Bulk observables in JT gravity

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Based on arXiv:1902.11194 with A. Blommaert and H. Verschelde
arXiv:1903.10485
arXiv:2005.13058 with A. Blommaert and H. Verschelde
WIP with J. Engelsöy

Outline

Jackiw-Teitelboim gravity

Jackiw-Teitelboim (JT) 2d dilaton gravity

$$S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\Phi(R+2) + \frac{1}{8\pi G}\int d\tau\sqrt{-\gamma}\Phi_{bdy}K$$

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Motivation:

- Dimensional reduction (s-wave) of 3d pure $\Lambda < 0$ gravity
- Appears as near-horizon theory of near-extremal higher-dimensional black holes
- Describes low-energy sector of all (known) SYK-like models
- Solvable including coupling to bulk matter fields
- Understanding of role of higher genus (non-pt) contributions

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Here: Discuss bulk QG physics

Path integrate over $\Phi \Rightarrow R = -2$:
Geometry fixed as AdS_2: $ds^2 = \frac{-dF^2+dZ^2}{Z^2}, \quad Z \geq 0$
Poincaré patch (frame) of AdS_2, boundary at $Z = 0$

Important frames in AdS_2 (1)

Lightcone coordinates $U = F + Z$ and $V = F - Z$
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- BH frame: $U(u) = \tanh\left(\frac{\pi}{\beta}u\right)$, $V(v) = \tanh\left(\frac{\pi}{\beta}v\right)$
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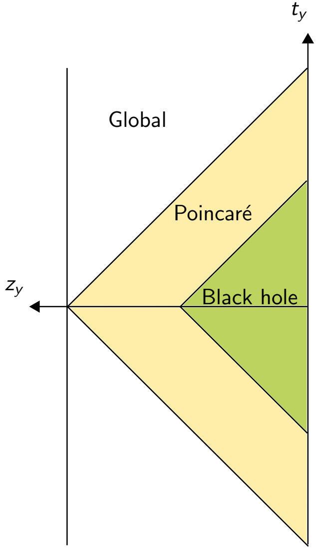
Important frames in AdS_2 (2)

Penrose diagram



Jackiw-Teitelboim gravity and the Schwarzian

Path-integrate out Φ:

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$$\boxed{\Rightarrow S = -C \int d\tau \{F, \tau\}}, \quad C = \frac{a}{16\pi G}, \ \left\{F, \tau\right\} = \frac{F'''}{F'} - \frac{3}{2}\left(\frac{F''}{F'}\right)^2$$

Almheiri-Polchinski '15, Jensen '16, Maldacena-Stanford-Yang '16, Engelsöy-TM-Verlinde '16

$F(\tau) =$ time reparametrization

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Compare to CS / WZW topological duality

Semi-classical regime: $C \to \infty \equiv G, \hbar \to 0$

Note: C has dimension length $\to$ quantum effects important in IR

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$\mathcal{M} = \mathrm{Diff}(S^1)/SL(2, \mathbb{R})$, $\qquad f(\tau + \beta) = f(\tau) + \beta, \; f' \geq 0$

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Q: What are the natural operators to consider?

Boundary two-point function

Take massive scalar field in bulk, asymptotic expansion (AdS_2/CFT_1):

$$\phi(Z,F) \quad \to Z^{1-\Delta}\tilde{\phi}_b(F) = \epsilon^{1-\Delta}F'^{1-\Delta}\tilde{\phi}_b(F(\tau)) = \epsilon^{1-\Delta}\phi_b(\tau)$$

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Generating functional:

$$I \sim \int dF_1 \int dF_2 \frac{1}{(F_1-F_2)^{2\Delta}}\tilde{\phi}_b(F_1)\tilde{\phi}_b(F_2)$$

$$= \int d\tau_1 \int d\tau_2 \frac{F'(\tau_1)^{\Delta}F'(\tau_2)^{\Delta}}{(F(\tau_1)-F(\tau_2))^{2\Delta}}\phi_b(\tau_1)\phi_b(\tau_2)$$

