PROBING THE EVH LIMIT OF SUPERSYMMETRIC ADS BLACK HOLES

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Workshop on Black Holes: BPS, BMS, and Integrability
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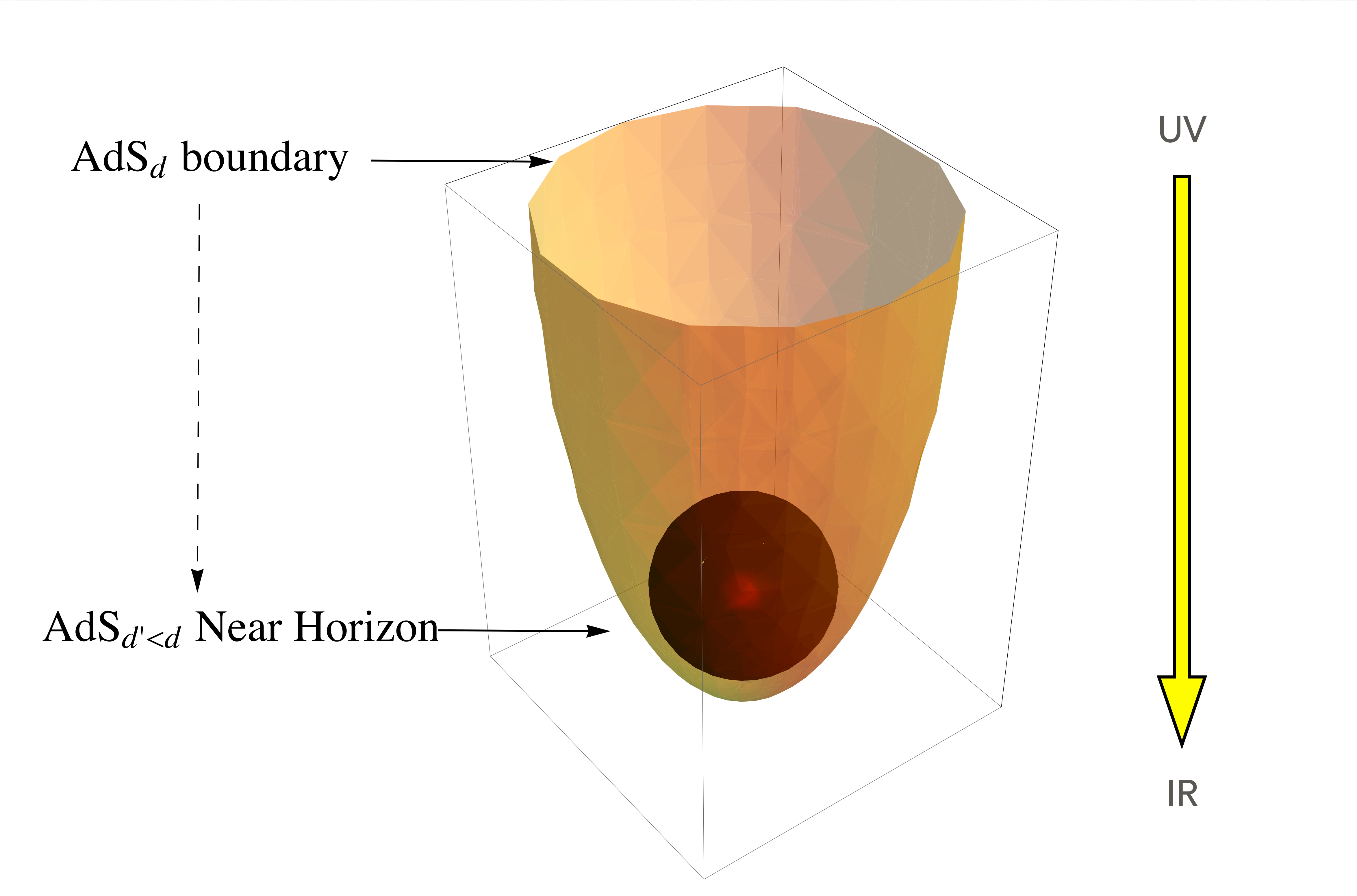
MOTIVATION



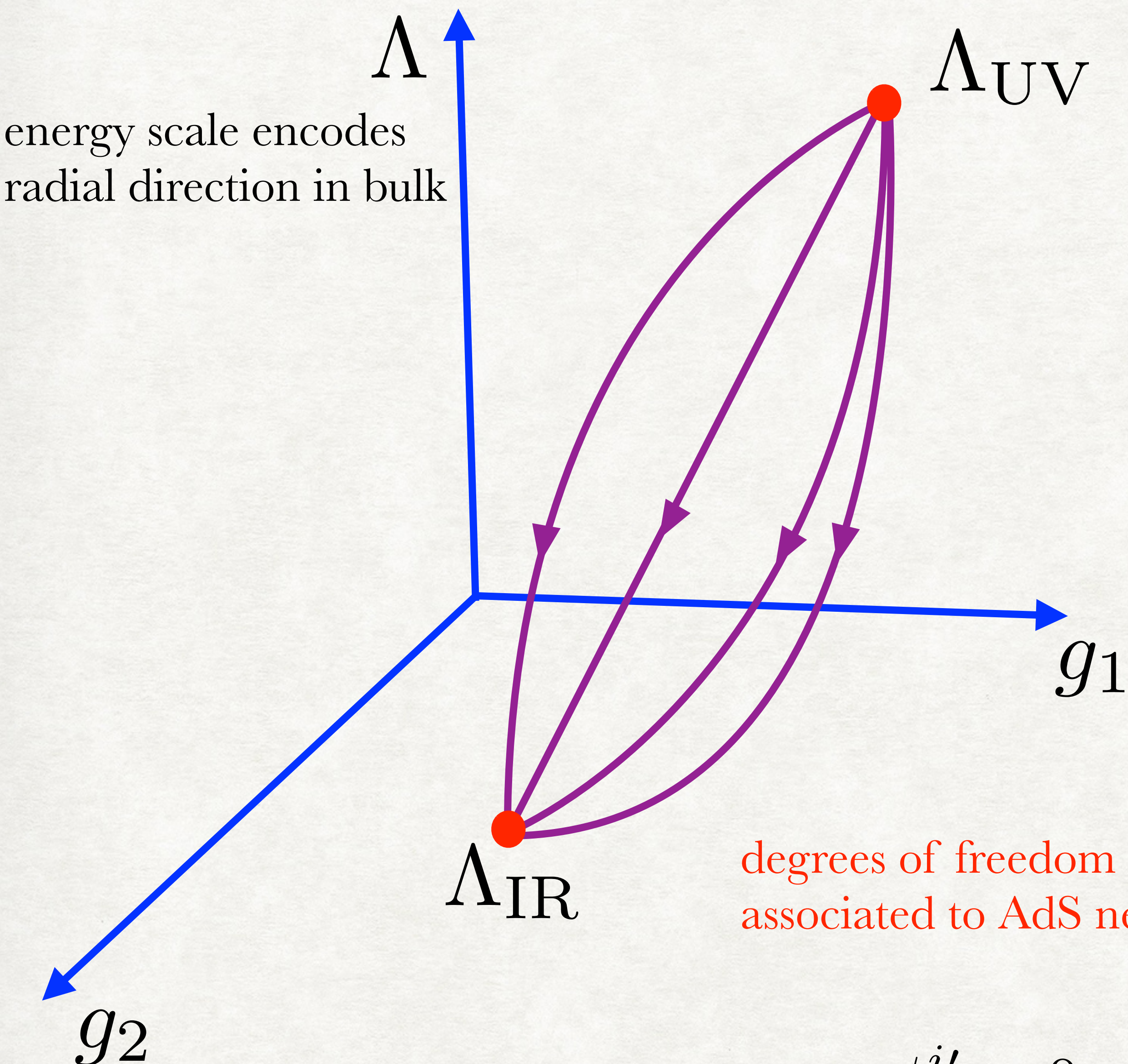
- Black hole entropy captured by both asymptotic and near-horizon CFTs

