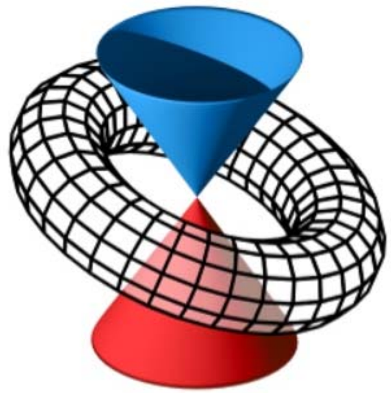
QM3 Quantum Matter meets Maths

Topological theory of non-Hermitian photonic systems

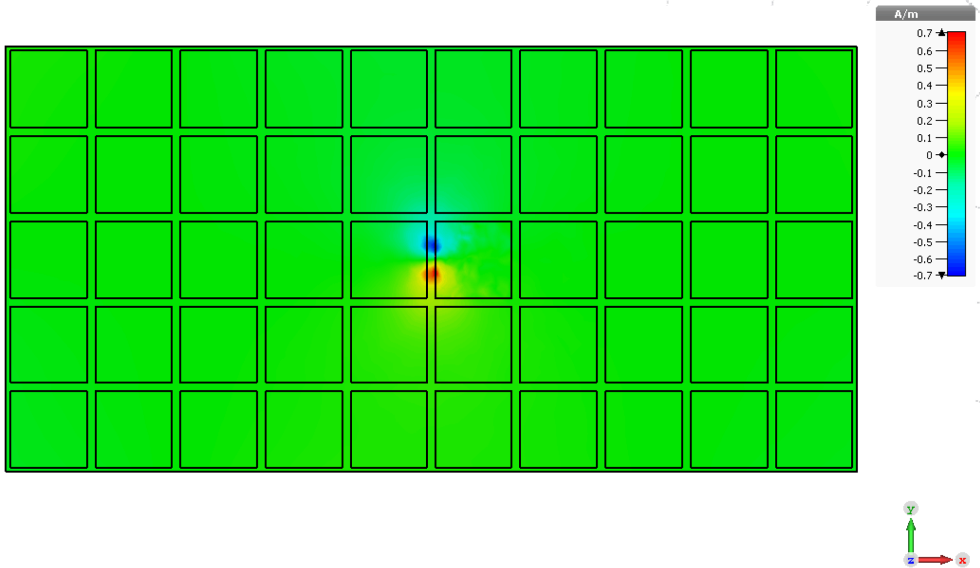
Mário G. Silveirinha



Topological photonics

Topological materials (Chern insulators)

No propagation in the bulk region

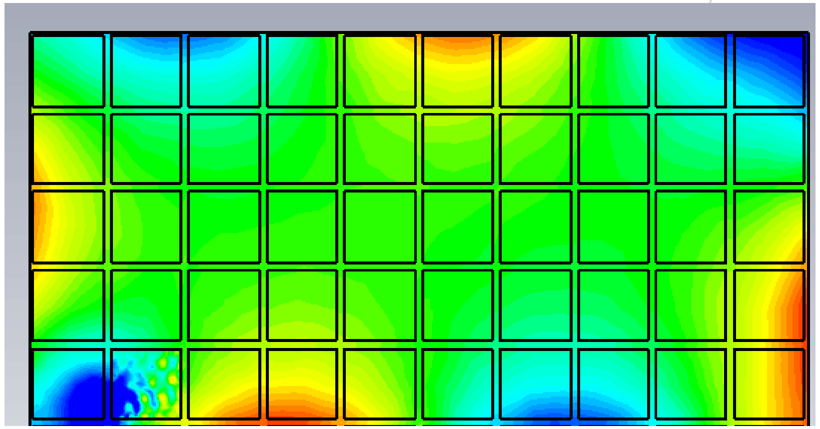


PRL **100**, 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

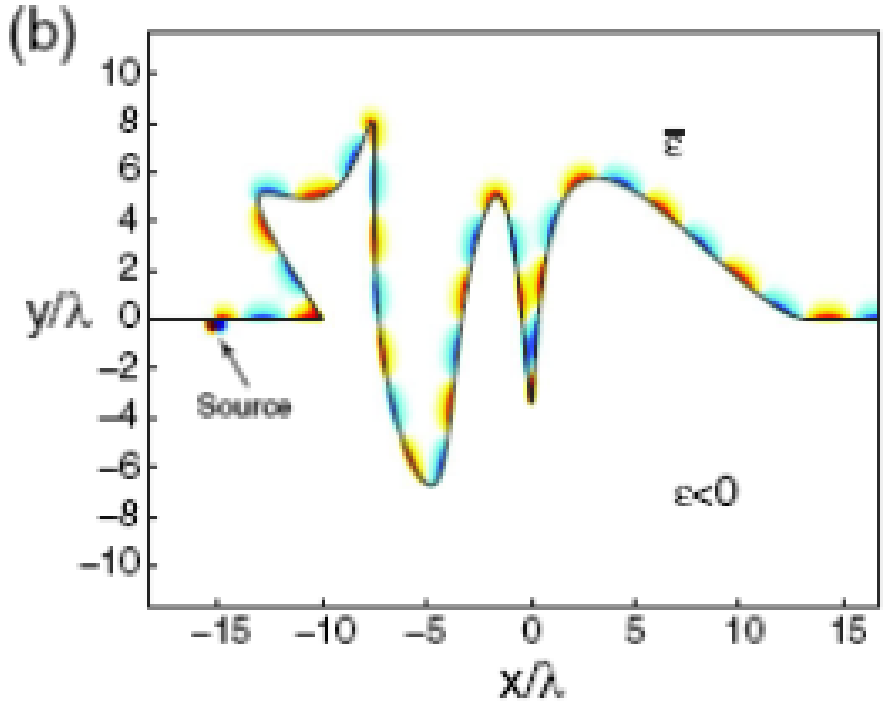
Topological materials (contd.)



There is no way to "close" a topological system (with a opaque-type) boundary without creating edge states

M. G. Silveirinha, "Proof of the bulk-edge correspondence through a link between topological photonics and fluctuation-electrodynamics", Phys. Rev. X, 9, 011037, 2019.

Do we really need a photonic crystal?



PRL **111,** 257401 (2013) PHYSICAL REVIEW LETTERS week ending 20 DECEMBER 2013

Theory of Wave Propagation in Magnetized Near-Zero-Epsilon Metamaterials: Evidence for One-Way Photonic States and Magnetically Switched Transparency and Opacity

Arthur R. Davoyan* and Nader Engheta†

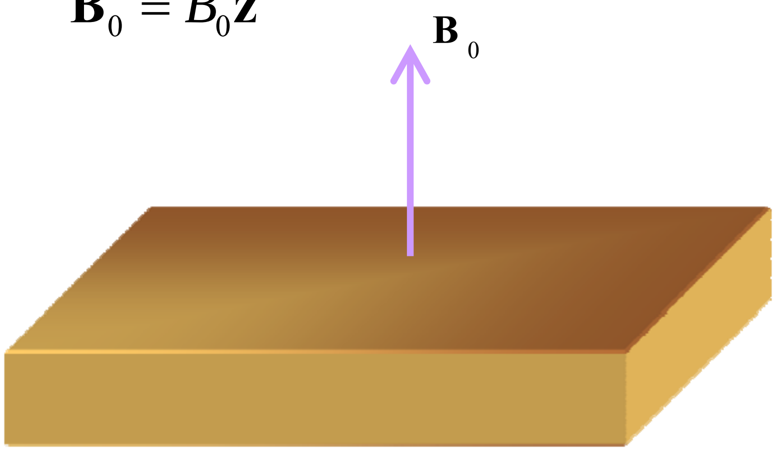
Topological photonics in a continuum

M. G. Silveirinha, "Chern Invariants for Continuous Media", Phys. Rev. B, 92, 125153, 2015.

Electromagnetic continua

$$\overline{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$$



Topological band theory

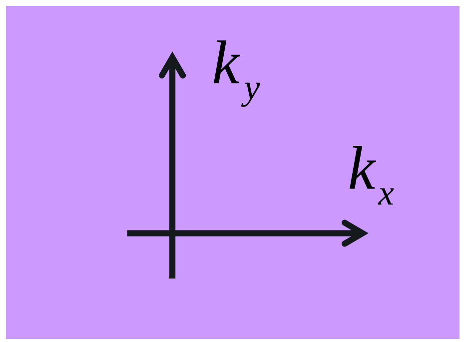
$$\mathcal{C} = \frac{1}{2\pi}\iint dk_x dk_y \, \mathcal{F}_{\mathbf{k}}$$

$$\mathcal{F}_{\mathbf{k}} = \frac{\partial \mathcal{A}_y}{\partial k_x} - \frac{\partial \mathcal{A}_x}{\partial k_y}$$

