R^2 corrected AdS_2 holography

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based on ongoing work with G. L. Cardoso, S. Nampuri

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Outline

Main goal

Determining the holographic dual of a specific subset of 4-derivative $\mathcal{N} = 2$ low energy effective actions of gravity in 4D.

Holographic principle postulates that d.o.f. of quantum gravity are encoded in a lower dimensional CFT.

- AdS/CFT - Quantum gravity in asymptotically AdS spacetimes given in terms of dual CFT on their boundaries.
- Extremal static near horizon backgrounds factorize into $\text{AdS}_2 \times S^2$.

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One way of proceeding is using **$nAdS_2/nCFT_1$** [Almheiri, Polchinski '15; Maldacena, Stanford, Yang '16] – breaking conformal symmetry and taking non-trivial profile for the scalar field.

$\hookrightarrow$ **Constant scalar field** [Cvetič, Papadimitriou '16]

- holographic stress-tensor vanishes identically
- dual operator to constant field is non-trivial and transforms anomalously
- microstates accounting for black hole entropy survive and should be related to the expectation value of the dual operator

Theory

Four derivative, $\mathcal{N} = 2$ low energy effective actions in 4D obtained from CY_3 compactification of superstring theory [Cardoso, de Wit, Mahapatra '07]

- Gravity coupled to Abelian gauge fields and scalars;
- Extremal static near horizon backgrounds;
- BPS configurations.

2D effective action

4D metric given by $ds^2 = {ds_2}^2 + v_2 d\Omega^2 \equiv h_{ij} dx^i dx^j + v_2 d\Omega^2$

$$v_2 = e^{-\psi} B^2 \,, \quad \frac{1}{{k_2}^2} \equiv \frac{B^2}{G_4} \,.$$

4D background supported by e^I, p^I together with constant fields $X^I, \hat{A} = -4\omega^2$ and a holomorphic function $F(X^I, \hat{A})$. We set

$${G_4}^{-1} = i \left(\bar{X}^I F_I - X^I \bar{F}_I \right) \,.$$

Ricci scalar splits into

$$R_4 = R_2 - \frac{2}{v_2} \,, \quad R_2 = \frac{2}{v_1} \,.$$

Locally, can use FG gauge:

$${ds_2}^2 = dr^2 + h_{tt} dt^2 \,, \quad \sqrt{-h} = \alpha(t) e^{r/\sqrt{v_1}} + \beta(t) e^{-r/\sqrt{v_1}} \,.$$

2D effective action

Obtain 2D bulk Lagrangian $\mathcal{F} = \mathcal{F}_1 + \mathcal{F}_2$ with $N_{IJ} = -i(F_{IJ} - \bar{F}_{IJ})$, $Y^I = \frac{1}{4} v_2 \bar{\omega} X^I$, $\Upsilon = -\frac{1}{4} {v_2}^2 \left|\omega\right|^4$

$$\begin{aligned}
\tfrac{1}{2}\mathcal{F}_1 =& \tfrac{1}{8} N_{IJ}\left[(\sqrt{-h_2}/v_2)^{-1} e^I e^J - \frac{\sqrt{-h_2}}{v_2}\, p^I p^J\right] - \tfrac{1}{4}(F_{IJ} + \bar{F}_{IJ}) e^I p^J \\
&+ \tfrac{1}{2} i e^I \left[F_I + F_{IJ} \bar{Y}^J - \mathrm{h.c.}\right] \\
&- \tfrac{1}{2} \frac{\sqrt{-h_2}}{v_2}\, p^I \left[F_I - F_{IJ} \bar{Y}^J + \mathrm{h.c.}\right] ,
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}\mathcal{F}_2 =& \frac{4i}{\sqrt{-\Upsilon}} (\bar{Y}^I F_I - Y^I \bar{F}_I)\, \frac{\sqrt{-h_2}}{v_2}\, (1 - \tfrac{1}{2} v_2\, R_2) \\
&+ i \frac{\sqrt{-h_2}}{v_2} \left[F - Y^I F_I - 2\Upsilon F_\Upsilon + \tfrac{1}{2} \bar{F}_{IJ} Y^I Y^J - \mathrm{h.c.}\right] \\
&+ i(F_\Upsilon - \bar{F}_\Upsilon)\, \frac{\sqrt{-h_2}}{v_2} \left[8\, v_2^2\, R_2^2 - 32 v_2\, R_2 + 32 - 8(\tfrac{1}{2} v_2\, R_2 + 1)\sqrt{-\Upsilon}\right] .
\end{aligned}$$