EXAMPLE

- Non-extremal, two R-charged black hole in $\mathrm{AdS}_5 \times S^5$
- Near horizon is $\mathrm{AdS}_3 \times S^3 \times T^4$
- Focusing limit gives BTZ
- Entropy is $S = \int dS_{\mathrm{BTZ}} = S_{\text{two-charge}}$
- Naïve state counting in CFT_4 in terms of extremal operators plus defects recovers entropy (quasi-precise in single R-charge case)
- Open question: How do the states connect?

Balasubramanian, de Boer, VJ, Simón

ATTRACTOR MECHANISM



degrees of freedom correspond to CFT_d
associated to AdS asymptopia

renormalization integrates out short distance physics;
c-function consequently flows downhill

scalar fields ϕ^i in bulk dual to operators
sourced by coupling constants on boundary

degrees of freedom correspond to CFT_n
associated to AdS near horizon region

c-function extremized at endpoints, $\phi^{i\prime} = 0$ as well

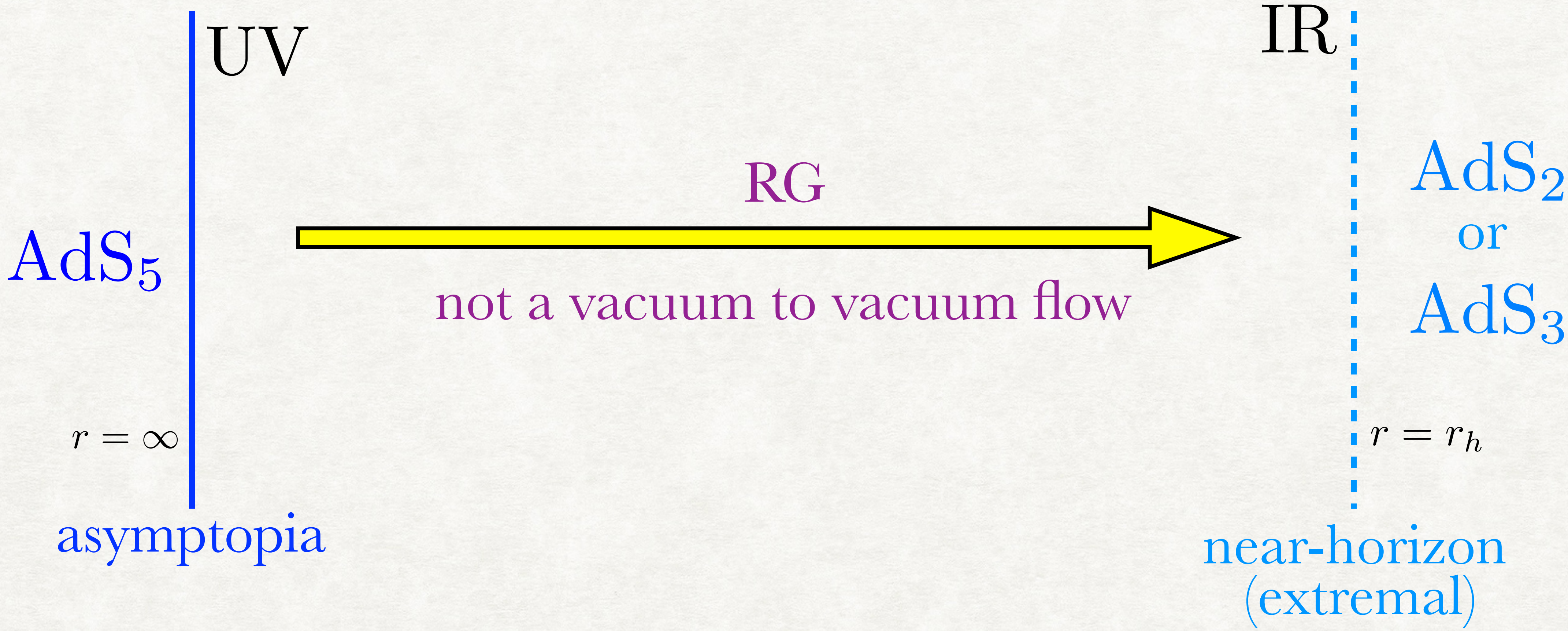
monotonically non-increasing flow

c-function not unique; non-uniqueness related to scheme independence/bulk diffeomorphism invariance

embed story in $\mathcal{N} = 2$ supergravity for extremal black holes, use attractor mechanism to construct c-function