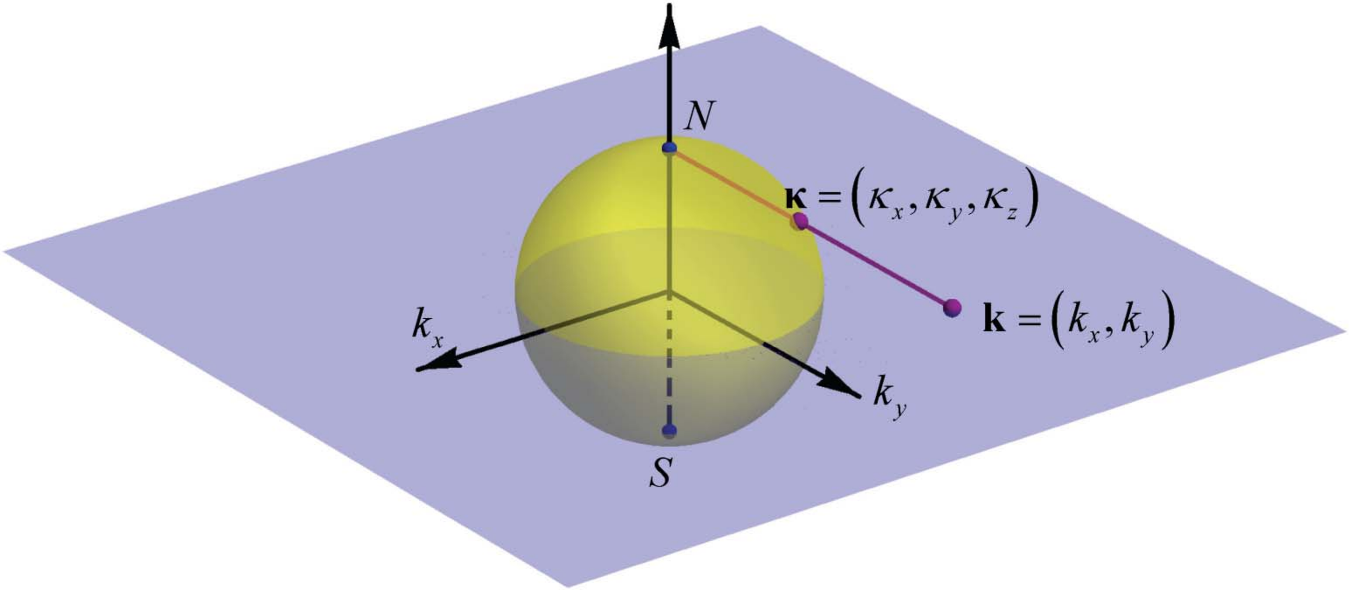
If the wave vector space is a closed surface with no boundary the Chern number is an integer

Photonic crystals (BZ is a torus):

Electromagnetic continuum



The Riemann sphere



Magneto-optical material

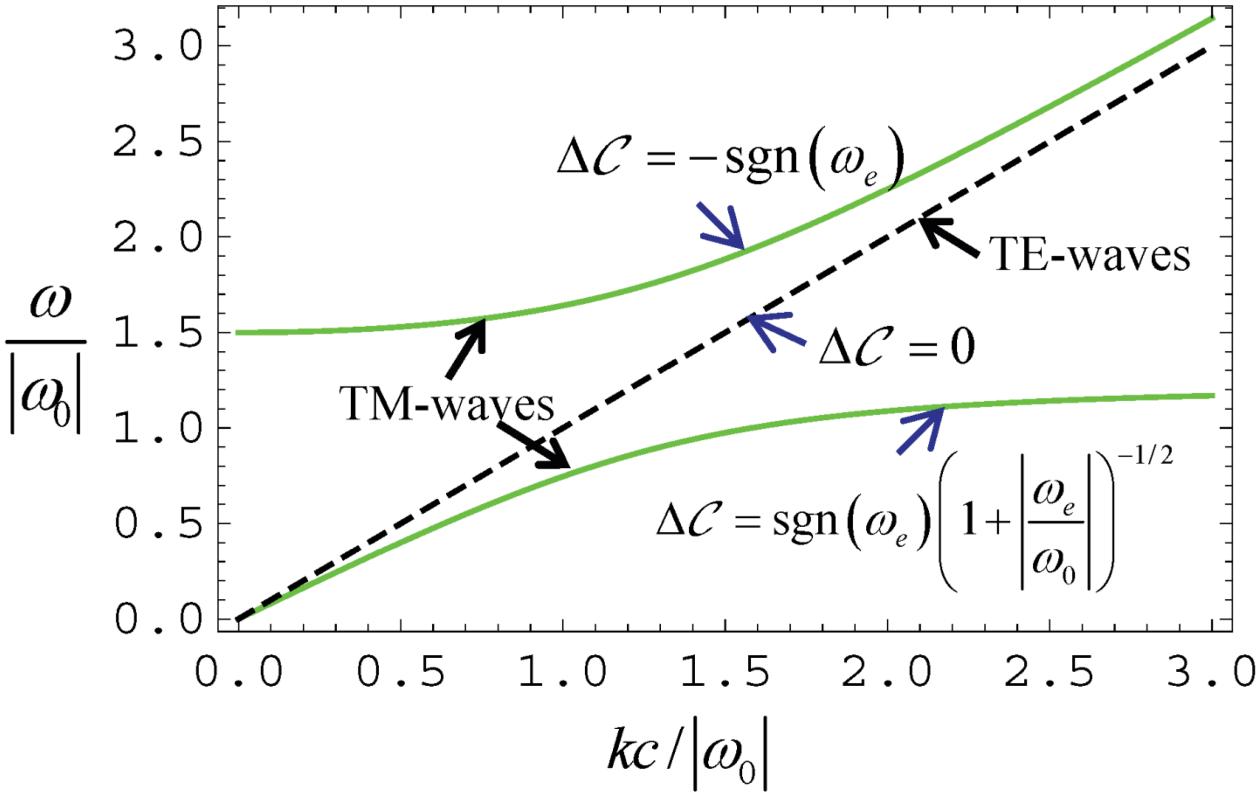
$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\varepsilon_t = 1 + \frac{\omega_0 \omega_e}{\omega_0^2 - \omega^2} \qquad \varepsilon_g = \frac{\omega_e \omega}{\omega_0^2 - \omega^2} \qquad \varepsilon_a = 1$$

TM waves: $k^2 = \dfrac{\varepsilon_t^2 - \varepsilon_g^2}{\varepsilon_t}\left(\dfrac{\omega}{c}\right)^2$

TE waves: $k^2 = \varepsilon_a \left(\dfrac{\omega}{c}\right)^2$

Band structure and Chern numbers



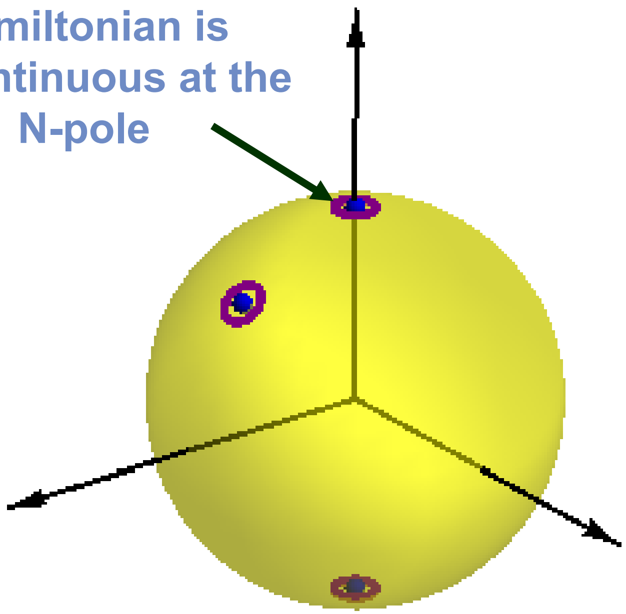
Should not the Chern number be an integer?

$$\mathcal{C} = \frac{1}{2\pi}\iint dk_x dk_y\, \hat{\mathbf{z}} \cdot \nabla \times \mathcal{A}$$

Stokes theorem:

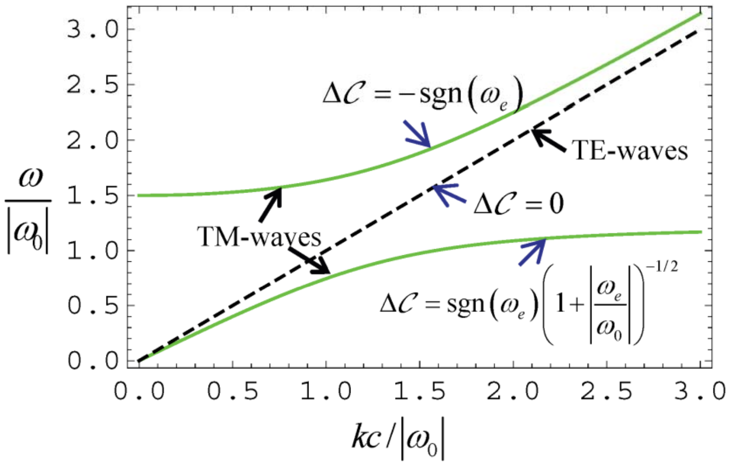
$$\mathcal{C}_n = -\frac{1}{2\pi}\sum_{\mathbf{k}_{s,i}} \oint_{C_{s,i}} \mathcal{A}_{n\mathbf{k}} \cdot \mathbf{dl}$$

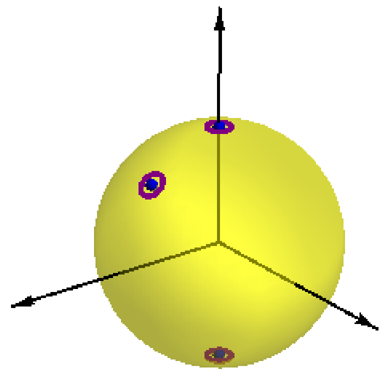
Hamiltonian is discontinuous at the N-pole



Contribution of a singular point is an integer when Hamiltonian varies smoothly with the wave vector!

