Chapter One
Map of the Territory

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Resumo

As lógicas da inconsistência formal (**LIF**s) são lógicas paraconsistentes que nos permitem internalizar os conceitos de consistência ou inconsistência em nossa linguagem obje[c]to, introduzindo novos operadores para falar sobre tais conceitos e tornando possível, em princípio, separar logicamente as noções de contraditoriedade e de inconsistência. Apresentamos as definições formais de tais lógicas no contexto da Lógica Abstra[c]ta Geral, sustentamos que elas representam na realidade a maior parte das lógicas paraconsistentes existentes até o momento, se não ao menos as mais excepcionais dentre elas, e demarcamos uma subclasse de tais lógicas, os chamados **C**-sistemas, como aquelas **LIF**s que são construídas sobre a base positiva de alguma dada lógica consistente. A partir de caracterizações precisas de alguns princípios lógicos estabelecidos, mostramos que o ponto fulcral da lógica paraconsistente repousa sobre o Princípio da Explosão, ao invés do Princípio da Não-Contradição, e também distinguimos claramente estes dois princípios do Princípio da Não-Trivialidade, considerando a seguir várias formulações mais fracas da explosão e investigando suas inter-relações. Em seguida, apresentamos as formulações sintá[c]ticas de alguns dos principais **C**-sistemas baseados na lógica clássica, mostramos como várias lógicas bem conhecidas da literatura podem ser reformuladas como **C**-sistemas e estudamos cuidadosamente as suas propriedades e limitações, mostrando por exemplo como tais sistemas podem ser usados para reproduzir inteiramente as inferências clássicas, apesar de constituírem eles próprios apenas fragmentos da lógica clássica, e aventuramos alguns comentários sobre as suas contrapartidas algébricas. Definimos ainda uma classe particular dos **C**-sistemas, os **dC**-sistemas, como aqueles nos quais os novos operadores de consistência e inconsistência podem ser dispensados. O escrutínio dos métodos gerais apropriados para fornecer interpretações adequadas para estas lógicas, tanto em termos de semânticas de valorações quanto em termos de semânticas de traduções possíveis, pode ser encontrado em outros artigos. O presente estudo se propõe tanto a apresentar e caracterizar do zero o campo no qual ele se insere, apontando evidentemente as conexões com o trabalho de vários autores e anotando algumas questões em aberto, quanto a apontar algumas direções para continuação, estabelecendo de passagem um arcabouço teórico unificador para a investigação ulterior por pesquisadores envolvidos com os fundamentos da lógica paraconsistente.

Contents

> These ambiguities, redundancies and deficiencies remind us of those which doctor Franz Kuhn attributes to a certain Chinese encyclopedia entitled 'Celestial Empire of Benevolent Knowledge'. On those remote pages it is written that animals are divided into (a) those that belong to the Emperor, (b) embalmed ones, (c) those that are trained, (d) suckling pigs, (e) mermaids, (f) fabulous ones, (g) stray dogs, (h) those that are included in this classification, (i) those that tremble as if they were mad, (j) innumerable ones, (k) those drawn with a very fine camel's hair brush, (l) others, (m) those that have just broken a flower vase, (n) those that resemble flies from a distance.
> —Jorge Luis Borges, *The analytical language of John Wilkins*, 1952.

I will briefly highlight in what follows some of the most significative motivations and results of the hereby included paper, 'A taxonomy of **C**-systems' —henceforth referred to as TAXONOMY.

