THE HOLOGRAPHIC LANDSCAPE OF SYMMETRIC PRODUCT ORBIFOLDS

Lisboa
2020

Workshop on black holes:
BPS, BMS & Integrability

Collaborators

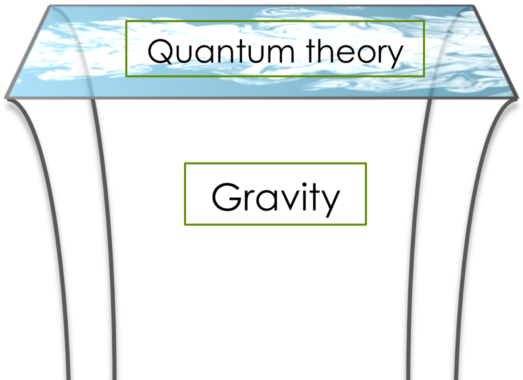
Based on
arXiv: 1611.04588 [hep-th]
arXiv: 1805.09336 [hep-th]
with A. Belin, J. Gomes and C. Keller

arXiv: 1910.05353 [hep-th]
arXiv: 1910.05342 [hep-th]
with A. Belin, C. Keller and B. Mühlmann

arXiv: 2002.07819 [hep-th]
with A. Belin, N. Benjamin, C. Keller and S. Harrison

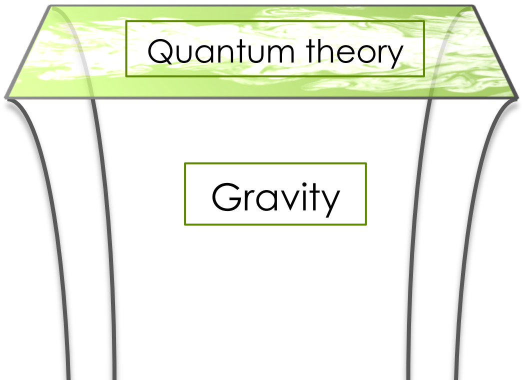
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

There are conditions on a CFT such that it captures semi-classical gravitational features.

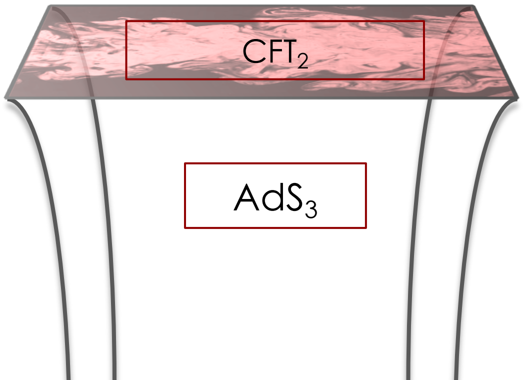


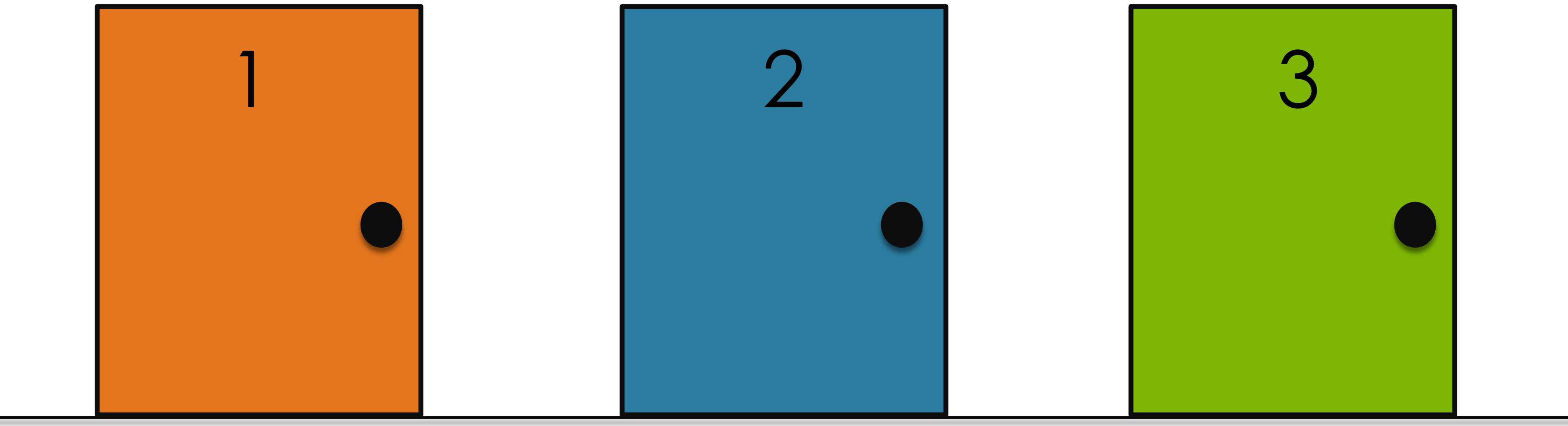
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

How to go about building CFTs with semi-classical gravitational features?



We will focus on the difficulties you encounter in AdS_3/CFT_2.
Not universal, but it illustrates the challenges.