Why is the Chern number an integer for the high-frequency band?





$$\left.\bar{\bar{\varepsilon}}\right|_{\substack{\omega\to\infty \\ k\to\infty}} \doteq \text{vacuum}$$

(eigenfunctions can be chosen real-valued)

Well-behaved material:

$$\left.\bar{\bar{\varepsilon}}\right|_{\text{north-pole}} \doteq \text{vacuum}$$

(or any dielectric material)

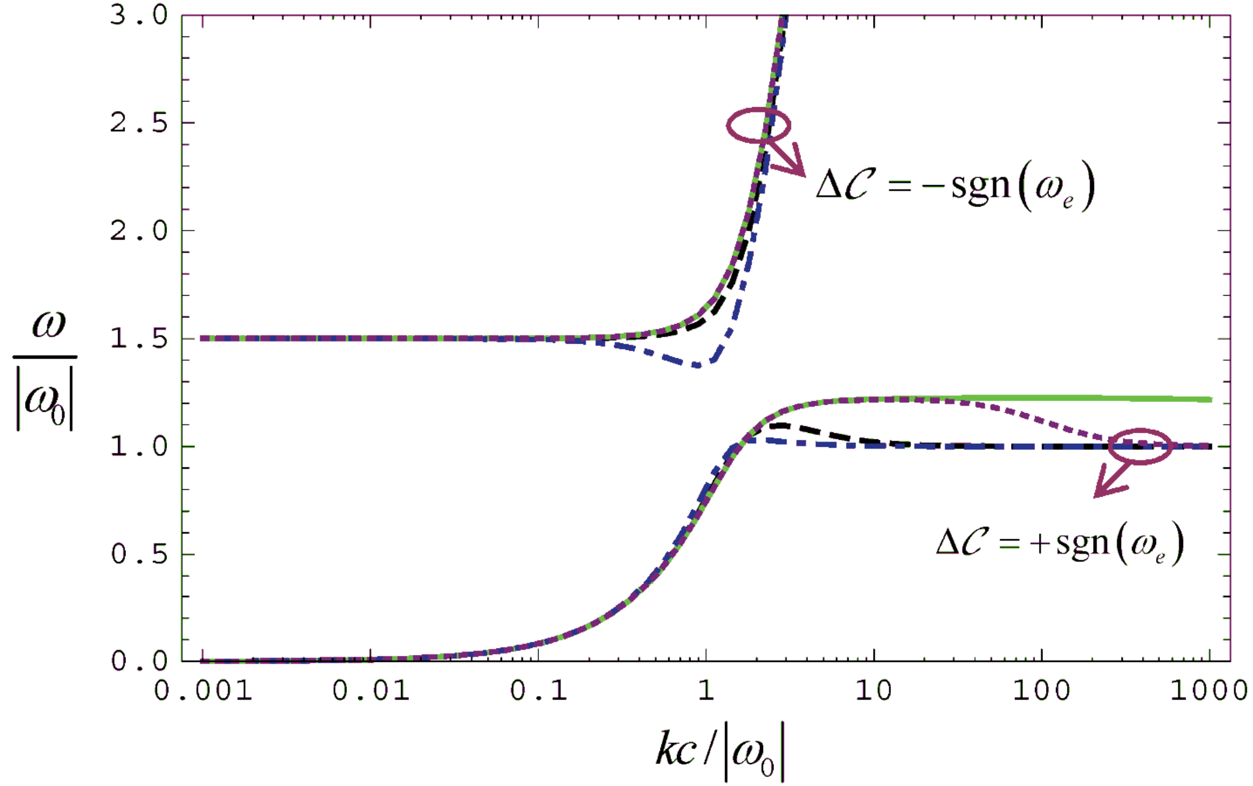
High Frequency Spatial Cut-off

$$\bar{\varepsilon} \to \varepsilon_0 \mathbf{1} + \frac{1}{1 + k^2 / k_{\max}^2} \left[\bar{\varepsilon}(\omega) - \varepsilon_0 \mathbf{1} \right]$$

$$k_{\max} = \text{spatial cut-off}$$

For large wave vectors the material response becomes asymptotically the same as the vacuum!

Effect of the spatial cut-off



$$k_{max} = 100|\omega_0|/c,$$
$$3|\omega_0|/c,$$
$$|\omega_0|/c$$

$$\omega_e = 0.5\omega_0$$

Another example: magnetized plasma

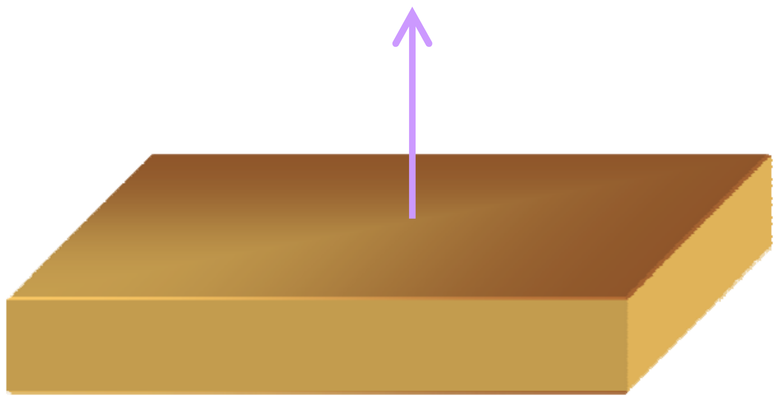
$$\overline{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\varepsilon_t = 1 - \frac{\omega_p^2 \left(1 + i\Gamma / \omega\right)}{\left(\omega + i\Gamma\right)^2 - \omega_0^2}$$

$$\varepsilon_g = \frac{1}{\omega} \frac{\omega_p^2 \omega_0}{\omega_0^2 - \left(\omega + i\Gamma\right)^2}$$

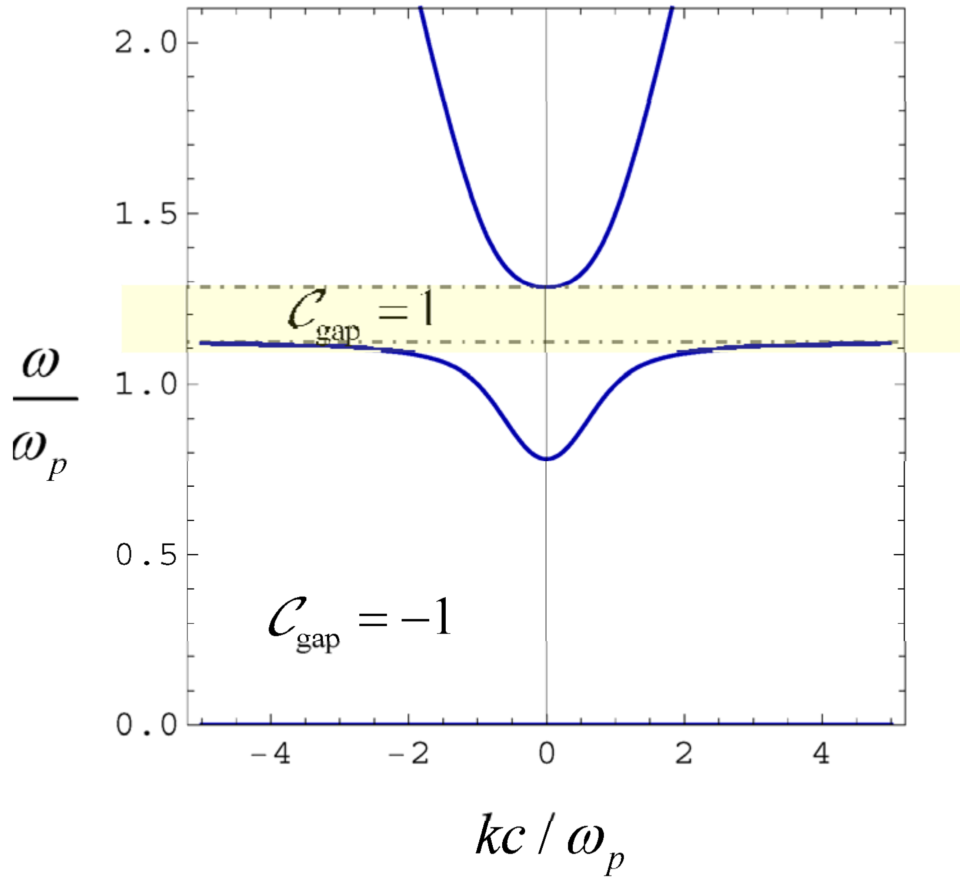
$$\varepsilon_a = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\Gamma\right)}$$