Byzantinisms

There are several inappropriate ways of depluming a biped, and several ways of rendering one's field of research harmless and uninteresting by way of an inappropriate definition or classification. Good ol' Diogenes would certainly have found rather amusing the classification of the paraconsistent logics produced by the school founded by Newton da Costa as 'Brazilian paraconsistent logics', 'positive-plus logics', 'non-truth-functional logics', and so on. For such 'paraconsistent definitions' are at the same time too restrictive and too general, and, even at an informal level, they leave too much of reality out, on the one hand, and put too much of it in, on the other. Our TAXONOMY aims at a methodic classification of several varieties of paraconsistency and purports to make a criterious selection of what should be inside the above kingdoms and phyla, but it also gets fine-grained enough so as to talk about some specific remarkable genera and species. The corresponding resulting class of 'Brazilian paraconsistent logics' —or at least the (arguably) most interesting among them, wherever they might be produced— will in the end comprehend those, and exactly those, logics that are able to express the notion of consistency, including the surprisingly large family of logics which have been (re)christened, in the TAXONOMY, **C**-systems.

The first label, 'Brazilian paraconsistent logics' (cf. [40, 35]), certainly sounds facetious. Or does anybody think that logics have nationalities? What next, requiring a visa for some Third World logics to travel from one place to another? Perhaps this is just an inner joke of the relevantist community, being already used to separate the world between 'US' (U.S.?) and 'them' (other improbable places, such as 'Australia'), and talking about the 'American Plan' on relevance logics in contrast to the 'Australian Plan'. By the way, given that Jaśkowski's logic **D2** can be characterized as a **C**-system on our current definition of the term, it is somewhat droll to realize then that 'Polish logics' are 'Brazilian'... Or is it the other way around?

The next label, 'positive-plus logics' (cf. [33, 23]), or, even more inelegantly, 'positive logic plus approach' (cf. [32]), is supposed to designate the "logics that augment classical or intuitionist [sic] positive logic with a non-truth-functional negation" (cf. [31], p.300). This classification seems to have gained quite a few adherents, perhaps because it made some people believe that they knew what 'Brazilian paraconsistent logics' were about. Most of the time, however, the denomination was simply used in order to refer to this other category of things that they call '**C**-systems', and by the term '**C**-systems' most people mean just the original daCostian hierarchy of paraconsistent logics C_n, $1 \leq n \leq \omega$ (cf. [15, 16]). From our present point of view, this label is a bad choice for various reasons, among them: (1) because there is nothing really special about the original C_n logics, and it is easy to imagine indeed several other similar hierarchies that could take their place (cf. [19, 28]); (2) because logics such as C_ω are indeed 'positive-plus' with respect to intuitionistic logic, but do not deserve to be called **C**-systems according to our present approach; (3) because there are logics that are perfectly truth-functional (see more about this below), such as $\mathbf{J}_3$ and $\mathbf{P}^1$, that have been proposed by the Brazilian school and are very different from the original C_n, but that do fit under our present definition of **C**-systems, being 'positive-plus' with respect to classical logic and being able to express consistency. To be sure, the Brazilian school is partly to be blamed for the confusion, as it never cared to make clear what it meant by the term '**C**-system', and used it always in a very loose way. It never ceases to amaze me that people will endlessly discuss the adequacy of the use of a certain term without even trying first to make clear what they mean by it! In the TAXONOMY we do our best so as to fix this situation, starting with a precise notion of an order of expressive entities to be called 'Logics of Formal Inconsistency'. Not by mere chance, the definition that we will offer for **C**-systems as a particular family of entities from that order will make sure that those systems (be they truth-functional or not) *are* in fact 'positive-plus' with respect to some previously given consistent logical basis —though intuitionistic and classical logics will certainly not be the only possible bases for that operation.

