BMS_2

Black holes — BMS & Integrability

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

Black holes — BPS, BMS & Integrability, IST Lisbon
September 2020

with Afshar, González, Vassilevich 1911.05739

Outline

Outline

Motivation

Kinematics

Dynamics

Relation to SYK/JT

Outlook

Asymptotic symmetries

Asymptotic symmetries = boundary condition preserving transformations modulo proper gauge transformations

Asymptotic symmetries

Asymptotic symmetries = boundary condition preserving transformations modulo proper gauge transformations

- AdS/CFT
  - basic ingredient of AdS/CFT tests based on symmetries
  - captures universal UV features of QFTs (conformal symmetries)
  - Brown–Henneaux precursor for AdS_3/CFT_2

Asymptotic symmetries

Asymptotic symmetries = boundary condition preserving transformations modulo proper gauge transformations

- AdS/CFT
  - basic ingredient of AdS/CFT tests based on symmetries
  - captures universal UV features of QFTs (conformal symmetries)
  - Brown–Henneaux precursor for $\mathrm{AdS}_3/\mathrm{CFT}_2$
- Flat space
  - basic ingredient of flat space holography tests based on symmetries
  - captures universal IR features of QFTs (Ward id's $\leftrightarrow$ soft theorems)
  - Barnich–Compére precursor for $\mathrm{FS}_3/\mathrm{CCFT}_2$

Asymptotic symmetries

Asymptotic symmetries = boundary condition preserving transformations modulo proper gauge transformations

- AdS/CFT
  - basic ingredient of AdS/CFT tests based on symmetries
  - captures universal UV features of QFTs (conformal symmetries)
  - Brown–Henneaux precursor for $\text{AdS}_3/\text{CFT}_2$
- Flat space
  - basic ingredient of flat space holography tests based on symmetries
  - captures universal IR features of QFTs (Ward id's $\leftrightarrow$ soft theorems)
  - Barnich–Compére precursor for $\text{FS}_3/\text{CCFT}_2$
- Holography beyond AdS/CFT
  - asymptotic holography beyond AdS/CFT?
  - near horizon holography?
  - asymptotic symmetries important input for structure of dual QFT

BMS_4

Brief history:

- general relativity in limit of low curvature $\stackrel{?}{=}$ special relativity

BMS_4

Brief history:

- general relativity in limit of low curvature $\stackrel{?}{=}$ special relativity
- if yes, expect Poincaré as asymptotic symmetries

BMS_4

Brief history:

- general relativity in limit of low curvature $\neq$ special relativity
- if yes, expect Poincaré as asymptotic symmetries
- Bondi, van der Burgh, Metzner and Sachs '62 (BMS):
  no, get infinite extension of Poincaré by super-translations $P(x)$

  x: angular coordinates

BMS_4

Brief history:

- general relativity in limit of low curvature $\neq$ special relativity
- if yes, expect Poincaré as asymptotic symmetries
- Bondi, van der Burgh, Metzner and Sachs '62 (BMS): no, get infinite extension of Poincaré by super-translations $P(x)$
- (extended) BMS_4 algebra ($J_a(x)$: diff S^2 or restriction thereof)

$$\begin{aligned}
\{J_a(x),\, J_b(x')\} &= \big(J_a(x')\partial_b - J_b(x)\partial'_a\big)\,\delta(x-x') \\
\{J_a(x),\, P(x')\} &= \Big(\frac{s}{2}\,P(x')\partial_a - P(x)\partial'_a\Big)\,\delta(x-x') \\
\{P(x),\, P(x')\} &= 0
\end{aligned}$$

s: spin of super-translations (in original BMS_4: $s=1$)

BMS$_4$

Brief history:

- general relativity in limit of low curvature $\neq$ special relativity
- if yes, expect Poincaré as asymptotic symmetries
- Bondi, van der Burgh, Metzner and Sachs '62 (BMS):
  no, get infinite extension of Poincaré by super-translations $P(x)$
- (extended) BMS$_4$ algebra ($J_a(x)$: diff S^2 or restriction thereof)

$$\{J_a(x),\, J_b(x')\} = \big(J_a(x')\partial_b - J_b(x)\partial'_a\big)\,\delta(x-x')$$
$$\{J_a(x),\, P(x')\} = \Big(\frac{s}{2}\,P(x')\partial_a - P(x)\partial'_a\Big)\,\delta(x-x')$$
$$\{P(x),\, P(x')\} = 0$$

  s: spin of super-translations (in original BMS$_4$: $s=1$)
- get same algebra as near horizon symmetries (in any dimension ≥ 3)
  Donnay, Giribet, González, Pino '15 $s=0$ ('scalar super-translations')
  DG, Perez, Troncoso, Sheikh-Jabbari, Zwikel '19 arbitrary s

BMS_3

- Barnich, Gomberoff, González '12 BMS_3 from CFT_2 by contraction

BMS_3

- Barnich, Gomberoff, González '12 BMS_3 from CFT_2 by contraction
- Change the Virasoro basis $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad\qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Barnich, Gomberoff, González '12 BMS_3 from CFT_2 by contraction
- Change the Virasoro basis $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad\qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- In this new basis Virasoro algebras reads

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = \mathcal{O}(1/\ell^2)$$

with central charges $c_L = c - \bar{c}$ and $c_M = \frac{1}{\ell}\left(c + \bar{c}\right)$

BMS_3

- Barnich, Gomberoff, González '12 BMS_3 from CFT_2 by contraction
- Change the Virasoro basis $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad\qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- In this new basis Virasoro algebras reads

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = \mathcal{O}(1/\ell^2)$$

  with central charges $c_L = c - \bar{c}$ and $c_M = \frac{1}{\ell}\left(c + \bar{c}\right)$
- Contraction means $\ell \to \infty$ and yields BMS_3 (M_n: super-translations)

BMS$_3$

- Barnich, Gomberoff, González '12 BMS$_3$ from CFT$_2$ by contraction
- Change the Virasoro basis $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad\qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- In this new basis Virasoro algebras reads