$$\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$$



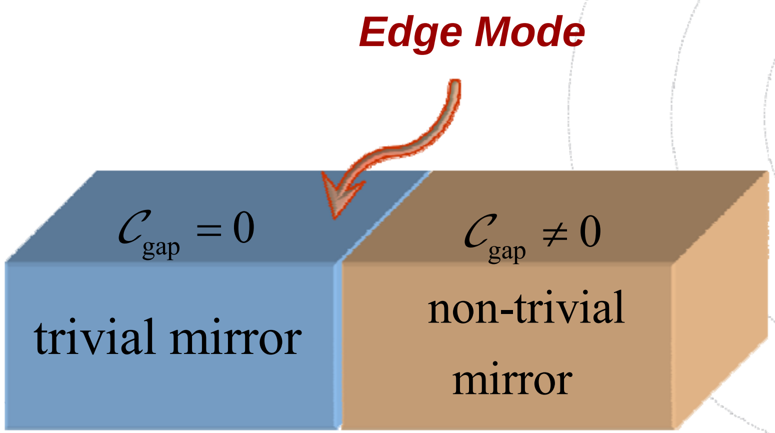
$$\omega_0 = -q B_0 / m$$

Band structure for a magnetized semiconductor (direction of propagation perpendicular to the bias field)



M. G. Silveirinha, "Chern Invariants for Continuous Media", Phys. Rev. B, 92, 125153, 2015.
M. G. Silveirinha, "Bulk edge correspondence for topological photonic continua", Phys. Rev. B, 94, 205105, 2016.

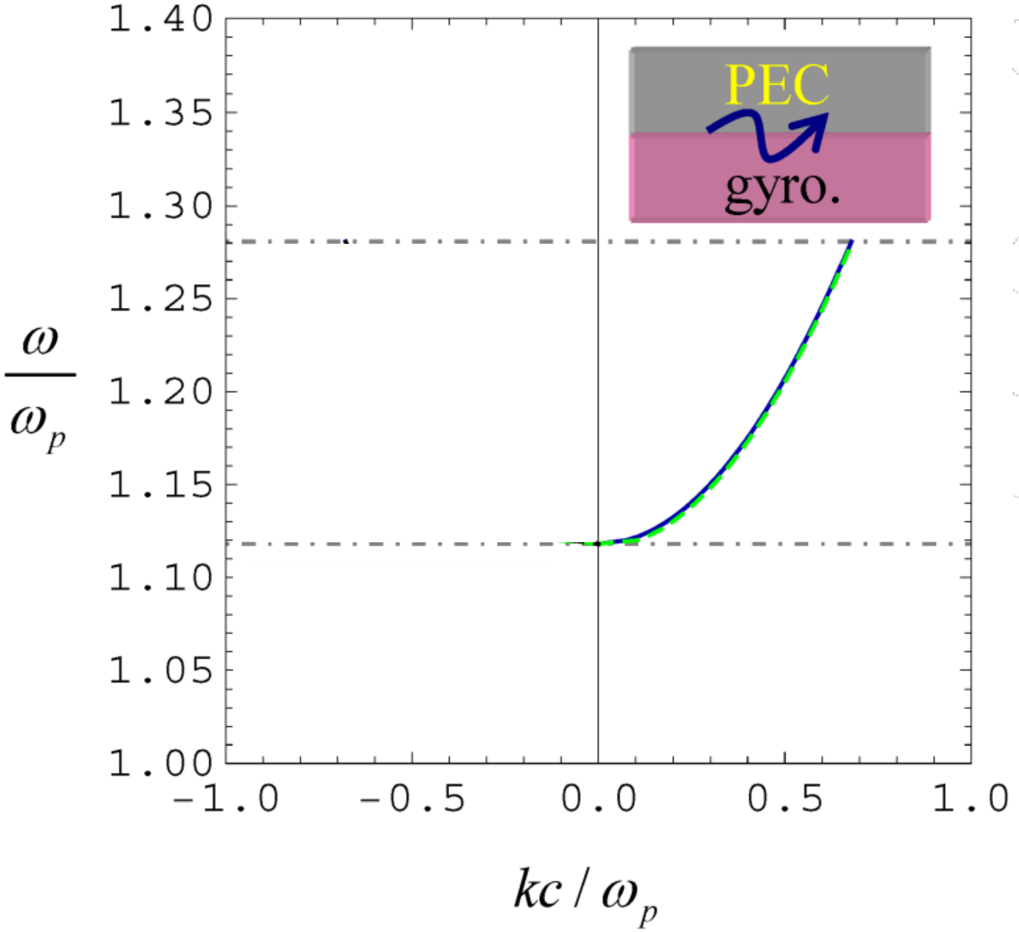
Bulk edge correspondence (Hatsugai)



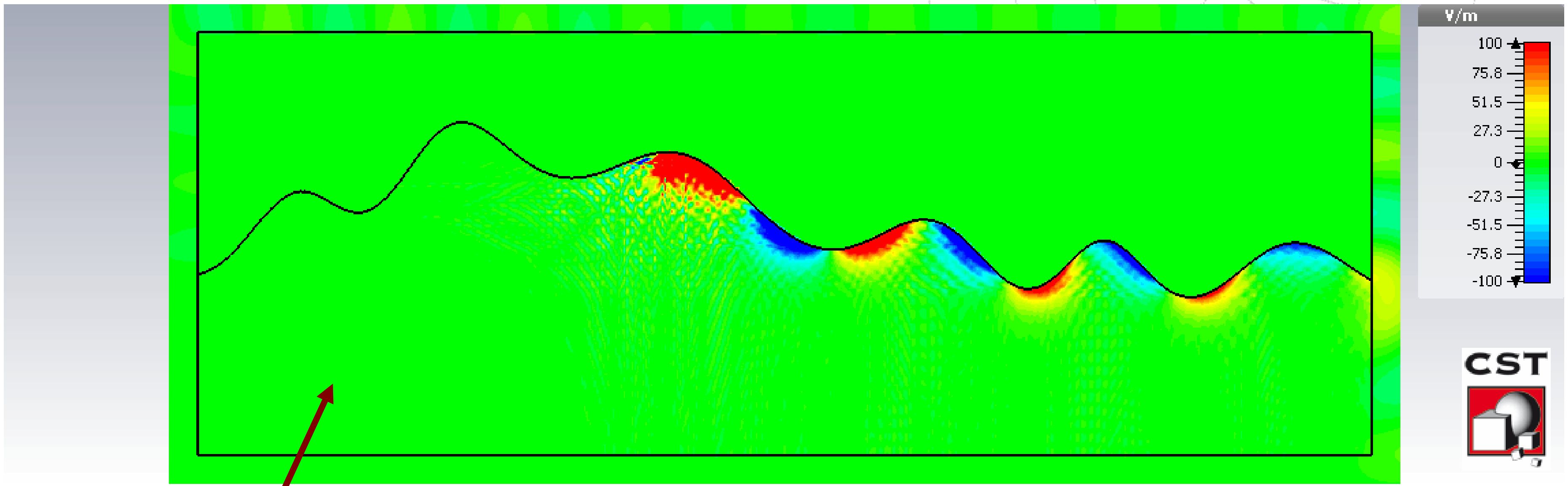
Number of <u>unidirectional</u> edge modes = gap Chern number

M. G. Silveirinha, "Bulk-edge correspondence for topological photonic continua", Phys. Rev. B, 94, 205105, 2016.
M. G. Silveirinha, "Proof of the bulk-edge correspondence through a link between topological photonics and fluctuation-electrodynamics", Phys. Rev. X, 9, 011037, 2019.

Gapless edge states



Unidirectional propagation



Gyrotropic Material

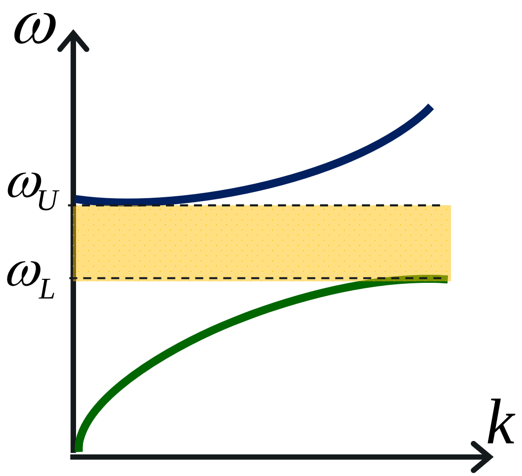
A first taste of the topological classification with the Green’s function

M. G. Silveirinha, “Topological classification of Chern-type insulators by means of the photonic Green function”, *Phys. Rev. B*, 97, 115146, 2018.

