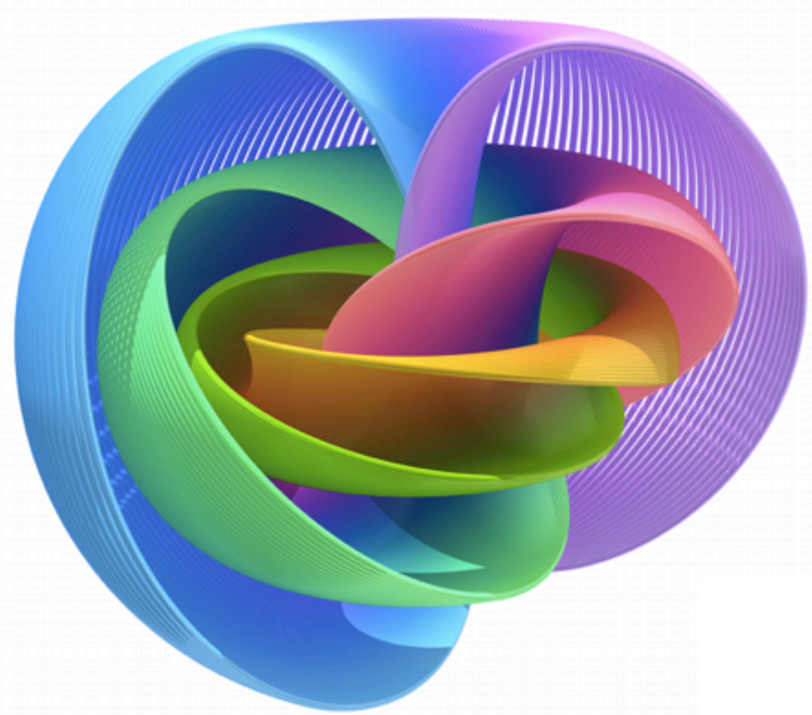
Bulk-edge dualities in topological matter

M. Asorey



CENTRO DE ASTROPARTÍCULAS
Y FISICA DE ALTAS ENERGÍAS

Departamento de
Física Teórica
Universidad Zaragoza

CENTRO DE CIENCIAS
DE BENASQUE
PEDRO PASCUAL

QM ^{3}- ITS - LISBOA

BULK-EDGE DUALITIES

**Den FestKörper hat Gott geschaffen,
die Oberfläche der Teufel**

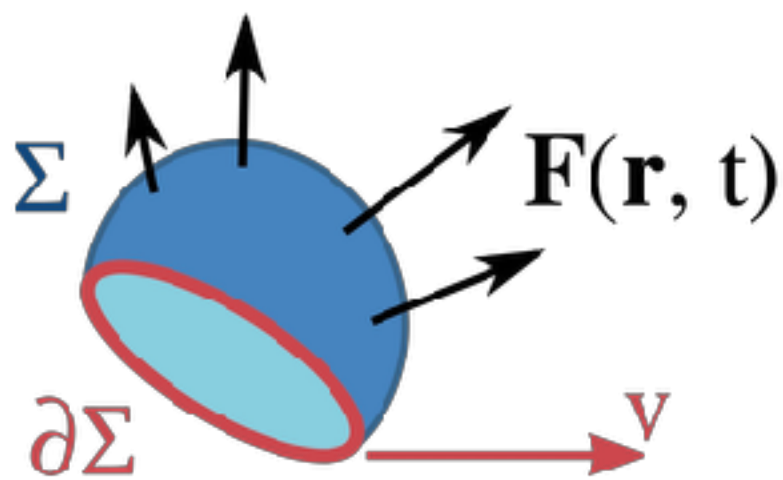
BULK-EDGE DUALITIES

God created the volumes, the Devil the boundaries

BULK-EDGE DUALITIES

Gauss-Ostrogradski theorem

$$\iint_{\Sigma} \vec{F} \cdot d\vec{\sigma} = \iiint_{V} \vec{\nabla} \cdot \vec{F}$$



Cauchy’s theorem

Green's theorem

Stokes’ theorem

BULK-EDGE DUALITIES

Casimir Force

Vacuum EM bulk Fluctuations

$$V(r) = \frac{\alpha}{d^7}$$

Relativistic van der Waals boundary forces

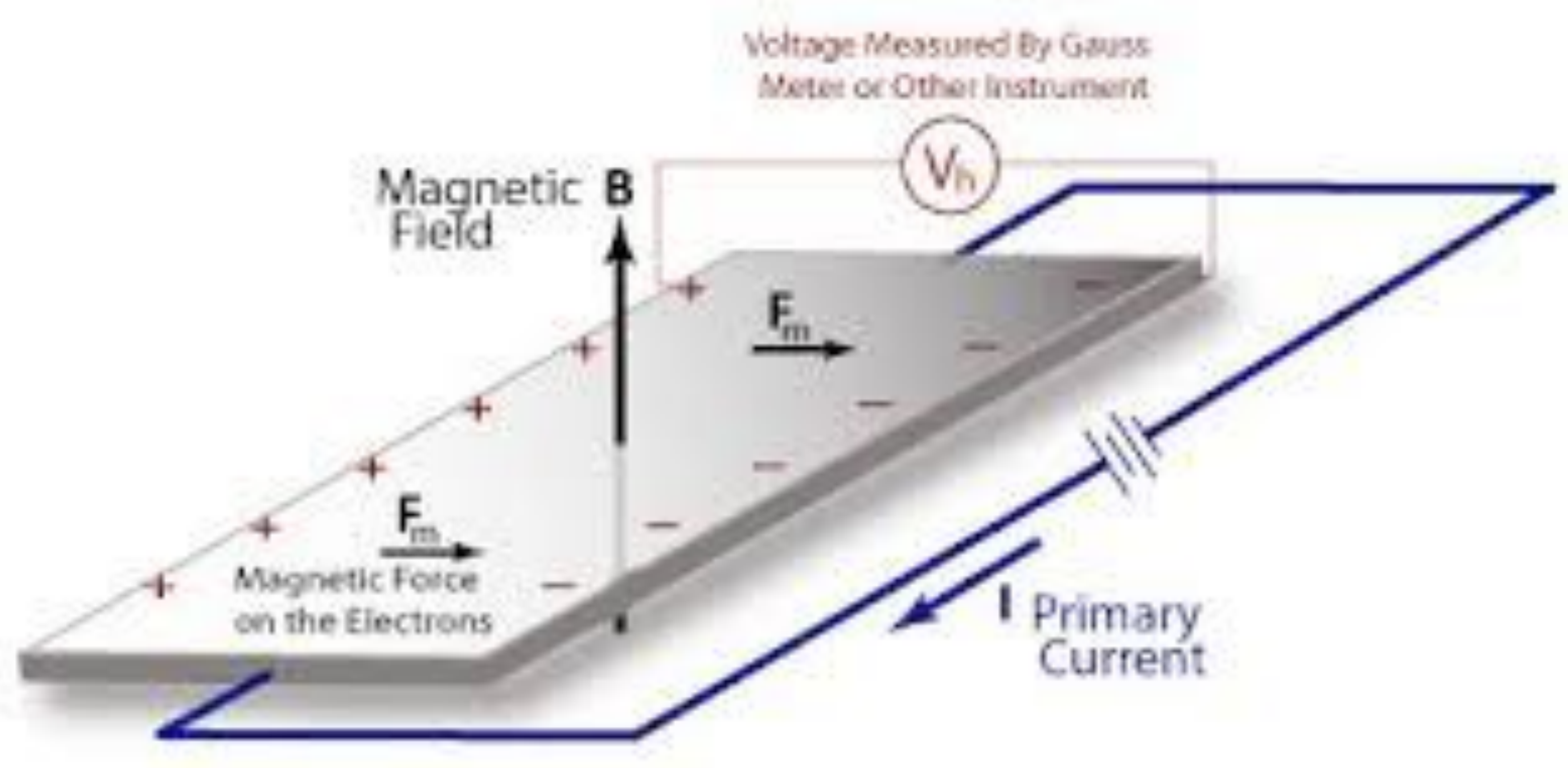
BULK-EDGE DUALITIES
TOPOLOGICAL MATTER

Quantum Hall Effect [1980]