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = \mathcal{O}(1/\ell^2)$$

  with central charges $c_L = c - \bar{c}$ and $c_M = \frac{1}{\ell}\left(c + \bar{c}\right)$
- Contraction means $\ell \to \infty$ and yields BMS$_3$ (M_n: super-translations)
- Example: Einstein gravity

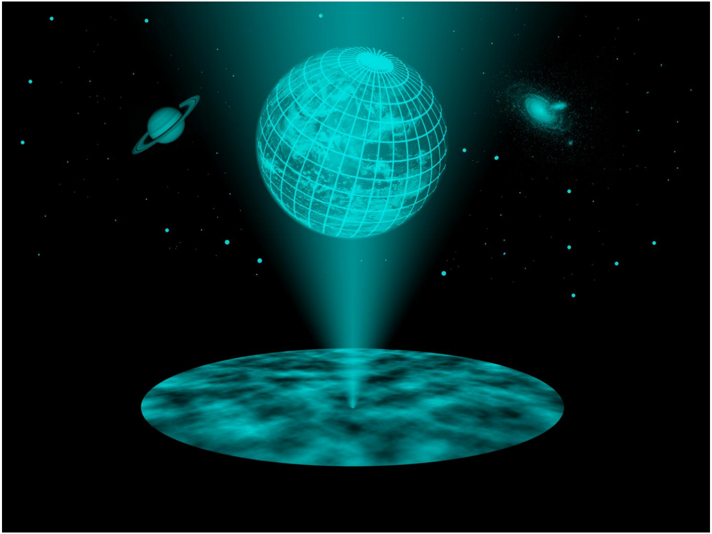
$$c = \bar{c} = \frac{3\ell}{2G} \qquad \Rightarrow \qquad c_L = 0 \quad c_M = \frac{3}{G}$$

Motivation for BMS_2

- because it is there (maybe)

Motivation for BMS_2

- because it is there (maybe)
- BMS_2 useful for toy models of flat space holography

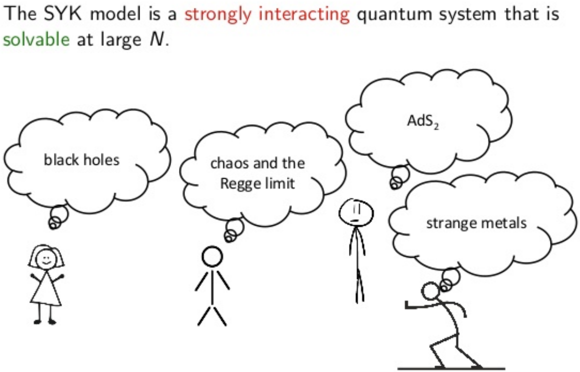


Motivation for BMS_2

- because it is there (maybe)
- BMS_2 useful for toy models of flat space holography
- BMS_2 perhaps useful for near horizon holography

Motivation for BMS_2

- because it is there (maybe)
- BMS_2 useful for toy models of flat space holography
- BMS_2 perhaps useful for near horizon holography
- construct SYK-like models with asymptotically flat gravity side



slide from Stanford's talk at Strings 2017

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless
  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\mathrm{BMS}_2/\mathrm{BMS}_3$ relation

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\mathrm{BMS}_2/\mathrm{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\text{BMS}_2/\text{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2
- super-rotations can only be in time direction

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\text{BMS}_2/\text{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2
- super-rotations can only be in time direction

  ok at least in Euclidean theory

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well

- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\mathrm{BMS}_2/\mathrm{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2

- super-rotations can only be in time direction

  ok at least in Euclidean theory

- super-translations can only be radial super-translations

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\text{BMS}_2/\text{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2
- super-rotations can only be in time direction

  ok at least in Euclidean theory
- super-translations can only be radial super-translations

  ok, why not?

Difficulties with BMS_2

- general point in 2d: co-dimension 2 structureless

  sort of ok in AdS_2, so maybe 'sort of' ok for BMS_2 as well
- AdS_2 algebra is half of AdS_3 algebra
  same cannot be true for $\mathrm{BMS}_2/\mathrm{BMS}_3$ relation

  just means we have no quick way of cheating towards BMS_2
- super-rotations can only be in time direction

  ok at least in Euclidean theory
- super-translations can only be radial super-translations

  ok, why not?

Ignore difficulties and proceed*

* van Nieuwenhuizen: task of theoretical physicists is to break no-go theorems

Outline

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\,\mathrm{d}u^2$$

Not obvious that this is possible with proper gauge trafos!

Same remark applies to *any* gauge fixing, e.g. in AdS_3

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\;\mathrm{d}u^2$$

- Demand Ricci-flatness

$$K(u,\,r) = 2\mathcal{P}(u)\,r + 2\mathcal{T}(u)$$

Note: for constant $\mathcal{P}$ and $\mathcal{T}$ Killing horizon

$$r_h = -\frac{\mathcal{T}}{\mathcal{P}}$$

Assume in most of talk constant $\mathcal{P}$ and $\mathcal{T}$

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\;\mathrm{d}u^2$$

- Demand Ricci-flatness

$$K(u,\,r) = 2\mathcal{P}(u)\,r + 2\mathcal{T}(u)$$

- Allow most general fluctuations $\delta\mathcal{P} \neq 0 \neq \delta\mathcal{T}$

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\;\mathrm{d}u^2$$

- Demand Ricci-flatness

$$K(u,\,r) = 2\mathcal{P}(u)\,r + 2\mathcal{T}(u)$$

- Allow most general fluctuations $\delta\mathcal{P} \neq 0 \neq \delta\mathcal{T}$
- Whatever the gravity theory is going to be, require the following boundary conditions for metric

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + \big(\underbrace{\mathcal{O}(r) + \mathcal{O}(1)}_{\text{state-dependent}} + o(1)\big)\;\mathrm{d}u^2$$