Bhattacharyya, Haque, VJ, Nampuri, Véliz-Osorio

ATTRACTOR MECHANISM



- Gravitational analogue for extremal black hole in $\mathcal{N} = 2$ supergravity
- Due to attractor mechanism $\phi^{i\prime}(r) = G^{ij}\partial_j \Upsilon$
- Proposal: $c = \lambda + \kappa\,\Upsilon$
- Gradient flow plus null energy condition

Bhattacharyya, Haque, VJ, Nampuri, Véliz-Osorio

QUESTIONS

- How do degrees of freedom of the UV CFT map to degrees of freedom in the IR CFT?
- If there is a microstate picture, can we follow the ensemble of states describing a black hole along RG trajectory?
- Extremal black holes have AdS_2 or AdS_3 in near horizon
- For black holes in AdS_5 we go from CFT_4 (finite number of generators) to CFT_1 or CFT_2 (infinite number of generators)
- How does $\mathrm{AdS}_3/\mathrm{CFT}_2$ appear in $\mathrm{AdS}_5/\mathrm{CFT}_4$?
- Generalize to non-extremal case

OUTLINE

- Motivation
- EVH black holes
- Index counting
- Origin of IR CFT in UV CFT
- Prospectus

PUNCHLINES

- Techniques for analyzing $\frac{1}{16}$-BPS black holes in AdS_5 extended to study $\mathrm{EVH/CFT}_2$
- Entropy calculation for EVH and near EVH in fast rotating limit
- Legendre transform of superconformal index, which gives black hole entropy, is equivalent to derivation of Cardy formula
- Shows how IR CFT is realized in UV CFT
- Hints of similar stories in higher dimensions

COLLABORATORS

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based on 1910.14293, 2009.nnnnn

WHY STRING THEORY?

- The Schwarzschild black hole is first non-trivial solution of GR

$$\mathrm{d}s^2 = -\left(1 - \frac{2G_N M}{r}\right) dt^2 + \left(1 - \frac{2G_N M}{r}\right)^{-1} dr^2 + r^2\, d\Omega_2^2$$

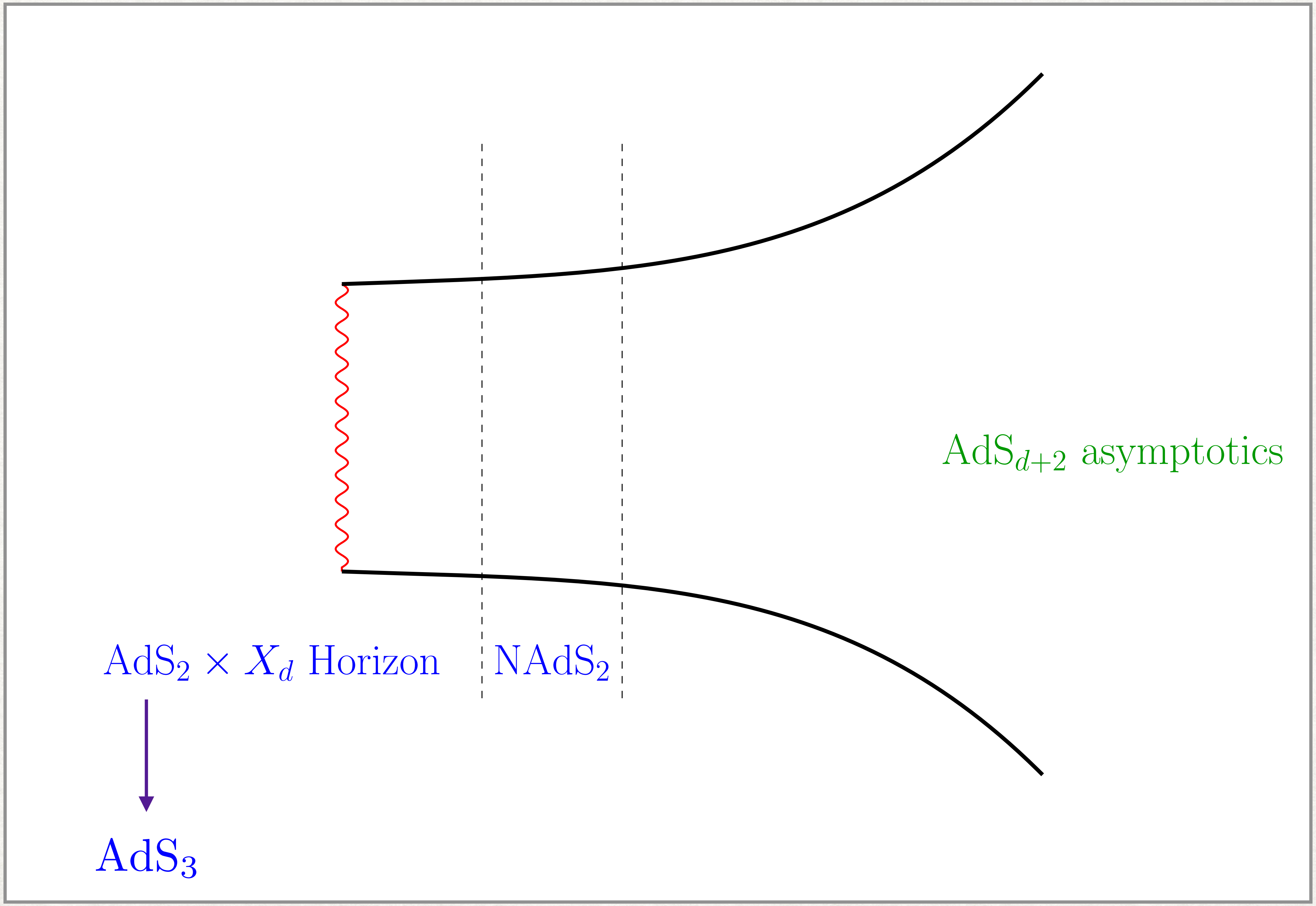
- The Schwarzschild radius is $r_h = 2G_N M$

- The size of a black hole grows with the gravitational coupling

- Most gravitating systems become smaller as interaction strength increases

- This is a test only black holes (and black hole microstates) in string theory pass (*cf.*, Horowitz–Polchinski correspondence principle)