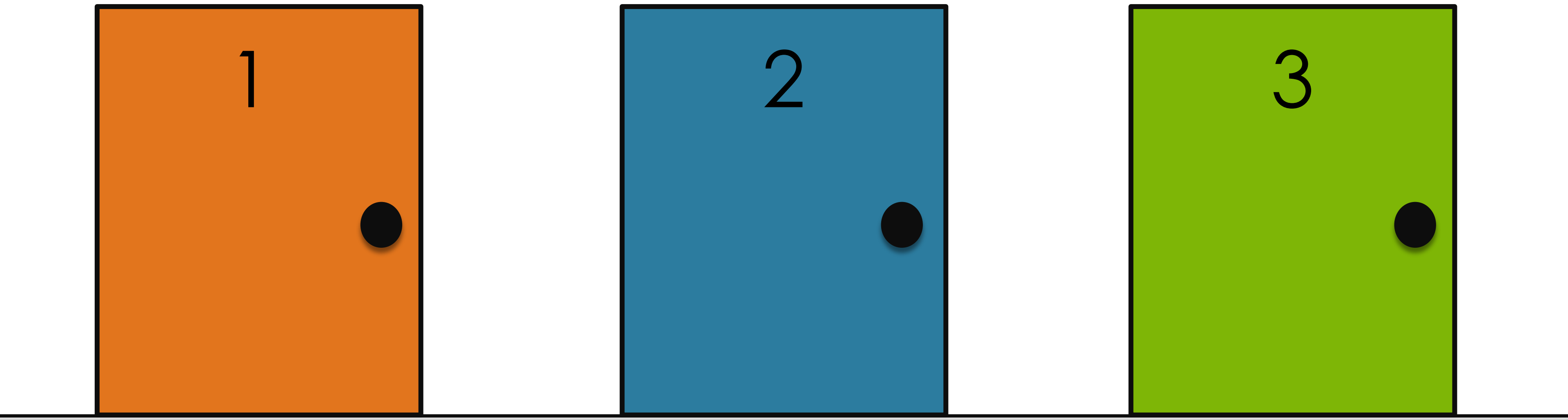




Holographic CFTs

Symmetric Product CFTs

Our Landscape



Holographic CFTs

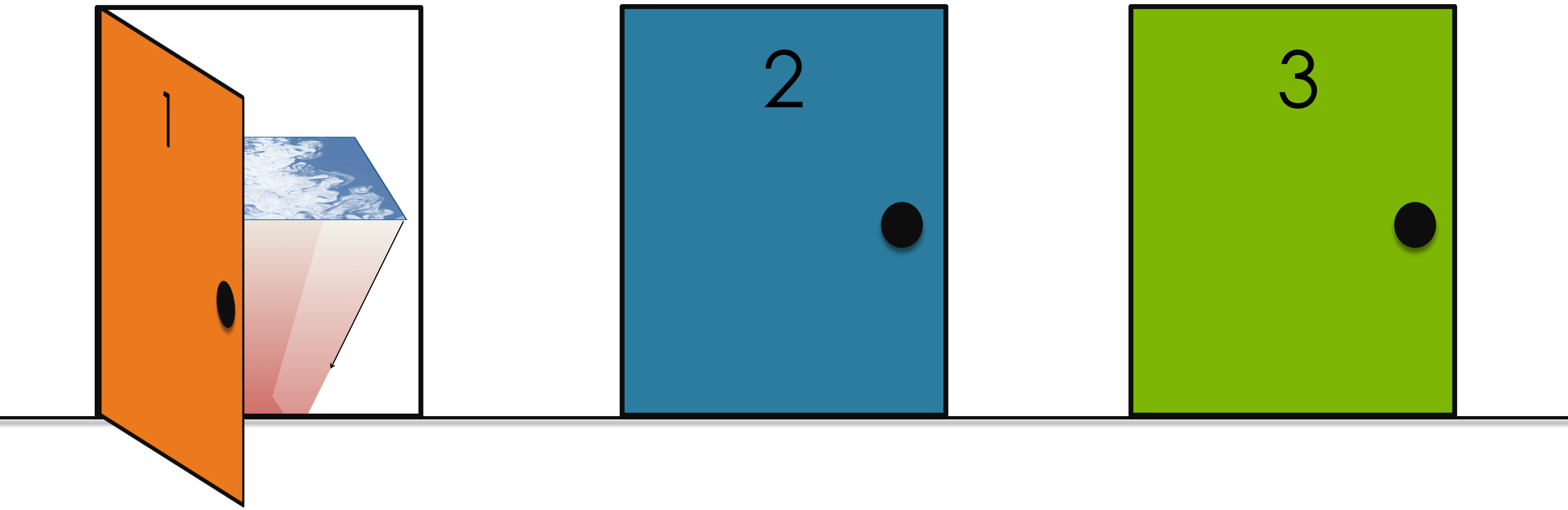
Universality

Symmetric Product CFTs

Strategy

Our Landscape

Output



Holographic
CFTs

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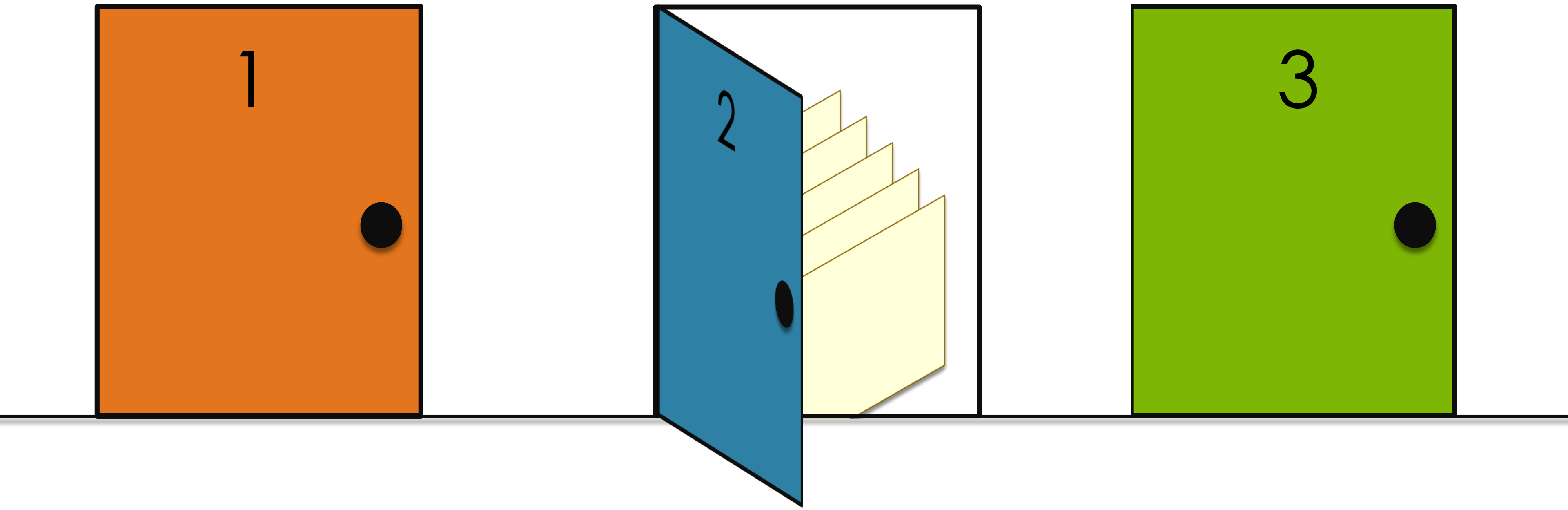
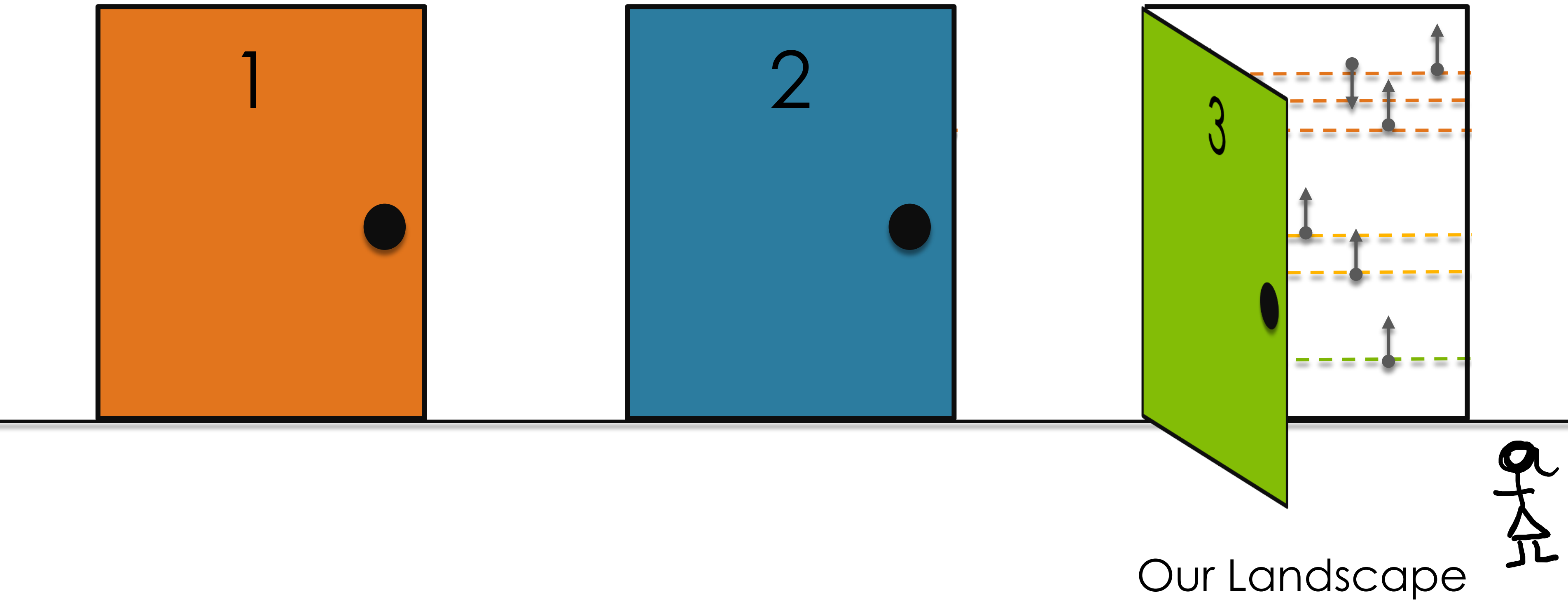
universality: Necessary conditions on the spectrum of CFT_2.