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\;\mathrm{d}u^2$$

- Demand Ricci-flatness

$$K(u,\,r) = 2\mathcal{P}(u)\,r + 2\mathcal{T}(u)$$

- Allow most general fluctuations $\delta\mathcal{P} \neq 0 \neq \delta\mathcal{T}$
- Whatever the gravity theory is going to be, require the following boundary conditions for metric

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + \big(\underbrace{\mathcal{O}(r) + \mathcal{O}(1)}_{\text{state-dependent}} + o(1)\big)\;\mathrm{d}u^2$$

Determine next asymptotic Killing vectors

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\left(\mathcal{P}(u)\,r + \mathcal{T}(u)\right)\,\mathrm{d}u^2$$

preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \left(\epsilon'(u)r + \eta(u)\right)\partial_r$$

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\big(\mathcal{P}(u)\,r + \mathcal{T}(u)\big)\,\mathrm{d}u^2$$

  preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

- $\epsilon(u)$ generates 'super-rotations'

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\,\big(\mathcal{P}(u)\,r + \mathcal{T}(u)\big)\,\mathrm{d}u^2$$

  preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

- $\epsilon(u)$ generates ‘super-rotations’
- $\eta(u)$ generates radial ‘super-translations’

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\left(\mathcal{P}(u)\,r + \mathcal{T}(u)\right)\,\mathrm{d}u^2$$

preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \left(\epsilon'(u)r + \eta(u)\right)\partial_r$$

- $\epsilon(u)$ generates 'super-rotations'
- $\eta(u)$ generates radial 'super-translations'
- Metric functions transform non-trivially

$$\mathcal{L}_\xi\mathcal{P} = \epsilon\mathcal{P}' + \epsilon'\mathcal{P} + \epsilon''$$
$$\mathcal{L}_\xi\mathcal{T} = \epsilon\mathcal{T}' + 2\epsilon'\mathcal{T} + \eta' - \eta\mathcal{P}$$

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\,\big(\mathcal{P}(u)\,r + \mathcal{T}(u)\big)\,\mathrm{d}u^2$$

preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

- $\epsilon(u)$ generates 'super-rotations'
- $\eta(u)$ generates radial 'super-translations'
- Metric functions transform non-trivially

$$\mathcal{L}_\xi \mathcal{P} = \epsilon\mathcal{P}' + \epsilon'\mathcal{P} + \epsilon''$$
$$\mathcal{L}_\xi \mathcal{T} = \epsilon\mathcal{T}' + 2\epsilon'\mathcal{T} + \eta' - \eta\mathcal{P}$$

- Looks promising!

$\mathcal{P}$ like $u(1)$ current
$\mathcal{T}$ like Virasoro generator

BMS_2 algebra

- Lie-bracket algebra of asymptotic Killing vectors

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big]=\xi\big(\epsilon_1\epsilon_2'-\epsilon_2\epsilon_1',\,(\epsilon_1\eta_2-\epsilon_2\eta_1)'\big)$$

BMS_2 algebra

- Lie-bracket algebra of asymptotic Killing vectors

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big] = \xi\big(\epsilon_1\epsilon_2' - \epsilon_2\epsilon_1',\,(\epsilon_1\eta_2 - \epsilon_2\eta_1)'\big)$$

- Algebra for Laurent modes $L_n = \xi(-u^{n+1}, 0)$, $M_n = \xi(0, u^{n-1})$

$$\begin{aligned}
[L_n,\,L_m] &= (n-m)\,L_{n+m} \\
[L_n,\,M_m] &= (-n-m)\,M_{n+m} \\
[M_n,\,M_m] &= 0
\end{aligned}$$

BMS_2 algebra

- Lie-bracket algebra of asymptotic Killing vectors

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big]=\xi\big(\epsilon_1\epsilon_2'-\epsilon_2\epsilon_1',\,(\epsilon_1\eta_2-\epsilon_2\eta_1)'\big)$$

- Algebra for Laurent modes $L_n=\xi(-u^{n+1},0)$, $M_n=\xi(0,u^{n-1})$

$$\begin{aligned}[L_n,\,L_m]&=(n-m)\,L_{n+m}\\ [L_n,\,M_m]&=(-n-m)\,M_{n+m}\\ [M_n,\,M_m]&=0\end{aligned}$$

- Witt subalgebra generated by L_n
  spin-0 super-translations generated by M_n

BMS_2 algebra

- Lie-bracket algebra of asymptotic Killing vectors

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big]=\xi\big(\epsilon_1\epsilon_2'-\epsilon_2\epsilon_1',\,(\epsilon_1\eta_2-\epsilon_2\eta_1)'\big)$$

- Algebra for Laurent modes $L_n=\xi(-u^{n+1},0)$, $M_n=\xi(0,u^{n-1})$

$$\begin{aligned}[L_n,\,L_m]&=(n-m)\,L_{n+m}\\ [L_n,\,M_m]&=(-n-m)\,M_{n+m}\\ [M_n,\,M_m]&=0\end{aligned}$$

- Witt subalgebra generated by L_n
  spin-0 super-translations generated by M_n

> Call this algebra BMS_2
> Can (and will) have non-trivial central extensions

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, J_m] = -m\, J_{n+m}$$
$$[J_n,\, J_m] = 0$$

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$
\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} \\
[L_n,\, J_m] &= -m\, J_{n+m} \\
[J_n,\, J_m] &= 0
\end{aligned}
$$

- Warped Witt algebra (J_n: spin-1 current)

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, J_m] = -m\, J_{n+m}$$
$$[J_n,\, J_m] = 0$$

- Warped Witt algebra (J_n: spin-1 current)
- Relation to old super-translation generators (M_n: spin-0 current)

$$J_n = n\, M_n \quad n \neq 0$$

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, J_m] = -m\, J_{n+m}$$
$$[J_n,\, J_m] = 0$$