On what concerns the third unfortunate label, 'non-truth-functional logics' (cf. [31, 34]), it is not even clear whether the people that use it know precisely what they are talking about. In [34] one can read that: "The study of non-truth-functional systems was initiated by da Costa (who has also produced several other kinds of system [sic]). The main idea here was to maintain the apparatus of some positive logic, say classical or intuitionistic, but to allow negation in an interpretation to behave non-truth-functionally." Now, even if we decide to overlook the fact that 'non-truth-functionality' certainly has nothing to do with 'positive logic', we are still left with the problem of determining what is the precise underlying notion of 'truth-functionality' that should here come into play. It is somewhat unfortunate that still nowadays people will believe, without any particular technical justification and even without a philosophical justification, that some given logic can be distinguished from another given logic by means of the semantics that might be circumstantially associated to them. This seems in fact to be one of the underlying beliefs of the important recent chapter [31] of the Handbook of Philosophical Logic, where paraconsistent logics are presented and contrasted from the point of view of some preferred semantic presentations. The first trouble with this ill-advised approach is that the *same* logic (under most definitions of what it means to say that two logics are 'the same') can often be characterized through several different semantic presentations. Consider a particular example. In [30] the same logic LP is presented twice, by way of two different sorts of 3-valued semantics. It is not difficult to see, anyhow, that this same logic can also be characterized in many other ways: by means of a non-truth-functional 2-valued semantics, a possible-translations semantics, a society semantics, a modal-like semantics, dialogues, tableaux, and so forth. Should 'truth-functional' mean that the logic has 'at least one many-valued adequate semantics'? One who defends such a definition should then recall some well-known adequacy results from General Abstract Logics (a.k.a. Universal Logic) that can be used to show that *any* 'tarskian logic', LP included, has an adequate many-valued semantics, in fact, even a 2-valued one (check **Chapter 2.1**, further on). And what does it mean for a connective to 'behave non-truth-functionally'? Does it mean that it does not have a canonical modal interpretation satisfying the replacement property? In that case, again, it should be recalled that there are many **C**-systems, in the present definition of the term, that satisfy that property (check **Chapters 3.2** and **3.3** for all usual normal modal logics recast as **dC**-systems). Perhaps the term 'non-truth-functional' should simply be avoided by those who do not feel comfortable with the topic of General Abstract Logics (otherwise, a diagonal reading of [41] or of [42] or of [38] is always recommendable).

Fortunately, at least one of the authors of [34] seems to have now adopted a more reasonable appellation: In an interesting recent paper (cf. [39]), the expression 'Brazilian school of paraconsistency' is used. It is still somewhat problematic, however, to talk about 'schools of paraconsistency' as if they had nationalities. While the expression 'Belgian school of paraconsistency' nowadays might bring to one's mind the inconsistency-adaptive logics developed by Batens and his disciples, notwithstanding the fact that there are other people in Belgium that do paraconsistency with no affiliation to nor coincidence of interests with that school, the expression 'Australian school of paraconsistency' used so as to refer to some specific developments by Priest and Routley/Sylvan is pretty abusive: In that case one might easily think instead of other 'Australian' paraconsistentists, say, those that deal more with relevance issues, such as Meyer, Slaney, and Brady. Besides, is there anyone else alive in Australia, or in the world, willing (or capable) to defend exactly the same 'dialetheist' views on logic as those systematically defended by Priest since many years? If not, why bother to talk about a 'school' that has no pupils? Finally, in the case of the so-called 'Brazilian school of paraconsistency', influenced by the work of da Costa and collaborators, the situation is even more disconcerting. The trouble is not so much that there are always people from the 'Brazilian school' that do not live and work in Brazil, but that there are many many people working on paraconsistency in Brazil at any given time, and they cannot be said to belong all to the daCostian school, or even to share common interests and tools with that school.

All that said and done, the reader will observe that the present thesis, benefiting a little from the work of each school on paraconsistency and departing freely from the received traditions when necessary, is as 'Brazilian' as it can be.

The meat

The **Section 1** of the TAXONOMY brings an extensive and detailed introduction to the contents of the paper, and it is better that you read it than that I try to further condense it here.

Section 2 contains the material that I consider to be of more immediate philosophical significance. If we shall have logics as objects of research rather than auxiliary tools that come to help on that research, we need a rich metalanguage to talk about the inferential mechanisms of these logics. This is to say that the study of logics as mother-structures in the sense of Bourbaki (cf. [8] and [5]) will need to be set in a more or less formal framework such as the one in [13], in such a way that we can schematically quantify, for instance, over theories and formulas of a given logic. The study of General Abstract Logic is anything but new, but syntactical and semantic-oriented approaches to logic certainly have collected more adherents in the present times. This section of the paper was born from my intuition that neither syntax nor semantics provide in general the right level of abstraction for a number of logical properties to be expressed.