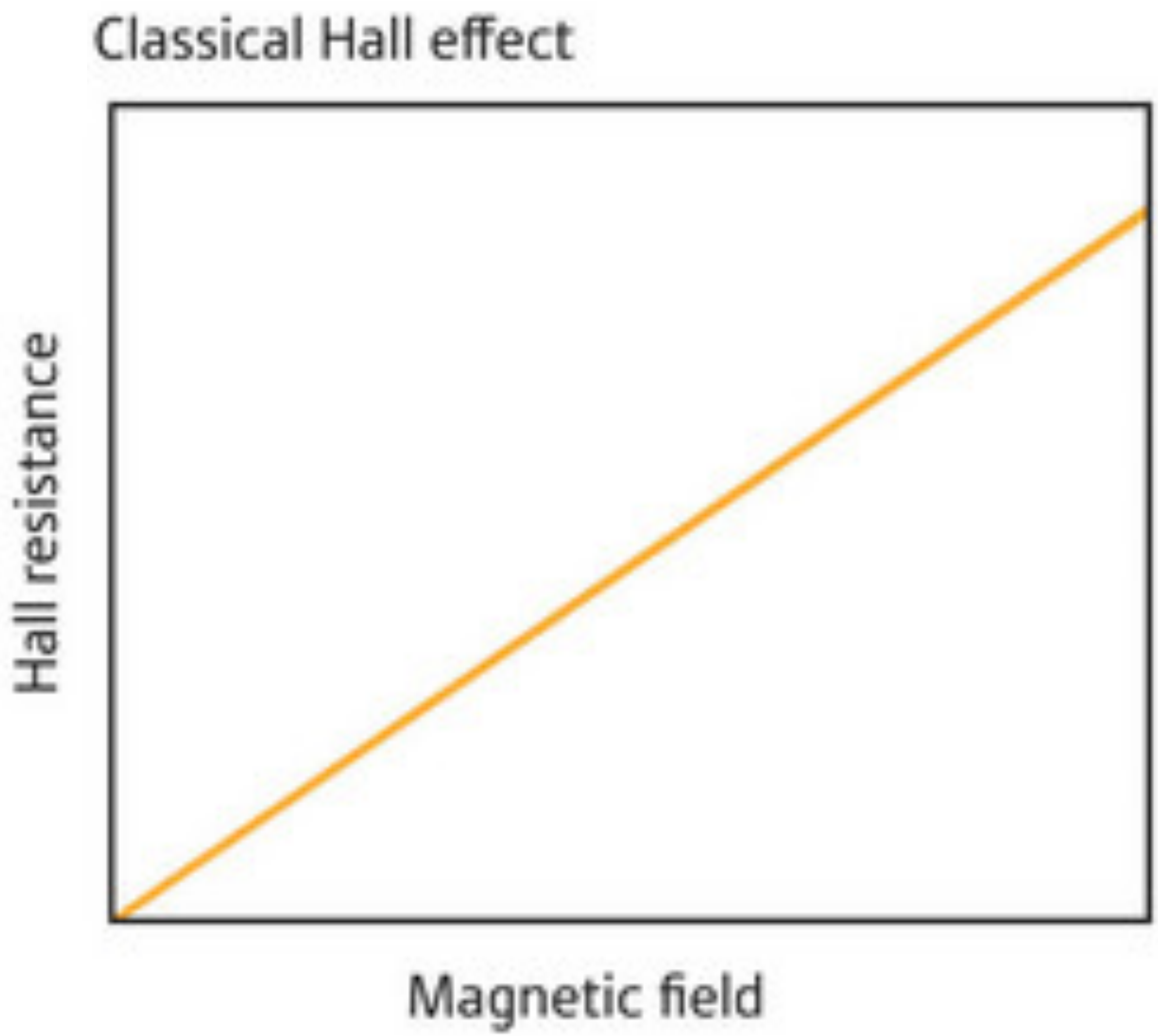
Topological Insulators [2005]

Weyl Semimetals [2015]

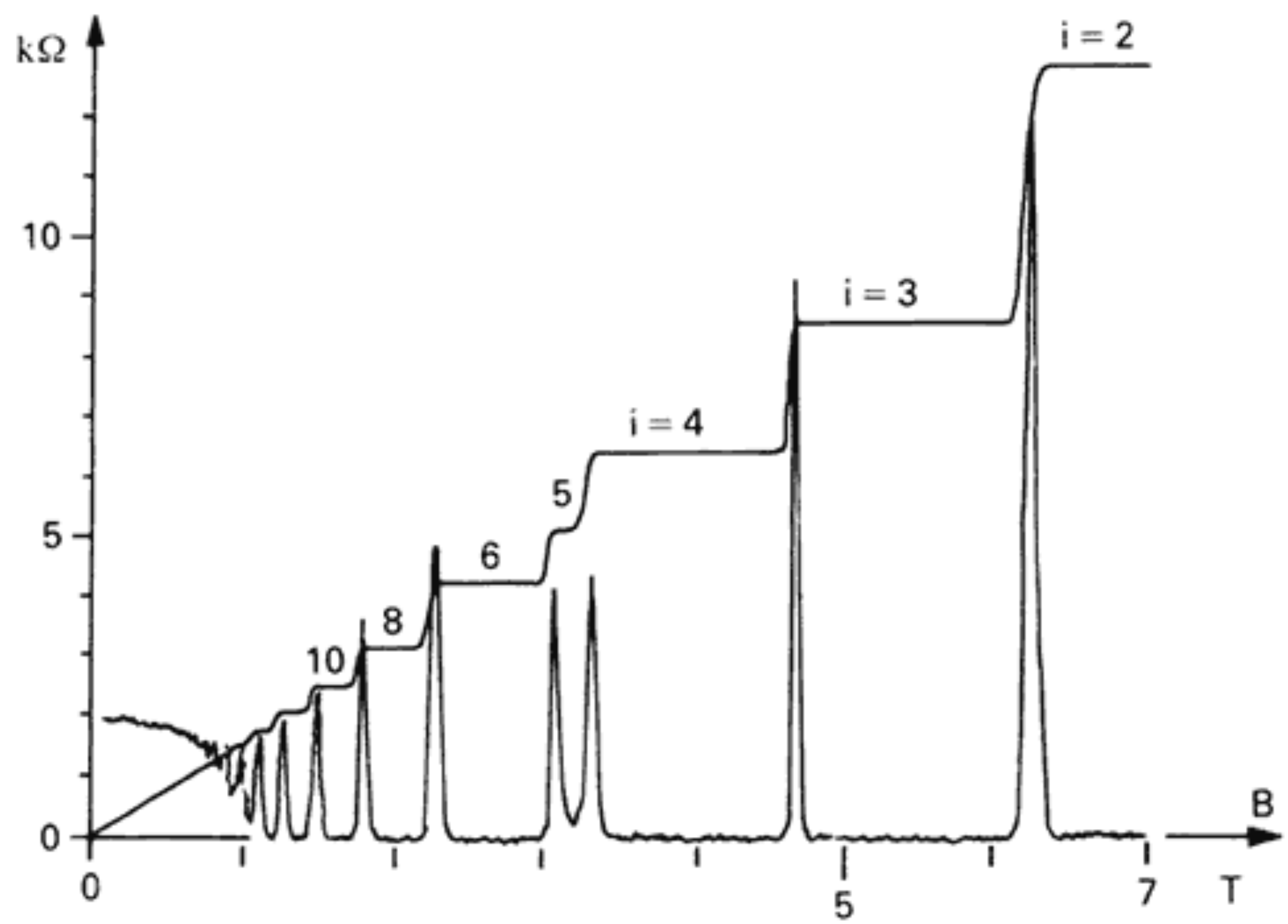
Hall effect


Voltage Measured By Gauss
Meter or Other Instrument
V_h
Magnetic Field
B
F_m
F_m
Magnetic Force
on the Electrons
I Primary
Current