Chern number calculated by means of the system Green's function

Eigenvalue problem: $\hat{L}_{\mathbf{k}} \cdot \mathbf{Q}_{n\mathbf{k}} = \omega_{n\mathbf{k}} \mathbf{M}_g \cdot \mathbf{Q}_{n\mathbf{k}}$

Green's function: $\mathcal{G}_{\mathbf{k}}(\omega) = i\left(\hat{L}_{\mathbf{k}} - \mathbf{M}_g \omega\right)^{-1}$

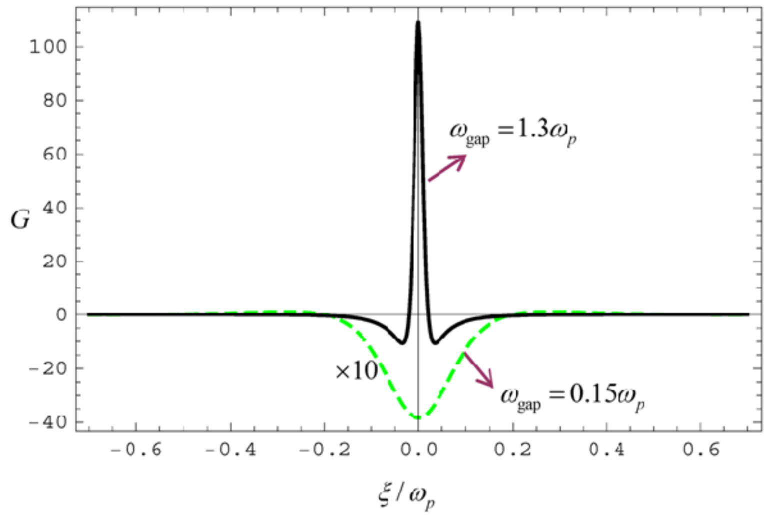


Chern number calculated by means of the system Green's function (cont.)

$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \mathrm{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}$$

$$\mathcal{G}_{\mathbf{k}}(\omega) = i\left(\hat{L}_{\mathbf{k}} - \mathbf{M}_g \omega\right)^{-1}$$

Numerical example (magnetized plasma)



$$\mathcal{C} = \int_{\omega_{gap}-i\infty}^{\omega_{gap}+i\infty} d\omega \underbrace{\frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k}\, \mathrm{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}}_{G}$$

Non-Hermitian Systems

M. G. Silveirinha, “Topological theory of non-Hermitian photonic systems”, Phys. Rev. B, 99, 125155, 2019.

Generalized eigenvalue problem

$$\hat{L}_{\mathbf{k}} \cdot \mathbf{Q}_{n\mathbf{k}} = \omega_{n\mathbf{k}} \mathbf{M}_g \cdot \mathbf{Q}_{n\mathbf{k}}$$

$\hat{L}_{\mathbf{k}}, \mathbf{M}_g$ do not need to be Hermitian

$\hat{L}_{\mathbf{k}}$ is parameterized by a real-valued wave vector $\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}}$

Green’s function operator

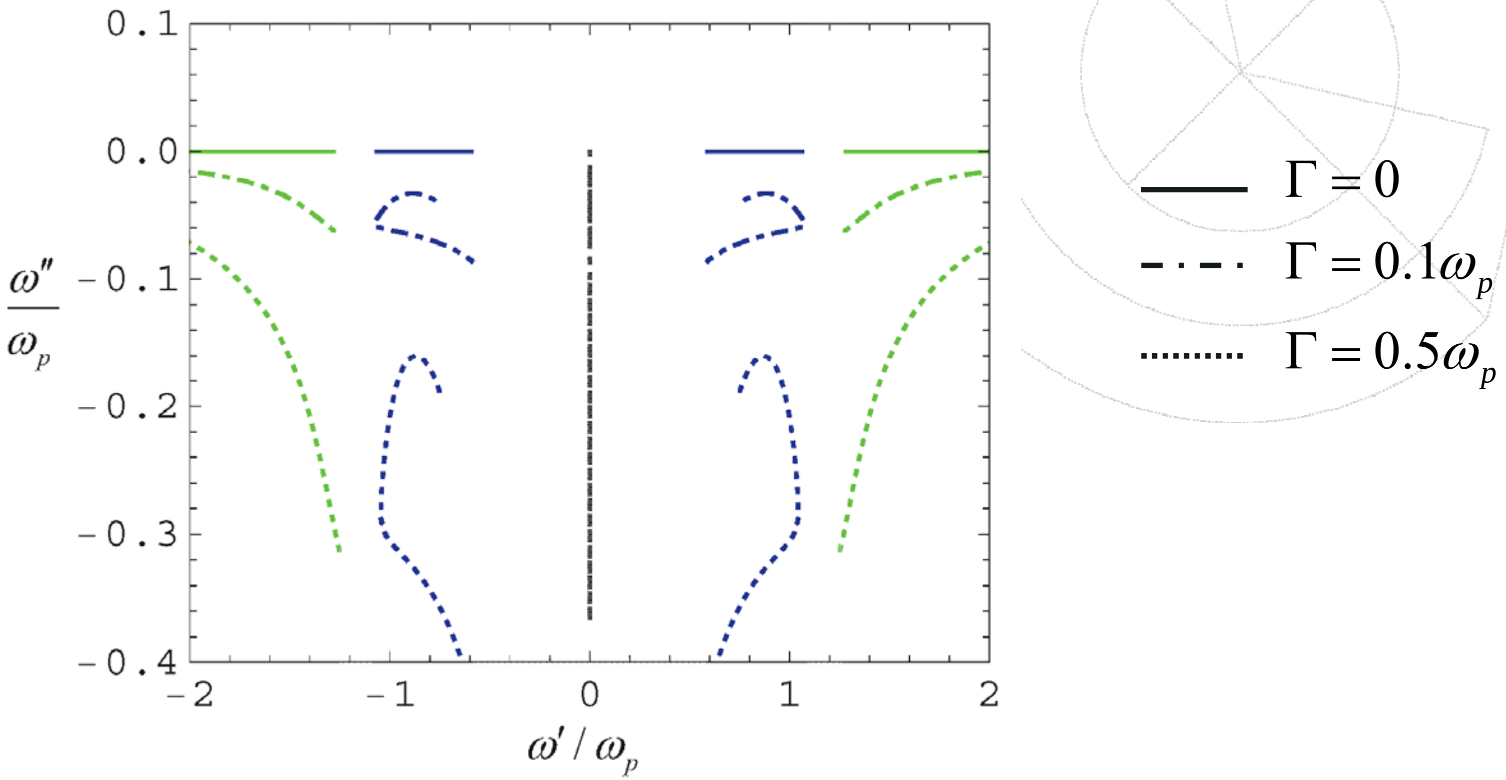
$$\mathcal{G}_{\mathbf{k}}(\omega) = i\left(\hat{L}_{\mathbf{k}} - \mathbf{M}_g \omega\right)^{-1}$$

Band gaps

Vertical strips in the complex frequency plane where the Green's function is analytic (no modes)

$$\omega_L < \mathrm{Re}\{\omega\} < \omega_U$$

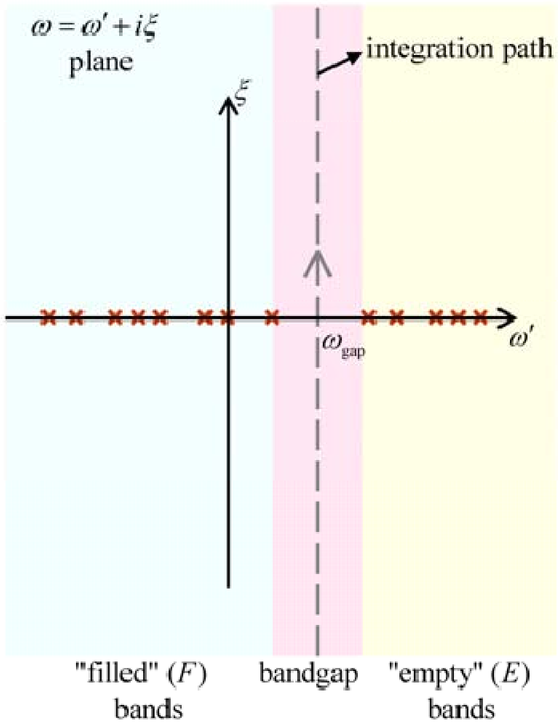
Illustration: Spectrum of a non-Hermitian material (magnetized plasma with loss)