There is an awful number of papers in the philosophical literature discussing 'logical principles' that you finish reading without having any clear idea of what the authors even *meant* by such and such a principle. This is because typically the principles are not defined or stated with any degree of precision. No wonder there is so much disagreement then on the import of such principles: It seems everybody has their private understanding of a given principle, and they will refuse to formalize it a single bit to help other people agreeing or disagreeing over that understanding. Hopefully, this hand-waving way of doing science and philosophy will become more the exception than the rule in the near future, as serious and well-trained new generations of logicians take the scene.

This section of the Taxonomy offers precise formulations of several logical principles, assuming a logic to be a schematic structure whose universe of discourse is a set of formulas in a signature containing a symbol for negation. The logical structure is also assumed to contain a relation that represents the notion of (multiple-premise-single-conclusion) consequence. All 'decent' logics are supposed to respect the Principle of Non-Triviality (PNT), and paraconsistency is equated to the failure of the Principle of Explosion (PPS), also referred to in the paper as Pseudo-Scotus or as *ex contradictione sequitur quodlibet*. I also offer here a very particular reading of the so-called Principle of Non-Contradiction (PNC), sharply distinguishing it from the Principle of Explosion: 'Dialectic' non-trivial logics are paraconsistent logics that fail (PNC) —being often non-structural logics, to that effect; the immense majority of paraconsistent logics in the literature however are not dialectic and do not disrespect (PNC). For logics that do respect (PPS), at any rate, the principles (PNC) and (PNT) are often interderivable (Fact 2.6). A few alternative abstract definitions of paraconsistent logics are surveyed and the conditions for their equivalence to be proved are emphasized (Facts 2.7 and 2.14). Several weaker varieties of 'explosion' that are compatible with paraconsistency are also surveyed. In particular, my formulation of '*ex falso sequitur quodlibet*' does not imply paraconsistency, as it is commonly assumed in the literature: *ex falso* and *ex contradictione* are simply two distinct principles, as it is (in the present framework) easy to check. A paraconsistent logic can also have —and often has— a contrary-forming negation operator. In that case, and in general in any case in which a negation with an explosive character is present, the logic is said to respect (sPPS), the 'supplementing' form of (PPS). The role that conjunction and implication might play on relating the previous principles is elucidated (Figures 2.1 and 2.2). Other important varieties of explosion are also formulated, including a 'partial' form (pPPS) according to which not all formulas of the logic are derived from a contradiction, but all formulas with a certain format (say, all negated formulas) are so derived. Usually, paraconsistent logics are required to be 'boldly paraconsistent', avoiding both the basic form of explosion and the other partial forms, that is, they are required to fail (pPPS). The **Errata** at the end of this chapter shows, among other things, that the logics we work with in the Taxonomy *are* boldly paraconsistent. Yet another form of explosion, a 'controllable' one (cPPS), says that at least some contradictions explode, if not all. This form of explosion is almost inevitable: Fact 3.32 later on will show that already very weak paraconsistent logics are controllably explosive, if only they are sufficiently expressive.

