Applied CFT: Fire Phenomenology

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  with

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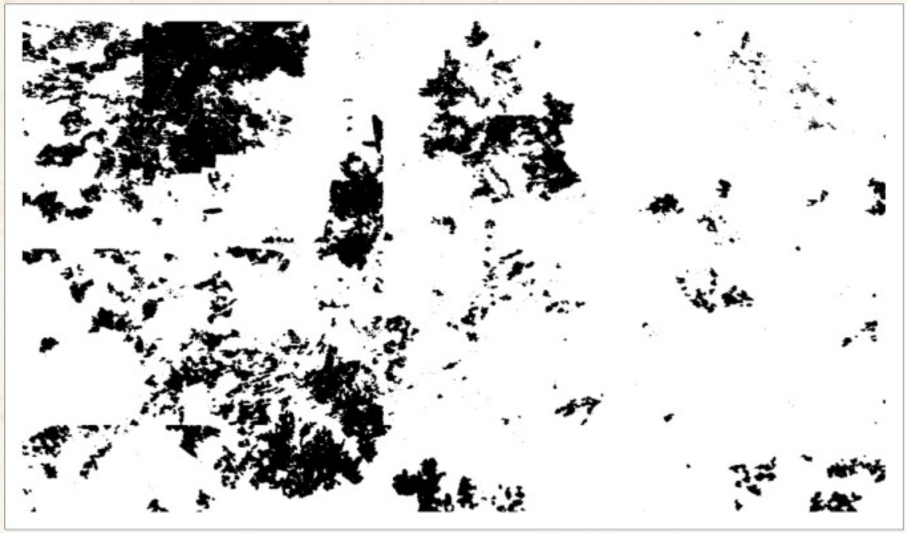
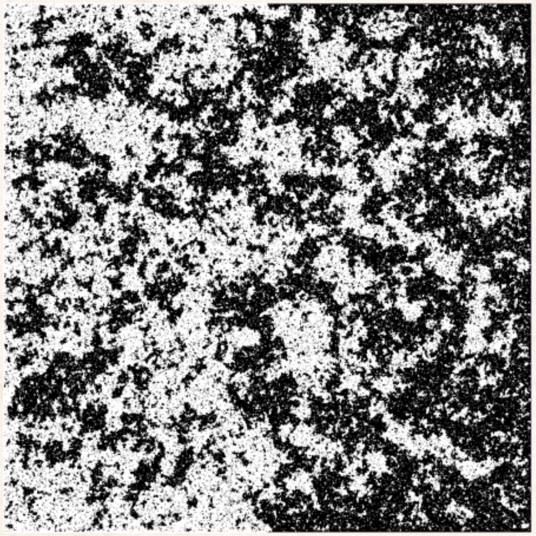
2D CFT & Fire





- Fire occurrences in Botswana (2011) compared with Ising model near critical temperature.
- Evidence that fire size distributions follow power law behaviour suggesting scale invariance [Hantson, Pueyo, Chuvieco]
- **Suggestion:** correlation functions natural tool to study scale & conformal invariance

Motivation

- Understanding fire propagation plays an important role in global climate modelling
  - as the climate changes, so does the probability of fire
- Interplay between climate, fire & vegetation
- Fire also has important ecological role
  - destroying habit but also releasing nutrients
- Extensive satellite imagery → scope to study spacial distribution of fires

Fire 101

- 3 basic stages
  - Ignition
  - Spread
  - Stopping
- Beyond that, many models for fire propagation
  - questions we propose:
    - 0^{th} order: is there evidence for scale invariance ?
    - 1^{st} order: will some basic tools from CFT be useful in analysing the data and accessing various models?
  - starting point: looked at correlation function

1/0 Fire field

$$\psi(\mathbf{x}, t) = \begin{cases} 1 & \text{if } \mathbf{x} \text{ on fire in year } t \\ 0 & \text{if } \mathbf{x} \text{ not on fire in year } t \end{cases}$$

- E[X] = average of X over the grid in a year
  - eg. the one point function $E[\psi(\mathbf{x},t)]$ tells us the density of fire in year

2 & 3 point correlation function

$$\langle\psi(\mathbf{x}_1,t)\psi(\mathbf{x}_2,t)\rangle \quad = \quad \mathrm{E}[\psi(\mathbf{x}_1,t)\psi(\mathbf{x}_2,t)] - \mathrm{E}[\psi(\mathbf{x},t)]^2$$

$$\langle\psi(\mathbf{x}_1,t)\psi(\mathbf{x}_2,t)\phi(\mathbf{x}_3,t)\rangle \quad = \quad \mathrm{E}[\psi(\mathbf{x}_1,t)\psi(\mathbf{x}_2,t)\psi(\mathbf{x}_3,t)] - \mathrm{E}[\psi(\mathbf{x},t)]^3$$

  - subtract off 1-pt function

- notation
  - $\mathrm{f}_{1/0}(x_{12},t) \quad = \langle\, \psi(\boldsymbol{x}_1,t)\ \psi(\boldsymbol{x}_2,t)\,\rangle$
  - $\mathrm{g}_{1/0}(x_{123},t) \quad = \langle\, \psi(\boldsymbol{x}_1,t)\ \psi(\boldsymbol{x}_2,t)\psi(\boldsymbol{x}_3,t)\,\rangle$

1 / –1 Fire field

$$\varphi(\mathbf{x},t) = \begin{cases} +1 & \text{if } \mathbf{x} \text{ on fire in year } t \\ -1 & \text{if } \mathbf{x} \text{ not on fire in year } t \end{cases}$$