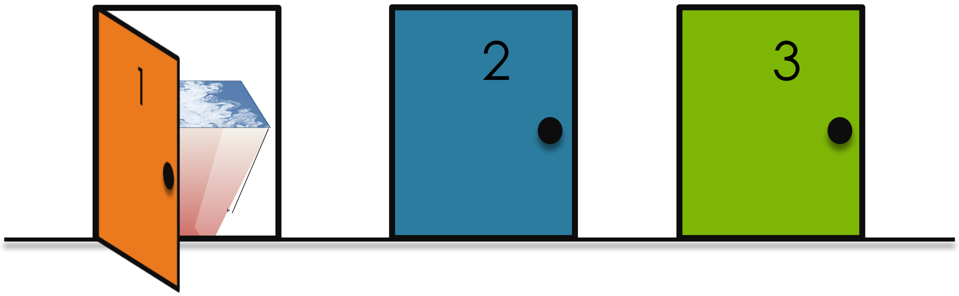
Symmetric Product
CFTs

Strategy: Describing a subspace of CFT_2.
Restrict the analysis to supersymmetric states.



Output: Finding the needles in the haystack. Minimal models come to rescue.

Holographic CFTs



AdS_3 Gravity

The theory:

$$I_{3\mathrm{D}} = \frac{1}{16\pi G_N}\int d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right) + \text{matter}$$

The spectrum:

1. Light States: Perturbative states
2. Heavy States: Black holes
3. Other stuff, e.g., multi-centered, conical defects (to be ignored today)

Universal entry in AdS_3/CFT_2:

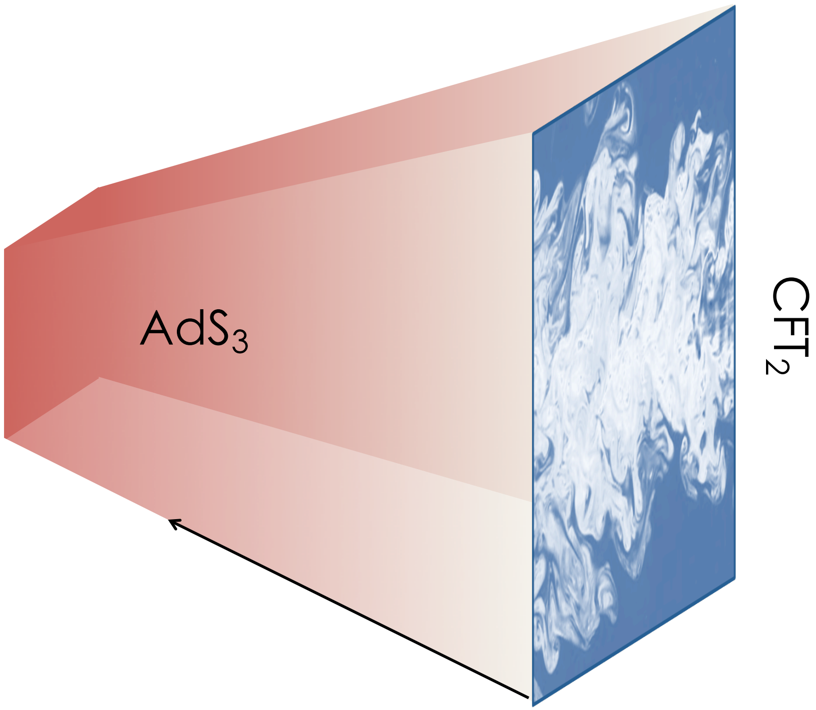
$$c = \frac{3\ell}{2G_N} \gg 1$$

Holographic CFT_2

We will discuss two conditions

1. Black hole regime

2. Perturbative regime



Holographic CFT_2

1. Black hole regime:

$$\boxed{A_H \gg G_N} \longrightarrow \boxed{E \sim c \gg 1}$$

$$\begin{aligned} S_{\rm BH} &= \ln d(c, E) \\ &= 2\pi\sqrt{\frac{cE}{6}} + \cdots \\ &= \frac{A_{\rm H}}{4G} + \cdots \end{aligned}$$

[Strominger+Vafa; Strominger]

Holographic CFT_2

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While the Cardy regime correctly accounts the entropy of very large BHs, we want CFTs with an extended Cardy regime that covers black hole that are large relative to the Planck scale.

Holographic CFT_2

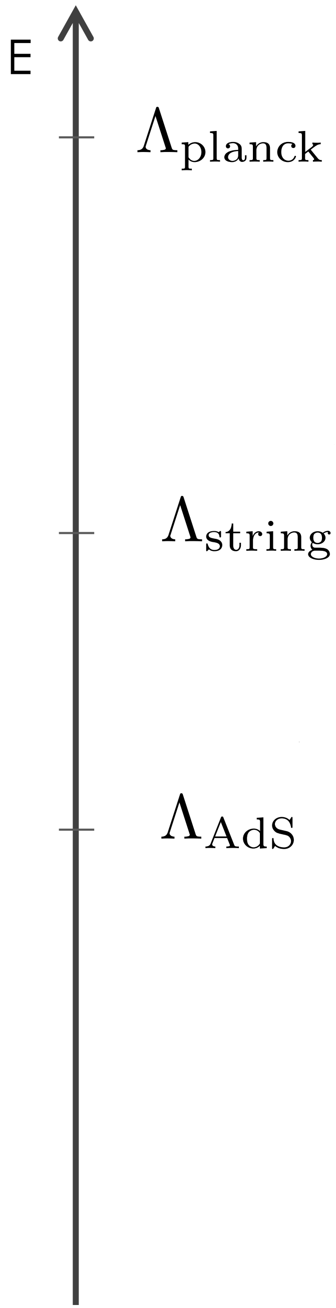
2. Perturbative regime:

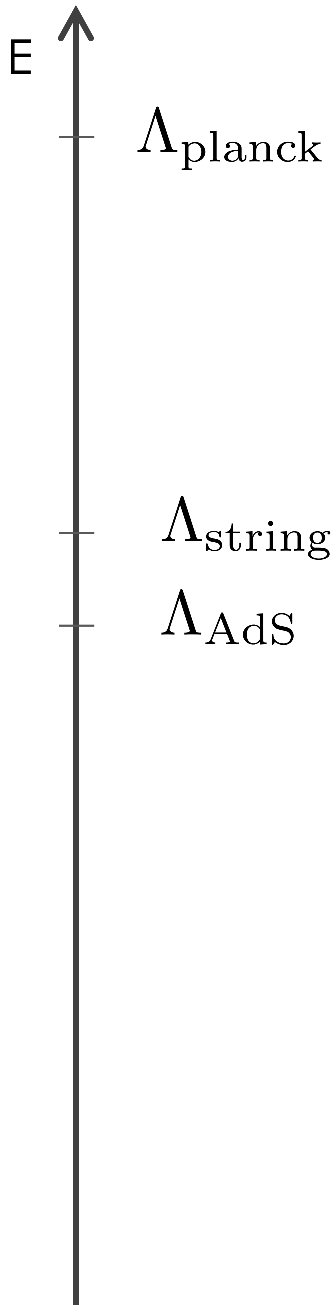
Light = Energy is O(1) in Planck units.
Perturbative excitations that do not form a black hole.