Magnetoresistence: Hall effect



Integer Quantum Hall effect



[von Klitzing]

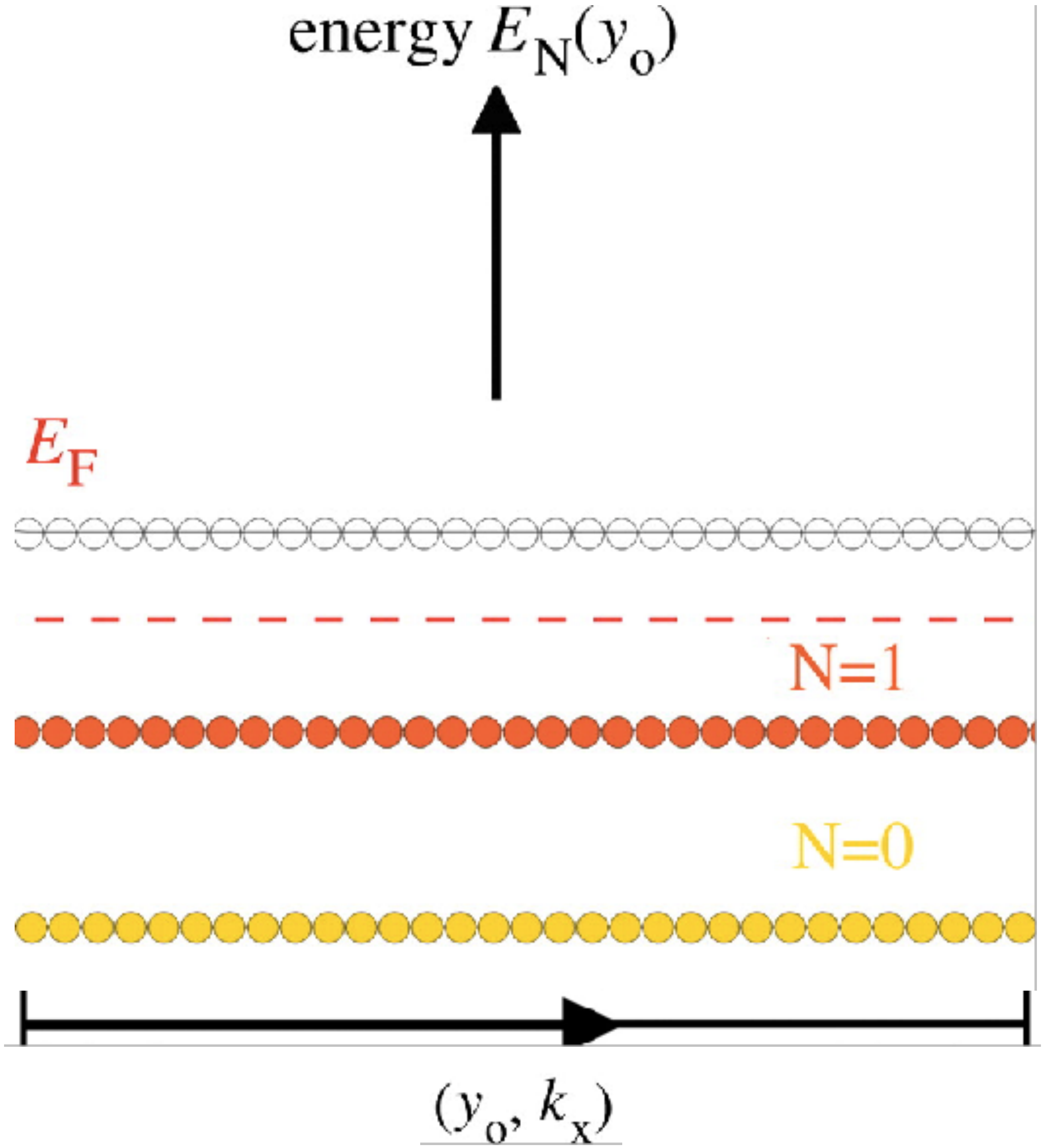
Quantum Hall effect

$$\mathbb{H} = -\frac{1}{2m}\left[\partial_2^2 + \left(\partial_1 + ieBx_2\right)^2\right]$$

$$E_n = \frac{eB}{2m}\left(n + \frac{1}{2}\right)$$

$$\psi_{n,k_1}(x) = e^{ik_1x_1}\,H_n(eBx_2)\,e^{-eBx_2^2/2}$$

Integer Quantum Hall effect



Quantum Hall effect and Fiber bundles

Floquet-Bloch

$$L^2(\mathbb{R}^2) = \oplus_{\lambda \in \widehat{\mathbb{T}}^2} L^2(\mathbb{T}^2)$$