- $\varphi = 2(\psi - ½)$
- convenient to subtract out 1-pt function

$$\phi(\mathbf{x},t) = \varphi(\mathbf{x},t) - \overline{\varphi}(t)\ , \qquad \overline{\varphi}(t) := \mathrm{E}[\varphi(\mathbf{x},t)]$$

2 & 3 point correlation function

$$\langle \phi(\mathbf{x}_1, t)\phi(\mathbf{x}_2, t)\rangle = \mathrm{E}[\phi(\mathbf{x}_1, t)\phi(\mathbf{x}_2, t)]$$

$$\langle \phi(\mathbf{x}_1, t)\phi(\mathbf{x}_2, t)\phi(\mathbf{x}_3, t)\rangle = \mathrm{E}[\phi(\mathbf{x}_1, t)\phi(\mathbf{x}_2, t)\phi(\mathbf{x}_3, t)]$$

- notation
  - $f_{1/-1}(x_{12},t) = \langle\, \phi(\boldsymbol{x}_1,t)\, \phi(\boldsymbol{x}_2,t)\, \rangle$
  - $g_{1/-1}(x_{123},t) = \langle\, \phi(\boldsymbol{x}_1,t)\, \phi(\boldsymbol{x}_2,t)\psi(\boldsymbol{x}_3,t)\, \rangle$

2 & 3 point correlation function

- 2-point function of 1/0 field, ψ
  - only non-zero when both points had a fire
  - manifestly tells us about fire correlations
- 2-point function of 1/–1 field, ϕ
  - tells us about anti-correlations
    - (fire) & (no fire)
  - and correlations
    - (fire & fire) or (no fire & no fire)
- naively should be a simple relationship between two point functions of 1/0 & 1/ –1 fields
  - finite lattice & non-periodic b.c. introduce differences

    → we looked at both fields

Fits

- 2 & 3 point functions fitted with

$$f(x_{12}, t) = \frac{Ae^{-x_{12}/\xi}}{x_{12}^{\alpha}}$$

$$g(x_{123}, t) = \frac{Be^{-x_{123}/\zeta}}{x_{123}^{\eta}}$$

  - $x_{12} = |x_1 - x_2|$ and $x_{123} = (x_{12} x_{23} x_{31})^{1/3}$
  - CFT $\rightarrow \frac{1}{2}\alpha = \frac{1}{3}\eta$
  - we introduced correlation lengths ξ and ζ in our fit to account for
    - finite size effects
    - breaking of conformal symmetry

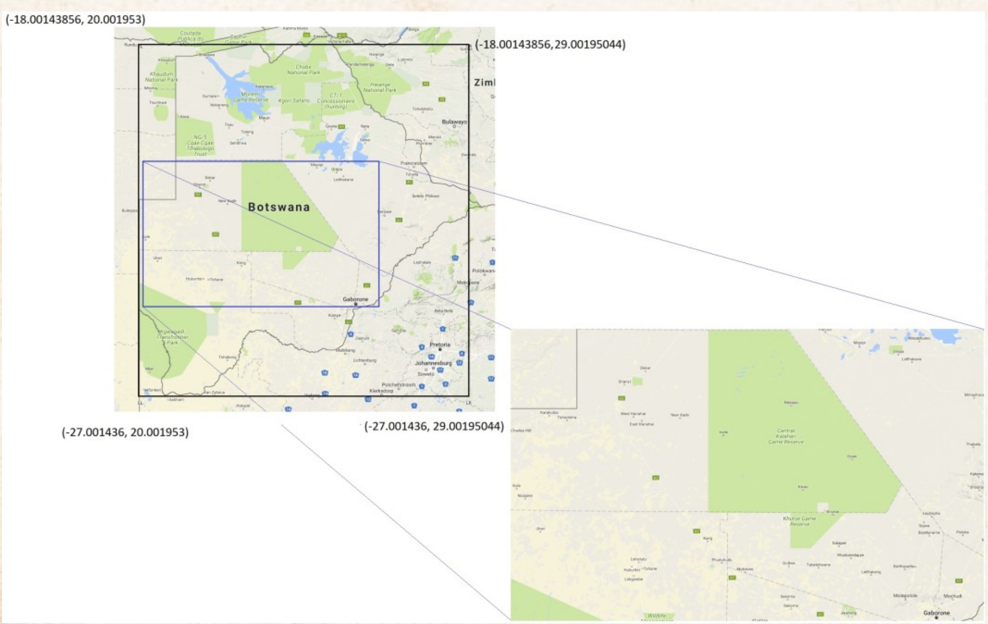
Conformal vs Scale invariance

- In 2D, scale invariance → conformal invariance
  [Polchinski]
  - Need:
    - Lorentz invariance ✗
    - **Quantum** field theory ✗
    - Unitarity ✗
      - fires do not “unburn”
- fire data we found
  - evidence for scale invariance
  - broken rotational symmetry
  - $\frac{1}{2}\alpha \neq \frac{1}{3}\eta$