- Warped Witt algebra (J_n: spin-1 current)
- Relation to old super-translation generators (M_n: spin-0 current)

$$J_n = n\, M_n \quad n \neq 0$$

- Almost basis change, but J_0 mapped to zero and nothing maps to M_0

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, J_m] = -m\, J_{n+m}$$
$$[J_n,\, J_m] = 0$$

- Warped Witt algebra (J_n: spin-1 current)
- Relation to old super-translation generators (M_n: spin-0 current)

$$J_n = n\, M_n \qquad n \neq 0$$

- Almost basis change, but J_0 mapped to zero and nothing maps to M_0
- Later: M_0 interpretable as winding mode of Maxwell field

Global aspects

- Redefine function generating super-translations, $\eta = \sigma'$
- Redefine corresponding generators $J_n = \xi(0,\, \sigma = u^n)$

$$
\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} \\
[L_n,\, J_m] &= -m\, J_{n+m} \\
[J_n,\, J_m] &= 0
\end{aligned}
$$

- Warped Witt algebra (J_n: spin-1 current)
- Relation to old super-translation generators (M_n: spin-0 current)

$$
J_n = n\, M_n \qquad n \neq 0
$$

- Almost basis change, but J_0 mapped to zero and nothing maps to M_0
- Later: M_0 interpretable as winding mode of Maxwell field

> Dismiss winding mode and focus on warped Witt algebra

Outline

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$
\begin{aligned}
I = \frac{1}{16\pi\,G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big] \\
- \frac{1}{8\pi\,G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\;\;[XK - S(X)] + I^{(m)}
\end{aligned}
$$

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\ \big[XK - S(X)\big] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\;\; [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\;\;[XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term for Dirichlet boundary problem

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|} \left[XR - U(X)(\nabla X)^2 - V(X)\right] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \left[XK - S(X)\right] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term for Dirichlet boundary problem
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y} \int^X V(y) e^{\int^y U(z)\,\mathrm{d}z}\,\mathrm{d}y$$

  and guarantees well-defined variational principle $\delta I = 0$ with fineprint

Dilaton gravity in two dimensions (review hep-th/0204253)

- Candidate for gravity theory realizing our bc's:
  Einstein–dilaton–Maxwell in 2d (see e.g. DG, McNees, Salzer '14)
- Second order action

$$I = \frac{1}{16\pi\,G_2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big]$$

$$I^{(m)} = \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,f(X)F^{\mu\nu}F_{\mu\nu} \qquad\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term for Dirichlet boundary problem
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y}\int^X V(y)e^{\int^y U(z)\,\mathrm{d}z}\,\mathrm{d}y$$

  and guarantees well-defined variational principle $\delta I = 0$ with fineprint
- Interesting option: couple 2d dilaton gravity to matter

Selected list of models (see review hep-th/0604049)

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten **Black Hole** (1991)	$-\frac{1}{X}$	$-2b^2X$
4. CGHS (1992)	0	-2Λ
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. **Black Hole** attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A\sinh X$
17. 2D type 0A string **Black Hole**	$-\frac{1}{X}$	$-2b^2X + \frac{b^2q^2}{8\pi}$
18. exact string **Black Hole** (2005)	lengthy	lengthy

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM
  Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)
- for our purposes: linear, non-abelian gauge theory sufficient

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)
- for our purposes: linear, non-abelian gauge theory sufficient
- non-abelian BF action

$$I_{\text{BF}}[B,\,\mathcal{A}] = \kappa\int\langle B,\,F\rangle \qquad F = \mathrm{d}\mathcal{A} + \mathcal{A}\wedge\mathcal{A}$$

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)
- for our purposes: linear, non-abelian gauge theory sufficient
- non-abelian BF action

$$I_{\mathrm{BF}}[B,\,\mathcal{A}] = \kappa \int \langle B,\, F\rangle \qquad F = \mathrm{d}\mathcal{A} + \mathcal{A}\wedge\mathcal{A}$$

- connection 1-form chosen as

$$\mathcal{A} = \omega\, J + e^a P_a + A\, Z$$

ω: (dualized) spin-connection, e^a: zweibein, A: Maxwell connection

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM
  Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)
- for our purposes: linear, non-abelian gauge theory sufficient
- non-abelian BF action

$$I_{\text{BF}}[B,\,\mathcal{A}]=\kappa\int\langle B,\,F\rangle \qquad F=\mathrm{d}\mathcal{A}+\mathcal{A}\wedge\mathcal{A}$$

- connection 1-form chosen as

$$\mathcal{A}=\omega\,J+e^aP_a+A\,Z$$

  ω: (dualized) spin-connection, e^a: zweibein, A: Maxwell connection
- scalar field chosen as

$$B=X\,Z+X^a\epsilon_a{}^bP_b+Y\,J$$

  X: dilaton, X^a, Y: auxiliary fields

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM
  Ikeda '93; Schaller, Strobl '94 (non-linear gauge theory)
- for our purposes: linear, non-abelian gauge theory sufficient
- non-abelian BF action

$$I_{\mathrm{BF}}[B,\,\mathcal{A}] = \kappa \int \langle B,\,F\rangle \qquad F = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$$

- connection 1-form chosen as

$$\mathcal{A} = \omega\, J + e^a P_a + A\, Z$$

  ω: (dualized) spin-connection, e^a: zweibein, A: Maxwell connection
- scalar field chosen as

$$B = X\, Z + X^a \epsilon_a{}^b P_b + Y\, J$$

  X: dilaton, X^a, Y: auxiliary fields
- still need to choose gauge algebra and bilinear form

Cangemi–Jackiw version of Callan–Giddings–Harvey–Strominger

- Choose Maxwell algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, Z \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

with bilinear form

$$\langle J,\, Z\rangle = -1 \qquad\qquad \langle P_a,\, P_b\rangle = \eta_{ab}$$

Cangemi–Jackiw version of Callan–Giddings–Harvey–Strominger

- Choose Maxwell algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, Z \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

with bilinear form

$$\langle J,\, Z\rangle = -1 \qquad\qquad \langle P_a,\, P_b\rangle = \eta_{ab}$$