EXTREMAL BLACK HOLES



Reall

DYNAMICS IN ADS$_2$

- Deviate from pure AdS_2 by breaking conformal symmetry
  - NAdS/SYK, random matrices, etc.
- Make UV CFT defined on non-compact space;
  - Spectrum is continuous; non-trivial dynamics at low energies
  - AdS/CMT
- Extremal vanishing horizon (EVH) limit
  - Decrease gap: $c \to \infty\,, \;\; \delta\Delta_{\mathrm{UV}} \sim c^{-1}$
  - Entropy: $S = \dfrac{A}{4G_N} \sim cA$
  - To keep this finite: $A \to 0$

EVH BLACK HOLES

- Low temperature expansion of black hole entropy

$$S(T,Q) = S_0(Q) + S_1(Q)T + \dots$$

- If $S \sim c_k T^k$, then CFT_{k+1} in IR pointing to AdS_{k+2} near horizon

- AdS_3 in case $k = 1$

- In limit $S \to 0$, $T \to 0$, $\frac{S}{T} = \text{finite}$

- Effective CFT_2 gives dynamics of EVH black hole

- EVH limit of this type for AdS_4 and AdS_5 black holes

Balasubramanian, de Boer, VJ, Simón
Fareghbal, Gowdigere, Mosaffa, Sheikh-Jabbari
Sheikh-Jabbari, Yavartanoo

STATIC ADS5 BLACK HOLE

$$ds_{10}^2 = \sqrt{\Delta} ds_5^2 + \frac{1}{\sqrt{\Delta}} \left(\sum_{i=1}^{3} H_i (d\mu_i^2 + \mu_i^2 [d\phi_i + a_i dt]^2) \right)$$

$$ds_5^2 = -\frac{f}{H_1 H_2 H_3} dt^2 + \frac{dr^2}{f} + r^2 d\Omega_3^2$$

$$H_i = 1 + \frac{q_i}{r^2}, \quad f = 1 - \frac{\mu}{r^2} + r^2 H_1 H_2 H_3$$

$$\Delta = H_1 H_2 H_3 \left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3} \right]$$

$$ds_{S^5}^2 = \sum_{i=1}^{3} (d\mu_i^2 + \mu_i^2 d\phi_i^2), \qquad \mu_1^2 + \mu_2^2 + \mu_3^2 = 1$$

Fareghbal, Gowdigere, Mosaffa, Sheikh-Jabbari
de Boer, Johnstone, Sheikh-Jabbari, Simón

CHARGES

- Compute Komar integral

$$\widetilde{q}_i = \sqrt{q_i(\mu + q_i)} \ , \quad M = \frac{N^2}{2}\left(\frac{3}{2}\mu + q_1 + q_2 + q_3 + \frac{3}{8}\right)$$

- Extremal without being supersymmetric
  - *cf.* attractor mechanism
  - μ measures deviation from extremality
  - μ_c critical value below which there is a timelike singularity

- Work with Reissner–Nordström AdS_5 and rotating AdS_5 black holes

- Take particular scaling limits

ROTATION

- Have some understanding of 4d field theory in supersymmetric case
- Static supersymmetric black holes have singularities (*e.g.*, superstars)

Myers, Tafjord

- So we include rotation
- Generically, two angular momenta and three R-charges

Gutowski, Reall
Cvetic, Gibbons, Lu, Pope
Chong, Cvetic, Lu, Pope
Kunduri, Lucetti, Reall
Wu

- Beyond AdS_5
  - The known AdS_4 rotating solutions have equal pair charges $Q_1 = Q_3$, $Q_2 = Q_4$.
  - AdS_6 black holes only have two rotations and one R-charge.
  - AdS_7 black holes have three rotations and two R-charges generically. The known solutions are either equal charge or equal angular momentum.

METRIC

- Metric is complicated!

$$ds_5^2 = H^{-\frac{4}{3}}\left[-\frac{X}{\rho^2}(dt - a\sin^2\theta\frac{d\phi}{\Xi_a} - b\cos^2\theta\frac{d\psi}{\Xi_b})^2 + \frac{C}{\rho^2}\left(\frac{ab}{f_3}dt - \frac{b}{f_2}\sin^2\theta\frac{d\phi}{\Xi_a} - \frac{a}{f_1}\cos^2\theta\frac{d\psi}{\Xi_b}\right)^2\right.$$
$$\left. + \frac{Z\sin^2\theta}{\rho^2}\left(\frac{a}{f_3}dt - \frac{1}{f_2}\frac{d\phi}{\Xi_a}\right)^2 + \frac{W\cos^2\theta}{\rho^2}\left(\frac{b}{f_3}dt - \frac{1}{f_1}\frac{d\psi}{\Xi_b}\right)^2\right] + H^{\frac{2}{3}}\left[\frac{\rho^2}{X}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2\right],$$
$$H = \frac{\tilde{\rho}^2}{\rho^2}, \quad \rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \quad \tilde{\rho}^2 = \rho^2 + q,$$
$$f_1 = a^2 + r^2, \quad f_2 = b^2 + r^2, \quad f_3 = (a^2 + r^2)(b^2 + r^2) + qr^2,$$
$$\Delta_\theta = 1 - a^2\cos^2\theta - b^2\sin^2\theta, \quad X = \frac{(a^2 + r^2)(b^2 + r^2)}{r^2} - 2m + (a^2 + r^2 + q)(b^2 + r^2 + q),$$
$$C = f_1 f_2(X + 2m - \frac{q^2}{\rho^2}), \quad \Xi_a = 1 - a^2, \quad \Xi_b = 1 - b^2,$$
$$Z = -b^2 C + \frac{f_2 f_3}{r^2}\left[f_3 - r^2(a^2 - b^2)(a^2 + r^2 + q)\cos^2\theta\right],$$
$$W = -a^2 C + \frac{f_1 f_3}{r^2}\left[f_3 + r^2(a^2 - b^2)(b^2 + r^2 + q)\sin^2\theta\right].$$