Bilocal operator:

$$\boxed{\mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{F'(\tau_1)F'(\tau_2)}{(F(\tau_1)-F(\tau_2))^2}\right)^\ell \equiv \left(\frac{f'(\tau_1)f'(\tau_2)}{\frac{\beta}{\pi}\sin^2\frac{\pi}{\beta}[f(\tau_1)-f(\tau_2)]}\right)^\ell}$$

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Other origin of this operator:

- Boundary-anchored Wilson line Blommaert-TM-Verschelde '18, Iliesiu-Pufu-Verlinde-Wang '19

Approaches to JT correlators: an overview

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Result for $\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle_\beta$:

$$\boxed{\frac{1}{Z} \int dE_2 e^{-\beta E_2} \rho_0(E_2) \int dE_1 \rho_0(E_1) \, e^{-\tau_{12}(E_1 - E_2)} \, \frac{\Gamma\left(\ell \pm i\sqrt{E_1} \pm i\sqrt{E_2}\right)}{\Gamma(2\ell)}}$$

$Z=$ Schwarzian disk partition function, $\rho_0(E) = \frac{1}{2\pi^2} \sinh 2\pi\sqrt{E}$

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Fixed energy E_2 (microcanonical) answer by stripping off the Laplace E_2-integral

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Goal of this talk: compute bulk observables

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In QG defining bulk observable (= diff-invariant operator) requires care. E.g. scalar field $\phi(x)$, but what is x here ?

Need to specify bulk location in a geometrically invariant way
Holography $\rightarrow$ preferably boundary-intrinsic way

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Radar definition of bulk point:
$U = F(u)$, $V = F(v)$ Blommaert-TM-Verschelde '19

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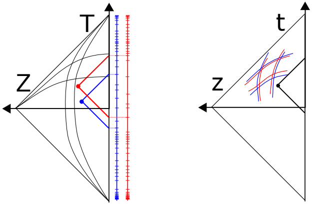

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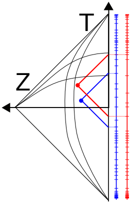

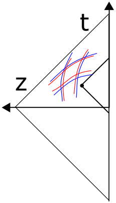

Observables $\mathcal{O}(F(u), F(v)) \rightarrow$ Contribution in correlator from implicit dependence on geometry F through this construction
Visible in e.g. commutator computations Donnelly-Giddings '15

Application: bulk matter two-point function (1)

Couple JT gravity to a bulk matter action, take massless scalar for simplicity:
$\frac{1}{2}\int d^2x\sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + S_{\text{JT}}[g,\Phi]$

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Matter two-point function in a fixed frame F:

$G_{bb}(x,x') = \langle\phi_1\phi_2\rangle_{\text{CFT}} = \ln\left|\frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(v)-F(u'))(F(u)-F(v'))}\right|$

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Integrate over frames: $\langle G_{bb}(x,x')\rangle = \int [\mathcal{D}F]\, G_{bb}(x,x')e^{-S[F]}$

Two-step process:

1. Integrate over matter to get a gravitational operator
2. Integrate over gravity with this operator insertion

Application: bulk matter two-point function (2)

Trick:

$$\ln\left|\frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(v)-F(u'))(F(u)-F(v'))}\right| = \int_v^u dt \int_{v'}^{u'} dt' \frac{F'(t)F'(t')}{(F(t)-F(t'))^2}$$

Coincides with HKLL prescription to map bulk operators into boundary observables Hamilton-Kabat-Lifschytz-Lowe '05

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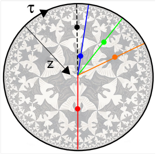

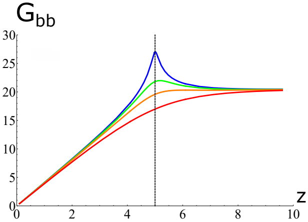

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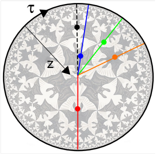