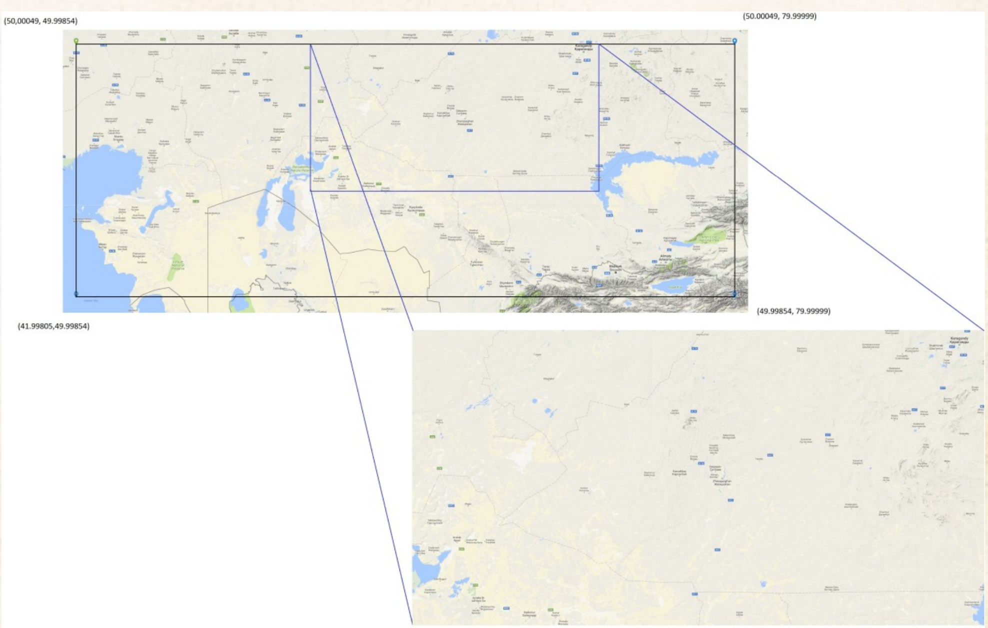
The data

- Looked at grass fires in Botswana & Kazakhstan
  - relatively homogeneous vegetation
  - each pixel is 50 m × 50 m

$$\psi(\mathbf{x}, t) = \begin{cases} 1 & \text{if } \mathbf{x} \text{ on fire in year } t \\ 0 & \text{if } \mathbf{x} \text{ not on fire in year } t \end{cases}$$

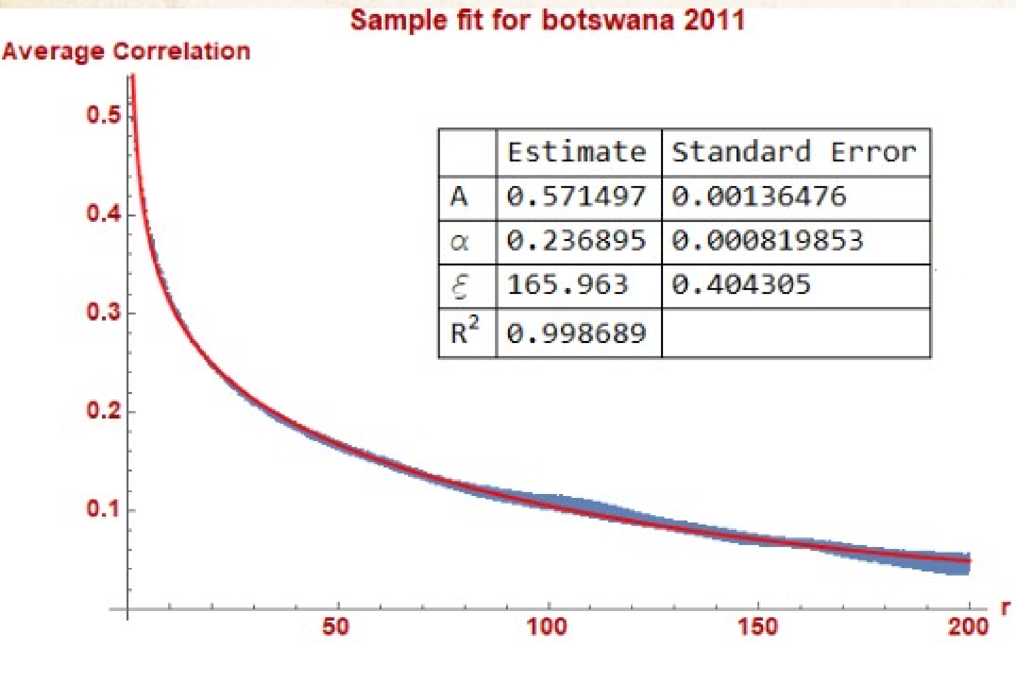


73 km × 41 km (1470×860 pixels)

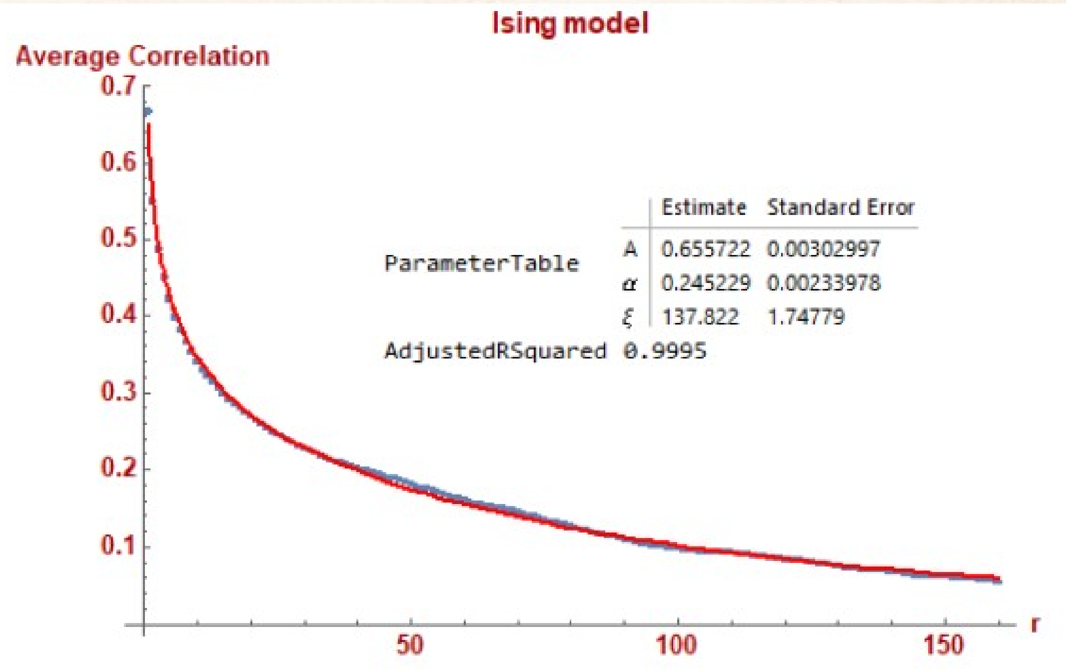


148 km × 51¼ km (2961 ×1025)