- corresponding action (after integrating our X^a and ω)

$$I[X,\, Y,\, g_{\mu\nu},\, A_\mu] = \frac{\kappa}{2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

Cangemi–Jackiw version of Callan–Giddings–Harvey–Strominger

- Choose Maxwell algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, Z \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

with bilinear form

$$\langle J,\, Z\rangle = -1 \qquad\qquad \langle P_a,\, P_b\rangle = \eta_{ab}$$

- corresponding action (after integrating our X^a and ω)

$$I[X,\, Y,\, g_{\mu\nu},\, A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- EOM

$$\begin{aligned} R &= 0 \quad \Rightarrow \quad \text{Ricci-flat} \\ \epsilon^{\mu\nu}\partial_\mu A_\nu &= 1 \\ \nabla_\mu\nabla_\nu X - g_{\mu\nu}\nabla^2 X &= g_{\mu\nu} Y \\ Y &= \Lambda = \text{const.} \end{aligned}$$

Cangemi–Jackiw version of Callan–Giddings–Harvey–Strominger

- Choose Maxwell algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, Z \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

with bilinear form

$$\langle J,\, Z\rangle = -1 \qquad\qquad \langle P_a,\, P_b\rangle = \eta_{ab}$$

- corresponding action (after integrating our X^a and ω)

$$I[X,\, Y,\, g_{\mu\nu},\, A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- EOM

$$R = 0 \quad \Rightarrow \quad \text{Ricci-flat}$$
$$\epsilon^{\mu\nu}\partial_\mu A_\nu = 1$$
$$\nabla_\mu\nabla_\nu X - g_{\mu\nu}\nabla^2 X = g_{\mu\nu} Y$$
$$Y = \Lambda = \text{const.}$$

- translate our bc's into BF-formulation

Boundary conditions in BF formulation

- Ansatz (worked nicely for Jackiw–Teitelboim; inspired by 3d)

$$\mathcal{A} = b^{-1}\big(\,\mathrm{d}+a\big)b \qquad\qquad B = b^{-1}xb$$

with group element $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)P_+ + P_- + \mathcal{P}(u)J\big)\;\mathrm{d}u$$
$$x = x^+(u)P_+ + x_1(u)P_- + YJ + x_0(u)Z$$

where $\delta\mathcal{T} \neq 0 \neq \delta\mathcal{P}$

Boundary conditions in BF formulation

- Ansatz (worked nicely for Jackiw–Teitelboim; inspired by 3d)

$$\mathcal{A} = b^{-1}\big(\mathrm{d}+a\big)b \qquad\qquad B = b^{-1}xb$$

  with group element $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)P_+ + P_- + \mathcal{P}(u)J\big)\ \mathrm{d}u$$
$$x = x^+(u)P_+ + x_1(u)P_- + YJ + x_0(u)Z$$

  where $\delta\mathcal{T} \neq 0 \neq \delta\mathcal{P}$

- yields metric shown before, dilaton

$$X = x_1(u)\,r + x_0(u)$$

  and Maxwell field $A = r\ \mathrm{d}u$

  get BMS_2 asymptotic symmetries!

Boundary conditions in BF formulation

- Ansatz (worked nicely for Jackiw–Teitelboim; inspired by 3d)

$$\mathcal{A} = b^{-1}\big(\,\mathrm{d}+a\big)b \qquad\qquad B = b^{-1}xb$$

  with group element $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)P_+ + P_- + \mathcal{P}(u)J\big)\;\mathrm{d}u$$
$$x = x^+(u)P_+ + x_1(u)P_- + YJ + x_0(u)Z$$

  where $\delta\mathcal{T} \neq 0 \neq \delta\mathcal{P}$
- yields metric shown before, dilaton

$$X = x_1(u)\,r + x_0(u)$$

  and Maxwell field $A = r\;\mathrm{d}u$
- Maxwell field preserved by combined diffeos and gauge trafos

$$\delta A_\nu = \xi^\mu\partial_\mu A_\nu + A_\mu\partial_\nu\xi^\mu + \partial_\nu\sigma \qquad \xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

  provided $\eta = \sigma'$
  either η has no 0-mode or σ not single-valued (winding modes)

Boundary conditions in BF formulation

- Ansatz (worked nicely for Jackiw–Teitelboim; inspired by 3d)

$$\mathcal{A} = b^{-1}\big(\mathrm{d}+a\big)b \qquad\qquad B = b^{-1}xb$$

with group element $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)P_+ + P_- + \mathcal{P}(u)J\big)\;\mathrm{d}u$$
$$x = x^+(u)P_+ + x_1(u)P_- + YJ + x_0(u)Z$$

where $\delta\mathcal{T} \neq 0 \neq \delta\mathcal{P}$

- yields metric shown before, dilaton

$$X = x_1(u)\,r + x_0(u)$$

and Maxwell field $A = r\;\mathrm{d}u$

- Maxwell field preserved by combined diffeos and gauge trafos

$$\delta A_\nu = \xi^\mu\partial_\mu A_\nu + A_\mu\partial_\nu\xi^\mu + \partial_\nu\sigma \qquad \xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

provided $\eta = \sigma'$

- focus on case $\delta_\sigma \oint A = 0$ (no winding modes) $\Rightarrow$ warped Witt algebra

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18
- defining $1/x_1 \sim \partial_t f$ and $x_0/x_1 \sim \partial_t g$ result is

$$I_{\text{tw}}[h, \, g] = \kappa \int\limits_0^\beta \mathrm{d}\tau \Big(\mathcal{T} h'^{\,2} - g' \Big(i\mathcal{P} h' + \frac{h''}{h'} \Big) \Big)$$

with $\tau := f(t)$, $h(\tau) := -f^{-1}(\tau)$ and $\tau \sim \tau + \beta$ (prime means $\mathrm{d}/\,\mathrm{d}\tau$)

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18
- defining $1/x_1 \sim \partial_t f$ and $x_0/x_1 \sim \partial_t g$ result is

$$I_{\text{tw}}[h, \, g] = \kappa \int_0^\beta \mathrm{d}\tau \Big(\mathcal{T} h'^{\,2} - g' \Big(i\mathcal{P} h' + \frac{h''}{h'} \Big) \Big)$$

twisted warped action is flat space analogue of Schwarzian action!