2D effective action

Take Legendre transfomation w.r.t p^I to make electric-magnetic duality manifest $H(e^I, f_I) = \mathcal{F}(e^I, p^I) + p^I f_I$

$$
\begin{aligned}
H =& \tfrac{1}{4}\left(\sqrt{-h_2}/v_2\right)^{-1}(e^I, f_I)\begin{bmatrix} N_{IJ} + R_{IK}N^{KL}R_{LJ} & -2R_{IK}N^{KJ} \\ -2N^{IK}R_{KJ} & 4N^{IJ}\end{bmatrix}\begin{pmatrix} e^J \\ f_J \end{pmatrix} \\
&+ (e^I, f_I)\left[2i\begin{pmatrix} F_I - \bar{F}_I \\ -(Y^I - \bar{Y}^I)\end{pmatrix} + 4\Upsilon\begin{pmatrix} \bar{F}_{IK}N^{KL}F_{\Upsilon L} \\ -N^{IJ}F_{\Upsilon J}\end{pmatrix} + 4\bar{\Upsilon}\begin{pmatrix} F_{IK}N^{KL}\bar{F}_{\Upsilon L} \\ -N^{IJ}\bar{F}_{\Upsilon J}\end{pmatrix}\right] \\
&- \sqrt{-h_2}\,F(R_2) \\
&+ \frac{\sqrt{-h_2}}{v_2}\Big\{\frac{8\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I) - 2i(\bar{Y}^I F_I - Y^I \bar{F}_I) \\
&\qquad -2i(\Upsilon F_\Upsilon - \bar{\Upsilon}\bar{F}_\Upsilon) + 8\Upsilon\bar{\Upsilon}\,\bar{F}_{\Upsilon I}N^{IJ}F_{\Upsilon J} \\
&\qquad + 2\Upsilon F_{\Upsilon I}N^{IJ}(F_J - \bar{F}_{JL}Y^L) + 2\bar{\Upsilon}\bar{F}_{\Upsilon I}N^{IJ}(\bar{F}_J - F_{JL}\bar{Y}^L) \\
&\qquad + 2\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\left(32 - 8\sqrt{-\Upsilon}\right)\Big\}\,,
\end{aligned}
$$

$$
\frac{F(R_2)}{4i} = \frac{(\bar{Y}^I F_I - Y^I \bar{F}_I)}{\sqrt{-\Upsilon}}\,R_2 - \frac{(F_\Upsilon - \bar{F}_\Upsilon)}{2v_2}\left[8\,v_2^2\,R_2^2 - 32v_2\,R_2 - 4v_2\,R_2\sqrt{-\Upsilon}\right].
$$

Outline

Dynamical fields: $h_{ij}, A_i^I, \tilde{A}_{i\,I}, Y^I, \Upsilon, v_2$

$$e^I \equiv F_{rt}^I = \partial_r A_t^I - \partial_t A_r^I\,, \quad f_I \equiv G_{rt\,I} = \partial_r \tilde{A}_{t\,I} - \partial_t \tilde{A}_{r\,I}\,.$$

Steps to take into account

1. Add counterterms to impose Dirichlet boundary conditions at $r \to \infty$.
2. Renormalize A_t^I and $\tilde{A}_{t\,I}$ to preserve symplectic structure.

[Papadimitriou '10]

[Cvetič, Papadimitriou '16]

[Castro, Larsen, Papadimitriou '18]

[Castro, Mühlmann '20]

Variational principle for A^I and $\tilde{A}_I$

Take $A^I_r = \tilde{A}_{r\,I} = 0$ gauge. E.O.M. imply

$$\begin{pmatrix} \tilde{A}_{tI} \\ -A^I_t \end{pmatrix} = \sqrt{v_1}\,\frac{\alpha(t)\,e^{r/\sqrt{v_1}}}{\sqrt{-h_2}}\left(1-\frac{\beta}{\alpha}\,e^{-2r/\sqrt{v_1}}\right)\begin{pmatrix} f_I \\ -e^I \end{pmatrix} + \begin{pmatrix} \tilde{\mu}_I(t) \\ -\mu^I(t) \end{pmatrix},$$

with $f_I, e^I \propto \sqrt{-h_2} \implies$ leading mode is the one $\propto \alpha(t)$.