Results – two point function



(A) *Two point correlation plot of fire occurrences.*

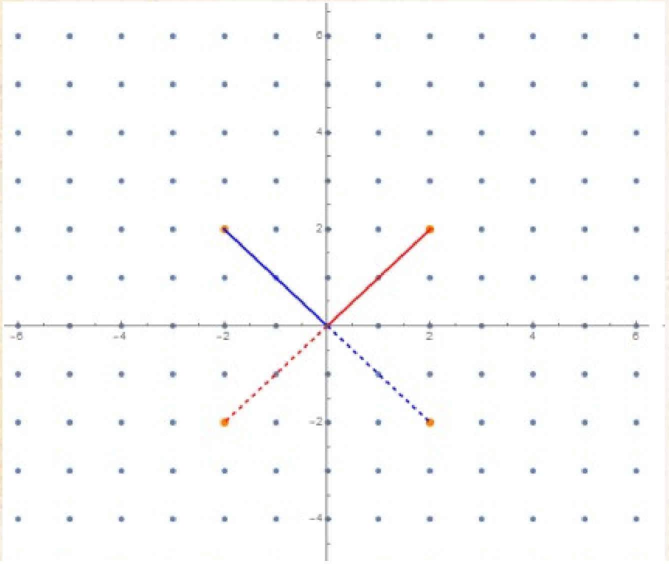


(B) *Two point correlation plot of the Ising model.*

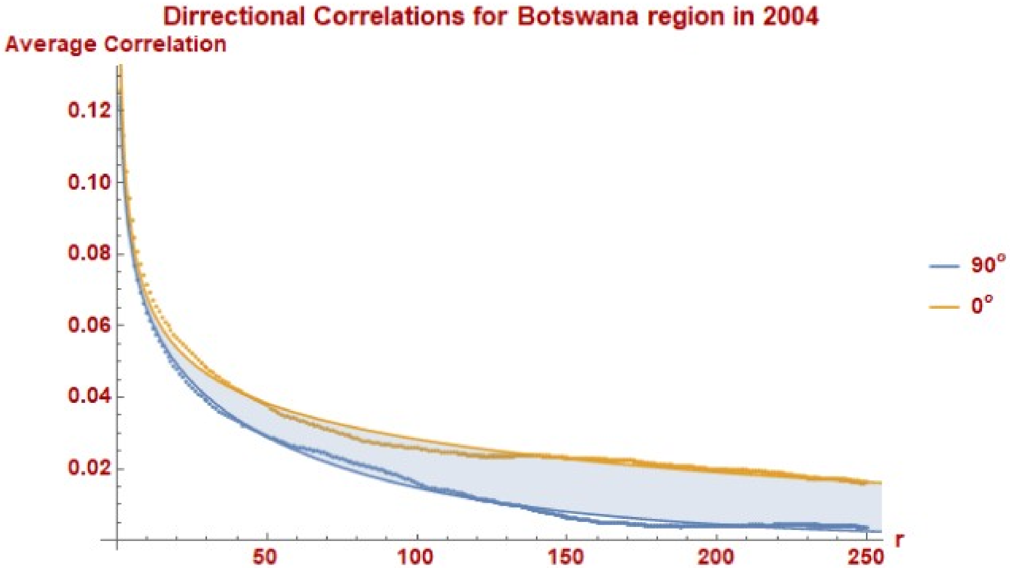
- Comparison of burn data & Ising model simulation

$$f(x_{12}, t) = \frac{A e^{-x_{12}/\xi}}{x_{12}^{\alpha}}$$

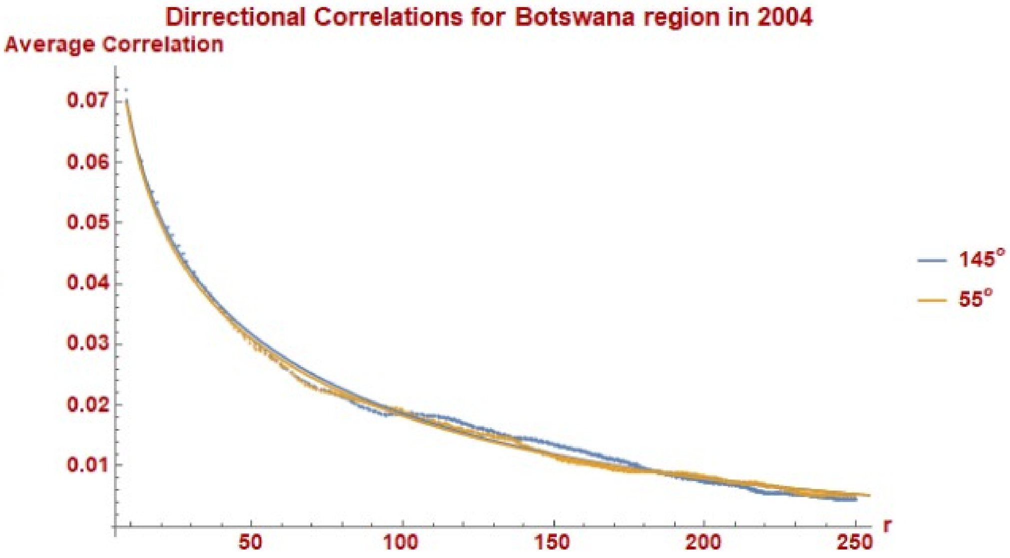
Directional correlation



- can test rotational invariance by comparing correlations along different angles