The black hole regime puts already restrictions on these states:

- Presence of Hawking-Page transition [Keller; Hartman, Keller, Stoica]
- Extended Cardy regime for BPS BHs [Benjamin, Cheng, Kachru, Moore, Paquette; Benjamin, Kachru, Keller, Paquette]

Expectations of the string/gravity spectrum





Expectations of the string/gravity spectrum

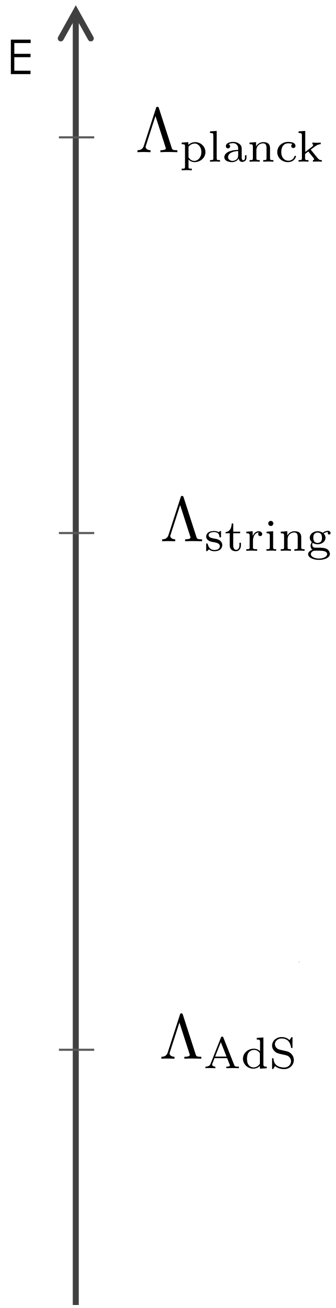
$$\Lambda_{\text{string}} \sim \Lambda_{\text{AdS}}$$

$$d(h) \sim e^{c_H h} \quad \text{where} \quad h \sim E/\Lambda_{\text{AdS}}$$

$$E \gg \Lambda_{\text{AdS}}$$

Hagedorn (fast) growth

Expectations of the string/gravity spectrum



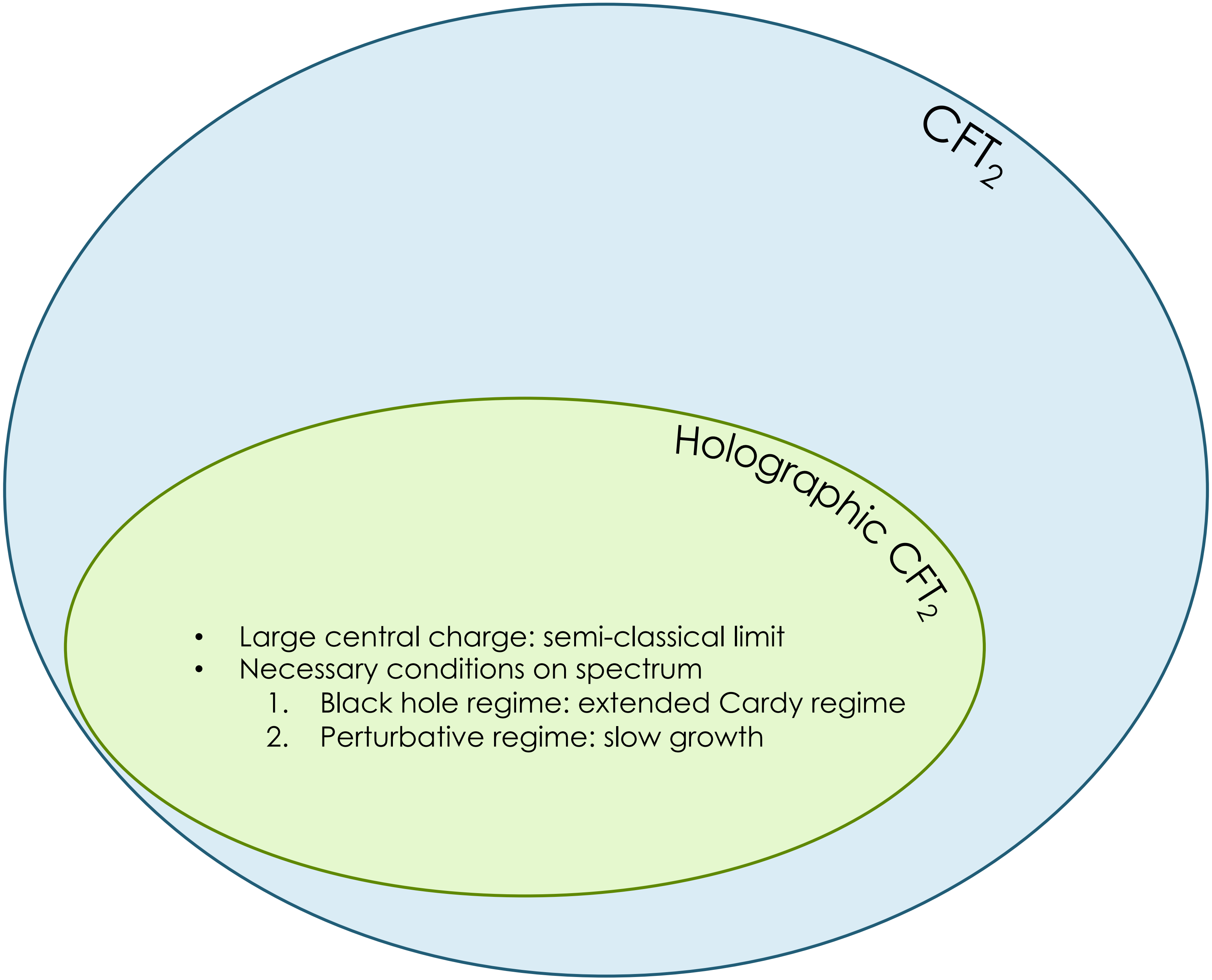
$$\Lambda_{\rm string} \gg \Lambda_{\rm AdS}$$

$$d(h) \sim e^{c s h^{\gamma}}$$

where $h \sim E/\Lambda_{\rm AdS}$ and $\gamma < 1$

$$\Lambda_{\rm string} > E \gg \Lambda_{\rm AdS}$$

Supergravity (slow) growth



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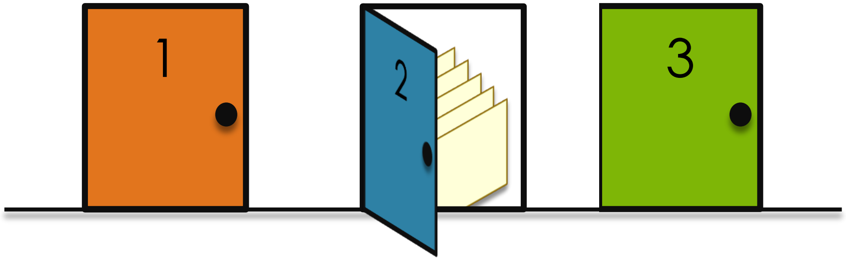
Symmetric Product Orbifolds



Strategy

We want CFTs with large central charge
We want control on the spectrum

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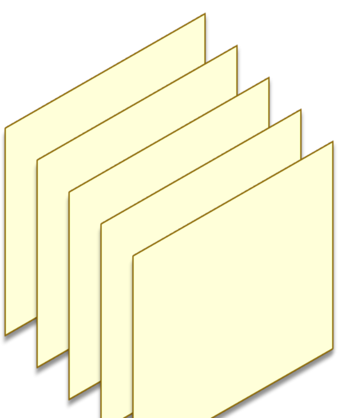
Search within symmetric product theories.

$$\mathcal{C}_m = \mathcal{C}^{\otimes m} / S_m$$



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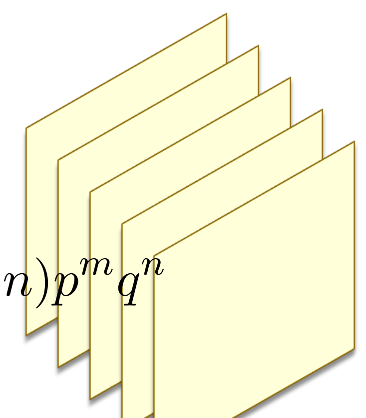
$$\mathcal{Z}(\tau,\rho) = \sum_m Z(\tau;\mathcal{C}_m)\, p^m = \prod_{m>0, n\in\mathbb{Z}} \frac{1}{(1-p^m q^n)^{d(mn)}} = \sum_{m,n} d_m(n) p^m q^n$$