For the canonical momenta we have $\pi_I = -q_I$ and $\tilde{\pi}^I = p^I$.

Add counterterms

$$-\int_{\partial M} dt\left(\pi_I A^I_t + \tilde{\pi}^I \tilde{A}_{t\,I}\right) + S'\left(\pi_I, \tilde{\pi}^I\right) + \int_{\partial M} dt\left(\pi_I A^{\mathrm{ren}\,I}_t + \tilde{\pi}^I \tilde{A}^{\mathrm{ren}}_{t\,I}\right),$$

inducing the canonical transformations

$$\begin{pmatrix} A^I_t \\ \pi_I \end{pmatrix} \to \begin{pmatrix} -\pi_I \\ A^{\mathrm{ren}\,I}_t \end{pmatrix} = \begin{pmatrix} -\pi_I \\ A^I_t - \frac{\delta S'}{\delta \pi_I} \end{pmatrix}, \quad \begin{pmatrix} \tilde{A}_{t\,I} \\ \tilde{\pi}^I \end{pmatrix} \to \begin{pmatrix} -\tilde{\pi}^I \\ \tilde{A}^{\mathrm{ren}}_{t\,I} \end{pmatrix} = \begin{pmatrix} -\tilde{\pi}^I \\ \tilde{A}_{tI} - \frac{\delta S'}{\delta \tilde{\pi}^I} \end{pmatrix}.$$

Variational principle for $F(R_2)$

Recall FG gauge

$$ds_2{}^2 = h_{ij}dx^i dx^j = h_{tt}dt^2 + dr^2\,.$$

Define $h_{ij} = \gamma_{ij} + n_i n_j$ with $n_i dx^i = dr$. Extrinsic curvature is given by

$$K \equiv h^{ij}K_{ij} = h^{ij}\gamma_i{}^k \nabla_k n_j = \partial_r \log\sqrt{-h_2} \sim \frac{1}{\sqrt{v_1}} + \mathcal{O}\left(e^{-2r/\sqrt{v_1}}\right)\,.$$

Using $F''(R_2) = 32i\left(F_\Upsilon - \bar{F}_\Upsilon\right)v_2$ we write the action+counterterms as

$$\int_M \sqrt{-h}d^2x\,[-F(R_2)] + \int_{\partial M} dt\sqrt{-\gamma}\left[\underbrace{2F'(R_2)K}_{\text{GHY}} + \frac{64i}{\sqrt{v_1}}\left(F_\Upsilon - \bar{F}_\Upsilon\right)\underbrace{(v_2R_2 - 2)}_{=0\text{ on}-\text{shell}}\right]$$

Boundary action

Our boundary action is

$$S_1 = \int_{\partial M} dt \sqrt{-\gamma} \Bigg\{ \underbrace{2F'(R_2)K}_{\text{GHY}} + \frac{64i}{\sqrt{v_1}} \left(F_\Upsilon - \bar{F}_\Upsilon\right) \underbrace{(v_2 R_2 - 2)}_{=0 \text{ on-shell}}$$

$$+ \underbrace{\frac{1}{4\sqrt{v_1}} \left(\pi_I, \tilde{\pi}^I\right) \begin{bmatrix} 4N^{IJ} & 2N^{IK}R_{KJ} \\ 2R_{IK}N^{KJ} & N_{IJ} + R_{IK}N^{KL}R_{LJ} \end{bmatrix} \begin{pmatrix} \pi_J \\ \tilde{\pi}^J \end{pmatrix}}_{\text{“}\pi^2\text{-term” – part of } S'}$$

$$+ \underbrace{\frac{4}{\sqrt{v_1}} \Re \left[\left(Y^I - 2i\Upsilon F_{\Upsilon J} N^{JI}, F_I - 2i\Upsilon F_{\Upsilon L} N^{LK} \bar{F}_{KI} \right) \right] \begin{pmatrix} \pi_I \\ \tilde{\pi}^I \end{pmatrix}}_{\text{“term linear in }\pi\text{” – part of } S'}$$

$$- \frac{i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{\sqrt{v_1}} \left(2 + \frac{8}{\sqrt{-\Upsilon}}\right) + \frac{8}{\sqrt{v_1}} \Upsilon \bar{\Upsilon} F_{\Upsilon J} N^{JI} \bar{F}_{\Upsilon I}$$

$$- \frac{2}{\sqrt{v_1}} \left[\Upsilon F_{\Upsilon J} N^{JI} \left(F_I - \bar{F}_{IK} Y^K\right) + \text{h.c.} \right] - \frac{16}{\sqrt{v_1}} i \left(F_\Upsilon - \bar{F}_\Upsilon\right) \left(\sqrt{-\Upsilon} - 8\right) \Bigg\}$$