Chong, Cvetic, Lu, Pope

- Non-BPS black holes has four parameters: a , b , m , q

- BPS solution has five parameters + constraint

BPS LIMIT

- Supersymmetry condition

$$E = J_a + J_b + Q_1 + Q_2 + Q_3$$

- Singularity free geometry when horizon is fixed size

$$r_+^2 = \frac{ab}{1+a+b} \,, \qquad q = \frac{(a+b)(1+a)(1+b)}{1+a+b}$$

- Chemical potentials generically complex

PARTITION FUNCTION

- $$Z = \sum e^{-\beta E + \beta \Omega Q} = \sum e^{-\beta(E-Q) - \beta(1-\Omega)Q}$$

  $$\longrightarrow \quad \lim_{\beta\to\infty} \sum_{E=Q} e^{-\beta(1-\Omega)Q}$$

- Using conjugate variables

  $$\Delta_I = \lim_{\beta\to\infty} \beta(1-\Phi_I) \ , \quad \omega_i = \lim_{\beta\to\infty} \beta(1-\Omega_i)$$

  $$\text{BPS :} \quad \Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i$$

Silva
Cabo-Bizet, Cassini, Martelli, Murthy

EVH LIMIT

- Start with non-extremal two charge superstar

- Take $q_1 = 0\ ,\ \mu = \mu_c = q_2 q_3$

- We get

$$ds^2 = \epsilon \left[R^2 (ds^2_{\text{AdS}} + d\Omega_3^2) + \frac{1}{R^2} ds^2_{\mathcal{M}_4} \right], \qquad R^2 = \hat{q}_2 \hat{q}_3$$

$$ds^2_{\text{AdS}} = -(\rho^2 - \gamma^2) dt^2 + \frac{d\rho^2}{\rho^2 - \gamma^2} + \rho^2 d\phi_1^2, \quad \gamma^2 = \frac{\mu - \mu_c}{\mu_c}$$

- The $\epsilon \sim \ell_s^2$ and $L^4 = 4\pi g_s N \ell_s^4$ is fixed

- Like generalized BMN limit:

$$i\partial_\tau = i\partial_t + i \sum_a \partial_{\phi_a} = E - Q_2 - Q_3 = \frac{N^2 \epsilon^2 \hat{u}}{4} \sim N^2 \epsilon^2 = N$$

Sheikh-Jabbari, Yavartanoo

EVH LIMIT

- Near horizon limit: $r = \epsilon\rho \ , \quad \epsilon \to 0$
- Entropy vanishes: $a \sim \epsilon^{\alpha} \ , \quad b \sim \epsilon^{\beta}$
- Singularity free: $\alpha + \beta = 2$
- Choose $\alpha = 0 \ , \quad \beta = 2$
- Now: $S \sim T \sim \epsilon$
- EVH/BPS condition puts $b = 0$
- Sending $\epsilon \to 0$: $ds^2_{10} = h(\theta)\left[-\frac{\mathbf{V}}{a^2+q}\epsilon^2\rho^2 dt^2 + \frac{a^2+q}{\mathbf{V}}\frac{d\rho^2}{\rho^2} + \frac{a^2+q}{a^2}\rho^2\epsilon^2 d\psi^2\right] + ds^2_{\mathcal{M}_7}$
- AdS_3 decouples from $\mathcal{M}_7$

NEAR EVH LIMIT

- Instead of $b = 0$ for EVH, leave $b = \lambda\epsilon^2$
- The effect is to replace AdS_3 in decoupled IR with (pinching extremal) BTZ
- Metric: $$ds^2 = -\frac{(\rho^2 - \rho_0^2)^2}{L_3^2\rho^2} d\tau^2 + \frac{L_3^2\rho^2 d\rho^2}{(\rho^2 - \rho_0^2)^2} + \rho^2 \left(d\chi - \frac{\rho_0^2}{L_3^2\rho^2} d\tau \right)^2$$
- Central charge: $$c = \frac{3L_3}{2G_3}\epsilon = 3\sqrt{2}\frac{a^2}{1-a^2}N^2\epsilon$$
- Entropy: $$S = \frac{2\pi\epsilon\rho_0}{4G_3} = \frac{\pi a}{1-a}\sqrt{\frac{\lambda a}{1+a}}N^2\epsilon$$
- In effect, we have double scaling limit: $\epsilon \to 0 \ , \quad N \to \infty \ , \quad N^2\epsilon = \text{fixed}$

de Boer, Johnstone, Sheikh-Jabbari, Simón
Johnstone, Sheikh-Jabbari, Simón

SCALINGS

- In EVH limit: $\frac{1}{2}N^2 J_a = Q_1^2$

  – dual operator ~ Fermi surface

Berkooz, Narayan, Zait

- In the near EVH limit: $J_a,\ Q_1 = Q_2 \sim N^2$ while $J_b,\ Q_3 \sim \epsilon^2 N^2$

$$\omega_a = -\frac{\pi i(1-a)}{1+a}\ ,\quad \omega_b = \frac{\pi}{\sqrt{\lambda}\,\epsilon}\sqrt{\frac{a}{1+a}}\ ,\quad \Delta_{1,2} = \Delta = \frac{\pi i a}{1+a}$$

Chiral Virasoro generator:

$$L_0 = \partial_\tau = \frac{1}{\sqrt{2}\,\epsilon}(E - J_a - Q_1 - Q_2) = \frac{1}{\sqrt{2}\,\epsilon}(J_b + Q_3) = \frac{1}{2\sqrt{2}}\frac{a}{1-a}\lambda\epsilon N^2$$

we will see how this arises in CFT_4

UV TO IR

- UV BTZ: $$ds^2 = -(r^2 - r_+^2)dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\phi^2$$
- EVH limit: $r = \epsilon\rho \ , \quad r_+ = \epsilon\rho_+ \ , \quad \tau = \epsilon t \ , \quad \tilde{\phi} = \epsilon\phi$
- IR BTZ: $$ds^2 = -(\rho^2 - \rho_+^2)d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_+^2} + \rho^2 d\tilde{\phi}^2$$

  $$\tilde{\phi} \sim \tilde{\phi} + 2\pi\epsilon$$
- Entropy: $$S = \frac{2\pi r_+}{4G_3} = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} \sim \epsilon$$
- Pinching orbifold

UV TO IR

- To have finite dynamics, we should scale central charge $c \to \frac{c}{\epsilon}$

- This requires $L_0 - \frac{c}{24} = \frac{c}{24}\left(\frac{r_+ + r_-}{\ell_3}\right)^2 \longrightarrow \epsilon^{\cancel{2}}\left(L_0 - \frac{c}{24}\right)$

$$S = \frac{2\pi r_+}{4G_3} = \text{fixed}$$

- We have $\text{CFT}_{\text{IR}} = \text{Sym}^{\frac{1}{\epsilon}}(\text{CFT}_{\text{UV}})$