[DMVV]

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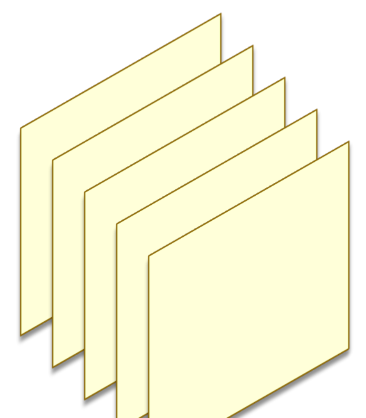

[Keller, 2011]

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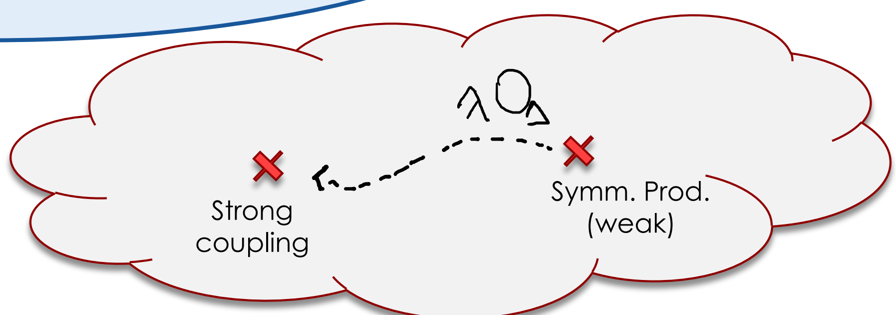

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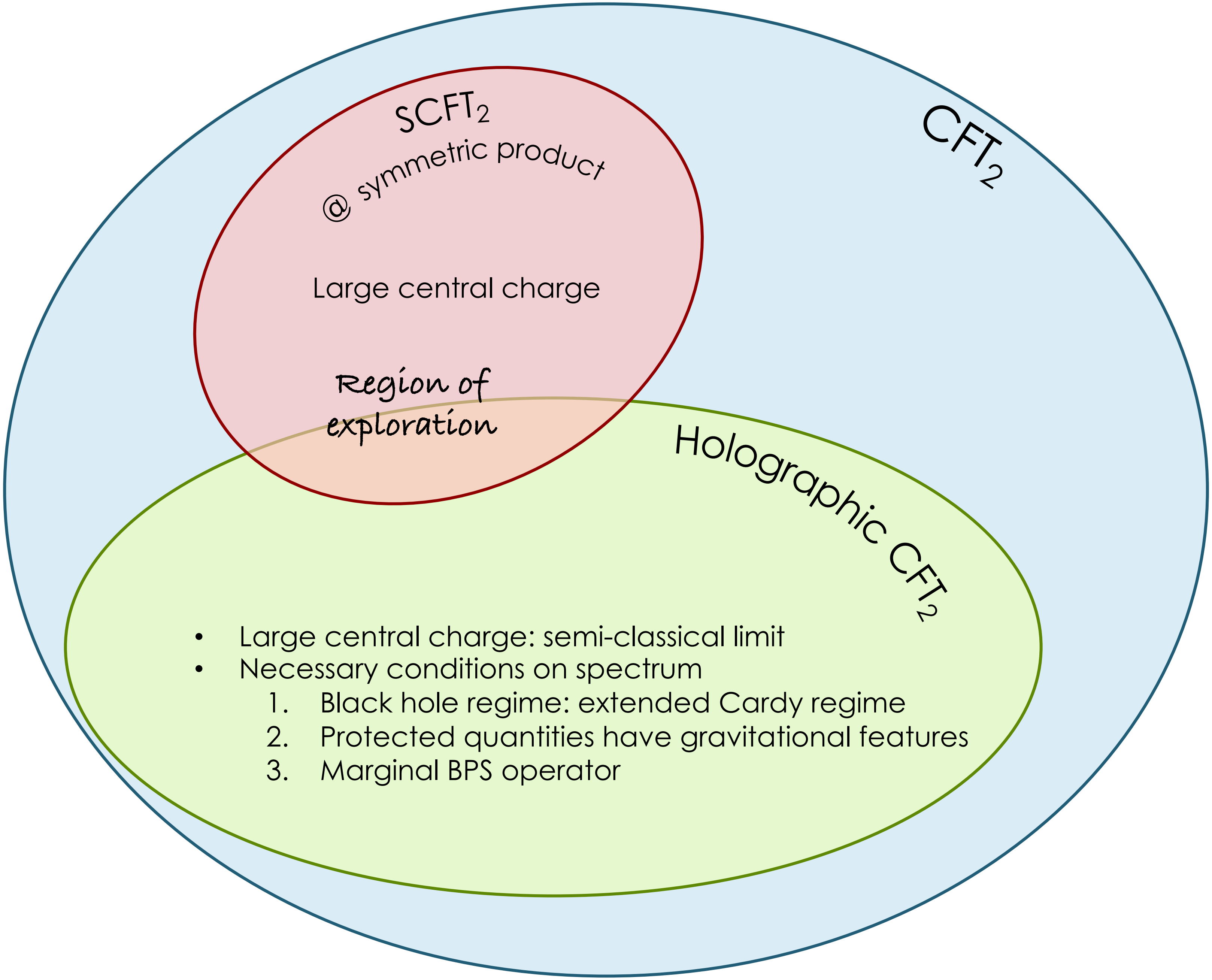
Only hope: Add marginal deformation to make the spectrum sparse

Focus on quantities that are protected.
Those should have gravitational features.



Warning: Sizes are not meaningful.

BPS states in $SCFT_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

The elliptic genera is related to a weak Jacobi form of index t.

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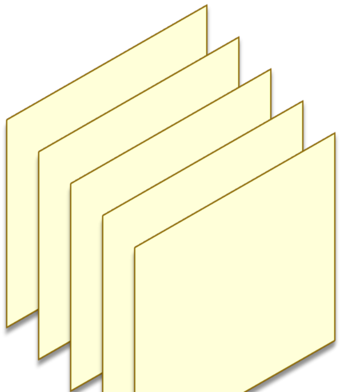
Focus on symmetric product orbifolds: easy to get large values of c.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r}$$



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Necessary condition on light states in NS sector →

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Note: The partition function of symmetric product CFT_2 has $\gamma=1$.

The elliptic genus can display cancellations that capture the spectrum away from the symmetric product point.