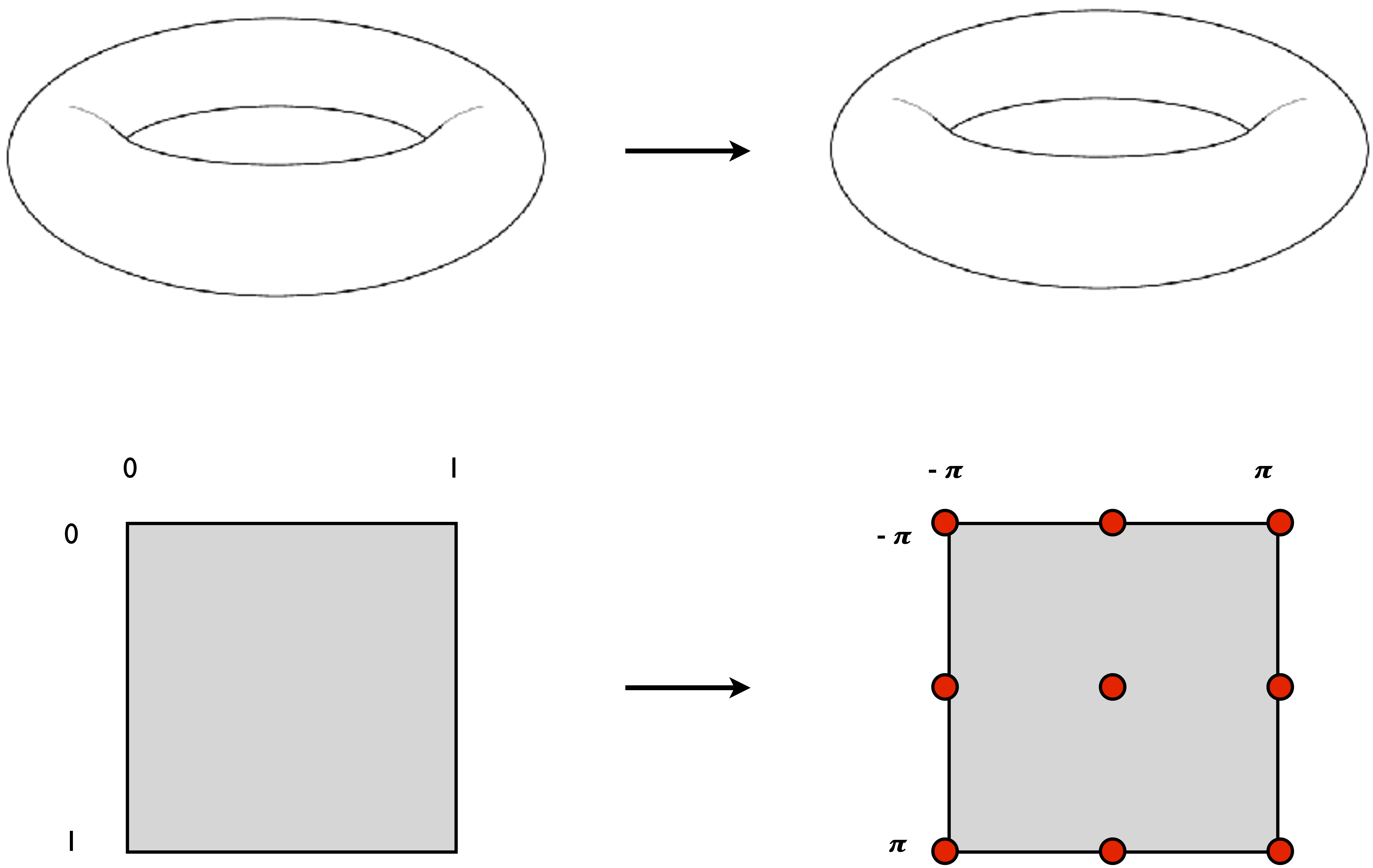
The space of states adquires a bundle structure

$$L^2(\mathbb{R}^2)\left(\widehat{\mathbb{T}}^2, L^2(\mathbb{T}^2)\right)$$

Spectral Floquet-Bloch theorem

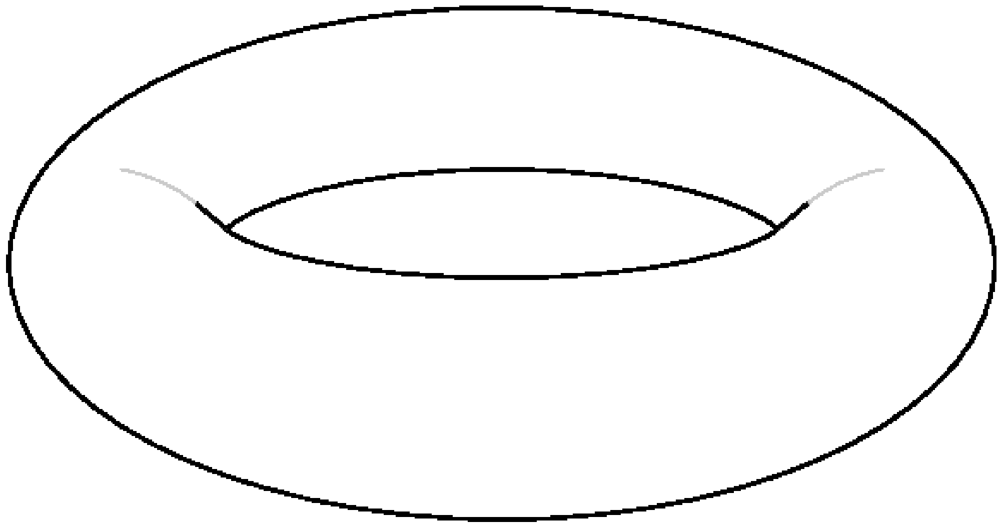
$$\mathbb{H}_{\mathbb{R}^2} = \oplus_{\lambda \in \widehat{\mathbb{T}}^2} \mathbb{H}^{\lambda}_{\mathbb{T}^2}$$

Real Space Brillouin Zone



Kramers points

Hall Effect in 2D Torus $\mathbb{T}$



$$\frac{e}{2\pi}\int_T \mathbf{F} = k \in \mathbb{Z} \qquad k = eB/2\pi$$

In complex coordinates:

$$\psi(\phi_1 + 2\pi, \phi_2) = e^{i\frac{k}{2}\phi_2}\psi(\phi_1, \phi_2)$$

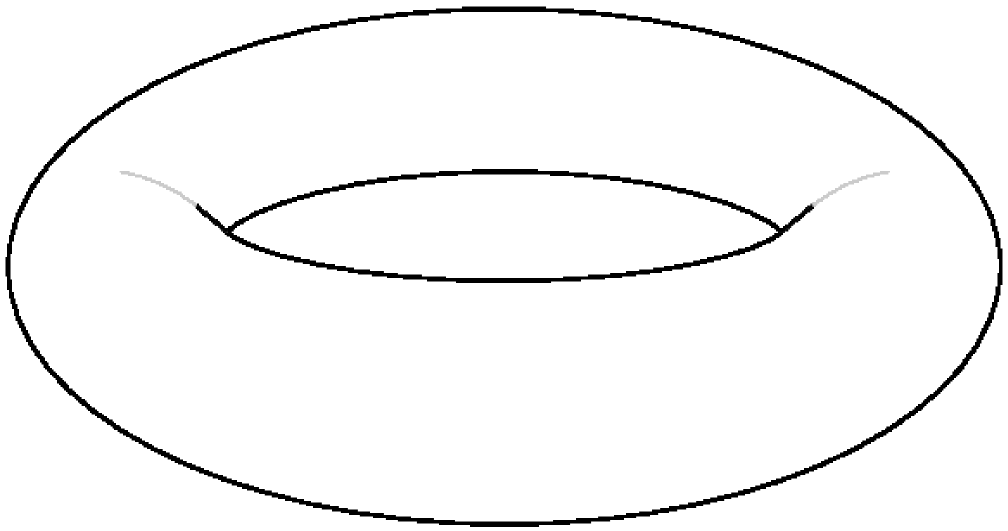
$$\psi(\phi_1, \phi_2 + 2\pi) = e^{-i\frac{k}{2}\phi_2}\psi(\phi_1, \phi_2)$$

$$\mathbb{H} = \frac{1}{2m}\left[\left(\partial_1 + i\frac{B}{2}(\phi_2 + \epsilon_2)\right)^2 + \left(\partial_2 + i\frac{B}{2}(-\phi_1 - \epsilon_1)\right)^2\right]$$

Energy levels (degeneracy: $|k|$) **[Landau]**

$$E_n = \frac{2\pi|k|}{m}\left(n + \frac{1}{2}\right)$$

Hall Effect in 2D Torus $\mathbb{T}$



$$\frac{e}{2\pi}\int_T \mathbf{F} = k \in \mathbb{Z} \qquad k = eB/2\pi$$

Ground State Eigenfunctions (degeneracy: $|k|$) :