Solution to the equations of motion

Recall $v_1 = v_2$ on-shell and

$$ds_2{}^2 = dr^2 + h_{tt}dt^2\,, \quad h_{tt} = -\left(\alpha(t)e^{r/\sqrt{v_1}} + \beta(t)e^{-r/\sqrt{v_1}}\right)^2 .$$

We focus on class of solutions which includes BPS black hole solutions:

$$Y^I - \bar{Y}^I = ip^I\,, \quad F_I - \bar{F}_I = iq_I\,, \quad \Upsilon = -64\,, \quad \frac{v_2}{G_4} = i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) .$$

These imply, on-shell

$$\begin{pmatrix} f_I \\ e^I \end{pmatrix} = \frac{\sqrt{-h_2}}{v_2}\begin{pmatrix} F_I + \bar{F}_I \\ Y^I + \bar{Y}^I \end{pmatrix} .$$

Outline

On-shell variation of the renormalized action

The on-shell variation of the **renormalized action** $S_{\rm ren} = I_{\rm bulk} + I_{\rm ct}$ yields

$$\delta S_{\rm ren} = \int_{\partial M} dt \left(\pi^{tt}_{\rm ren} \delta h_{tt} + \pi_I \delta A_t^{{\rm ren}\, I} + \tilde{\pi}^I \delta \tilde{A}^{\rm ren}_{t\, I} + \pi^{\rm ren}_{v_2} \delta v_2 \right)$$

We consider variations in the space of asymptotic solutions

$$\delta h_{tt} = e^{2r/\sqrt{v_1}} \delta \left(-\alpha^2 \right) , \quad \delta A_t^{{\rm ren}\, I} = \delta \mu^I , \quad \delta \tilde{A}^{\rm ren}_{t\, I} = \delta \tilde{\mu}_I$$

Moreover, we add a source $\nu(t)$ for the irrelevant operator dual to v_2

$$\delta v_2 = e^{r/\sqrt{v_1}} \delta \nu$$

Recall $Y^I \propto v_2$ and $\Upsilon \propto {v_2}^2 \implies \pi^{\rm ren}_{v_2}$ has contributions from these terms.

On-shell variation in terms of the sources

$$\delta S_{\text{ren}} = \int_{\partial M} dt \left(\pi^{tt}_{\text{ren}} \delta h_{tt} + \pi_I \delta A^{\text{ren}\, I}_t + \tilde{\pi}^I \delta \tilde{A}^{\text{ren}}_{t\, I} + \pi^{\text{ren}}_{v_2} \delta v_2 \right) .$$

In terms of the sources we then have

$$\delta S_{\text{ren}} = \int_{\partial M} dt\, \alpha \left[\hat{\pi}^{tt}_{\text{ren}} \delta \left(-\alpha^2 \right) + \hat{\pi}_I \delta \mu^I + \hat{\tilde{\pi}}^I \delta \tilde{\mu}_I + \hat{\pi}_{v_2} \delta \nu \right] .$$

where we have defined the one-point functions

$$\hat{\pi}^{tt} \equiv \lim_{r\to\infty} \left(\frac{e^{3r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\text{ren}}}{\delta \gamma_{tt}} \right) \qquad \hat{\pi}_I \equiv \lim_{r\to\infty} \left(\frac{e^{r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\text{ren}}}{\delta A^{\text{ren}\, I}_t} \right)$$

$$\hat{\pi}_{v_2} \equiv \lim_{r\to\infty} \left(\frac{e^{2r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\text{ren}}}{\delta v_2} \right) \qquad \hat{\tilde{\pi}}^I \equiv \lim_{r\to\infty} \left(\frac{e^{r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\text{ren}}}{\delta \tilde{A}^{\text{ren}}_{t\, I}} \right)$$

$$\hat{\pi}^{tt} = 0\,, \quad \hat{\pi}_I = -\frac{q_I}{\alpha}\,, \quad \hat{\tilde{\pi}}^I = \frac{p^I}{\alpha}\,, \quad \hat{\pi}_{v_2} = -\frac{2}{G_4\sqrt{v_1}}\frac{\beta}{\alpha}\,,$$

where we recall

$$\frac{v_2}{8G_4} = i\,\frac{\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{\sqrt{-\Upsilon}}\,.$$