- Schwarzian action: group action for Virasoro coadjoint orbits
- twisted warped action: group action for twisted warped coadjoint orbits

$$[L_n, \, L_m] = (n-m) \, L_{n+m}$$
$$[L_n, \, J_m] = -m \, J_{n+m} - i\kappa \big(n^2 - n \big) \, \delta_{n+m, \, 0}$$
$$[J_n, \, J_m] = 0$$

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18
- defining $1/x_1 \sim \partial_t f$ and $x_0/x_1 \sim \partial_t g$ result is

$$I_{\text{tw}}[h, \, g] = \kappa \int_0^\beta \mathrm{d}\tau \Big(\mathcal{T} h'^{\,2} - g' \Big(i\mathcal{P} h' + \frac{h''}{h'} \Big) \Big)$$

- asymptotic symmetries: h, g boundary scalars under diffeos and g phase under $u(1)$ trafos

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18
- defining $1/x_1 \sim \partial_t f$ and $x_0/x_1 \sim \partial_t g$ result is

$$I_{\text{tw}}[h, \, g] = \kappa \int\limits_0^\beta \mathrm{d}\tau \Big(\mathcal{T} h'^{\,2} - g' \Big(i\mathcal{P} h' + \frac{h''}{h'} \Big) \Big)$$

- asymptotic symmetries: h, g boundary scalars under diffeos and g phase under $u(1)$ trafos

Twisted warped action resembles effective action for complex SYK

Outline

Hamiltonian formulation

- twisted warped Hamiltonian action

$$I_{\text{tw}} = -\kappa \int\limits_0^\beta \mathrm{d}t \left(p_i \dot{q}_i - p_1 p_2 - e^{q_1} p_3\right) \qquad i = 1, 2, 3$$

where $q_3 = \exp(i\mathcal{P}h)$ and $q_2 = g + ih\mathcal{T}/\mathcal{P}$ (rest: auxiliary fields)

Hamiltonian formulation

- twisted warped Hamiltonian action

$$I_{\text{tw}} = -\kappa \int\limits_0^\beta \mathrm{d}t\, \big(p_i \dot q_i - p_1 p_2 - e^{q_1} p_3\big) \qquad i = 1,2,3$$

  where $q_3 = \exp(i\mathcal{P}h)$ and $q_2 = g + ih\mathcal{T}/\mathcal{P}$ (rest: auxiliary fields)
- similar to Schwarzian Hamiltonian action

$$I_{\text{sch}} = -\kappa \int\limits_0^\beta \mathrm{d}t\, \big(p_i \dot q_i - p_1^2 - e^{q_1} p_3\big)$$

Hamiltonian formulation

- twisted warped Hamiltonian action

$$I_{\text{tw}} = -\kappa \int\limits_0^\beta \mathrm{d}t \left(p_i \dot{q}_i - p_1 p_2 - e^{q_1} p_3\right) \qquad i = 1, 2, 3$$

where $q_3 = \exp(i\mathcal{P}h)$ and $q_2 = g + ih\mathcal{T}/\mathcal{P}$ (rest: auxiliary fields)

- similar to Schwarzian Hamiltonian action

$$I_{\text{sch}} = -\kappa \int\limits_0^\beta \mathrm{d}t \left(p_i \dot{q}_i - p_1^2 - e^{q_1} p_3\right)$$

- solutions

$$q_3 = h_0 + h_1 e^{i\tau/\tau_0} \qquad\qquad q_2 = g_0 - ig_1\,\tau + g_2\,e^{i\tau/\tau_0}$$

five integration constants g_0, g_1, g_2, h_0, h_1; periodicity $\tau_0 = \beta/(2\pi)$

Hamiltonian formulation

- twisted warped Hamiltonian action

$$I_{\text{tw}} = -\kappa \int\limits_0^\beta \mathrm{d}t \, \big(p_i \dot{q}_i - p_1 p_2 - e^{q_1} p_3\big) \qquad i = 1, 2, 3$$

  where $q_3 = \exp(i\mathcal{P}h)$ and $q_2 = g + ih\mathcal{T}/\mathcal{P}$ (rest: auxiliary fields)
- similar to Schwarzian Hamiltonian action

$$I_{\text{sch}} = -\kappa \int\limits_0^\beta \mathrm{d}t \, \big(p_i \dot{q}_i - p_1^2 - e^{q_1} p_3\big)$$

- solutions

$$q_3 = h_0 + h_1 e^{i\tau/\tau_0} \qquad\qquad q_2 = g_0 - ig_1 \, \tau + g_2 \, e^{i\tau/\tau_0}$$

  five integration constants g_0, g_1, g_2, h_0, h_1; periodicity $\tau_0 = \beta/(2\pi)$
- on-shell action $I_{\text{tw}}|_{\text{EOM}} = -2\pi\kappa g_1$

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

- Using result for dilaton $g_1 = X\big|_{\text{horizon}}$ yields

$$S = 2\pi\kappa\, X\big|_{\text{horizon}}$$

  standard result (Wald entropy applied to 2d dilaton gravity)