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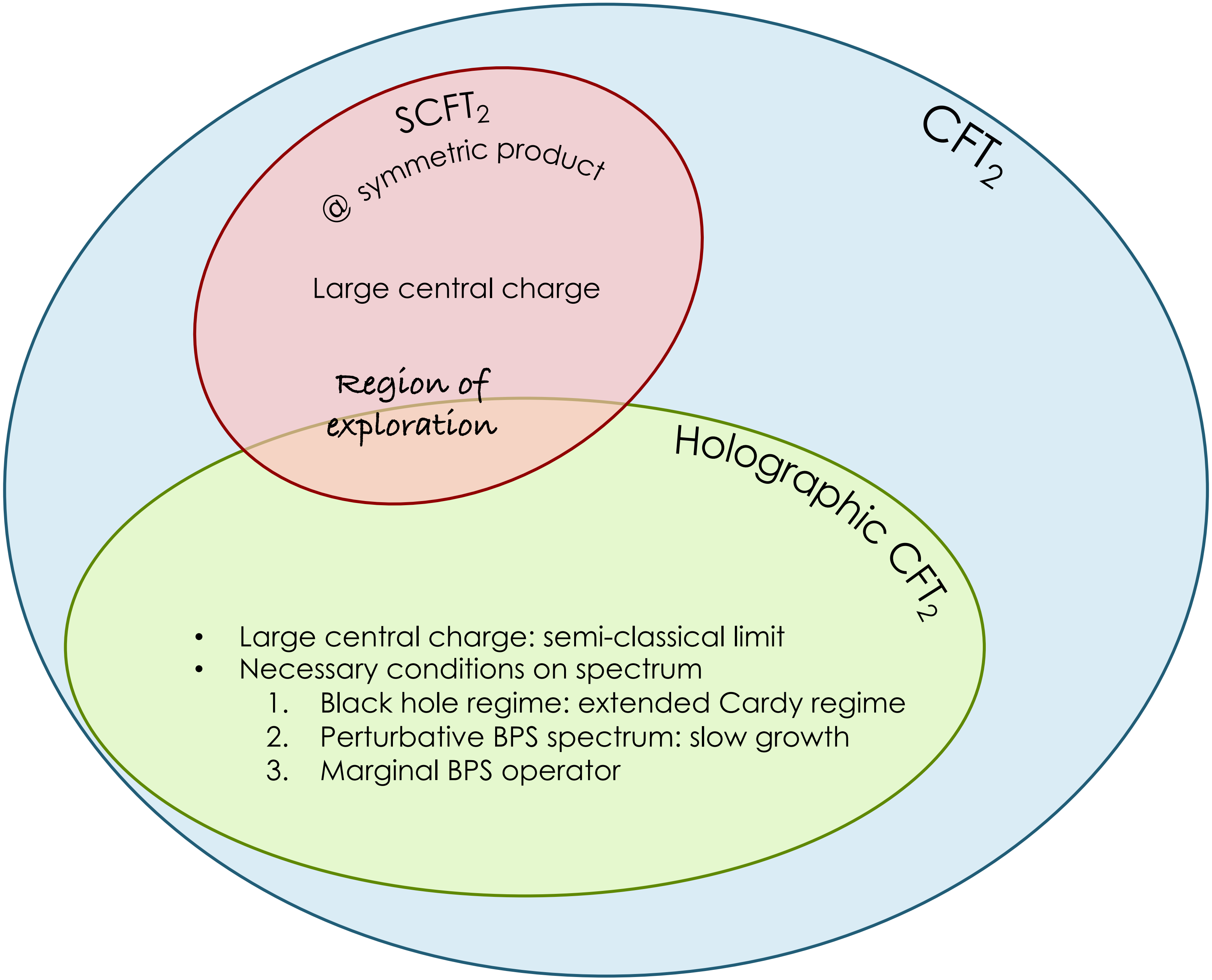
Spoiler!

We can tell you unambiguously which wJFs are holographic, i.e γ<1.
New examples are unveiled.



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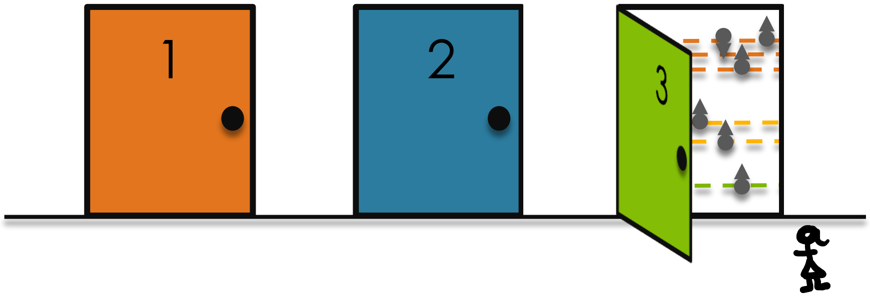
Landscape

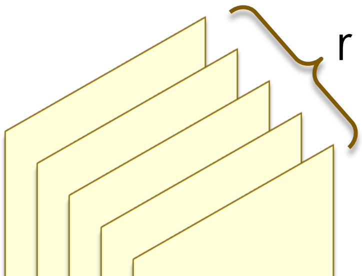


Our procedure in a few steps:

1. Select one seed theory: select a Jacobi form.
2. Perform symmetric product orbifold: increases c
3. Relate generating function to Siegel paramodular form: gives the mathematical control to extract spectrum.
4. Build a modular form that capture the light part of the CFT spectrum

$$\chi_{\mathrm{NS},\infty} \equiv \sum_{h,l} d_{\infty}^{\mathrm{NS}}(h,l) q^h y^l = \prod_{\substack{h\geq 0, l\in\mathbb{Z} \\ (h,l)\neq(0,0)}} \frac{1}{(1-q^h y^l)^{f(h,l)}}$$

Generating functional for light (perturbative) states at infinite r



Classification

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Generating functional for light states at infinite c

Stringy Examples

$$f(h) \sim e^{2\pi\nu_0 h}$$

$$\Rightarrow \quad d_{\infty}^{\mathrm{NS}}(h) \sim e^{2\pi\nu_0 h}$$

Promising (semi-classical gravity) Examples

$$f(h) \sim \delta_{h,h_0}$$

$$\Rightarrow \quad d_{\infty}^{\mathrm{NS}}(h) \sim e^{\sqrt{h}}$$

Promising Examples

t	b	dim	t	b	dim	t	b	dim	t	b	dim
1	1	1	8	1	0	12	1	0	16	1	0
2	1	1	8	2	2	12	2	2	16	2	1
3	1	1	9	1	0	12	3	3	16	3	2
4	1	1	9	2	1	13	1	0	16	4	4
4	2	2	9	3	3	13	2	0	17	1	0
5	1	0	10	1	0	13	3	1	17	2	0
5	2	1	10	2	1	14	1	0	17	3	0
6	1	1	10	3	2	14	2	0	17	4	2
6	2	2	11	1	0	14	3	1	18	1	0
7	1	0	11	2	0	15	1	0	18	2	0
7	2	1	11	3	1	15	2	1	18	3	3
						15	3	2	18	4	3

Good news!

We can easily diagnose if a Symmetric Product Orbifold obeys holographic conditions.

Promising Examples

t	b	dim	t	b	dim	t	b	dim	t	b	dim
1	1	1	8	1	0	12	1	0	16	1	0
2	1	1	8	2	2	12	2	2	16	2	1
3	1	1	9	1	0	12	3	3	16	3	2
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4	2	2	9	3	3	13	2	0	17	1	0
5	1	0	10	1	0	13	3	1	17	2	0
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6	2	2	11	1	0	14	3	1	18	1	0
7	1	0	11	2	0	15	1	0	18	2	0
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Needles in a haystack!

Promising Examples

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What is the physics behind this classification!?

Constructing the CFT

From the mathematical classification of Jacobi forms, there are two necessary properties on the seed:

1. SCFT with at least N=(2,2) or more. Restricts b to divide t,

2. Central charge is bounded,

$$c = 6\frac{b^2}{t} \leq 6$$

Note:

$c \leq 6$ is a necessary condition for both the requirement of slow growth and marginal operators in the spectrum.

N=2 Minimal Models

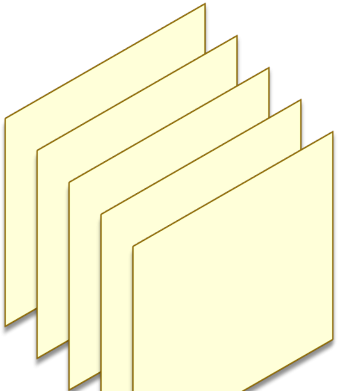
What you need to know about SUSY Minimal Models:

- CFT_2 whose spectrum is built from a finite number of irreducible representations of the N=2 super-Virasoro algebra.

- Obey ADE classification.