Holomorphic sections of $E(T^2, \mathbb{C})$

$$\psi_0^l(\epsilon,\phi) = \frac{\mathrm{e}^{i\frac{k}{4\pi}(\phi_1+2\epsilon_1)\phi_2}}{(8\pi^4 k)^{\frac{1}{4}}} \sum_{m=-\infty}^{\infty} \mathrm{e}^{im\left(\phi_1+\epsilon_1+2\pi\frac{l}{k}\right)-\frac{1}{4\pi k}(2\pi m+k\phi_2+k\epsilon_2)^2}$$

$$l \;=\; 0, 1, 2\ldots, |k| - 1.$$

Hall effect and Dirac operator

$$\not{D}_A = i\sigma_1(\partial_1 + ieA_1) + i\sigma_2(\partial_2 + ieA_2)$$

$$\not{D}_A^2 = -(\partial_1 + ieA_1)^2 - (\partial_2 + ieA_2)^2 + e\sigma_3 B$$

$$H = -D_A^2 = \not{D}_A^2 - eB\sigma_3$$

$$E_n = eB(2n+1) \quad (n = 1, 2, \ldots)$$ 2ν degeneracy

$$E_0 = eB$$ ν degeneracy

Index Theorem: $$\nu = \frac{1}{2\pi}\int_{T^2} F = 2\pi B$$

TKKN and the Hidden topology

The states with energies below the Fermi level define a vector bundle over the Brillouin zone torus.

$$E\left(\widehat{\mathbb{T}}^2, \mathbb{C}^N)\right)$$

In this bundle there are gauge fields defined by the Berry phases of the different states

$$\mathcal{A}_{j\,n}^{l,l'}(\epsilon) = -i \int_{\mathbb{T}^2} \psi_n^{l\,*} \partial_{\epsilon_j} \psi_n^{l'}$$

$$\mathcal{F}_n^{l,l'}(\epsilon) = \partial_{\epsilon_1} \mathcal{A}_{2\,n}^{l,l'} - \partial_{\epsilon_2} \mathcal{A}_{1\,n}^{l,l'} + [\mathcal{A}_{1\,n}, \mathcal{A}_{2\,n}]^{l,l'}$$

TKKN and the Bloch bundle

First Chern class of Bloch bundle

$$C_1 = -\frac{i}{4\pi} \sum_{n=0}^{\nu} \sum_{l=0}^{|k|-1} \int_{\widehat{\mathbb{T}}^2} \mathcal{F}_n^{l,l} = \nu$$

TKKN formula:

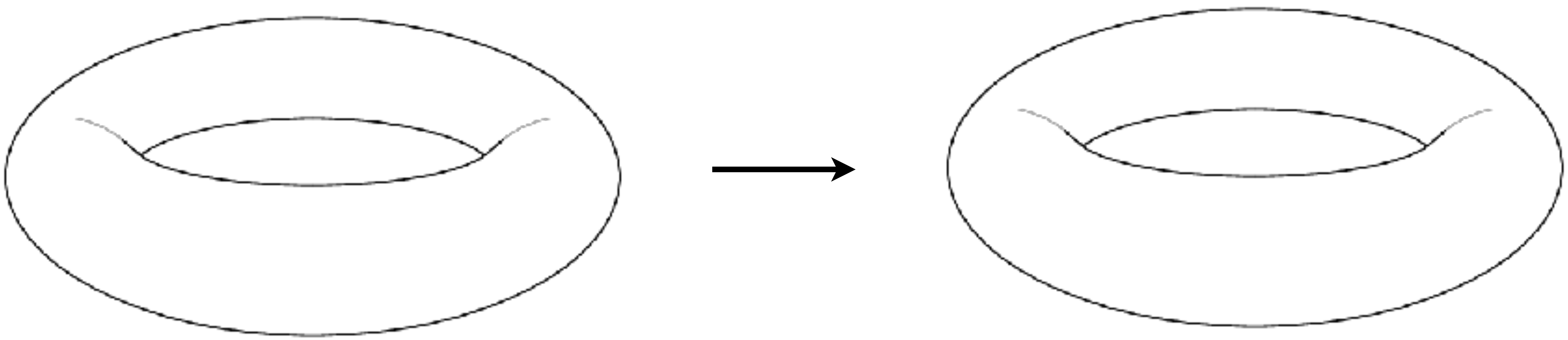
$$\sigma_{xy} = \frac{e^2}{2\pi}\nu$$

Quantization of Hall conductivity

Fourier-Mukai transform

Real Space

Brillouin Zone



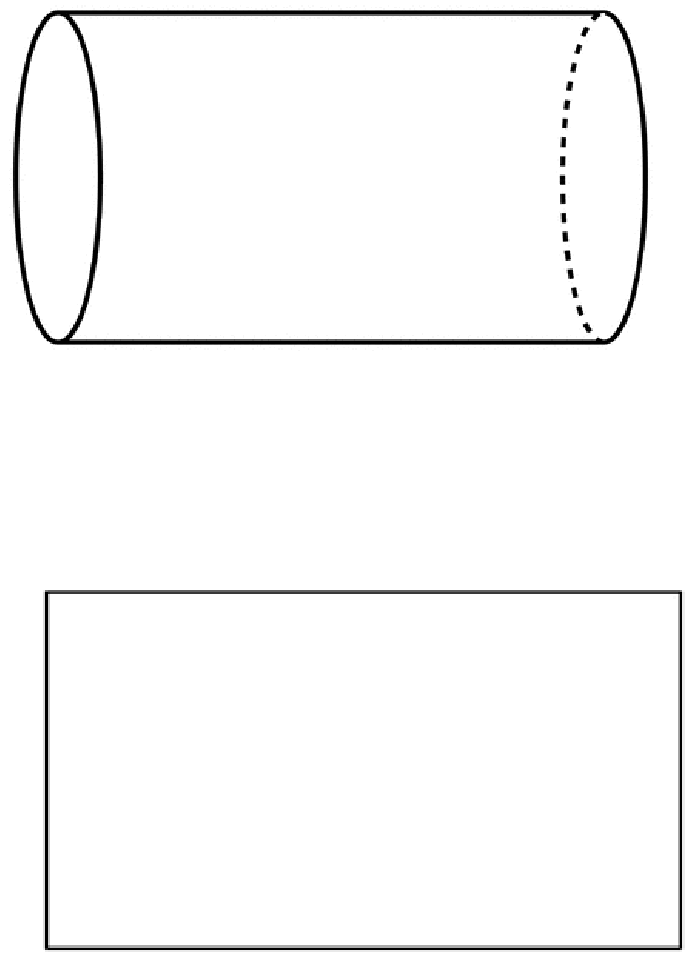
U(N) gauge field A_μ
with c_1 (A)=k

U(k) gauge field A_μ
with c_1 (A)=N

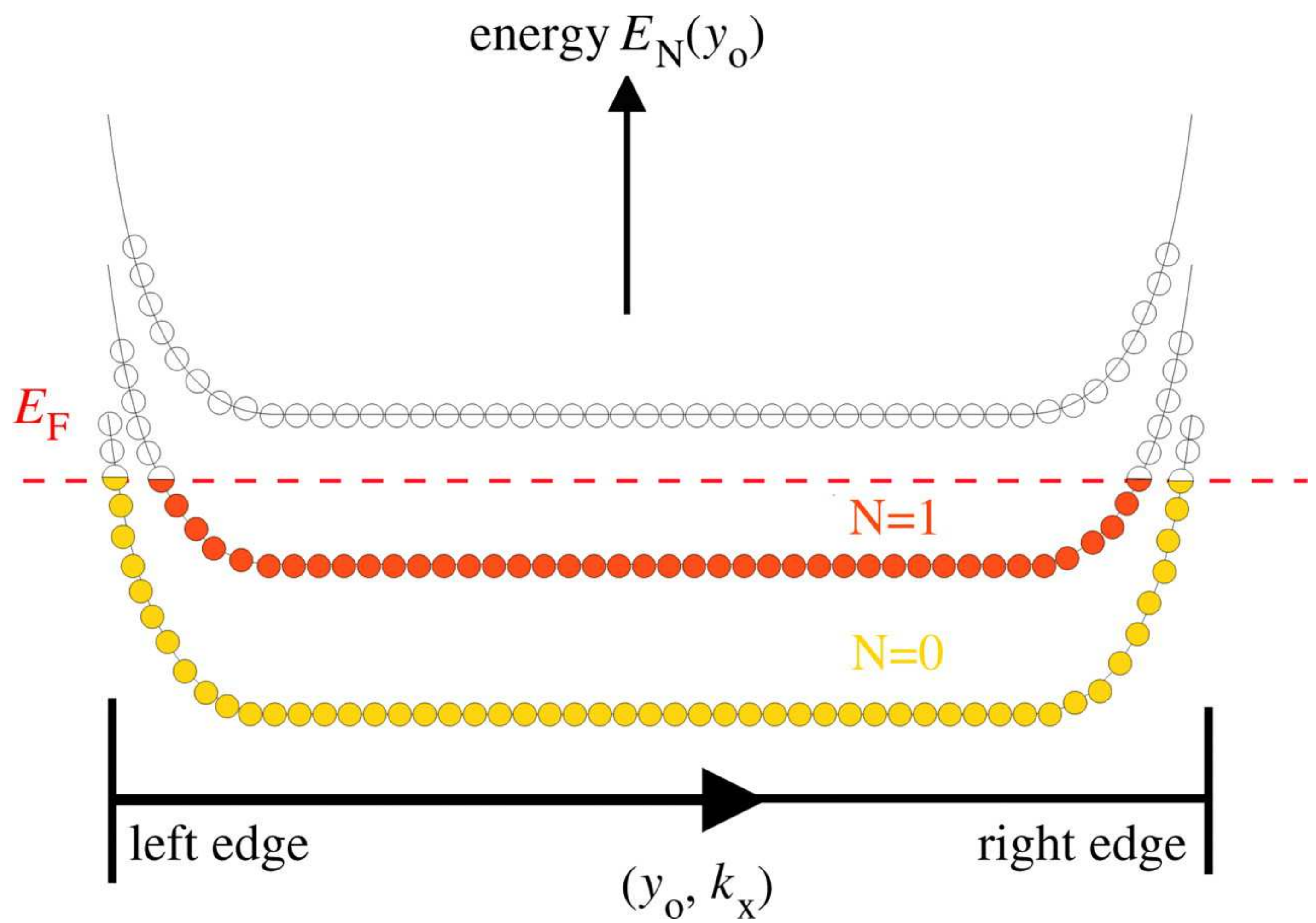
Nahm transform

[M.A. , Nature Phys.]