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

- Using result for dilaton $g_1 = X\big|_{\text{horizon}}$ yields

$$S = 2\pi\kappa\, X\big|_{\text{horizon}}$$

  standard result (Wald entropy applied to 2d dilaton gravity)
- assumption above derivable from regularity condition

$$\mathcal{P} = 2\pi T \qquad\qquad \mathcal{T} \text{ arbitrary}$$

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

- Using result for dilaton $g_1 = X\big|_{\text{horizon}}$ yields

$$S = 2\pi\kappa\, X\big|_{\text{horizon}}$$

  standard result (Wald entropy applied to 2d dilaton gravity)
- assumption above derivable from regularity condition

$$\mathcal{P} = 2\pi T \qquad\qquad \mathcal{T} \text{ arbitrary}$$

- on-shell $\mathcal{P} = Y$ is $u(1)$ charge, while $\mathcal{T}$ is mass

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

- Using result for dilaton $g_1 = X\big|_{\text{horizon}}$ yields

$$S = 2\pi\kappa\, X\big|_{\text{horizon}}$$

  standard result (Wald entropy applied to 2d dilaton gravity)
- assumption above derivable from regularity condition

$$\mathcal{P} = 2\pi T \qquad\qquad \mathcal{T} \text{ arbitrary}$$

- on-shell $\mathcal{P} = Y$ is $u(1)$ charge, while $\mathcal{T}$ is mass
- peculiarity: inverse specific vanishes (well-known property of CGHS)

$$C^{-1} = \frac{1}{T}\,\frac{\mathrm{d}T}{\mathrm{d}S}\Big|_{\delta\mathcal{P}=0} = 0$$

Thermodynamics

- Assuming g_1 independent from temperature get entropy

$$S = -I_{\text{tw}}\big|_{\text{EOM}} = 2\pi\kappa g_1$$

- Using result for dilaton $g_1 = X\big|_{\text{horizon}}$ yields

$$S = 2\pi\kappa\, X\big|_{\text{horizon}}$$

  standard result (Wald entropy applied to 2d dilaton gravity)
- assumption above derivable from regularity condition

$$\mathcal{P} = 2\pi T \qquad\qquad \mathcal{T} \text{ arbitrary}$$

- on-shell $\mathcal{P} = Y$ is $u(1)$ charge, while $\mathcal{T}$ is mass
- peculiarity: inverse specific vanishes (well-known property of CGHS)

$$C^{-1} = \frac{1}{T}\,\frac{\mathrm{d}T}{\mathrm{d}S}\bigg|_{\delta\mathcal{P}=0} = 0$$

- useful property for scaling limit from complex SYK

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\mathrm{cSYK}}[h,\, g] = \frac{NK}{2} \int\limits_0^\beta \mathrm{d}\tau \, \Big(g' + \frac{2\pi i\mathcal{E}}{\beta}\, h'\Big)^2 - \frac{N\gamma}{4\pi^2} \int\limits_0^\beta \mathrm{d}\tau \, \Big\{ \tan\Big(\frac{\pi}{\beta} h\Big);\, \tau \Big\}$$

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\mathrm{cSYK}}[h,\,g]=\frac{NK}{2}\int\limits_0^\beta \mathrm{d}\tau\,\Big(g'+\frac{2\pi i\mathcal{E}}{\beta}\,h'\Big)^2-\frac{N\gamma}{4\pi^2}\int\limits_0^\beta \mathrm{d}\tau\,\Big\{\tan\Big(\frac{\pi}{\beta}h\Big);\,\tau\Big\}$$

$\{f;\,\tau\}:=f'''/f'-\frac{3}{2}(f''/f')^2$ Schwarzian derivative
N (large) number of complex fermions
$N\gamma$ specific heat at fixed charge
K zero temperature compressibility
$\mathcal{E}$ spectral asymmetry parameter
$h(\tau)$ time-reparametrization field (quasi-periodic, $h(\tau+\beta)=h(\tau)+\beta$)
$g(\tau)$ phase field (periodic in absence of winding)

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\text{cSYK}}[h,\, g] = \frac{NK}{2} \int_0^\beta \mathrm{d}\tau \, \Big(g' + \frac{2\pi i\mathcal{E}}{\beta}\, h'\Big)^2 - \frac{N\gamma}{4\pi^2} \int_0^\beta \mathrm{d}\tau \, \Big\{ \tan\Big(\frac{\pi}{\beta} h\Big);\, \tau \Big\}$$

  $\{f;\, \tau\} := f'''/f' - \frac{3}{2}(f''/f')^2$ Schwarzian derivative
  N (large) number of complex fermions
  $N\gamma$ specific heat at fixed charge
  K zero temperature compressibility
  $\mathcal{E}$ spectral asymmetry parameter
  $h(\tau)$ time-reparametrization field (quasi-periodic, $h(\tau+\beta) = h(\tau) + \beta$)
  $g(\tau)$ phase field (periodic in absence of winding)
- according to our thermodynamics need limit $N\gamma \to \infty$ (infinite specific heat)

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\text{cSYK}}[h,\, g] = \frac{NK}{2}\int_0^\beta \mathrm{d}\tau\,\Big(g' + \frac{2\pi i\mathcal{E}}{\beta}\,h'\Big)^2 - \frac{N\gamma}{4\pi^2}\int_0^\beta \mathrm{d}\tau\,\Big\{\tan\Big(\frac{\pi}{\beta}h\Big);\,\tau\Big\}$$

- according to our thermodynamics need limit $N\gamma\to\infty$ (infinite specific heat)
- turns out additionally need limit $K\to 0$ (vanishing zero temperature compressibility)

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\text{cSYK}}[h,\,g] = \frac{NK}{2}\int\limits_0^\beta \mathrm{d}\tau\,\big(g' + \frac{2\pi i\mathcal{E}}{\beta}\,h'\big)^2 - \frac{N\gamma}{4\pi^2}\int\limits_0^\beta \mathrm{d}\tau\,\Big\{\tan\Big(\frac{\pi}{\beta}h\Big);\,\tau\Big\}$$