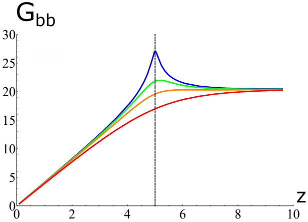

- Singularities only at lightlike separated points
- Log-divergences close to the lightcones

Generalizations: CFT primaries and massive fields

Generalization to matter CFT primaries:

$$G_{h,\bar{h}}(u,u',v,v') = \left(\frac{F'(u)F'(u')}{(F(u)-F(u'))^2}\right)^h \left(\frac{F'(v)F'(v')}{(F(v)-F(v'))^2}\right)^{\bar{h}} - (u' \leftrightarrow v')$$

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$$G_{h,\bar{h}}(u,u',v,v') = \left(\frac{F'(u)F'(u')}{(F(u)-F(u'))^2}\right)^h \left(\frac{F'(v)F'(v')}{(F(v)-F(v'))^2}\right)^{\bar{h}} - (u' \leftrightarrow v')$$

Generalization to massive bulk fields :

$G(x,x') \sim \frac{1}{\sigma^\Delta}\, {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \frac{2\Delta+1}{2}; \frac{1}{\sigma^2}\right)$ with invariant distance function $\sigma = 1 - 2\frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(u)-F(v))(F(u')-F(v'))}$ and $m^2 = \Delta(\Delta-1)$

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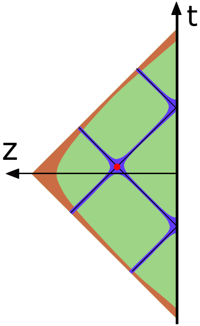
Generic picture:



blue: UV singularities

red: IR region where strong QG fluctuations appear

Bulk locality

Local operators: should commute for spacelike separation

$[\phi(t_1, z_1), \phi(t_2, z_2)] = 0, \qquad (t_1, z_1)$ and (t_2, z_2) spacelike

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See also Lin-Maldacena-Zhao '19 for other construction of diff-invariant operators that turn out to be local

$\Rightarrow$ JT gravity is more local than generically expected in QG (as in e.g. Donnelly-Giddings '15)

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Near-horizon region is IR region, similar to late time regime $\Rightarrow$ strong quantum gravitational effects, deviations from Rindler correlators

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Reservations:

- for this specific model $\leftrightarrow$ universality JT
- for these specific bulk operators $\rightarrow$ (less natural) bulk operators (presumably) exist that do not have this property

Unruh heat bath: bulk detector (1)

Now: spectral content of the bulk two-point function
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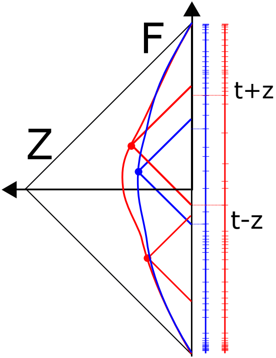
Define detector trajectory of Unruh-DeWitt detector operationally:



- Use radar definition to define entire trajectory $(Z(t), F(t))$ of detector worldline
- Along worldline, introduce interaction $H_{\text{int}} = \mu(t)\phi(\mathbf{x}(t))$ coupling the bulk quantum field ϕ to a detector QM system μ Unruh '79, DeWitt '80

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Transition probability for detector to go from ground state $|0_{\text{det}}\rangle$ to $|\omega_{\text{det}}\rangle$, without any information on the excitation of the QFT matter state:

$$P(\omega) = \sum_{\phi_{\text{QFT}}} \left| \langle \omega_{\text{det}}, \phi_{\text{QFT}} | - i \int_{-\infty}^{+\infty} dt H_{\text{int}}(t) \, |0_{\text{det}}, 0_F \rangle \right|^2$$