Plane waves with a real-valued wave vector

Definition

$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \text{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}$$

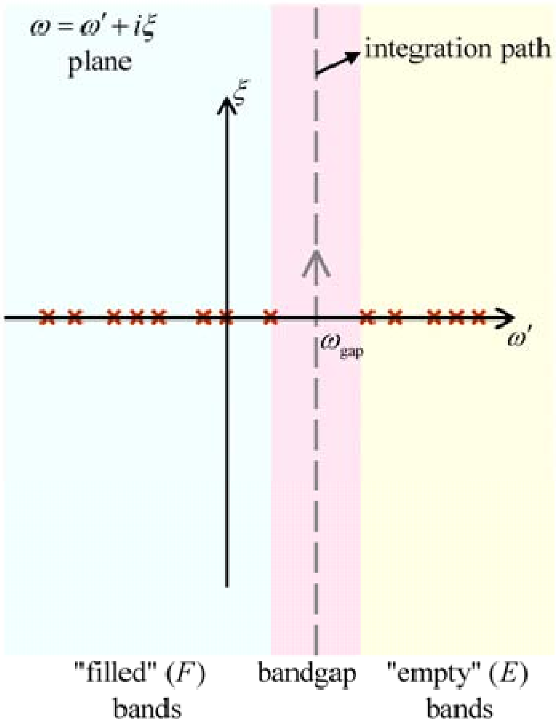


"Deformation" of the system

$$\hat{L}_{\mathbf{k}} = \hat{L}_{\mathbf{k}}(\alpha)$$

The key result

$$\frac{\partial \mathcal{C}}{\partial \alpha} = \frac{1}{(2\pi)^2} \frac{1}{2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \varepsilon^{ijl} \partial_i \operatorname{Tr}\left\{ \partial_\alpha \mathcal{G}_{\mathbf{k}} \cdot \partial_j \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_l \mathcal{G}_{\mathbf{k}}^{-1} \right\}$$



$i, j, l = 0, 1, 2$

$$\partial_0 = \frac{\partial}{\partial \omega}, \quad \partial_1 = \frac{\partial}{\partial k_1}, \quad \partial_2 = \frac{\partial}{\partial k_2}$$

$$\frac{\partial \mathcal{C}}{\partial \alpha} = 0$$

B. A. Bernervig, T. Hughes, Topological Insulators and Topological Superconductors, Princeton University Press, 2013.

"Chern theorem in non-Hermitian systems": Poor's man proof

$\hat{L}_{\mathbf{k}}(\alpha)$ $\quad\quad$ $0 \le \alpha \le 1$

$\hat{L}_{\mathbf{k}}(0)$ is Hermitian

$\hat{L}_{\mathbf{k}}(1) = \hat{L}_{\mathbf{k}}$ possibly non-Hermitian

$\mathcal{C}(\alpha = 0)$ is an integer

$$\frac{\partial \mathcal{C}}{\partial \alpha} = 0 \quad \longrightarrow \mathcal{C} = \mathcal{C}(\alpha = 1) \text{ is an integer}$$

Limitation: the proof only holds true if the band gap is not closed by the deformation

General proof

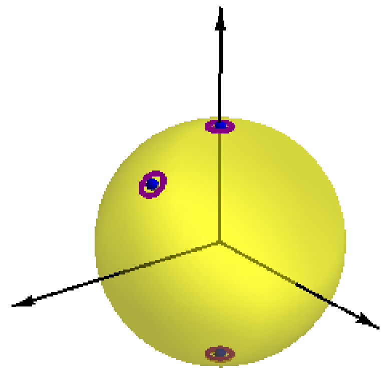
Assumptions: $\hat{L}_{\mathbf{k}}$ are diagonalizable

[or weak perturbations (that do not close gaps) of diagonalizable operators]

Key step:

$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}} - i\infty}^{\omega_{\text{gap}} + i\infty} d\omega \, \mathrm{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}$$

$$= \frac{1}{2\pi} \iint_{B.Z.} d^2\mathbf{k} \left(\partial_1 \mathcal{A}_2 - \partial_2 \mathcal{A}_1 \right)$$



Link with the Berry potential

Reduction to a standard eigenvalue problem:

$$\hat{\mathcal{L}}_{\mathbf{k}} = \mathbf{M}_g^{-1/2} \hat{L}_{\mathbf{k}} \mathbf{M}_g^{-1/2}$$

$$\hat{\mathcal{L}}_{\mathbf{k}} \cdot \mathcal{Q}_{n\mathbf{k}} = \omega_{n\mathbf{k}} \mathcal{Q}_{n\mathbf{k}} \qquad \mathcal{Q}_{n\mathbf{k}} = \mathbf{M}_g^{1/2} \cdot \mathbf{Q}_{n\mathbf{k}}$$

$n = 1, 2, ... N_F$, "filled" bands

$n = N_F + 1, ...$ "empty" bands

See also: H. Shen, B. Zhen, and L. Fu, Topological Band Theory for Non-Hermitian Hamiltonians, Phys. Rev. Lett. 120, 146402 (2018).

Link with the Berry potential (contd.)

$\mathcal{Q}_{n\mathbf{k}}$ – spans the entire space (operator is diagonalizable)

$\hat{\mathcal{L}}_{\mathbf{k}} \longrightarrow$ represented by the diagonal matrix $\Omega_{\mathbf{k}} = \left[\omega_{n\mathbf{k}}\delta_{m,n}\right]_{m,n=1,2,...}$ in the basis $\mathcal{Q}_{n\mathbf{k}}$

$\mathbf{e}_1, \mathbf{e}_2, ... \longrightarrow$ some fixed basis of the vector space

Change of basis matrix ($\mathcal{Q}_{n\mathbf{k}} \rightarrow \mathbf{e}_n$): $S_{\mathbf{k}}$ (gauge dependent)

$\hat{\mathcal{L}}_{\mathbf{k}} \longrightarrow$ represented by $S_{\mathbf{k}} \cdot \Omega_{\mathbf{k}} \cdot S_{\mathbf{k}}^{-1}$ in the basis $\mathbf{e}_n$

Link with the Berry potential (contd.)

$$\mathcal{G}_{\mathbf{k}} = i\left(\hat{\mathcal{L}}_{\mathbf{k}} - \omega \mathbf{1}\right)^{-1}$$

Replace in:

$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \text{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}$$

$$\int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \frac{1}{(\omega_0 - \omega)^3} = 0$$

After simplifications:

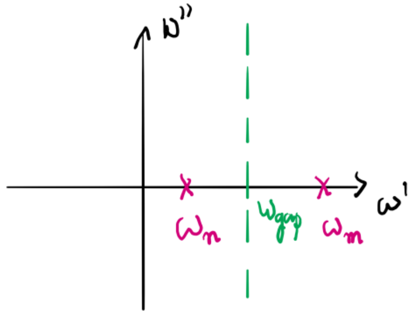
$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint_{B.Z.} d^2\mathbf{k} \int_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \text{Tr}\left\{ \left(\left[S_{\mathbf{k}}^{-1} \cdot \partial_1 S_{\mathbf{k}}, \Omega_{\mathbf{k}} \right] + \partial_1 \Omega_{\mathbf{k}} \right) \cdot \left(\Omega_{\mathbf{k}} - \mathbf{1}\omega \right)^{-1} \cdot \left(\left[S_{\mathbf{k}}^{-1} \cdot \partial_2 S_{\mathbf{k}}, \Omega_{\mathbf{k}} \right] + \partial_2 \Omega_{\mathbf{k}} \right) \cdot \left(\Omega_{\mathbf{k}} - \mathbf{1}\omega \right)^{-2} \right\}$$

Link with the Berry potential (contd.)