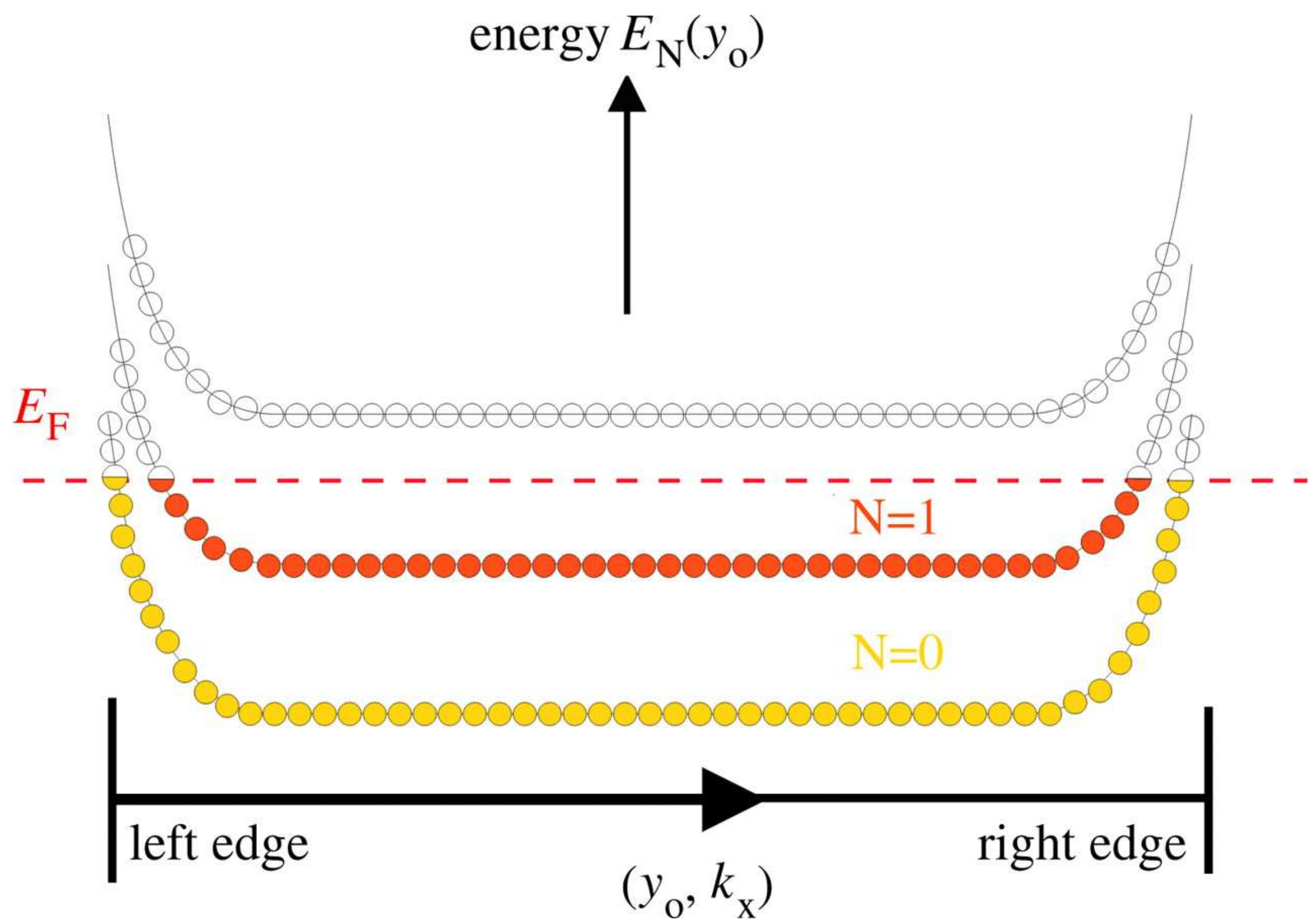
Hall effect with boundaries



Finite size effects



Finite size effects



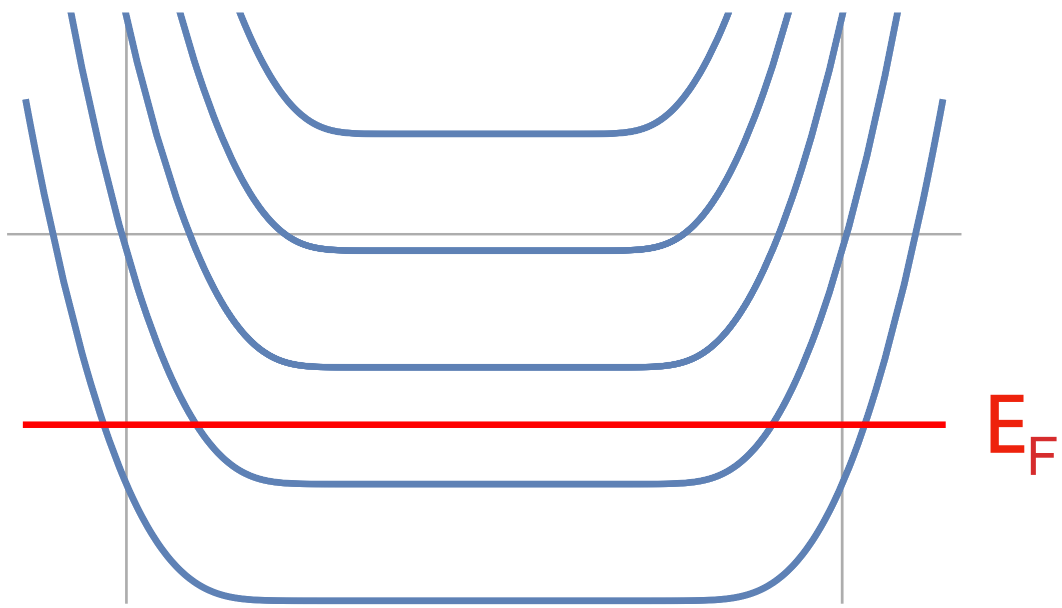
$$\sigma_{xy} = \frac{e^2}{2\pi}\nu$$

[Halperin, Laughlin]

Finite size Hall effects

$$\not{D}_A^2 = -(\partial_1 + ieA_1)^2 - (\partial_2 + ieA_2)^2 + e\sigma_3 B$$

Chiral Boundary conditions: Dirichlet

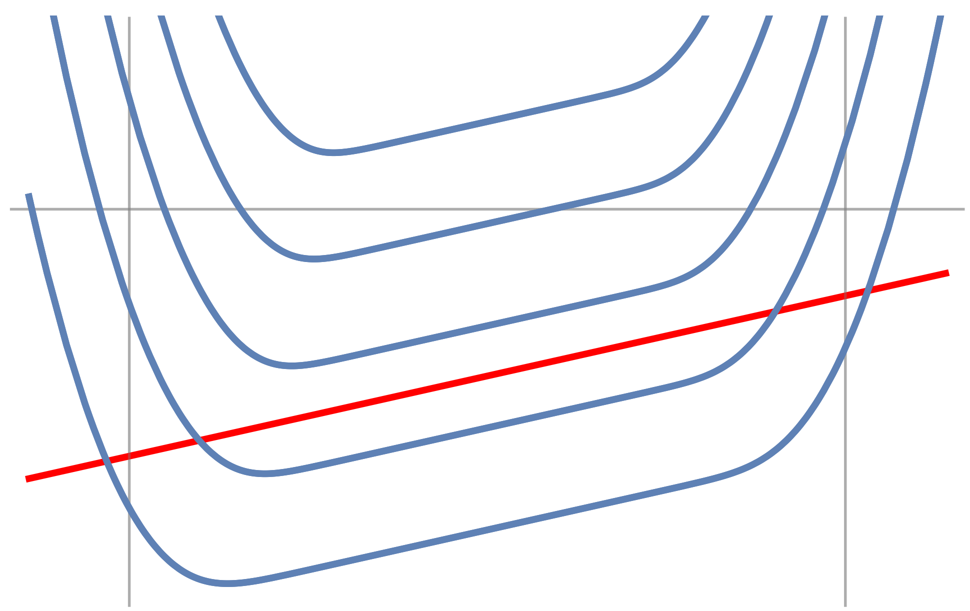


Non-chiral Edge states

[M.A.-Balachandran-Pérez-Pardo, JHEP]

External Electric Field E

$$\not{D}_A^2 + eEx_1 = -(\partial_1 + ieA_1)^2 - (\partial_2 + ieA_2)^2 + e\sigma_3 B + eEx_1$$

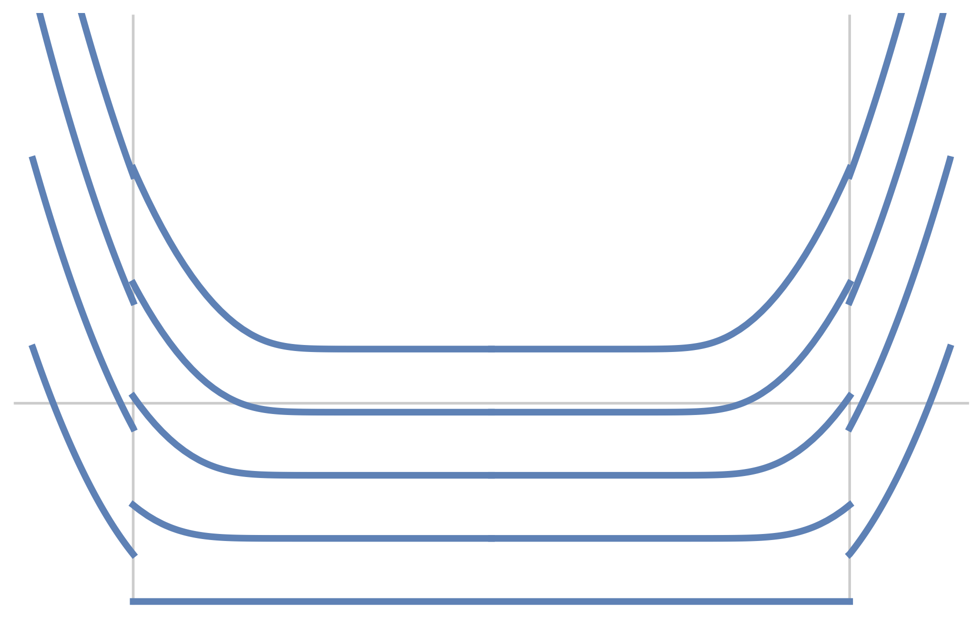


$$\sigma_{xy} = \frac{I_\varphi}{2\pi E} = \sum_{n=0}^{\nu} \int_{\epsilon_n^-}^{\epsilon_n^+} d\epsilon_1 \frac{\partial_{\epsilon_1} E_n}{2\pi E} = \frac{1}{2\pi E} \sum_{n=0}^{\nu} \frac{E_n(\epsilon_n^+) - E_n(\epsilon_n^-)}{2\pi E} = \frac{e^2}{2\pi} \boldsymbol{\nu}$$

[M.A. , Class. Quant. Phys.]