- Central charge

$$c = \frac{3k}{k+2} < 3 \quad \text{where } k = 1, 2, \ldots$$



Constructing the CFT

t	b	$c = \frac{6b^2}{t}$	dim	CFTs
1	1	6	1	$K3$ sigma model
2	1	3	1	
3	1	2	1	D_4
4	1	$\frac{3}{2}$	1	A_3
4	2	6	2	Unwrapped $K3$; T^4/G
6	1	1	1	A_2
6	2	4	2	
8	2	3	1	$(A_3)^2$
9	3	6	3	Unwrapped $K3$
10	2	$\frac{12}{5}$	1	D_6
12	2	2	2	A_5; unwrapped D_4
12	3	$\frac{9}{2}$	3	
15	3	$\frac{18}{5}$	2	$(A_4)^2$
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18	3	3	2	Kazama-Suzuki theory at $M = 2, k = 3$

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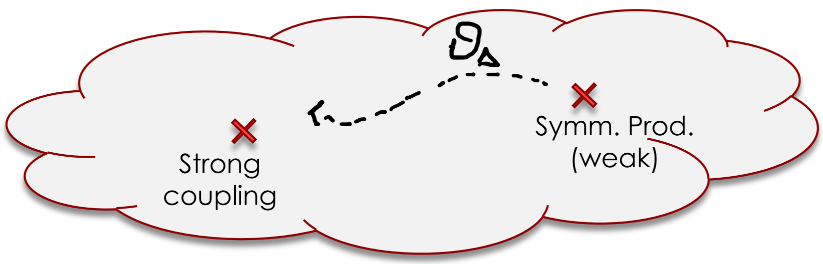
Moduli

Marginal operators are crucial!
Symmetric product orbifold is the weakly coupled description.
These operators should drive the CFT_2 to strong coupling.



Moduli:

- marginal operators: $(h, \bar{h}) = (1, 1)$
- SUSY: $G^{\pm}_{-1/2}$ descendants of (anti-)chiral primaries in NS sector with

$$Q = 1(-1)$$

Moduli

Series	k	untwisted moduli	twisted moduli	single trace twisted
A_2	1	1	28	1 twist 5, 1 twist 7
A_3	2	3	26	1 twist 3, 1 twist 4, 1 twist 5
A_5	4	9	24	1 twist 2, 1 twist 3, 1 twist 4
A_{k+1}	odd, ≥ 3	$P(k+2)-2$	9	1 twist 3
A_{k+1}	even, ≥ 6	$P(k+2)-2$	$10+\sum_{r=1}^{\frac{k}{2}+2} P(r)$	1 twist 2, 1 twist 3
D_4	4	6	20	1 twist 2, 2 twist 3, 1 twist 4
$D_{\frac{k}{2}+2}$	0 mod 4, ≥ 8	$P(\frac{k}{2}+1)+P(\frac{k}{4}+1)$	$8+\sum_{r=1}^{\frac{k}{4}+1} P(r)$	1 twist 2, 1 twist 3
$D_{\frac{k}{2}+2}$	2 mod 4, ≥ 6	$P(\frac{k}{2}+1)$	7	1 twist 3
E_6	10	4	5	1 twist 2
E_7	16	6	5	1 twist 2
E_8	28	6	5	1 twist 2

Seed data

$$c = \frac{3k}{k+2} < 3 \qquad k = 1, 2, \ldots$$

$$Q_r = \frac{r}{k+2} - \frac{1}{2}\,, \qquad r = 1, \ldots, k+1$$

SymN of ½ BPS states

$$\sum_{N=0}^{\infty} Z_{cc}^{N}(y,\bar{y}) p^N = \prod_{L=1}^{\infty} \prod_{Q,\bar{Q}} \frac{1}{(1 - p^L y^{Q+cL/6} \bar{y}^{\bar{Q}+cL/6})^{d(Q,\bar{Q})}}$$

$$Q = \sum_i \frac{2r_i - 2 + k(m_i - 1)}{2(k+2)} \stackrel{!}{=} 1$$

Moduli

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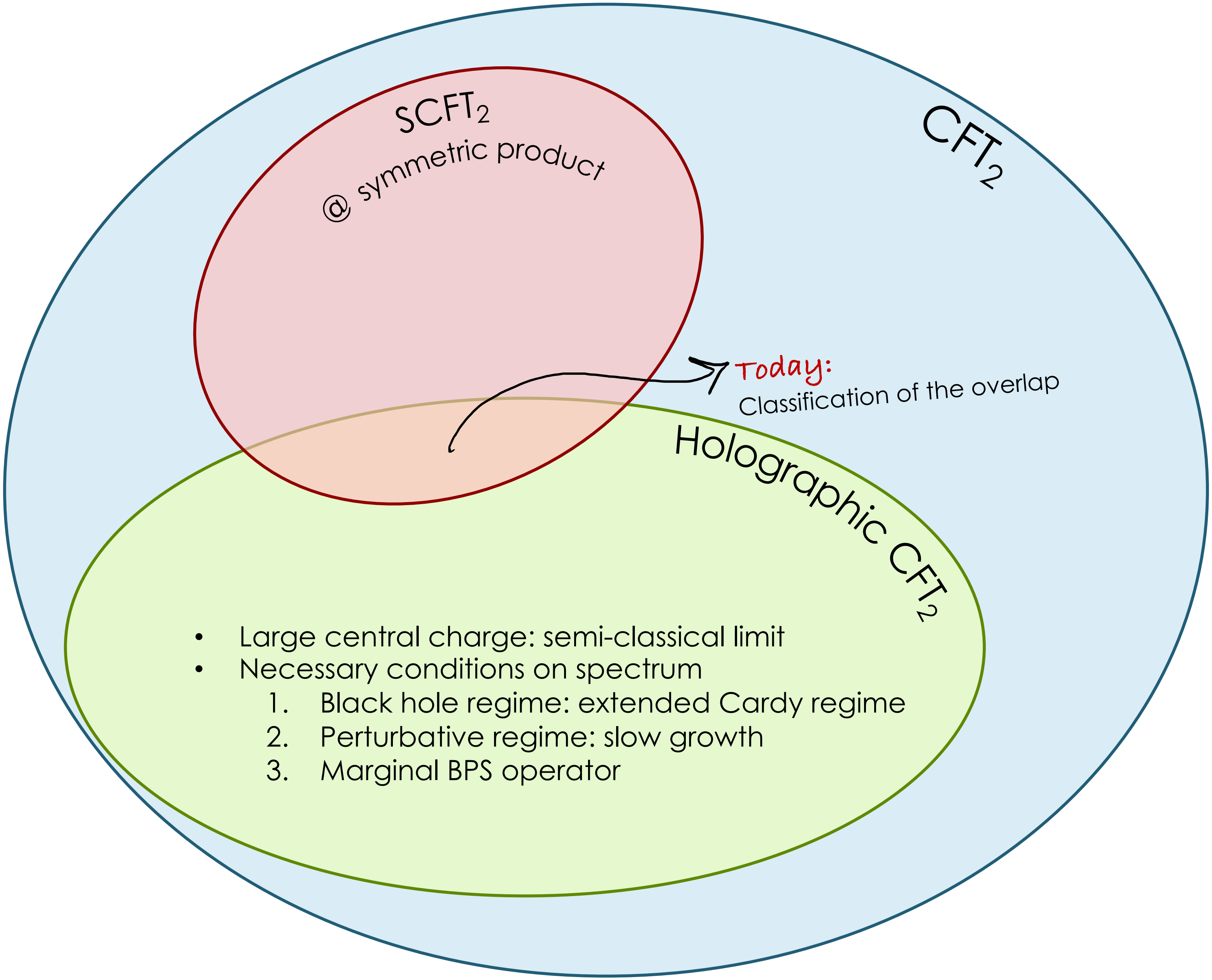
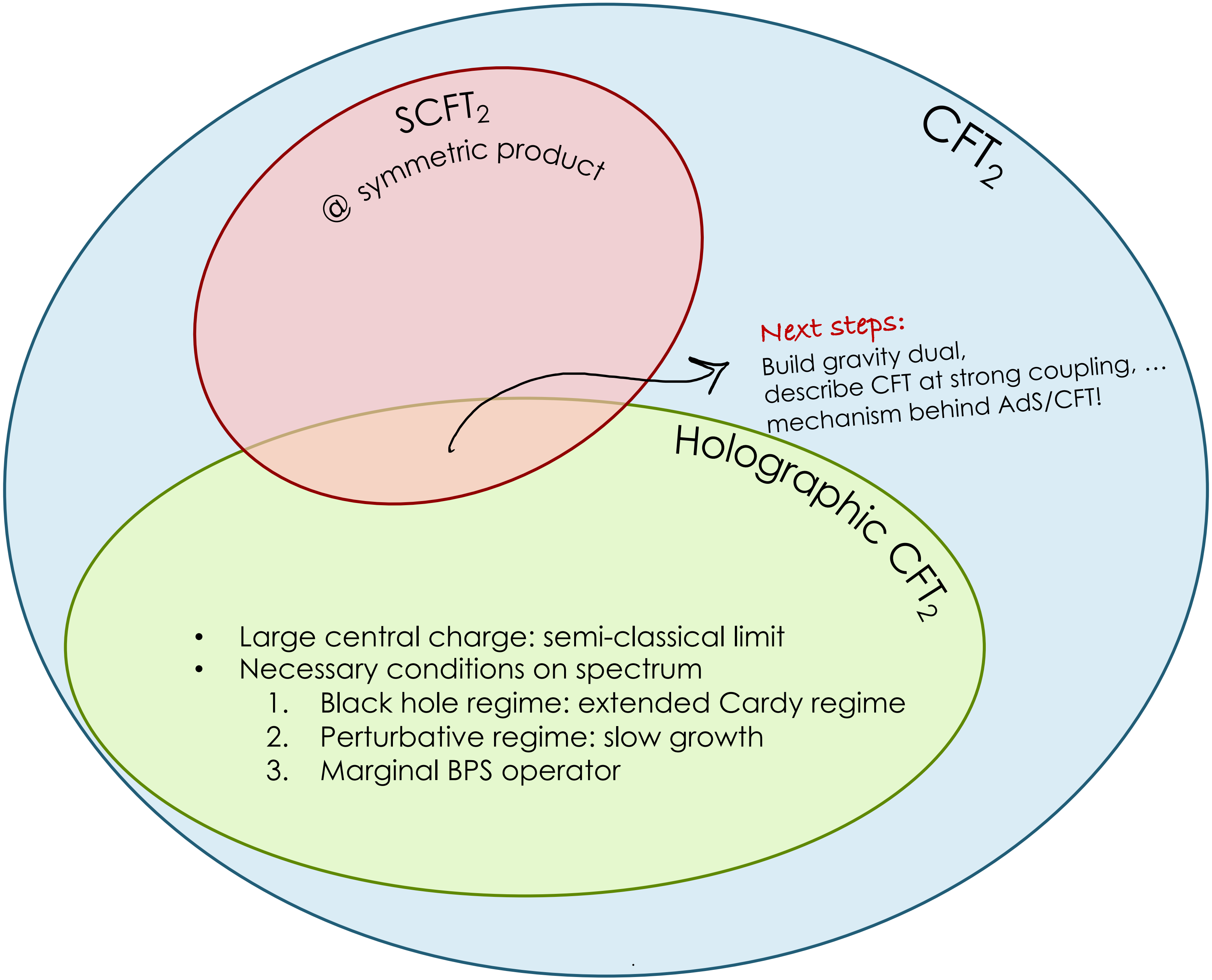
Summary