$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint\limits_{B.Z.} d^2\mathbf{k} \int\limits_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \,\text{Tr}\left\{ \left[S_{\mathbf{k}}^{-1}\cdot\partial_1 S_{\mathbf{k}}, \Omega_{\mathbf{k}} \right]\cdot\left(\Omega_{\mathbf{k}} - \mathbf{1}\omega\right)^{-1}\cdot\left[S_{\mathbf{k}}^{-1}\cdot\partial_2 S_{\mathbf{k}}, \Omega_{\mathbf{k}} \right]\cdot\left(\Omega_{\mathbf{k}} - \mathbf{1}\omega\right)^{-2} \right\}$$

Auxiliary result:

$$\int\limits_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \frac{1}{(\omega-\omega_m)^2}\frac{1}{\omega-\omega_n} = \begin{cases} \dfrac{2\pi i}{(\omega_m-\omega_n)^2}\,\text{sgn}\left(\omega_{\text{gap}} - \text{Re}\{\omega_n\}\right) & \text{different sides of the gap} \\ 0, & \text{same side of the gap} \end{cases}$$



Link with the Berry potential (conclusion)

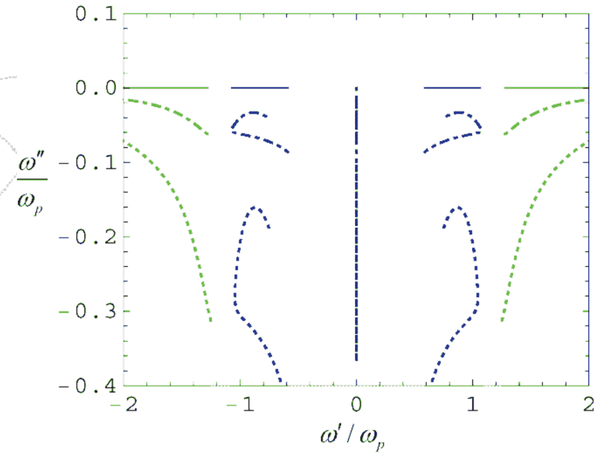
$$\mathcal{C} = \frac{1}{2\pi} \iint_{B.Z.} d^2\mathbf{k}\; i\,\mathrm{Tr}\left\{ S_{\mathbf{k}}^{-1} \cdot \partial_1 S_{\mathbf{k}} \cdot \mathbf{1}_F \cdot S_{\mathbf{k}}^{-1} \cdot \partial_2 S_{\mathbf{k}} - 1 \leftrightarrow 2 \right\}$$

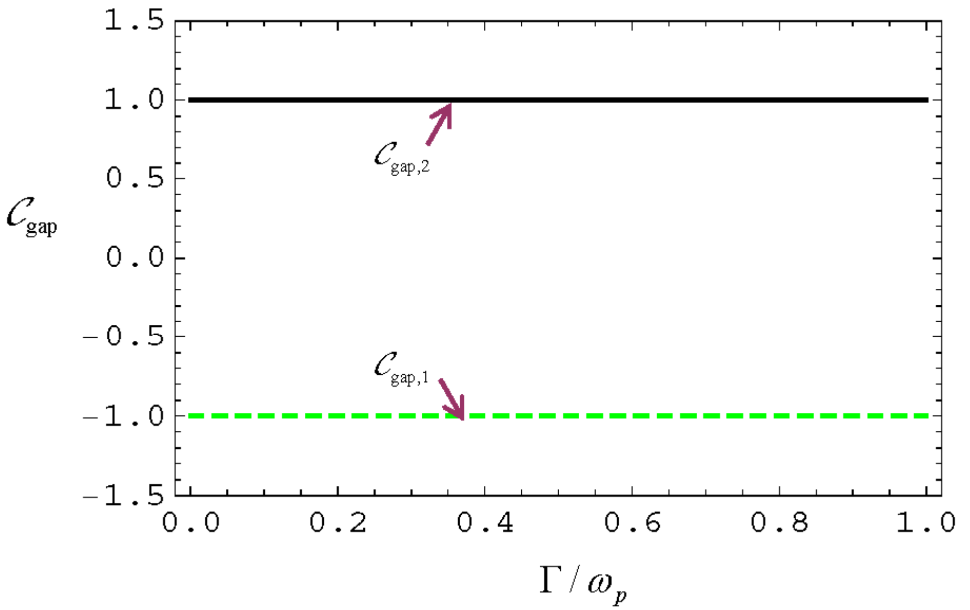
$$\mathbf{1}_F = \sum_{\mathrm{Re}\{\omega_{n\mathbf{k}}\} < \omega_{\mathrm{gap}}} \hat{\mathbf{u}}_n \otimes \hat{\mathbf{u}}_n$$

$$\mathcal{C} = \frac{1}{2\pi} \iint_{B.Z.} d^2\mathbf{k}\; \left(\partial_1 \mathcal{A}_2 - \partial_2 \mathcal{A}_1 \right)$$

$$\mathcal{A}_{\mathbf{k}} = i\,\mathrm{Tr}\left\{ S_{\mathbf{k}}^{-1} \cdot \partial_{\mathbf{k}} S_{\mathbf{k}} \cdot \mathbf{1}_F \right\}$$

Example: lossy magnetized plasma



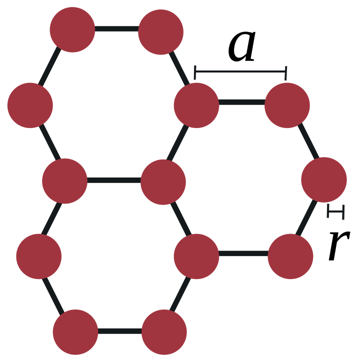


First Principles Calculations of the Topological Phases of Non-Hermitian Photonic crystals

F. R. Prudêncio, M. G. Silveirinha, First Principles Calculation of Topological Invariants of non-Hermitian Photonic Crystals, arXiv:2003.01539

Ferrite Photonic Crystal

Hexagonal array of ferrite cylinders



Ferrite constitutive relations

$$\begin{cases} \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \\ \mathbf{B} = \varepsilon_0 \overline{\mu} \cdot \mathbf{H} \end{cases}$$

$$\overline{\mu} = \begin{pmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Band structure

$$\hat{L}(-i\nabla)\cdot E_z = \mathcal{E}\mathbf{M}_g \cdot E_z \qquad \mathcal{E} = (\omega / c)^2$$

Operators:

$$\mathbf{M}_g \cdot E_z \equiv \varepsilon E_z$$

$$\hat{L}\cdot E_z \equiv -\partial_x\left(\mu_{ef}^{-1}\partial_x E_z - i\chi\partial_y E_z\right) - \partial_y\left(\mu_{ef}^{-1}\partial_y E_z + i\chi\partial_x E_z\right)$$

$$\mu_{ef} = \left(\mu^2 - \kappa^2\right)/\mu$$

$$\chi = \kappa \big/ \left(\mu^2 - \kappa^2\right)$$

Plane wave method

Bloch modes: $E_z = e_z(x,y)e^{i\mathbf{k}\cdot\mathbf{r}}$

Plane wave expansion: $e_z = \sum_{\mathbf{J}} c_{\mathbf{J}}^E e^{i\mathbf{G}_{\mathbf{J}}\cdot\mathbf{r}}$

Secular equation: $\hat{L}_{\mathbf{k}} \cdot e_z = \mathcal{E}\mathbf{M}_g \cdot e_z$

$\hat{L}_{\mathbf{k}} \equiv \hat{L}(-i\nabla + \mathbf{k})$

$\mathbf{G}_{\mathbf{J}} \equiv j_1\mathbf{b}_1 + j_2\mathbf{b}_2$ **is a generic reciprocal lattice primitive vector**

Operators:

$$\hat{L}_{\mathbf{k}} = \left[L_{\mathbf{I},\mathbf{J}}\right] = \left[(\mathbf{k}+\mathbf{G}_{\mathbf{I}})\cdot(\mathbf{k}+\mathbf{G}_{\mathbf{J}})\, p_{\mu_{ef}^{-1},\mathbf{I}-\mathbf{J}} + i\left[(\mathbf{k}+\mathbf{G}_{\mathbf{J}})\times(\mathbf{k}+\mathbf{G}_{\mathbf{I}})\right]\cdot\hat{\mathbf{z}}\, p_{\chi,\mathbf{I}-\mathbf{J}}\right]$$

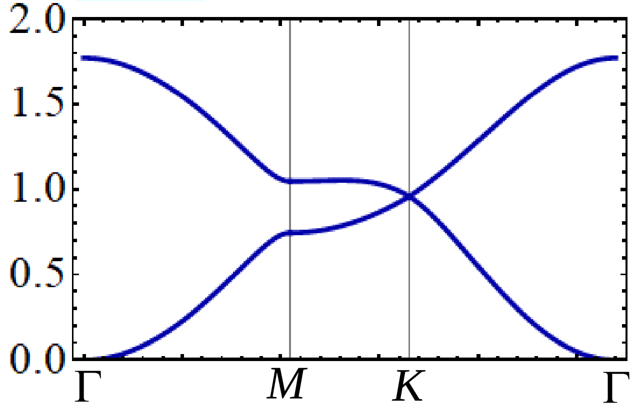
$$\mathbf{M} = \left[M_{\mathbf{I},\mathbf{J}}\right] = p_{\varepsilon,\mathbf{I}-\mathbf{J}}$$

We use 49 plane waves.