Renormalized action written in terms of the sources

$$S_{\text{ren}} = \int dt \left(-\frac{2}{G_4\sqrt{v_1}}\beta\,\nu - q_I\mu^I + p^I\tilde{\mu}_I + \mathcal{O}\left(\nu^2\right)\right)\,.$$

Our prescription here was equivalent to using scalar fields $(v_2, X^I, \hat{A})$ and sourcing only v_2

- Obtained result similar to the one in [Cvetič, Papadimitriou '16];
- Functional form of result does not depend on R^2 corrections.

Outline

Residual gauge symmetries

We are interested in the (PBH) diffeos $\xi^i \partial_i$ and gauge transformations $\Lambda^I, \tilde{\Lambda}_I$ which preserve FG gauge and $A_r^I = \tilde{A}_{r\,I} = 0$

$$\mathcal{L}_\xi h_{rr} = \mathcal{L}_\xi h_{rt} = 0\,, \quad \mathcal{L}_\xi A_r^I + \partial_r \Lambda^I = 0\,, \quad \mathcal{L}_\xi \tilde{A}_{r\,I} + \partial_r \tilde{\Lambda}_I = 0\,.$$

Solution is [Cvetič, Papadimitriou '16]

$$\xi^t = \varepsilon(t) + \partial_t \sigma(t) \int_r^\infty h^{tt}\left(r', t\right) dr'\,, \quad \xi^r = \sigma(t)\,,$$

$$\Lambda^I = \varphi^I(t) - \partial_t \sigma(t) \int_r^\infty h^{tt}(r', t)\, A_t^I(r', t)\, dr'\,,$$

$$\tilde{\Lambda}_I = \tilde{\varphi}_I(t) - \partial_t \sigma(t) \int_r^\infty h^{tt}(r', t)\, \tilde{A}_{t\,I}(r', t)\, dr'\,.$$

Asymptotic symmetries

PBH transformations act on the sources

$$\delta_{\rm PBH}\alpha = \frac{\sigma}{\sqrt{v_1}}\,\alpha + \partial_t(\varepsilon\,\alpha) \qquad \delta_{\rm PBH}\,\nu = \varepsilon\,\partial_t\nu + \frac{\sigma}{\sqrt{v_1}}\nu$$

$$\delta_{\rm PBH}\,\mu^I = \partial_t(\varepsilon\,\mu^I + \varphi^I) \qquad \delta_{\rm PBH}\,\tilde{\mu}_I = \partial_t(\varepsilon\,\tilde{\mu}_I + \tilde{\varphi}_I)$$

$$\delta_{\rm PBH}\,\beta = \partial_t\left(\varepsilon\,\beta\right) - \frac{\sigma}{\sqrt{v_1}}\,\beta - \frac{\sqrt{v_1}}{2}\,\partial_t\left(\frac{\partial_t\sigma}{\alpha}\right)$$

Focusing on asymptotic symmetries $\implies \delta\alpha = \delta\mu^I = \delta\tilde{\mu}_I = 0$

$$\varepsilon = \frac{\zeta(t)}{\alpha} \qquad \sigma = -\sqrt{v_1}\frac{\partial_t\zeta}{\alpha}$$

$$\varphi^I = -\varepsilon\,\mu^I + k^I \qquad \tilde{\varphi}_I = -\varepsilon\,\tilde{\mu}_I + \tilde{k}_I$$

Define $dx^+ = \alpha\, dt$. Using

$$\varepsilon = \frac{\zeta(t)}{\alpha}\,, \quad \sigma = -\sqrt{v_1}\partial_+\zeta\,, \quad \varphi^I = -\varepsilon\,\mu^I + k^I\,, \quad \tilde{\varphi}_I = -\varepsilon\,\tilde{\mu}_I + \tilde{k}_I\,,$$

and

$$\delta_{\rm sym}\,\beta = \partial_t\left(\varepsilon\,\beta\right) + \partial_+\zeta\;\beta + \frac{\alpha\,v_1}{2}\,\partial_+^3\zeta\,,$$

we obtain

$$\delta_{\rm sym}\hat{\pi}_{v_2} = \zeta\,\partial_+\hat{\pi}_{v_2} + 2\partial_+\zeta\,\hat{\pi}_{v_2} - \frac{\sqrt{v_1}}{G_4}\partial_+^3\zeta\,,$$