A final fundamental variety of explosion introduced in this section should be highlighted: the 'gentle' explosion (gPPS). Inconsistent logics can be either trivial (absolutely inconsistent) or non-explosive (paraconsistent). Paraconsistent logics are thus non-trivial logics having a negation that lacks the 'consistency presupposition'. But some paraconsistent logics —those respecting (gPPS)— are expressive enough so as to internalize the very notion of consistency at the object language level. Such logics are called Logics of Formal Inconsistency (**LFI**s). As a consequence, despite constituting fragments of consistent logics, such **LFI**s can canonically be used to faithfully reproduce all admissible consistent inferences, just by adding to them, in each case, a convenient set of 'consistency assumptions' (recall the *Fundamental Feature of* **LFI**s mentioned at the **Prolegomena** of this thesis, or at the section 2 of **Chapter 3.3**, further on). Other ways in which our **LFI**s can recover consistent reasoning by way of direct grammatical translations are illustrated in the whole of **Section 3.7** and in Theorems 3.61 and 3.67 of the present TAXONOMY. **C**-systems are defined in the end of **Section 2** as those **LFI**s that can be constructed from the positive part of given consistent logics by the addition of a single new connective to represent consistency. With the exception of one **LFI** mentioned in **Section 3.10** —the logic that constitutes the deductive limit to da Costa's hierarchy or paraconsistent logics C_n, $1 \leq n < \omega$ (cf. [11])—, all the remaining **LFI**s studied in the TAXONOMY are **C**-systems based on classical logic. The near-ubiquity of **LFI**s among paraconsistent logics is illustrated at Fact 2.19.

A few other abstract definitions can be found elsewhere on the paper that would theoretically belong to the present section. Thus, the definition of **dC**-systems as a variety of **C**-systems in which the consistency connective can be introduced through a definition in terms of more usual connectives is to be found in **Section 3.8**. Well-known examples of **dC**-systems include the C_n, $1 \leq n < \omega$, and the logic $\mathbf{P}^1$. A well-known example of a **C**-system that is not a **dC**-system is given by the logic $\mathbf{J}_3$ (or **LFI1**). A well-known example of a logic that used to be informally included among the '**C**-systems' (or '**C**-logics') and that now falls outside this class is given by da Costa's logic C_ω. It should be mentioned that almost all of the above mentioned definitions are novel, at least at the present level of precision.

As not everybody seems to have this clear in mind, there are some further small generic results about paraconsistent logics that are worth mentioning: (1) That not all contradictions are equivalent in a paraconsistent logic (Fact 2.8); (2) that disjunctive syllogism in general cannot hold good (Fact 3.19); (3) that contraposition also fails, in general (Theorem 3.20).

Section 3 is much more practically-minded. I work there in the old-fashioned way, with Hilbert-style characterizations of a few simple **C**-systems, starting with **bC** in **Section 3.2**, and then I add more and more axioms until I arrive to a class of maximal paraconsistent fragments of classical logic in **Section 3.11**. There are several results (3.14, 3.17, and many others) showing how classical reasoning can be recovered from inside **C**-systems if only a sufficient number of 'consistency assumptions' are in each case added to the premises of our inferences. Several independence results related to the axioms that we consider are stated, and in spite of the fact that they are in general not that easy to prove if a decision procedure is not available, they are not really worth mentioning here. There are many results related to the failure of the 'intersubstitutivity of provable equivalents' (IpE) (or 'replacement property') inside many of the present paraconsistent logics, and some other results (such as Theorem 3.41 and Fact 3.81) that show that a partial form of (IpE) occasionally holds at least for some particular formulas of some of our logics. On what concerns the failure of (IpE), Theorem 3.51 is noteworthy, for summarizing results from several papers together with new ones and setting some very general conditions for (IpE) to be disrespected by paraconsistent logics. The problems with (IpE) eventually evolve into serious trouble in producing non-degenerate algebraizations for many of our present logics. A survey of what was known by then and a partial classification of our **LFI**s from the point of view of Abstract Algebraic Logic in the manner of Blok-Pigozzi (cf. [7, 22]) is done in **Section 3.12**. We are now sure, however, that replacement is not really out of reach, as there are indeed many paraconsistent logics that satisfy full (IpE) —again, on that issue, remember to check **Chapters 3.2** and **3.3**.

For the interested researchers, several open problems and directions for further investigation are listed in **Section 4**.