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Transition rate:

$$R(\omega) \equiv \lim_{T\to+\infty} P(\omega)/T =$$

$$|\langle\omega| \, \mu(0) \, |0\rangle_{\text{det}}|^2 \lim_{T\to+\infty} \frac{1}{T} \int_{-T}^{+T} dt dt' e^{-i\omega(t-t')} \langle \phi(\mathbf{x}(t))\phi(\mathbf{x}(t'))\rangle_{\text{CFT}}$$

in terms of CFT bulk matter two-point function

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in terms of CFT bulk matter two-point function

Strategy: insert in Schwarzian path integral and Fourier transform
For simplicity, consider the microcanonical ensemble for a fixed energy M black hole state

Unruh heat bath: bulk detector (3)

Answer:

$$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\sinh 2\pi\sqrt{M-\omega}}{2\pi^2}\,\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})\,\Theta(M-\omega)$$

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Interpretation:

- $2\left(\frac{\sin z\omega}{\omega}\right)^2$ is interference factor from the image charges across the AdS_2 boundary
- $\frac{\sinh 2\pi\sqrt{M-\omega}}{2\pi^2}\,\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})\,\Theta(M-\omega)$ is the matter emission probability
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In semi-classical regime $M \gg 1$, $M \gg \omega$, we approximate:
$R(\omega) \approx 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\omega}{e^{\beta\omega}-1}$ in terms of the Bose-Einstein (Planckian) black body spectrum

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Including higher topology in gravity is important:

Typically strongly suppressed by $\sim e^{-(2g-1)S_0}$, coming from the EH term $S_0 \frac{1}{4\pi} \int d^2x \sqrt{g} R$ with S_0 the extremal entropy

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Higher genus expansion is asymptotic, requires non-perturbative completion
$\Rightarrow$ For JT gravity, a double-scaled random matrix integral completes the genus expansion Saad-Shenker-Stanford '19

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$$\rho_0(E_1)\rho_0(E_2) \;\to\; \rho_{JT}(E_1, E_2), \qquad \rho_0(E) = \frac{e^{S_0}}{2\pi^2}\sinh 2\pi\sqrt{E}$$

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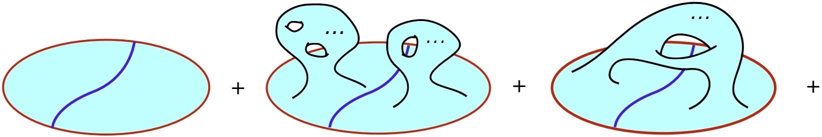
Important features:

- level repulsion: $\rho(E_1, E_2) \overset{E_1 \approx E_2}{\approx} (E_1 - E_2)^2 + \ldots$
- high-frequency wiggles: spacing $\sim e^{-S_0}$

Higher topology (3)

Geometrically:



Interpretation:

First two diagrams: disk topology + disconnected higher topology on each side of the line: $\rho(E_1)\rho(E_2)$

Last diagram: connected higher topology across the line: $\rho_{\text{conn}}(E_1, E_2)$

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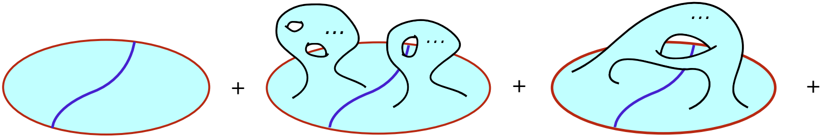

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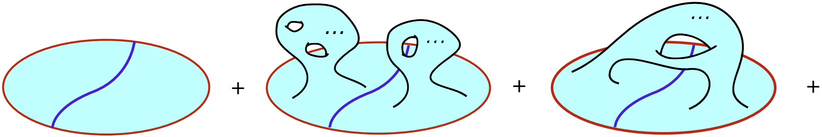

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We obtain for the detector response rate:

$$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\rho(M, M-\omega)}{\rho(M)} \Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})$$

Higher topology (4)

