO(4)×SO($d+1$)-symmetric gauge fields is now given by

$$\hat{A}(t)=\frac{1}{2}\left[\sum_{p,q=1}^{\mu} B^{pq}(t)T^{(N)}_{3+\nu d+p\,3+\nu d+q}+\sum_{r,s=1}^{\mu} C^{pq}(t)\tilde{T}^{(N)}_{rs}\right]dt+\frac{1}{4}\sum_{i,j}^{3} T^{(N)}_{ij}\omega^{ij}+\frac{1}{4}\sum_{m,n}^{3+d}\sum_{s=1}^{\mu} T^{(N)}_{m+(s-1)d\,n+(s-1)d}\tilde{\omega}^{m-3\,n-3}$$
$$+\sum_{i=1}^{3}\left[\frac{1}{4}f_0(t)\sum_{j,k=1}^{3}\varepsilon_{jik}T^{(N)}_{jk}+\frac{1}{2}\sum_{p=1}^{\mu} f_p(t)T^{(N)}_{i\,3+\nu d+p}\right]\omega^i+\sum_{m=4}^{d+3}\left[\frac{1}{2}\sum_{p=1}^{\mu}\sum_{s=1}^{\nu}\tilde{g}_{ps}(t)T^{(N)}_{m\,d+3+q}\right]\omega^m , \qquad (4.6)$$

where $\tilde{T}^{(N)}_{rs}=\sum_{k=1}^{d}T^{(N)}_{3+(r-1)d+k\,3+(s-1)d+k}$.

By substituting (2.11)–(2.13) and (4.6) into action (2.2) we obtain an effective action which differs from (2.17) only in the part corresponding to the internal gauge field components:

$$S^{\text{eff}}_{\text{IGF}}=16\pi^2\int_{t_1}^{t_2}dt\,Na^3\left[e^{-2\beta\psi}\frac{d}{4e^2}\frac{1}{b_0^2}\frac{1}{2}\text{Tr}\frac{\mathcal{D}_tG}{N}\frac{(\mathcal{D}_tG)^T}{N}\right.$$
$$\left.+e^{-(d+4)\beta\psi}\frac{1}{b_0^4}\frac{d(d-1)}{8e^2}V_2(G)+e^{-2\beta\psi}\frac{1}{a^2b_0^2}\frac{3d}{4}\frac{1}{8e^2}\sum_{s=1}^{\nu}\sum_{p,q=1}^{\mu}f_pf_q\tilde{g}_{ps}\tilde{g}_{qs}\right] \qquad (4.7)$$

where $G=(\tilde{g}_{ps})_{p=1,\ldots,\mu;\ s=1,\ldots,\nu}$ parametrizes the internal components of the gauge field,

$$\mathcal{D}_tG=\frac{d}{dt}G+\hat{B}(t)G+\hat{C}(t)G \qquad (4.8)$$

with $\hat{B}(t)=[B^{pq}(t)]_{p,q=1,\ldots,\mu}$, $\hat{C}(t)=[C^{pq}(t)]_{p,q=1,\ldots,\nu}$ and

$$V_2(G)=\frac{1}{8}[\text{Tr}(1_\mu-GG^T)^2+\nu-\mu] , \qquad (4.9)$$

$1_\mu$ being the $\mu\times\mu$ identity matrix.

By neglecting the direct interaction between **f** and $G$ associated with the last term in the RHS of (4.7)—this interaction gives as in (2.24c) terms $O(16\pi kT^2)$—we can conclude that the dynamics of the internal components of the gauge field is essentially ruled by $V_2(G)$.

The potential (4.9) is as expected positive for $\nu>\mu$:

$$V_2(G)\geq\frac{\nu-\mu}{8}>0 . \qquad (4.10)$$

The absolute minimum of $V_2(G)=v_2=(\nu-\mu)/8$ is attained at $G^{(0)}$ such that

$$G^{(0)}G^{(0)^T}=1_\mu , \qquad (4.11)$$

which can be easily obtained explicitly.

We have therefore solved the problem of finding stable extrema (minima) $G^{(0)}$ of $V_2$ for which $V_2(G^{(0)})>0$.

## V. CONCLUSIONS

In this paper we have studied the stability of compactifying solutions in multidimensional Einstein-Yang-Mills theories after inflation. Aiming to study the cosmological setting upon which compactification of the extra dimensions occurs, we have imposed that the relevant fields were spatially homogeneous. Since we were interested in analyzing the stability of solutions after inflation, i.e., after reheating, we have not set the external-space components of the Yang-Mills gauge field to vanish. It is precisely through this procedure that we are able to introduce temperature into our analysis and generalize the effective potential (2.29) for the dilaton field—see the effective potential (3.5). Thus, our discussion is complementary to the one of Refs. [9,16,17] in which the external-space components of the Yang-Mills gauge field were set to vanish.

We find that at zero temperature there exist compactifying solutions of the Einstein-Yang-Mills system together with an inflaton field provided $\Lambda<c_2/16\pi k$ ($c_2=[(d+2)^2(d-1)/(d+4)]e^2/16v_2$)—see the discussion at the end of Sec. II. For $c_1<16\pi k\Lambda<c_2$ [$c_1=d(d-1)e^2/16v_2$] the compactifying solutions are classically stable but semiclassically unstable. However, if $\Lambda=c_1/16\pi k$ the solution corresponding to compactification is both classically and semiclassically stable. At a nonvanishing temperature the static dilaton field configuration for which the internal-space dimensions are compactified is the absolute minimum of the effective potential (3.5). Furthermore, we find that at a nonzero temperature the solutions corresponding to decompactification are semiclassically unstable. Nevertheless, despite this instability, the latter solutions are undesirable as their transition rate to the compactified vacuum is fairly slow. This implies that if after inflation the Universe had fallen trapped in the decompactified vacuum a major disruption in the cosmological standard scenario would ensue.

The conclusions above rely crucially on the assumption that $v_2=V_2(\mathbf{g}^v)>0$. For the simplest embedding (2.15) of SO($d$) into the SO($N$) gauge group this condition can be satisfied only for the unstable local maximum of $V_2(\mathbf{g}^v=0)$. However, as discussed in Sec. IV the condition $v_2=V_2(\mathbf{g}^v)>0$ can be achieved for nontrivial embeddings of SO($d$) in the gauge group SO($N$) or for internal spaces with nonsemisimple isotropy groups. For the former case we have shown that by choosing an embedding of the isotropy group of the internal space to the gauge group as defined by the branching rule (4.2) satisfying the condition (4.3), one can easily obtain stable minima of $V_2$ for which $V_2>0$.

As shown in Ref. [16] the classically stable but semiclassically unstable compactifying solutions of Ref. [9] do survive an inflationary period of whatever type. In addition to these results our analysis reveals that the reheating process that follows inflation is not a source of instabilities in the compactification process either. On the contrary, we have shown that the introduction of temperature turns the solutions corresponding to compactification classically as well as semiclassically stable.

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