Parts that were promised and are missing, things that will change

The Taxonomy was intended to be entirely self-contained (and it seems to have been reasonably successful on that), but it also aimed at exhaustiveness, if that is at all feasible. Thus, we also intended to deal, for example, with the semantics of the **LFI**s thereby presented, and we left there the promise to do that in a future paper. That paper is now unlikely to ever exist, having been superseded by a number of better conceived papers. Of course, the semantics of several **LFI**s (among them the 3-valued ones from **Section 3.11**) is already presented in the Taxonomy, but many other systems were left untreated. At any rate, semi-automated algorithms for defining adequate bivalent semantics for all our **LFI**s exist at least since [6], so we would not have much to contribute here. Many of the more convoluted **LFI**s from the Taxonomy had already received adequate possible-translations semantics in [28], and the paper on **Chapter 2.2** of the present thesis now shows how several of our weakest **LFI**s, none of them finite-valued, can also be interpreted in terms of a combination of specific 3-valued scenarios. The papers on **Chapter 3** show how the consistency connective can be given an adequate canonical modal interpretation, and putting that together with a modal interpretation of negation we can now talk about fully modal **LFI**s.

The maximality of the 8K 3-valued logics from **Section 3.11** with respect to classical logic was also hinted at, yet the corresponding paper, [27], is still not ready. The reader can have a very good idea of how the maximality proof works, however, if he only consults the Ap. $\omega+\omega$ of [28] or else the paper [29], where the proof is done in detail for a few logics from the above mentioned class.

Some very interesting extremely weak **LFI**s that were mentioned only in the (final) **Section 4** of the TAXONOMY are the logics **mbC** and **mCi**. They were now, however, carefully taken into account as our most basic examples of **C**-systems based on classical logic plus 'excluded middle', studied in the handbook chapter [10], an important offspring of the present dissertation. The main axiomatic and semantic properties of those logics are also studied here, in **Chapter 2.2**.

The present single-conclusion approach to consequence relations is, in a sense, 'biased towards truth' and it does not permit one to take full profit of the above mentioned general abstract definitions. Further on, in **Chapter 4**, in a multiple-conclusion framework, I will show for instance how the above Principle of Non-Triviality can be generalized so as to regulate not just one but four degenerate examples of logics. Moreover, I will also show how that framework allows us to distinguish Pseudo-Scotus from *ex contradictione*, the former principle to be failed by non-trivial inconsistent logics in general, and the latter to be failed by the 'decent' paraconsistent logics among them.

Brief history

The TAXONOMY has a somewhat winding history. During the writing of my Master's Thesis (cf. [28]), in between 1998 and 1999, I had the chance of acquiring a significant knowledge of the literature on paraconsistent logics, and specially of the variants of such logics that had been produced in Brazil in the last 40 years or so. I had no particular interest on paraconsistent logics from the start —my interest at the time laid more on formal semantics and all-purpose logical tools, and the things you can do with them. It was on and about paraconsistency, however, that I found a wealth of notable logical problems to attack with the tools I had at the time, and thus I dug into it.