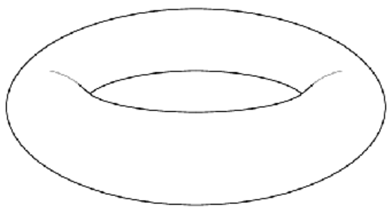
Finite size Hall effects

Atiyah-Patodi-Singer Boundary conditions



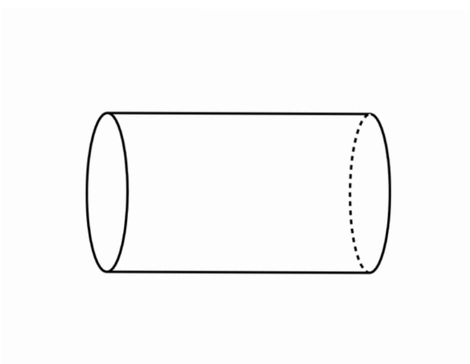
$$\sigma_{xy} \neq \frac{e^2}{2\pi}\nu$$

Atiyah-Patodi-Singer theorem



$$\nu_+ - \nu_- = \frac{1}{2\pi}\int_{T^2} F \in \mathbb{Z}$$

Edge - Bulk



$$\nu_+ - \nu_- = \frac{1}{2\pi}\int_C F - \frac{1}{2}\eta(A)$$

$$\eta(A) = 2\int_{\partial C} A - 2\,\mathrm{Int}\left[\int_{\partial C} A\right]$$

spectral asymmetry

[M.A., J.L. Lopez]

Edge states and Bulk-Edge correspondence

Chern-class on the cylinder C_1 is not any more an integer but

$$[C_1] = \nu_+ - \nu_- = \nu$$

is an integer quantum number

Edge states are chiral, due to the TR violation introduced by the magnetic field

Can edge states survive without magnetic field?

Yes adding SPIN and having spin-orbit couplings

TOPOLOGICAL INSULATORS

TOPOLOGICAL INSULATORS

Kane-Mele Hamiltonian on a torus

$$H_{\mathrm{KM}} = -i\sigma_1(\partial_{\varphi_1} + i\epsilon_1) - i\sigma_2(\partial_{\varphi_2} + i\epsilon_2) + m\sigma_3\tau_3$$

Time reversal symmetry

$$\mathbf{T} = i(\sigma_2 \otimes \tau_2)\mathcal{K},$$

$$[\mathbf{T}, \mathbb{H}] = 0$$

$$\mathbf{T}^2 = -\mathbb{I} \otimes \mathbb{I}$$

$$\mathbf{T}\epsilon_1 \to -\epsilon_1$$

TOPOLOGICAL INSULATORS

Kramers theorem:

All energy levels are double-degenerate

$$\mathbf{T}\psi = \lambda\psi, \qquad \mathbf{T}^2\psi = |\lambda|^2\psi = -\psi$$

Fu-Kane $\mathbb{Z}_2$ bulk topological invariant

Truncating the Hilbert space to filled states energy levels $-E_m < E < E_m$

$$\Theta_{mn} =< \psi_m|\mathbf{T}|\psi_n >$$

$$(-1)^\nu = \frac{Pf(\Theta)}{\det\Theta} = \pm 1$$

TOPOLOGICAL INSULATORS

Kane-Mele Hamiltonian on a cylinder

$$H_{\rm KM} = -i\sigma_1(\partial_\varphi + i\epsilon) - i\sigma_2\partial_x + m\sigma_3\tau_3$$

Special chiral boundary conditions

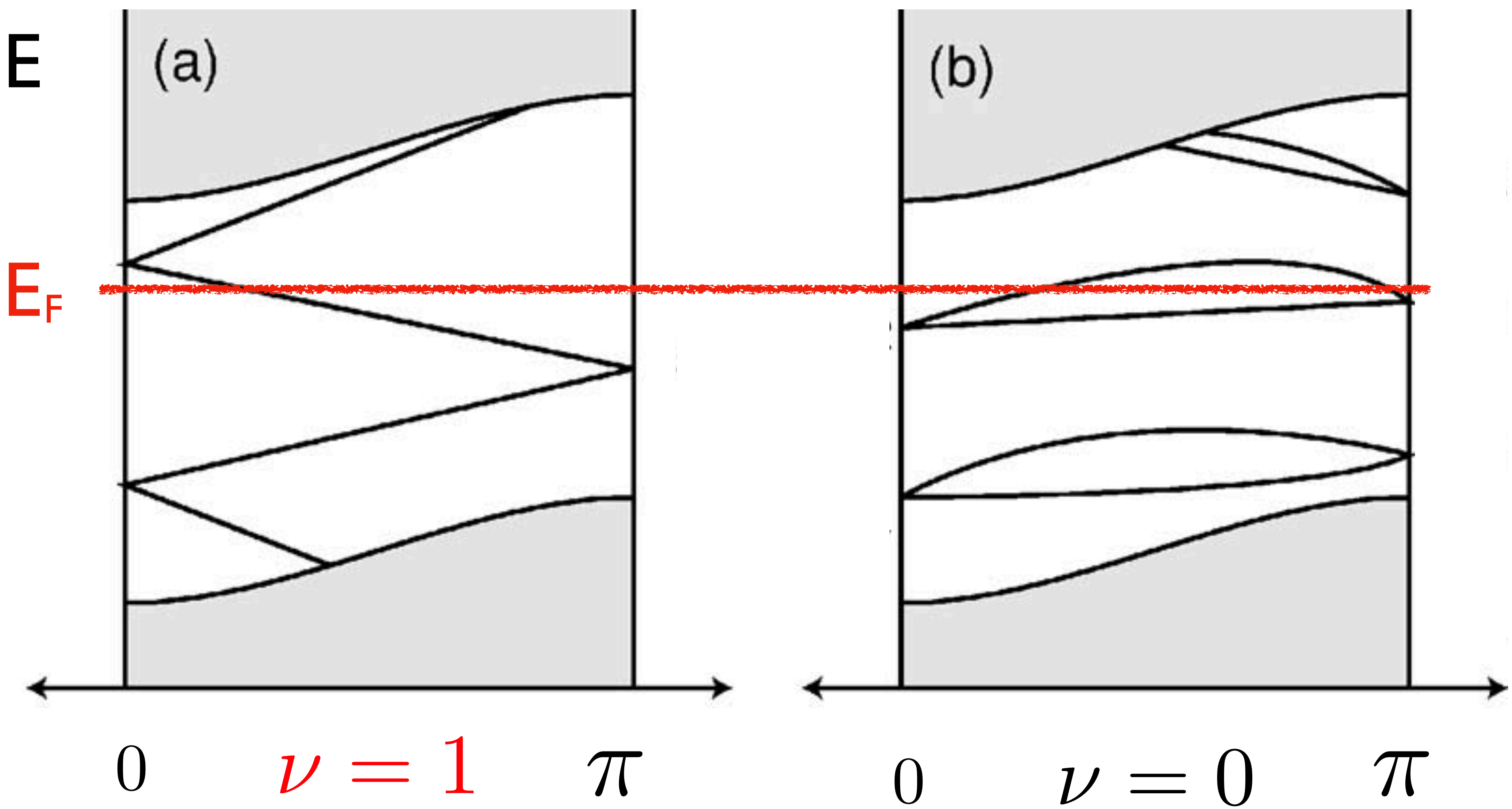
$$\Psi(\varphi, \pm L) = \pm i\sigma_2 \mathrm{e}^{\theta\sigma_3\tau_3}\sigma_3\tau_3\Psi(\varphi, \pm L)$$

$$\Psi(\varphi, x) = \Psi(\varphi + 2\pi, x)$$

preserve time reversal invariance

TOPOLOGICAL INSULATORS

Edge states $\mathbb{Z}_2$ index



Topological insulator

Normal insulator

TOPOLOGICAL INSULATORS

Momentum-spin locking

Bulk-edge correspondence

$$\nu_B = \nu_b$$

For any boundary condition except

$$\theta = \pm\infty$$

No edge states in that case

CONCLUSIONS

- Bulk-edge correspondence is robust in topological materials but depends on boundary conditions
- The correspondence breaks down with APS boundary conditions in the IQHE
- Bulk-edge dualities in topological matter go beyond APS and holography
- In topological insulators some boundary conditions suppress edge states and break the bulk-edge correspondence