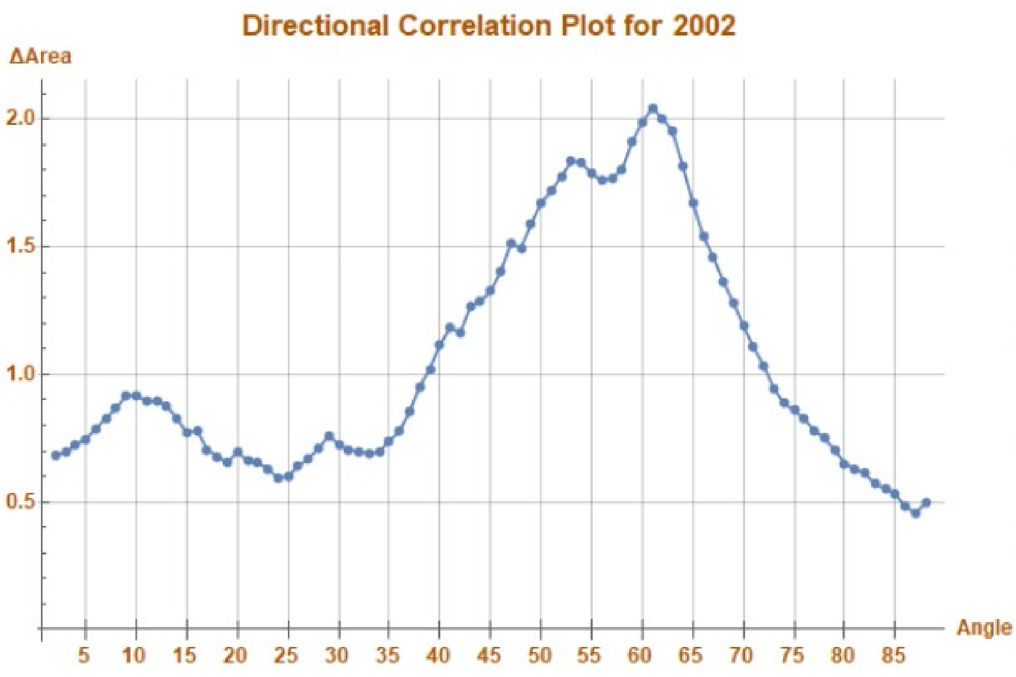
(A) *Correlations along angular directions of* 0^o *and* 90^o.



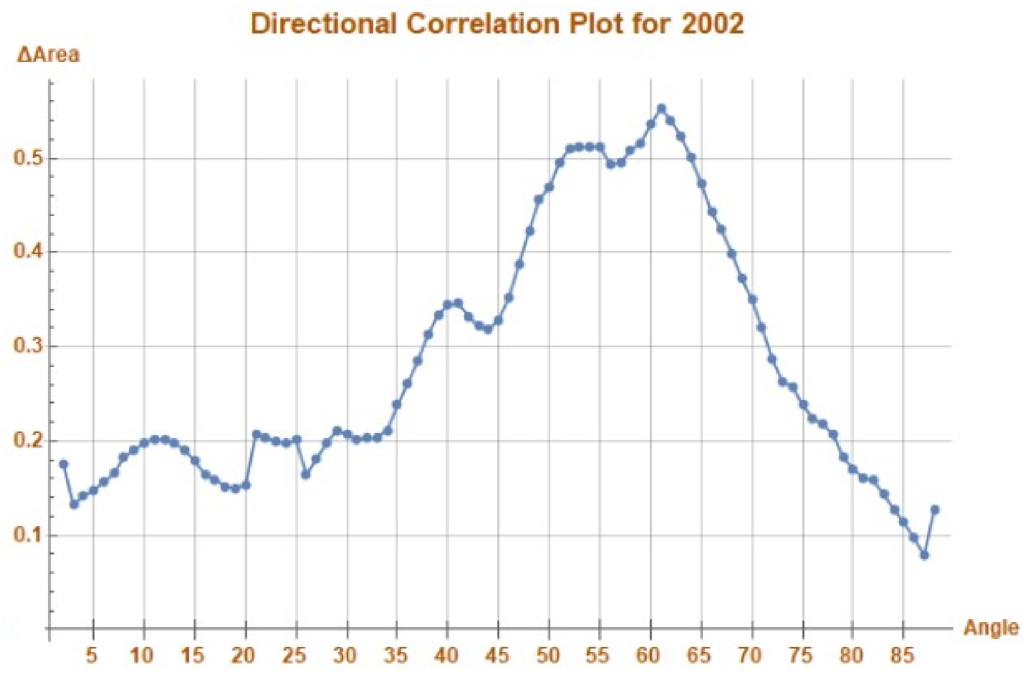
(B) *Correlations along angular directions of* 55^o *and* 145^o.