Anomalous transformation with

$$c \sim \frac{\sqrt{v_1}}{G_4} = \frac{\sqrt{v_1}}{2\sqrt{-h_2}}\left(p^I f_I - q_I e^I\right) \sim \frac{Q^2}{R}$$

[Castro, Grumiller, Larsen, McNees '08]

[Hartman, Strominger '09]

[Castro, Song '14]

Outline

Composite scalar field

Recall

$$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2 F(Y, \Upsilon)\,, \quad \frac{v_2}{G_4} = \frac{8i}{\sqrt{-\Upsilon}} \left(\bar{Y}^I F_I - Y^I \bar{F}_I \right) .$$

Consider a composite scalar field $\hat{\Omega}$ with asymptotic variation

$$\delta_{\hat{\Omega}} = e^{r/\sqrt{v_1}} \delta\Omega\, \mathcal{D}\,, \quad \mathcal{D} \equiv Y^I \partial_{Y^I} + 2\Upsilon \partial_{\Upsilon} + \mathrm{h.c.}$$

Note that

$$\mathcal{D} F(Y, \Upsilon) = 2F(Y, \Upsilon)\,, \quad \mathcal{D} {G_4}^{-1} = {G_4}^{-1}\,.$$

S_{ren} transforms as

$$\delta S_{\mathrm{ren}} = \int_{\partial M} dt\, e^{r/\sqrt{v_1}} \left(Y^I \Pi_I + 2\Upsilon \Pi_{\Upsilon} + \mathrm{h.c.} \right) \delta\Omega\,.$$

Composite scalar field

In terms of the sources

$$\delta S_{\rm ren} = \int_{\partial M} dt\, \alpha\, \hat{\Pi}\, \delta\Omega\,,$$

with $\hat{\Pi} = Y^I \hat{\Pi}_I + 2\Upsilon \hat{\Pi}_\Upsilon + \mathrm{h.c.}$

$$\hat{\Pi}_I = \lim_{r\to\infty} \left(\frac{e^{2r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\rm ren}}{\delta Y^I} \right)\,, \quad \hat{\Pi}_\Upsilon = \lim_{r\to\infty} \left(\frac{e^{2r/\sqrt{v_1}}}{\sqrt{-\gamma}} \frac{\delta S_{\rm ren}}{\delta \Upsilon} \right)\,.$$

Recall

$$F'(R_2) = \frac{4i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{\sqrt{-\Upsilon}} - 2i\left(F_\Upsilon - \bar{F}_\Upsilon\right)\left(16 v_2 R_2 - 32 - 4\sqrt{-\Upsilon}\right)\,,$$

from which we find $\mathcal{D}F'(R_2) = F'(R_2)$ on-shell. Consequently

$$\hat{\Pi} = -\frac{4}{\sqrt{v_1}} F'(R_2) \frac{\beta}{\alpha}\,, \quad \delta_{\rm sym}\hat{\Pi} = \zeta\, \partial_+ \hat{\Pi} + 2\partial_+\zeta\, \hat{\Pi} - 2\sqrt{v_1} F'(R_2)\partial_+^3 \zeta\,.$$

Composite scalar field

Setting $\alpha = 1$ and $\beta = 0$ the transformation is

$$\delta_{\text{sym}} \hat{\Pi} = -\sqrt{v_1}\, 2F'(R_2)\, \partial_t^3 \varepsilon \,.$$

Compare with 4D BPS entropy $\mathcal{S}$ [Cardoso, de Wit, Mohaupt '99]

$$\frac{\mathcal{S}}{\pi} = i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) + 128i\left(F_\Upsilon - \bar{F}_\Upsilon\right) = 2F'(R_2)\,.$$

Outline

Reduced theory in units of ${k_2}^2$

Restrict to 4D solutions with charges (q_0, p^A) with $A = 1, \ldots, n$. These solutions can be lifted to solutions of 5D $\mathcal{N} = 2$ supergravity with R^2 terms [Castro, Davis, Kraus, Larsen '07].