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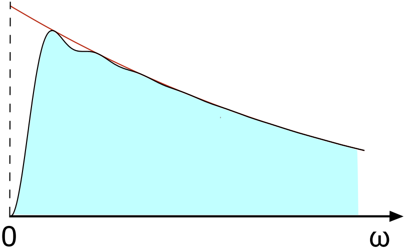

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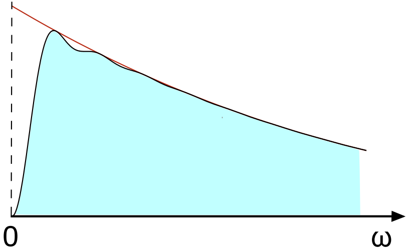

Interpretation as product of probabilities:

- Probability of black hole system containing two levels spaced by ω, $\sim \frac{\rho(M,M-\omega)}{\rho(M)\rho(M-\omega)}$
- Probability of matter emission from such a system $\sim \rho(M-\omega)\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})$
- AdS_2 interference factor

Energy flux densities:

As coincident limit of two-point function

$\langle : T_{uu}(u) : \rangle_{\text{CFT}} = \lim_{u' \to u} \langle : \partial_u \phi(u) \partial_u \phi(u') : \rangle_{\text{CFT}}$

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Unruh bath energy

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In Schwarzian path integral:

$\Rightarrow \langle : T_{uu}(u) : \rangle_{\beta} = \langle : T_{vv}(v) : \rangle_{\beta} = c \frac{\pi}{12\beta^2} + c \frac{1}{16\pi\beta C}$

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Total bath energy $E_{\text{bath}} = \int_0^{+\infty} du \, \langle : T_{uu} : \rangle + \langle : T_{vv} : \rangle$
defined operationally by summing local energy densities defined through radar definition

Unruh spectral energy density (1)

From two-point function, we can extract the energy occupation number $\omega N_\omega[f] \equiv \langle 0_F | \omega a_\omega^\dagger a_\omega | 0_F \rangle$ by Fourier transforming from the two-point function to the oscillators:

$$-\frac{1}{8\pi^2}\int du_1 \int du_2 e^{-i\omega(u_1-u_2)}\left[\frac{F'(u_1)F'(u_2)}{(F_1-F_2+i\epsilon)^2}-\left(\frac{1}{u_{12}+i\epsilon}\right)^2\right]+(\epsilon\to-\epsilon)$$

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Technical remark:

The above formula corresponds to writing

$N_\omega = \frac{1}{2}(a_\omega^\dagger a_\omega + a_\omega a_\omega^\dagger) - \frac{1}{2}\delta(0)$

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Trivial manipulation in QFT on curved spacetime, but not after coupling to quantum gravity

We need this form to match with the bulk bath energy

Unruh spectral energy density (2)

Insert this expression in Schwarzian path integral (only disk)

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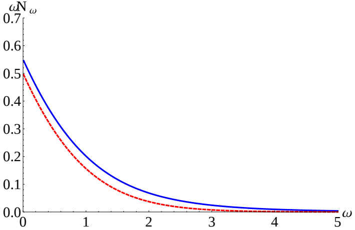
$\Rightarrow$ Canonical ensemble result:



red: Planckian black-body spectrum in 1+1d with $\beta = 4C$
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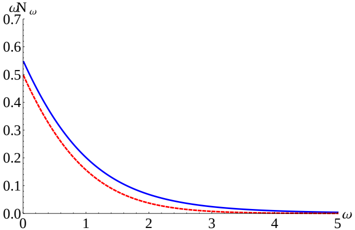

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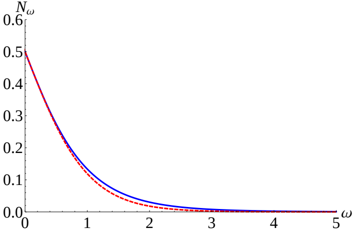
$\rightarrow$ Slightly higher population

Check: $\int_0^{+\infty} d\omega\, \omega \left\langle N_\omega \right\rangle_\beta = \int_0^{+\infty} du \left\langle :T_{uu}: \right\rangle_\beta + \left\langle :T_{vv}: \right\rangle_\beta$