Directional correlation

- clearly rotational symmetry broken
- possible mechanism = wind

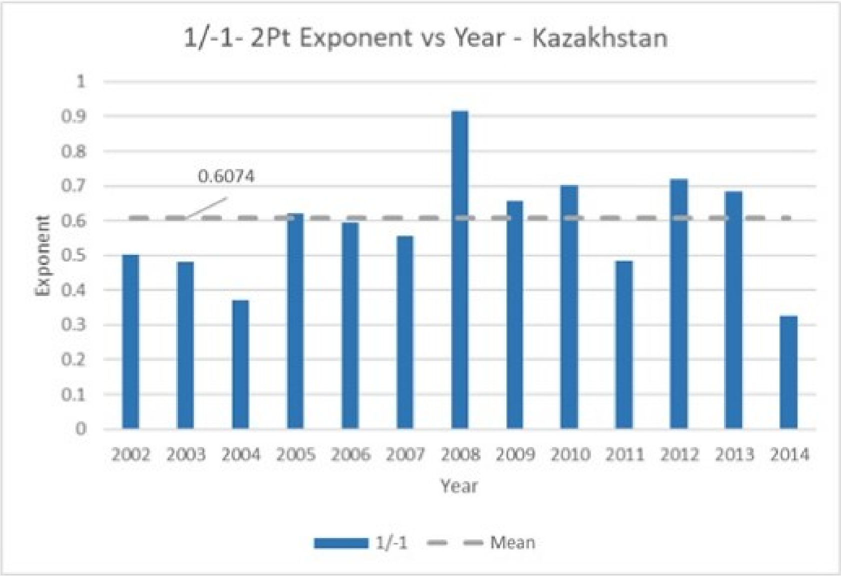


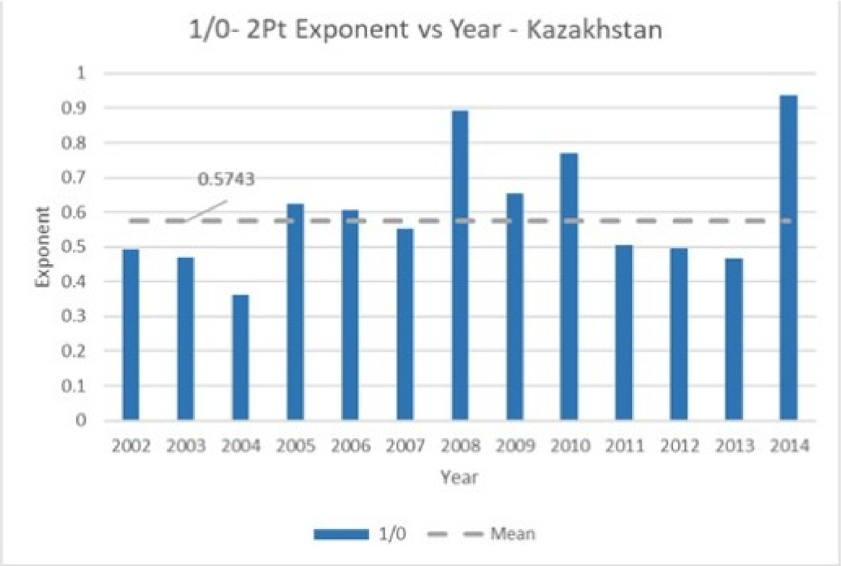
(A) *Directional correlations using* $1/-1$ *format.*

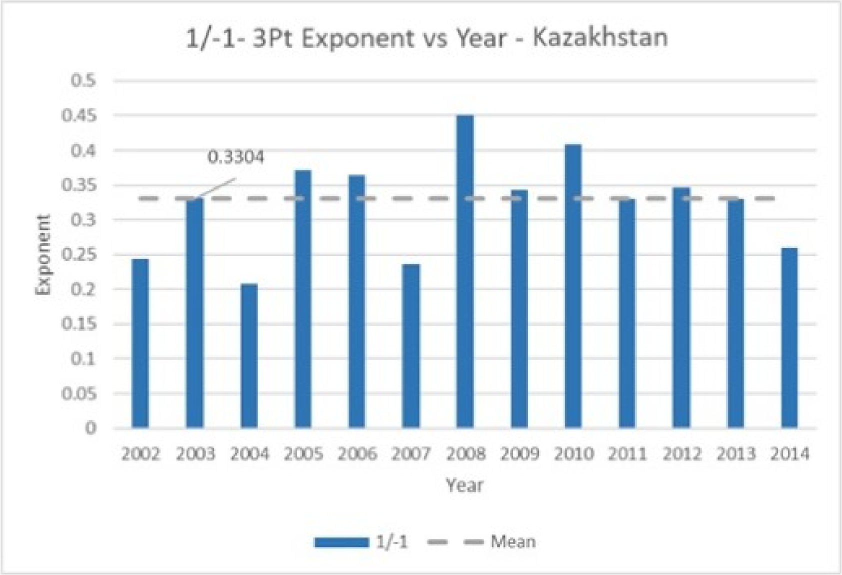


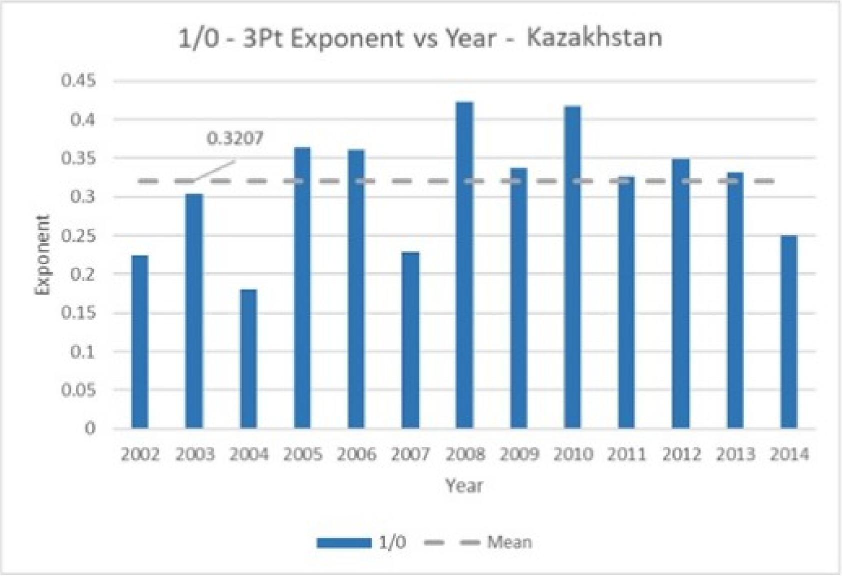
(B) *Directional correlations using* $1/0$ *format.*