- Central charges: $c_{\text{UV}} = \frac{3L_3}{2G_3} \sim N \to \infty\ ,\quad c_{\text{phys}} = \frac{3L_3}{2G_3}\epsilon \sim \epsilon N = \text{fixed}$

- Virasoro: $\ell_n = \epsilon L_{\frac{n}{\epsilon}} = \frac{1}{N} L_{nN}$

SUPERCONFORMAL INDEX

- The partition function of $\mathcal{N}=4$ SYM is

$$Z(\beta,\Delta_I,\omega_i) = \mathrm{Tr}_{\mathcal{H}}\left[e^{-\sum_{I=1}^{3}\Delta_I Q_I} e^{-\omega_a J_a} e^{-\omega_b J_b} e^{-\beta(E-\sum_I Q_I - J_a - J_b)}\right]$$

- Restrict this to hypersurface $\Delta_1+\Delta_2+\Delta_3-\omega_a-\omega_b=2\pi i$

- The partition function localizes to $\frac{1}{16}$-BPS states

- This is the most supersymmetric black hole with a finite horizon area

- Take $\beta\to\infty$ limit, impose chemical potential constraint to get

$$\begin{aligned} Z &= \mathsf{Tr}\left[(-1)^F e^{-\omega_a(J_a+Q_3)-\omega_b(J_b+Q_3)-(\Delta_1+\Delta_2)(Q-Q_3)-(\Delta_1-\Delta_2)f}\right] \\ &= \mathsf{Tr}\left[(-1)^F p^{J_a+Q_3} q^{J_b+Q_3} t^{Q-Q_3} a^f\right] \qquad Q,f=\frac{1}{2}(Q_1\pm Q_2) \end{aligned}$$

$\mathcal{N}=1$ superconformal index

Cabo-Bizet, Cassini, Martelli, Murthy
Choi, Kim, Kim, Nahmgoong
Benini, Milan

N=4 SYM

- The theory has 6 scalars, 4 Weyl fermions, vector gauge field plus descendants from acting with covariant derivatives

- Single letter partition function in BPS limit $E = \sum_I Q_I + J_a + J_b$

$$
\begin{aligned}
I_{\mathrm{st}}(p,q,t,a) &= \mathrm{Tr}_{\mathrm{BPS,st}}\left[(-1)^F p^{J_a+Q_3} q^{J_b+Q_3} t^{R-Q_3} a^f\right] \\
&= \frac{1}{1-p}\left(\sqrt{ta}+\sqrt{\frac{t}{a}}-p-t\right) \\
&\quad + \frac{1}{1-p}\frac{q}{1-q}\left(\sqrt{ta}+\sqrt{\frac{t}{a}}-t+\frac{p}{t}+p-1-\frac{p}{\sqrt{ta}}-p\sqrt{\frac{a}{t}}\right)
\end{aligned}
$$

- Equivalent to restricting to $SU(1,2|3)$ subsector

$\frac{1}{16}$-BPS states

BPS OPERATORS IN N=4 SYM

$S^1 \times S^3$ dual to global coordinates

Name in 0707.1621	$SO(4)[J_a, J_b]$	Name in 0510251	Q	Q_3	E_0	E
Z	$[0,0]$	Z	$\frac{1}{2}$	0	1	1
X	$[0,0]$	X	$\frac{1}{2}$	0	1	1
W	$[0,0]$	Y	0	1	1	1
F_+	$[1,1]$	F_{++}	0	0	2	2
χ_1	$[\frac{1}{2}, -\frac{1}{2}]$	$\psi_{0,+,+++}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
χ_2	$[-\frac{1}{2}, \frac{1}{2}]$	$\psi_{0,-,+++}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_3$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,-++}$	0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_5$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,+-+}$	0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_7$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,++-}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
d_1	$[1,0]$	∂_{++}	0	0	1	1
d_2	$[0,1]$	∂_{+-}	0	0	1	1

red / blue — $SU(1,1|2)$ subsector ($\frac{1}{8}$-BPS states)

blue — $SU(1,1)$ fermionic subsector

satisfy $E = E_0 = \sum_I Q_I + J_a + J_b$

$$J_{a,b} = J_1 \pm J_2$$

Harmark, Kristjansson, Orselli
Kinney, Maldacena, Minwalla, Raju

PLETHYSTICS

- Consider function $f(z) = \sum_{n=0}^{\infty} a_n x^n$

- Plethystic exponential $\mathrm{PE}[f(x)] = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n}\left(f(x^n) - f(0)\right)\right) = \prod_{n=1}^{\infty} \frac{1}{(1-x^n)^{a_n}}$

- Tool for counting GIOs

Feng, Hanany, He

- Gives Hilbert series

$$g^{N_f=N_c}(t) = \frac{1-t^{2N_f}}{(1-t^2)^{N_f^2}(1-t^{N_f})^2}$$

1 CONSTRAINT OF WEIGHT $2N_f$

N_f^2 MESONS OF WEIGHT 2

2 BARYONS OF WEIGHT N_f

Gray, Hanany, He, VJ, Mekareeya

PLETHYSTICS

- Single particle partition function $z(x)$ sums over oscillators
- Single trace states with k oscillators $Z_k \sim \dfrac{z(x)^k}{k}$
- Partition function of single trace states is

$$Z_{\text{st}} = \sum_k Z_k = -\sum_{q=1}^{\infty} \frac{\varphi(q)}{q} \ln(1 - z(x^q))$$

- Multitrace partition function is

$$\log Z = \sum_{n=1}^{\infty} \frac{1}{n} Z_{\text{st}}(x^n)$$

Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk

MULTILETTER PARTITION FUNCTION

- Applying plethystic program, we get full partition function

$$Z = \frac{1}{N!}\int \prod_{i=1}^{N}\frac{d\alpha_i}{2\pi}\prod_{i<j}\left(2\sin\frac{\alpha_{ij}}{2}\right)^2 \times \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1-\frac{\prod_{I=1}^{3}2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_a}{2}2\sinh\frac{n\omega_b}{2}}\right)\sum_{i,j=1}^{N}e^{in\alpha_{ij}}\right]$$