- according to our thermodynamics need limit $N\gamma \to \infty$ (infinite specific heat)
- turns out additionally need limit $K \to 0$ (vanishing zero temperature compressibility)
- inserting these limits into $I_{\text{cSYK}}[h,\,g]$ yields twisted warped action

$$\lim_{N\gamma\to\infty, K\to 0} I_{\text{cSYK}}[h,\,g] = I_{\text{tw}}[h,\,g] = \kappa\int\limits_0^\beta \mathrm{d}\tau\Big(\mathcal{T}h'^2 - g'\Big(i\mathcal{P}h' + \frac{h''}{h'}\Big)\Big)$$

with $\kappa^2 \sim N^a\gamma K$ kept finite

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras
- twisted warped Virasoro algebra (warped Witt with all cocycles)

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right)\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

c: Virasoro central charge; κ: twist; $\hat{K}$: level of $u(1)$ current

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras
- twisted warped Virasoro algebra (warped Witt with all cocycles)

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right)\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

  c: Virasoro central charge; κ: twist; $\hat{K}$: level of $u(1)$ current
- complex SYK: warped Virasoro algebra ($c \neq 0 \neq \hat{K}$; $\kappa = 0$)

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras
- twisted warped Virasoro algebra (warped Witt with all cocycles)

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right)\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

  c: Virasoro central charge; κ: twist; $\hat{K}$: level of $u(1)$ current
- complex SYK: warped Virasoro algebra ($c \neq 0 \neq \hat{K}$; $\kappa = 0$)
- our model: twisted warped algebra ($c = 0 = \hat{K}$; $\kappa \neq 0$)

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras
- twisted warped Virasoro algebra (warped Witt with all cocycles)

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right)\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

  c: Virasoro central charge; κ: twist; $\hat{K}$: level of $u(1)$ current
- complex SYK: warped Virasoro algebra ($c \neq 0 \neq \hat{K}$; $\kappa = 0$)
- our model: twisted warped algebra ($c = 0 = \hat{K}$; $\kappa \neq 0$)
- map first between twisted warped Virasoro and warped Virasoro

$$c \to c - \frac{24\kappa^2}{\hat{K}}$$

  by change of basis $L_n \to L_n + i\frac{2\kappa}{\hat{K}}\, n\, J_n$ and shift of 0-modes

Asymptotic symmetries

- can see same singular limit at level of asymptotic symmetry algebras
- twisted warped Virasoro algebra (warped Witt with all cocycles)

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right)\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

c: Virasoro central charge; κ: twist; $\hat{K}$: level of $u(1)$ current

- complex SYK: warped Virasoro algebra ($c \neq 0 \neq \hat{K}$; $\kappa = 0$)
- our model: twisted warped algebra ($c = 0 = \hat{K}$; $\kappa \neq 0$)
- map first between twisted warped Virasoro and warped Virasoro

$$c \to c - \frac{24\kappa^2}{\hat{K}}$$

by change of basis $L_n \to L_n + i\frac{2\kappa}{\hat{K}}\, n\, J_n$ and shift of 0-modes

- then take limit $\hat{K} \to 0$, $c \to \infty$ keeping fixed $\kappa = \sqrt{-\frac{c\hat{K}}{24}}$

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\, Y,\, g_{\mu\nu},\, A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\,Y,\,g_{\mu\nu},\,A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- asymptotically flat boundary conditions yield BMS_2

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\,Y,\,g_{\mu\nu},\,A_\mu]=\frac{\kappa}{2}\int_{\mathcal{M}}\mathrm{d}^2x\sqrt{|g|}\,\big(XR-2Y+2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- asymptotically flat boundary conditions yield BMS_2
- boundary action is twisted warped action

$$I_{\text{tw}}[h,\,g]=\kappa\int\limits_0^\beta\mathrm{d}\tau\Big(\mathcal{T}h'^{\,2}-g'\Big(i\mathcal{P}h'+\frac{h''}{h'}\Big)\Big)$$

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\,Y,\,g_{\mu\nu},\,A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}}\mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- asymptotically flat boundary conditions yield BMS_2
- boundary action is twisted warped action

$$I_{\mathrm{tw}}[h,\,g] = \kappa\int\limits_0^\beta\mathrm{d}\tau\Big(\mathcal{T}h'^{\,2} - g'\Big(i\mathcal{P}h' + \frac{h''}{h'}\Big)\Big)$$

- follows as singular limit from complex SYK
  (large specific heat, small compressibility)

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\,Y,\,g_{\mu\nu},\,A_\mu]=\frac{\kappa}{2}\int_{\mathcal{M}}\mathrm{d}^2x\sqrt{|g|}\,\big(XR-2Y+2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- asymptotically flat boundary conditions yield BMS_2
- boundary action is twisted warped action

$$I_{\mathrm{tw}}[h,\,g]=\kappa\int\limits_0^\beta\mathrm{d}\tau\Big(\mathcal{T}h'^2-g'\Big(i\mathcal{P}h'+\frac{h''}{h'}\Big)\Big)$$

- follows as singular limit from complex SYK (large specific heat, small compressibility)
- asymptotic symmetries also from singular limit of warped Virasoro

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\,Y,\,g_{\mu\nu},\,A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

- asymptotically flat boundary conditions yield BMS_2
- boundary action is twisted warped action

$$I_{\text{tw}}[h,\,g] = \kappa\int_0^\beta \mathrm{d}\tau\Big(\mathcal{T}h'^{\,2} - g'\Big(i\mathcal{P}h' + \frac{h''}{h'}\Big)\Big)$$

- follows as singular limit from complex SYK (large specific heat, small compressibility)
- asymptotic symmetries also from singular limit of warped Virasoro
- could be useful toy model for flat space holography

Outline

Motivation

Kinematics

Dynamics

Relation to SYK/JT

Outlook

Future developments

- Flat space holography in 2d

Future developments

- Flat space holography in 2d
- Cardyology

Future developments

- Flat space holography in 2d
- Cardyology
- Chaos bound saturation

Future developments

- Flat space holography in 2d
- Cardyology
- Chaos bound saturation
- Experimental realization

Future developments

- Flat space holography in 2d
- Cardyology
- Chaos bound saturation
- Experimental realization

Thanks for your attention!