Variation of parameters









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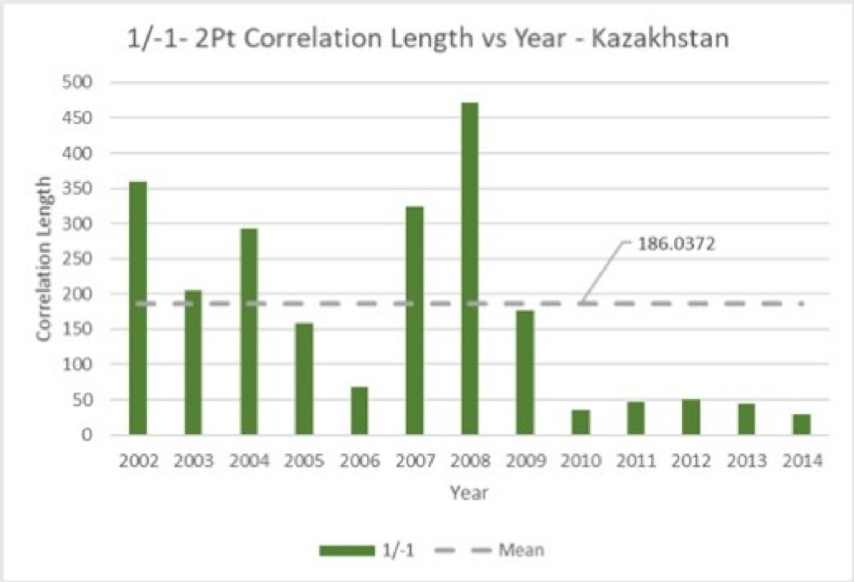
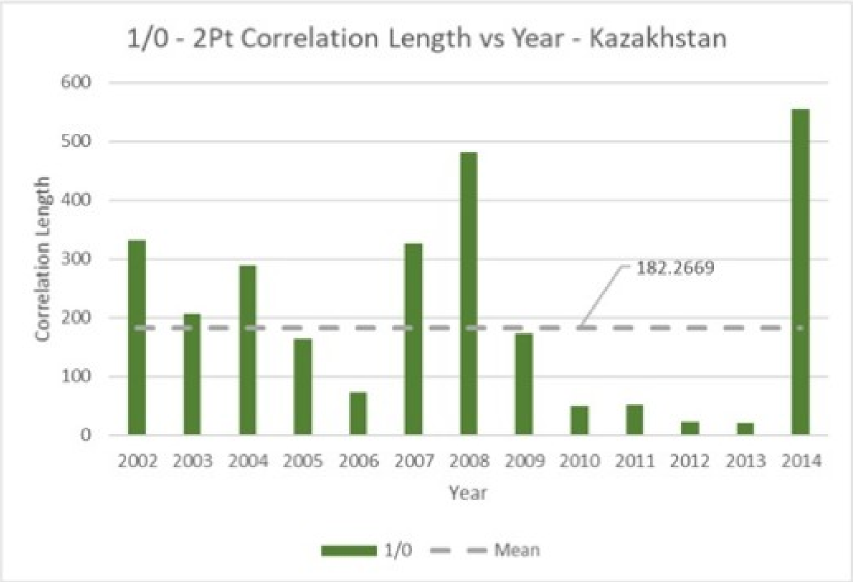
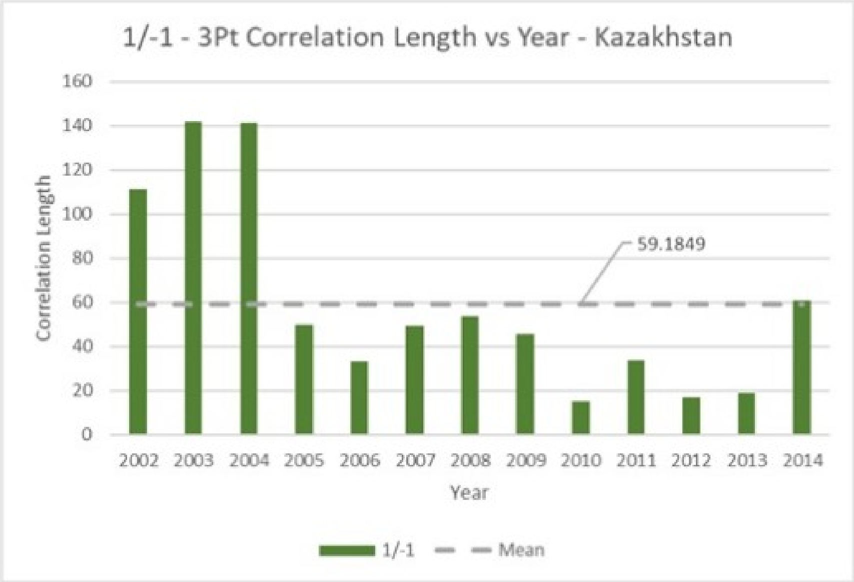
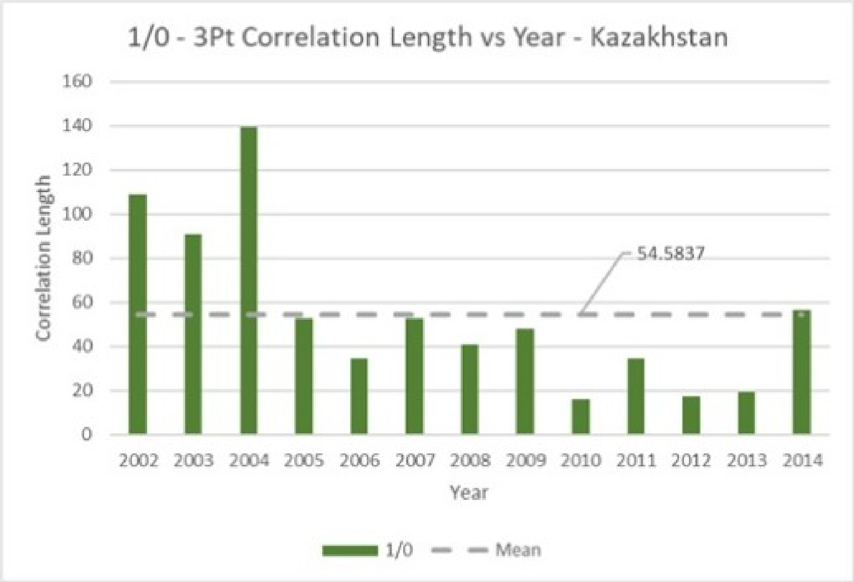