α_i = weights of $SU(N)$

$\alpha_{ij} = \alpha_i - \alpha_j$ = roots of $SU(N)$

- Valid in free field limit

Dolan, Osborne
Choi, Kim, Kim, Nahmgoong

LARGE-N LIMIT

- Large-N limit of $\log Z$

  - Kinney, Maldacena, Minwalla, Raju leading order result is $\mathcal{O}(1)$

    free field result cannot capture black hole entropy

  - Hosseini, Hritsov, Zaffaroni supersymmetric black hole entropy is Legendre transform of $\mathcal{O}(N^2)$ quantity interpreted as supersymmetric Casimir energy $\log Z = \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_a \omega_b}$

  - Cabo-Bizet, Cassani, Martelli, Murthy show the difference between partition function and superconformal index is supersymmetric Casimir energy; calculation where bulk dual is minimal 5d supergravity

  - Choi, Kim, Kim, Nahmgoong and Benini, Milan show using matrix model integral and Bethe ansatz that if chemical potential is complex, the superconformal index can have a leading N^2 term

LARGE-N LIMIT

- Large-N limit of $\log Z$
  - Kim, Kim, Song generalize to $\mathcal{N}=1$ with putative $\mathrm{AdS}_5 \times Y^{p,p}$ dual matrix model integral in Cardy limit
  - Lezcano, Pando Zayas perform $\mathcal{N}=1$ microstate counting with Bethe ansatz
  - Murthy talk: more recent developments on this theme

ZEITGEIST

- Illustrate the spirit of the calculation using matrix model methods

- Work in Cardy limit $\omega_a \; , \; \omega_b \ll 1$

- First, $\prod_{I=1}^{3} \sinh \frac{n\Delta_I}{2} e^{i\alpha_{ab}} \sim \sum_{s_I=\pm 1} e^{n\left(\frac{s\cdot\Delta}{2}+i\alpha_{ab}\right)}, \quad \sinh \frac{n\omega_i}{2} \sim \frac{n\omega_i}{2}$

- Then,

$$
\begin{aligned}
Z &= \frac{1}{N!}\int \prod_{i=1}^{N} \frac{d\alpha_i}{2\pi} \prod_{i<j} \left(2\sin\frac{\alpha_{ij}}{2}\right)^2 \\
&\quad \times \exp\left[\sum_{n=1}^{\infty} \frac{1}{n}\left(1-\frac{\prod_{I=1}^{3} 2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_a}{2}\, 2\sinh\frac{n\omega_b}{2}}\right) \sum_{i,j=1}^{N} e^{in\alpha_{ij}}\right] \\
&\sim \frac{1}{N!}\int \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} \exp\left[-\frac{1}{\omega_a\omega_b}\sum_{s}\sum_{a\neq b}\left(\mathrm{Li}_3(e^{\frac{s\cdot\Delta}{2}+i\alpha_{ab}}) - \mathrm{Li}_3(e^{-\frac{s\cdot\Delta}{2}-i\alpha_{ab}})\right)\right]
\end{aligned}
$$

$$\mathrm{Li}_3(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^3}$$

ZEITGEIST

- Apply math tricks such as:

$$\text{Li}_3(-e^x) - \text{Li}_3(-e^{-x}) = -\frac{x^3}{6} - \frac{\pi^2 x}{6}\,, \qquad -\pi < \text{Im}(x) < \pi$$

- Approximate α_a integrals by saddle point method

- Derivative of α_a zero if $\alpha_1 = \ldots = \alpha_N$

- Dominance of this saddle is an assumption

- Find: $\log Z \sim \frac{N^2}{6\omega_a\omega_b}\sum_s \left[\left(\frac{s\cdot\Delta}{2}\right)^3 + \pi^2\left(\frac{s\cdot\Delta}{2}\right)\right] = \frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_a\omega_b}$

- Relation $\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i$ used to simplify expression

ENTROPY

- We have: $S = \frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_a\omega_b} + \sum \Delta_I Q_I + \omega_a J_a + \omega_b J_b$
- Evaluate its saddle point $\frac{\partial S}{\partial \Delta_I} = \frac{\partial S}{\partial \omega_i} = 0$
- Entropy becomes

$$S = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_a + J_b)}$$

- Generically find complex entropy
- Demand $\mathrm{Im}(S) = 0$
- New constraint:

$$Q_1Q_2Q_3 + \frac{N^2}{2}J_aJ_b = \left(\frac{N^2}{2} + Q_1 + Q_2 + Q_3\right)\left(Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_a + J_b)\right)$$

EVH LIMIT

- In the EVH limit, $Q_3 = J_b = 0$
- Black hole is $\frac{1}{8}$-BPS so entropy vanishes
- Index is

$$Z = \mathrm{Tr}\Big[(-1)^F e^{-\Delta_1(Q_1+J_b)-\Delta_2(Q_2+J_b)-\Delta_3(Q_3+J_b)-\omega_a(J_a-J_b)}\Big]$$

- This is so called Macdonald index where $\Delta_3 \to \infty$, $\Delta_3 - \omega_b = \text{finite}$
- Leading term in partition function is $\log Z = \frac{N^2}{2}\frac{\Delta_1\Delta_2}{\omega_a}$
- Saddle point evaluation of entropy gives

$$S = \frac{N^2}{2}\frac{\Delta_1\Delta_2}{\omega_a} + \omega_a J_a + \Delta_1 Q_1 + \Delta_2 Q_2 + \Lambda(\Delta_1 + \Delta_2 - \omega_a - 2\pi i)$$

- Evaluates to $S = 0$; same chemical potentials as in gravity calculation

NEAR EVH LIMIT

- We have $\Delta_1 = \Delta_2 = \Delta$ (*i.e.*, $Q_1 = Q_2,\ J_a \sim N^2;\ Q_3, J_b \sim \epsilon^2 N^2$)

- Degeneracy is

$$d = \int d\Delta\, d\omega_a\, d\omega_b\ \exp\left(\frac{N^2}{2}\frac{\Delta^2\Delta_3}{\omega_a\omega_b} + \omega_a(J_a+Q_3) + \omega_b(J_b+Q_3) + 2\Delta(Q-Q_3) + 2\pi i Q_3\right)$$

- Saddle point evaluation of Δ , ω_a integrals

- Get

$$d = \int d\omega_b \exp\left[\frac{N^2}{2}\frac{\widehat{\Delta}^2}{\omega_b}\left(\frac{2\widehat{\Delta}}{\widehat{\omega}_a} - 1\right) + 2\pi i Q_3\right] e^{\omega_b(J_b+Q_3)}$$