Just when I finished the thesis and had all those results in hands, we were starting to organize the II World Congress on Paraconsistency (WCP'2000), that would be dedicated to Newton da Costa and that was to congregate a very international audience in Brazil in the following year. Together with Walter Carnielli, my supervisor in my then initial doctoral developments, we decided to offer at the WCP'2000 a kind of survey of the paraconsistent logics 'made in Brazil', to wit, those logics directly developed by da Costa and collaborators or at least inspired by their approach. What could be the unifying framework for reconstructing decades of variegated work in the area in just 50 minutes? We frankly had no real idea of where to start. We would certainly like to recall and generalize some fundamental ideas by da Costa (and collaborators): his 'Tolerance Principle' (cf. [14]), his initial requisites on paraconsistency (cf. [16]), the theory of valuations and bivalued semantics (cf. [18]), the intuitions on duality with paracompleteness (cf. [26]), the agnostic perspective on the existence of 'true contradictions' (cf. [17]). On the top of that, we would also like to add some new results: a couple of interesting new logics that I had been tinkering with, their possible-translations semantics, and some recent notes on troubles related to the algebraization of such logics. The lecture was announced as 'The **C**-systems: Paleontology and Futurology', and was chosen to close the congress. But when it finally came about, in May 2000, we had already chosen some very specific paths to tread. We had decided to capitalize on the notion of 'consistency' as a primitive object language notion, generalizing da Costa's notion of 'good behavior' to a whole new dimension. The idea, from the start, was that of exploring the possibility of having paraconsistent fragments of classical logic that would nevertheless be capable of recapturing classical reasoning in a very natural way. The ability to express consistency helped neatly on that. We were content as we seemed to had attained by then the right level of generality: The chosen framework was able to put together in the same class of **C**-systems logics so diverse as da Costa's 1963 logics C_n, $1 \leq n < \omega$ (cf. [15, 16]), Sette's 3-valued logic $\mathbf{P}^1$ (cf. [37]), and D'Ottaviano & da Costa's 3-valued logic $\mathbf{J}_3$ (cf. [21]), besides, as we saw later on, also Jaśkowki's 1948–49 logic **D2** (cf. [24, 25]), and Schütte-Batens logic **CLuNs** (cf. [36] and [4]); at the same time, other less expressive logics were definitively excluded from that framework, such as da Costa's logic C_ω (cf. [20, 15]), Asenjo-Priest's logic *LP* (cf. [30] and [1]), or Batens-Avron's logic *Pac* (cf. [2] and [3]).

By early September 2000 the above ideas had been much more thoroughly developed, and I gave a detailed account on them to the group of Newton da Costa at the Faculty of Philosophy, Languages and Human Sciences of the State University of São Paulo (BR). There, the outlines of the Taxonomy were first appreciated and warmly welcomed to the world. Anyhow, it was not before I went to live in Germany, a few days later, with a Capes / DAAD grant for a ProBrAl project, that the serious development of those ideas jump-started. It would in the end take me at least 7 months of hard work and require much more reading and research that I would have dreamt of. About 40% of the paper was written during that first period, and the remainder was written after March 2001, when I took up a research position in Belgium, under a Dehousse doctoral grant. My boss during this second period was Diderik Batens, and it was only with his gentle permission and the generosity of his extremely careful reading of the final version of the paper that the job got finally accomplished. I am also grateful to the editors of [9], the volume in which the paper was to appear, for their willingness to consider this very late contribution and for their rewarding choice of referees. The present version of the paper would not have completely fulfilled Pindar's injunction and 'become what it is' had it not been for the help of a few careful commentators, including Chris Mortensen, Jean-Yves Béziau, Carlos Caleiro and Marcel Guillaume. My most sincere thanks to all of them.

On coauthorship

Given that the present chapter contains the most fundamental and the longest paper from the thesis, and given that I sign the paper only as its 'second author', the question of coauthorship has been raised.

For one thing, the paper would surely not have been possible without the continuous support (and the pressure) of Walter Carnielli, my coauthor in it and my supervisor in the present thesis. I am much obliged to his help and encouragement, to the countless discussions and comments he patiently exchanged with me on the subject by e-mail, to the many attempts he made on helping me to complete the paper's writing, and to his firmness in making me put a stop to the seemingly endless task. I am glad to have had someone like him as a coauthor, always encouraging as an enthusiast of the underlying project. I am also grateful, of course, to have now someone to share the responsibility for the mistakes that have been committed in the paper and that have been found so far (check the **Errata** at the end of this chapter).

It should be clear, at any rate, that failing to acknowledge my contribution in organizing and writing the paper, setting forth its main ideas, painstakingly double-checking the related literature, proposing its definitions and theorems, and finding out all the corresponding proofs is to risk being seriously unfair. That would not be too different from failing to acknowledge, say, the work of Paul Bernays in the 2 volumes of Hilbert & Bernays's *Grundlagen der Mathematik*, or failing to acknowledge the work of Bertrand Russell in the 3 volumes of Whitehead & Russell's *Principia Mathematica*. You wouldn't like to commit that mistake.

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