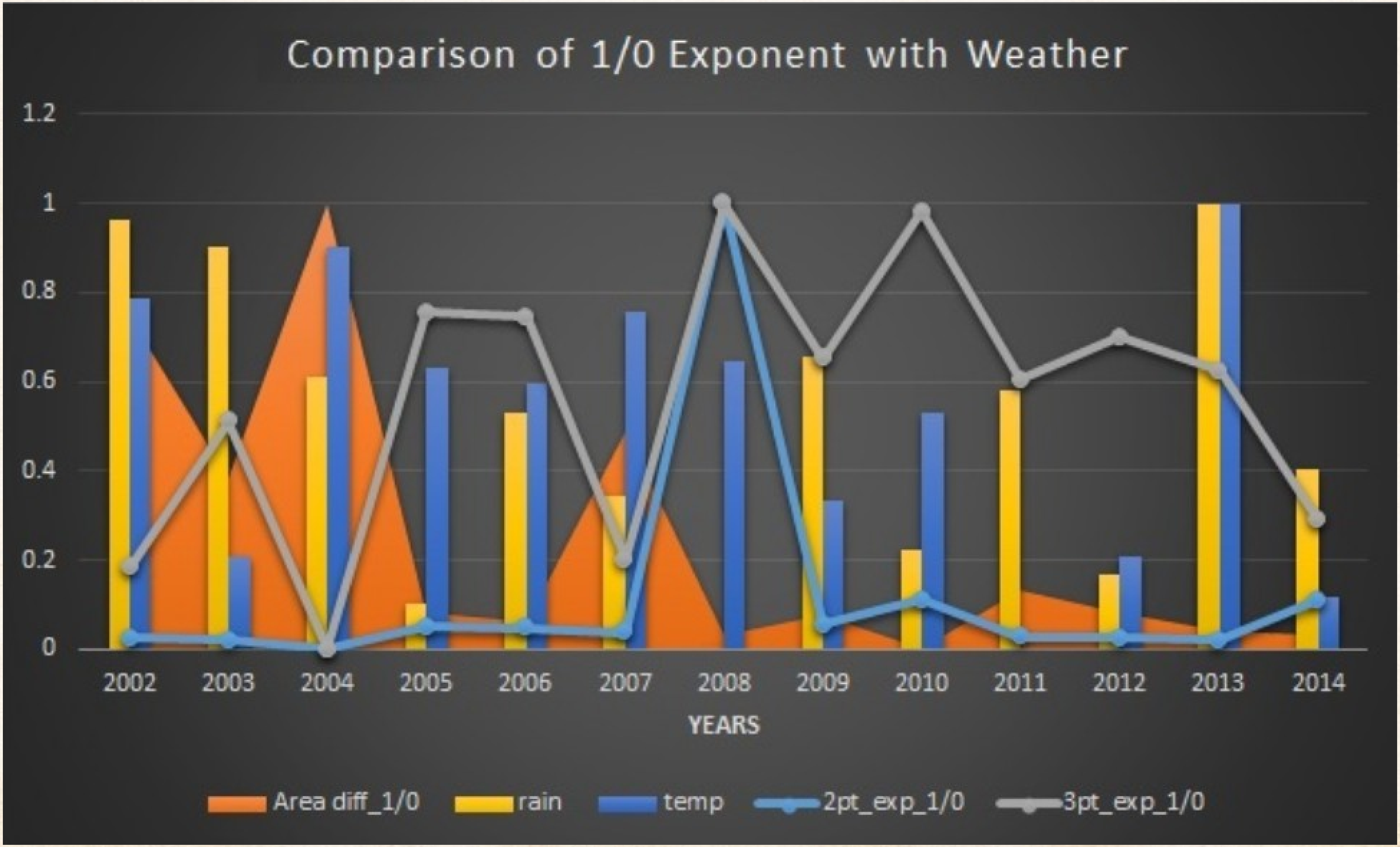






Correlation with weather

- no clear patterns emerged



Summary

- studied spacial correlation in fire phenomenology
- correlation functions and possibly other CFT techniques hold promise as an additional useful tool in this arena
- future
    - differentiate various models
    - extract relationship between exponents, models & data