Band structure of a ferrite photonic crystal

Reciprocal ferrite photonic crystal

$\kappa = 0$

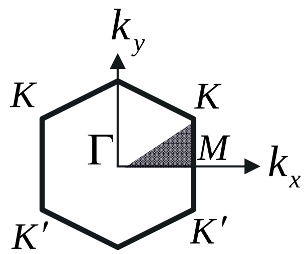


$$\left(\frac{\omega a}{c}\right)^2$$

$\varepsilon = 12$

$\mu = 1$

$r = 0.2\sqrt{3}a$



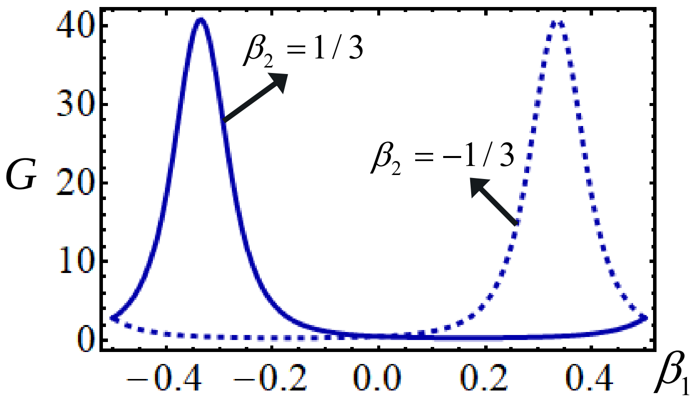
Chern number

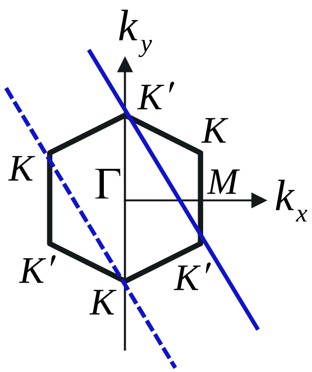
$$\mathcal{C} = \frac{1}{(2\pi)^2} \iint\limits_{B.Z.} d^2\mathbf{k} \int\limits_{\omega_{\text{gap}}-i\infty}^{\omega_{\text{gap}}+i\infty} d\omega \, \mathrm{Tr}\left\{ \partial_1 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \mathcal{G}_{\mathbf{k}} \cdot \partial_2 \mathcal{G}_{\mathbf{k}}^{-1} \cdot \partial_\omega \mathcal{G}_{\mathbf{k}} \right\}$$

$$\mathcal{G}_{\mathbf{k}}(\omega) = i\left(\hat{L}_{\mathbf{k}} - \mathbf{M}_g \omega\right)^{-1}$$

The operators L and M are the ones obtained with the plane wave expansion method (matrices)

Numerical example

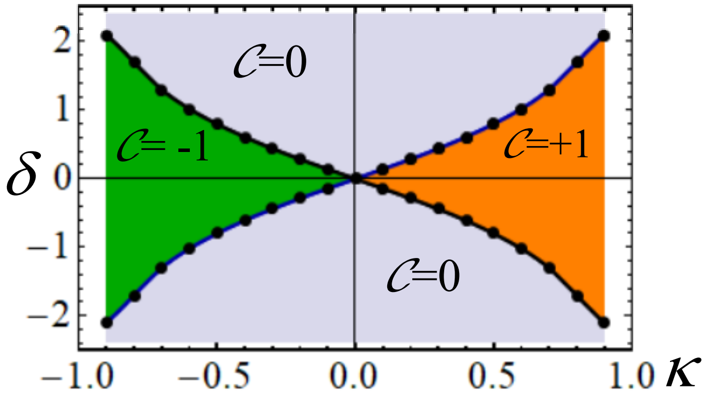




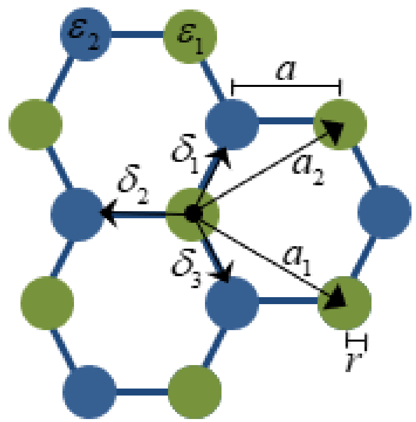
$$\mathbf{k} \equiv \beta_1 \mathbf{b}_1 + \beta_2 \mathbf{b}_2$$

The topological charge is concentrated near the two Dirac points

Phase diagram



Hexagonal array of ferrite cylinders



$\varepsilon_1 = 12 + \delta \qquad \mu_1 = \mu_2 = 1$

$\varepsilon_2 = 12 - \delta \qquad \kappa_1 = \kappa_2 = \kappa$

$r = 0.2\sqrt{3}a$

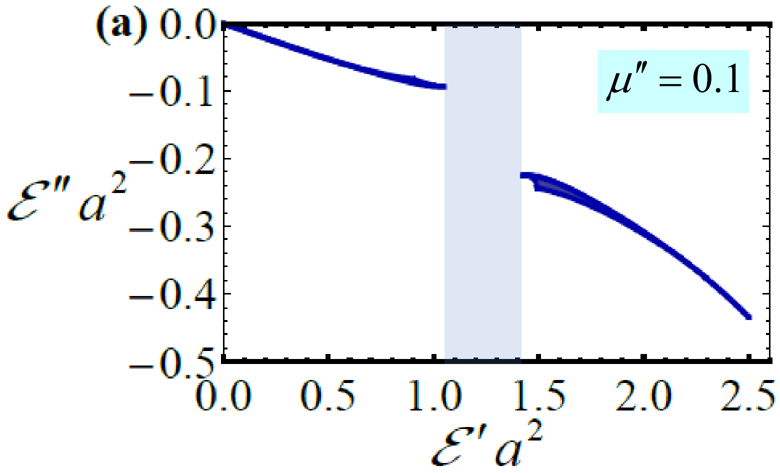
Impact of losses on the band structure

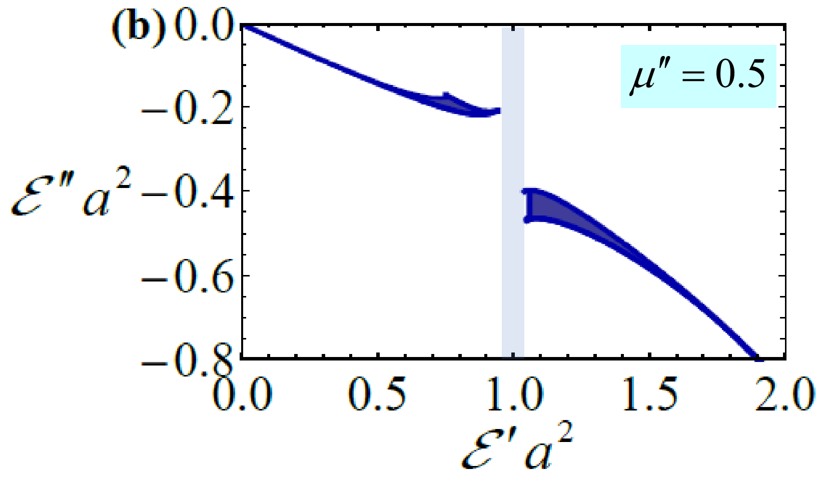
Permeability tensor:

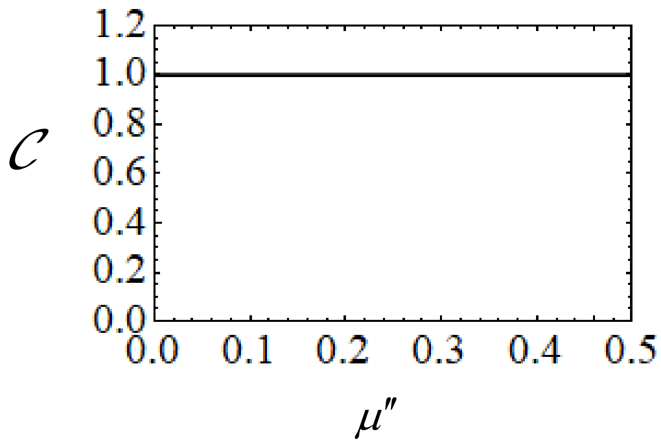
$$\bar{\mu} = \begin{pmatrix} \mu + i\mu'' & i\kappa & 0 \\ -i\kappa & \mu + i\mu'' & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Bloch modes with a <u>real-valued</u> wave vector

Chern number



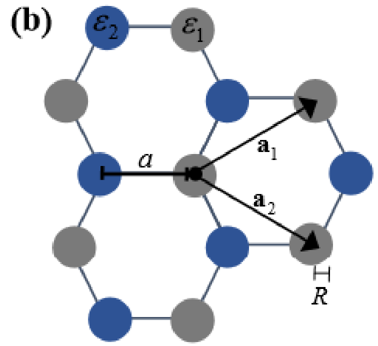




$\varepsilon = 12 \quad \kappa = 0.9$

$\mu = 1 \quad r = 0.2\sqrt{3}a$

PT-symmetric system



$$\mu_1 = 1 + i\mu''$$
$$\mu_2 = 1 - i\mu''$$

$$\varepsilon_1 = \varepsilon_2 = 12$$

Summary

•The topological classification of an electromagnetic continuum requires a spatial cut-off.

• We proposed a Green's function method to find the topological phases of non-Hermitian photonic systems

• Our formalism does not require the calculation of the photonic band-structure, and can be easily implemented using the operators obtained with a standard plane-wave expansion.

Thank you very much!