Unruh spectral energy density: fermions

Generalization to bulk massless Majorana fermion field TM '19:



red: Fermi-Dirac spectrum in 1+1d with $\beta = 4C$
blue: Exact result

Interpretation: low-energy spectrum has competition between gravity and Pauli-exclusion preventing any major modifications to these highly occupied levels

Extensions: charged and SUSY systems

Charged black hole: additional U(1) free boson action

$$S = -C \int_0^\beta dt \left\{ \tan \frac{\pi}{\beta} f(t), t \right\} - \frac{K}{2} \int_0^\beta dt \left(\Lambda'(t) - i\mu f'(t) \right)^2 ,$$

Bulk charged matter field also has to be dressed by Wilson line to make it small U(1) gauge invariant

Bilocal correlators of this action already determined in TM-Turiaci '19

Extensions: charged and SUSY systems

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$\tilde{\tau} = f(\tau + \theta\eta(\tau)), \qquad \tilde{\theta} = \sqrt{\partial_\tau f} \left(\theta + \eta(\tau) + \frac{1}{2}\theta\eta(\tau)\partial_\tau\eta(\tau) \right)$

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$\rightarrow$ Operational definition of bulk point in superspace

Given two such times τ, θ and τ', θ', we fully fix the small Sdiff gauge symmetry: $\Phi(\tilde{z}, \tilde{\theta}, \bar{\tilde{z}}, \bar{\tilde{\theta}})$ in terms of τ_i, θ_i

$ds^2 = \frac{1}{(\tilde{z} - \bar{\tilde{z}} - \tilde{\theta}\bar{\tilde{\theta}})^2} |d\tilde{z} + \tilde{\theta} d\tilde{\theta}|^2 = \frac{(D\tilde{\theta})^2(\bar{D}\bar{\tilde{\theta}})^2}{(\tilde{z} - \bar{\tilde{z}} - \tilde{\theta}\bar{\tilde{\theta}})^2} |dz + \theta d\theta|^2$

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$\Rightarrow$ Bulk correlators in terms of super-Schwarzian correlators

$\Rightarrow$ Differs from bosonic JT in the quantum corrections only (since classical bulk solution has no fermions) $\rightarrow$ super-Unruh effect

Unruh spectral energy density: beyond the disk (1)

Going beyond the disk, we choose to work microcanonically and refer our energy density w.r.t. the $M = 0$ energy density

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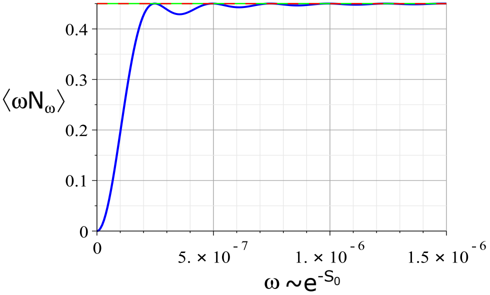
Results in level repulsion and high-frequency wiggles:



Green: semi-classical result, Red: Schwarzian result, Blue: full result

$M = 2\,(= 1/C)$, $S_0 = 10$

Conclusion

Jackiw-Teitelboim gravity is toy model of quantum gravity, striking the ideal balance between relevance and solvability

- Relevance: low-energy sector of all SYK-type models
  Most basic non-trivial holographic 2d gravity model
  Universal in near-extremal near-horizon regimes
- Solvability: gravitational dofs reduce to boundary time reparametrizations F, with explicit analytic solution for correlators, non-perturbatively in G_N. Explicit understanding of higher topology and resulting random matrix effects

Computed bulk two-point functions (strongly dependent on the definition of our bulk observables) that exhibit:

- Bulk microcausality
- Gravitational corrections to the Unruh heat bath and detector response, with level repulsion at ultra-low emission energies

JT gravity is ideal test case to study conceptual questions about quantum gravity

Thank you!