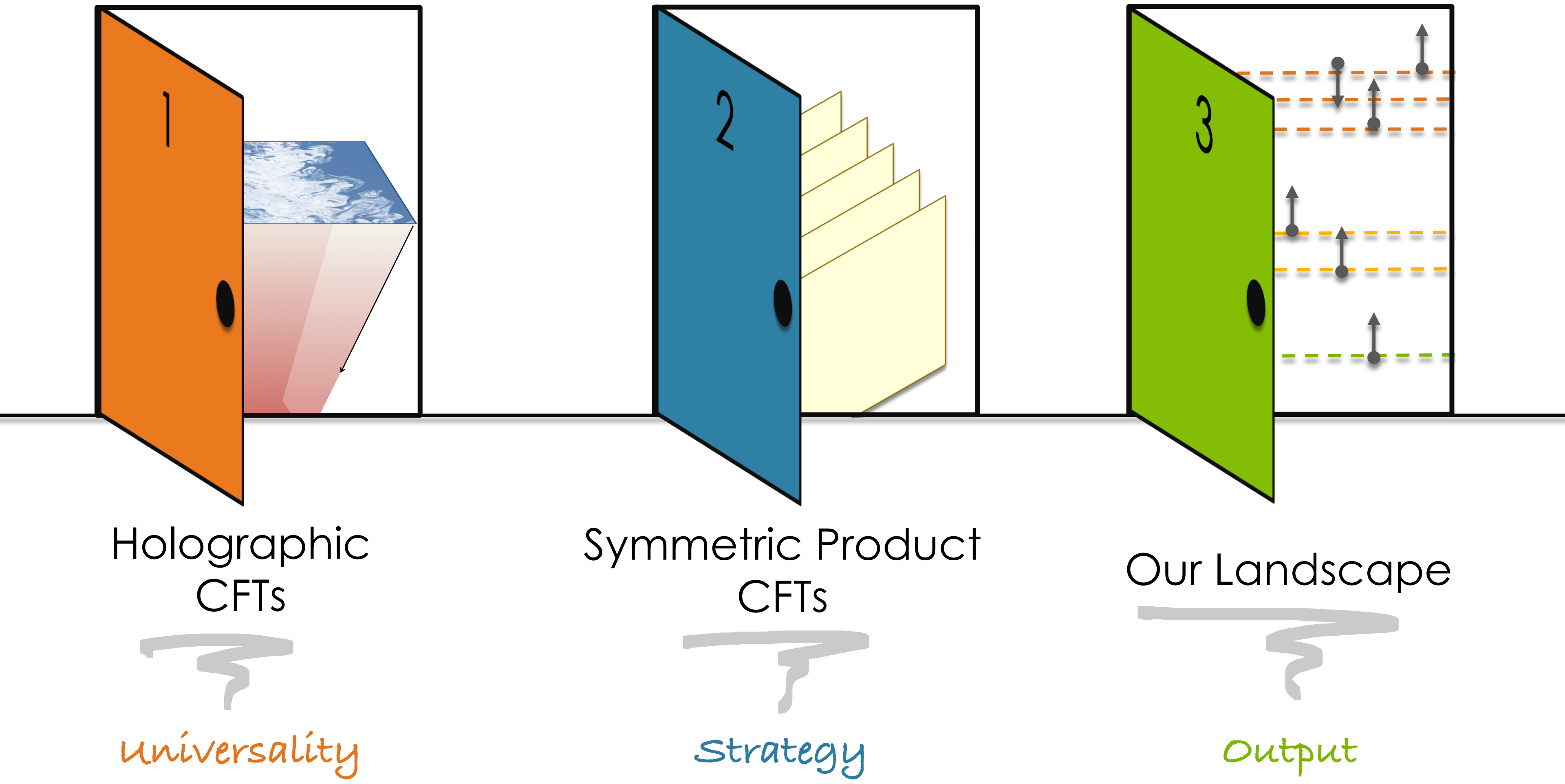


Warning: Sizes are not meaningful.

Summary



Warning: Sizes are not meaningful.



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Holographic CFTs
Symmetric Product CFTs
Our Landscape
Universality
Strategy
Output