NEAR EVH LIMIT

- Saddle point evaluation requires: $\epsilon N^2 \gg 1$ or $a \to 1$
- Either way, we find $S = \frac{\pi a}{1-a}\sqrt{\frac{\lambda a}{1+a}}\,\epsilon N^2$
- This reproduces gravity answer for near EVH entropy
- Also, near EVH limit of $\frac{1}{16}$-BPS solution in ϵ- expansion

CARDY FORMULA

- Partition function enjoys $Z(\beta) = Z(\frac{4\pi^2}{\beta})$

- Low temperature expansion gives $Z(\beta) \approx \exp\left(\frac{\pi^2 c}{6\beta}\right)$

- Saddle point evaluation of inverse Laplace transform yields

$$d \approx \int d\beta \exp\left[\frac{\pi^2 c}{6\beta} + \beta\left(L_0 - \frac{c}{24}\right)\right]$$

- In near EVH limit: $c = 3\sqrt{2}\frac{a^2}{1-a^2}\epsilon N^2\,,\quad L_0 - \frac{c}{24} = \frac{1}{2\sqrt{2}}\frac{a}{1-a}\lambda\epsilon N^2$

- Saddle at $\widehat{\beta} = \pi\sqrt{\frac{c}{6(L_0 - \frac{c}{24})}} = \pi\sqrt{\frac{2}{\lambda}\frac{a}{1+a}}$

CONNECTION TO CARDY

- Can write degeneracy equation as

$$d = \int d\omega_b \exp\left[\frac{N^2}{2}\frac{\pi^2 a^2}{1-a^2}\frac{1}{\omega_b} + \frac{N^2\epsilon^2}{2}\frac{\lambda a}{1-a}\omega_b + \mathcal{O}(\epsilon)\right]$$

$$= \int d\omega_b \exp\left[\frac{c\pi^2}{6\sqrt{2}}\frac{1}{\epsilon\omega_b} + \sqrt{2}\epsilon\left(L_0 - \frac{c}{24}\right)\omega_b + \mathcal{O}(\epsilon)\right]$$

- Compare to

$$d \approx \int d\beta \exp\left[\frac{\pi^2 c}{6\beta} + \beta\left(L_0 - \frac{c}{24}\right)\right]$$

- We have $\widetilde{\omega}_b = \sqrt{2}\epsilon\omega_b \sim \beta$

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from relation between time coordinates of AdS_3 & AdS_5

like factor of $\frac{3}{4}$ in thermal free energy; *i.e.*, strong coupling effect?

CONICAL DEFICIT

- $$d = \int d\omega_b \exp\left[\frac{N^2}{2}\frac{\pi^2 a^2}{1-a^2}\frac{1}{\omega_b} + \frac{N^2\epsilon^2}{2}\frac{\lambda a}{1-a}\omega_b + \mathcal{O}(\epsilon)\right]$$
- Two terms in integrand $\widetilde{c} = 3\frac{a^2}{1-a^2}N^2 \ , \ \widetilde{L}_0 - \frac{\widetilde{c}}{24} = \frac{1}{2}\frac{a}{1-a}\lambda\epsilon^2 N^2$
- In near EVH limit $\widetilde{L}_0 - \frac{\widetilde{c}}{24} = 0$
- Conical deficit $\sim \epsilon^{-1}$
- Rescale: $c = \epsilon\widetilde{c} \ , \quad L_0 - \frac{c}{24} = \frac{1}{\epsilon}\left(\widetilde{L}_0 - \frac{\widetilde{c}}{\widetilde{24}}\right)$

de Boer, Sheikh-Jabbari, Simón

- Cardy formula invariant, but fractionated spectrum

LONG STRINGS

- Suppose we have strings of size R
- If they form a long string, length is NR
- Energy gap goes from $\frac{1}{R} \to \frac{1}{NR}$
- Lot of low energy states in spectrum
- Long string spectrum/fractionation in IR 2d CFT

Maldacena, Susskind
Dijkgraaf, Moore, Verlinde, Verlinde

OPERATOR DUAL

- In $SU(1,1)$, we have χ_1 , $\overline{\chi}_7$, ∂_{++}
- Satisfy $\frac{N^2}{2} J_a = Q_1^2$ BPS condition
- Write $\text{Sym}\left[\prod_{a,b=1}^{N^2} \prod_{j=0}^{\frac{K}{2}-1} \psi_j^a \prod_{m=\frac{K}{2}}^{K-1} \bar{\psi}_m^b\right]$ with $\psi_k = \partial_{++}^k \chi_1$, $\bar{\psi}_k = \partial_{++}^k \overline{\chi}_7$
- These satisfy $J_b = Q_3 = 0$ for EVH black hole
- In $a \to 1$ limit, this gives explicit description of EVH CFT_2

Berkooz, Narayan, Zait

- In free limit we have $SU(N)_N \oplus SU(N)_N$ chiral WZW model
- Thermodynamics of near horizon BTZ as IR limit of thermodynamics of AdS_5

Johnstone, Sheikh-Jabbari, Simón, Yavartanoo

- Near EVH story is more complicated

OTHER DIMENSIONS

- AdS_4: EVH BPS limit has naked singularity
- AdS_6: can have $S \sim T^2$ EVH so AdS_4 near horizon
- AdS_7 : with equal charges, $S \sim T^3$ EVH so AdS_5 near horizon
- Details in paper

OTHER DIMENSIONS

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SUMMARY

- Techniques for analyzing $\frac{1}{16}$-BPS black holes in AdS_5 extended to study $\mathrm{EVH/CFT}_2$
- Entropy calculation for EVH and near EVH in fast rotating limit
- Legendre transform of superconformal index, which gives black hole entropy, is equivalent to derivation of Cardy formula
- Shows how IR CFT is realized in UV CFT
- Hints of similar stories in higher dimensions

OPEN QUESTIONS

- More precise statement of origin of infinite dimensional conformal symmetry in subsectors of $\mathcal{N} = 4$ SYM

- Entropy matches, but how we do this in a microcanonical picture sending states to states

- Mechanism for fractionation in 4d CFT, especially in near EVH case

- In generic setting, the near horizon $\mathrm{AdS}_d \times X$ mixes $\mathrm{AdS}_5,\ S^5$ coordinates

- Cardy & Cardy–Verlinde

OBRIGADO!