Exercise 1 – WS supersymmetry

Show that under an $N = 1$ supersymmetry transformation the WS current

$$G_a = \frac{i}{\sqrt{2\alpha'}}\,\rho^b\,\rho_a\psi^\mu\,\partial_b X_\mu$$

transforms into the WS energy-momentum stress tensor $T_{ab} = T^X_{ab} + T^\psi_{ab}$, i.e. $\delta_\epsilon G_a = -iT_{ab}\rho^b\epsilon$.

Hint: Use the Fierz rearrangement formula for spinors in two dimensions (with $\bar{\rho} \equiv \rho_0\rho_1$)

$$\left(\bar{\lambda}\psi\right)\chi_\alpha = -\tfrac{1}{2}\left[(\bar{\lambda}\chi)\psi_\alpha + (\bar{\lambda}\bar{\rho}\chi)(\bar{\rho}\psi)_\alpha + (\bar{\lambda}\rho^a\chi)(\rho_a\psi)_\alpha\right]$$

and the equation of motion for ψ^μ_α.

Exercise 2 – GSO projection: the fermion number $(-1)^F$

Consider the fermion number operator

$$(-1)^F \equiv \begin{cases} e^{i\pi\hat{F}} & \text{in the NS sector} \\ b^1_0\cdots b^8_0\, e^{i\pi\hat{F}} & \text{in the R sector} \end{cases}$$

where $(i = 1, \ldots, 8)$

$$\hat{F} = \begin{cases} \sum_{r\in\mathbb{N}_0+\frac{1}{2}} b^i_{-r}b^i_r & \text{in the NS sector} \\ \sum_{r\in\mathbb{N}} b^i_{-r}b^i_r & \text{in the R sector} \end{cases}$$

Show that

$$\{b^i_r, (-1)^F\} = 0$$

for all the modes in the NS sector, and for all the modes in the R sector.

Exercise 3 – RNS open string

Consider the RNS open string with $\sigma \in [0, \pi]$. For the open string, the variation

$$(\psi_- \cdot \delta\psi_- - \psi_+ \cdot \delta\psi_+)\Big|_{\sigma=0}^{\sigma=\pi} = 0$$

has to be cancelled at each of the boundaries separately. NN boundary conditions are defined by

$$\psi^\mu_+(0) = \psi^\mu_-(0) \quad , \quad \psi^\mu_+(\pi) = \eta\,\psi^\mu_-(\pi) \quad , \quad \eta = \pm 1 \quad , \quad \mu = 0, \ldots, D-1\ .$$

The choice $\eta = 1$ is called Ramond sector, while the choice $\eta = -1$ is called Neveu-Schwarz sector. Obtain the mode expansion for ψ for both these sectors. You may want to define a suitable extension of ψ^μ_+ to the interval $\sigma \in [0, 2\pi]$.