1. rewrite reduced theory in units of ${k_2}^2 \equiv G_4 B^{-2}$;
2. adapt our results to the notation of [Cvetič, Papadimitriou '16] ;
3. Perform lift to 3D;
4. Compare with AdS_3 results of [Castro, Davis, Kraus, Larsen '07].

We define

$$v_2 = e^{-\psi} B^2 \,, \quad Y^I = e^{-\psi} \tilde{Y}^I \,, \quad \Upsilon = e^{-2\psi} \tilde{\Upsilon} \,, \quad F'(R_2) = e^{-\psi} \tilde{F}'(R_2)$$

Take also ${ds_2}^2 = e^{-\psi} d{\tilde{s}_2}^2$ and redefine $\alpha = e^{-\psi/2} \tilde{\alpha}$ and $\beta = e^{\psi/2} \tilde{\beta}$ to obtain

$$\mathcal{O}_\psi = -\hat{\pi}_\psi = -\frac{2}{B {k_2}^2} \frac{\tilde{\beta}}{\tilde{\alpha}} \,.$$

From the 5D point of view

$$ds_5{}^2 = d\tilde{s}_2 + B^2 d\Omega_2{}^2 + e^{-2\psi}\left(dx^5 - A^0\right)^2 .$$

In 4D we are considering

$$F(Y, \Upsilon) = -\frac{1}{6}\frac{C_{ABC}Y^A Y^B Y^C}{Y^0} - \frac{1}{24}\frac{1}{64}c_{2A}\frac{Y^A}{Y^0}\Upsilon ,$$

with C_{ABC}, c_{2A} constants associated with CY3 data. Take $q_0 > 0$ and $p^A < 0$. Solving the attractor equations for Y^I yields

$$\frac{1}{Y^0} = \frac{1}{2}\sqrt{\frac{|q_0|}{p_L^3}} , \quad Y^A = \frac{i}{2}p^A \implies \frac{v_2}{G_4} = p^A F_A - q_0 Y^0 = \frac{p_R^3}{Y^0} ,$$

where we used [Castro, Davis, Kraus, Larsen '07]

$$p_L^3 = \frac{1}{6}\left(C \cdot p^3 + c_2 \cdot p\right) > 0 , \quad p_R^3 = \frac{1}{6}\left(C \cdot p^3 + \frac{1}{2}c_2 \cdot p\right) > 0 .$$

From [Castro, Davis, Kraus, Larsen '07] we have

$$e^{-\psi} = p_R \sqrt{\frac{|q_0|}{p_L^3}} \implies \frac{1}{{k_2}^2} = \frac{1}{2} p_R^2 \,.$$

Using [Cvetič, Papadimitriou '16] , we obtain for the 2D CFT stress tensor ${k_3}^3 \tau_{++} \propto \mathcal{O}_\psi$, which transforms with the anomalous term

$$\delta\tau_{++} = \cdots - \frac{B}{{k_3}^2} \partial_+^3 \zeta \implies \frac{c}{24\pi} = \frac{B}{{k_3}^2} \,.$$

Since ${k_3}^2 = 2\pi R_5 {k_2}^2$

$$\frac{c}{24\pi} = \frac{B}{2\pi\, R_5\, {k_2}^2} = \frac{\sqrt{G_4}}{4\pi\sqrt{2}\, R_5} p_R^3 \,,$$

finally, since $6 p_R^3 = c_R$ [Castro, Davis, Kraus, Larsen '07]

$$c = \frac{\sqrt{G_4}}{\sqrt{2} R_5} c_R \,.$$

Conclusions and outlook

1. Obtained renormalized variational principle for constant scalar fields;
2. Identified the expectation value of irrelevant operators dual to scalar fields. Using asymptotic symmetries we found
   - Composite operator with $c \sim \mathcal{S}$;
   - Under a lift $AdS_2 \to AdS_3$, $\hat{\pi}_\psi$ has $c \propto c_R$ of AdS_3.
3. Hints that holographic dual of 2D QG encodes data that is embedded in chiral half of 2D CFT.

Outlook

- Introduce dynamics ($nAdS_2/nCFT_1$) by perturbing away from the IR fixed point keeping G_4 fixed

$$G_4{}^{-1} = \frac{8}{\sqrt{-\Upsilon}} \frac{i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{v_2} .$$

- CFT_1 as chiral half of CFT_2 [Hartman, Strominger '09; Castro, Mühlmann '20].