Numerical Simulation of Heat and Mass Transfer in Fluids Flow

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Outline I

Outline II

4. Numerical Investigation for Natural Convection of a Nanofluid in an Inclined L-Shaped Cavity in the Presence of an Inclined Magnetic Field
   - Problem definition and mathematical formulation
   - Boundary conditions
   - Numerical method and validation
   - Some results

5. MHD Mixed Convection in a Lid-driven Cavity Filled by a Nanofluid with Sinusoidal Temperature Distribution on the both Vertical Walls using Buongiorno's Nanofluid Model
   - Problem definition and mathematical model
   - Boundary condition
   - Numerical method and validation

Navier-Stokes equations (Claude-Louis Navier - 1822 and George Gabriel Stokes - 1845)

- The Navier-Stokes equations are partial differential equations, where the unknowns are the velocity field and the pressure of a Newtonian fluid.
- N-S equations are useful because they describe the physics of many phenomena of scientific and engineering interest.
  - model weather, ocean currents, flow around an airfoil and motion of stars inside a galaxy, in the design of aircrafts and cars, the study of blood flow, the design of power stations, the analysis of the effects of pollution, etc.
- The problem of existence of regular solutions for the three-dimensional equations, properly formulated, is part of the problems selected by the Clay Mathematics Institute, which awards the prize of one million dollars to its resolution.

Navier-Stokes equations for an incompressible fluid

Given a bounded, fixed domain $\Omega \subset \mathbb{R}^n (n \geq 2)$, $t_f > 0$, a density ρ , a viscosity μ, an external force $\mathbf{f}$, $\mathbf{v}_*$ and $\mathbf{v}_0$, find the velocity $\mathbf{v} = \mathbf{v}(t; \mathbf{x})$ and the pressure $p = p(t, \mathbf{x})$ of the fluid, defined in $[0; t_f] \times \Omega$, satisfying

$$\begin{cases} \left.\begin{array}{c} \rho\left(\overbrace{\partial_t \mathbf{v}}^{\text{Variation}} + \underbrace{(\mathbf{v}.\nabla)\,\mathbf{v}}_{\text{Convection}}\right) = \underbrace{\mu\Delta\mathbf{v}}_{\text{Diffusion}} - \overbrace{\nabla p}^{\text{Pressure}} + \overbrace{\rho\mathbf{f}}^{\text{Ext. Force}} \\ \nabla.\mathbf{v} = \mathbf{0} \ (\text{Continuity}) \end{array}\right\} \text{in } (0, t_f) \times \Omega \\ \mathbf{v}(t,\mathbf{x}) = \mathbf{v}_*(t,\mathbf{x}) \ \textit{on} \ (0,t_f) \times \partial\Omega \qquad \text{(boundary condition)} \\ \mathbf{v}(0,\mathbf{x}) = \mathbf{v}_0(\mathbf{x}), \mathbf{x} \in \Omega \qquad \text{(initial condition)} \end{cases}$$

* More Models for special cases can be found in [1]

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Existence and uniqueness results

Define

$$\mathcal{V} = \{u \in W_0^{1,2}(\Omega);\ \text{div}\ u = 0 \text{ in } \Omega\}$$

$$\mathcal{H} = \{u \in L^2(\Omega);\ \text{div}\ u = 0 \text{ in } \Omega,\ \ u.N = 0 \text{ on } \partial\Omega\}$$

Theorem (weak solutions)

For f and v_0 given;

$$f \in L^2(0, t_f; \mathcal{V}'),\ \ v_0 \in \mathcal{H}$$

There exists a weak solution u to the N-S equations satisfying

$$u \in L^2(0, t_f; \mathcal{V}') \cap L^\infty(0, t_f; \mathcal{H})$$

Furthermore, if $n = 2$, u is unique and

$$u \in \mathcal{C}([0, t_f]; \mathcal{H}), \quad \partial_t u \in L^2(0, t_f; \mathcal{V}')$$

If $n = 3$, u is weakly continuous from $[0, t_f]$ into $\mathcal{H}$;

$$u \in \mathcal{C}([0, t_f]; \mathcal{H}_w), \quad \partial_t u \in L^{4/3}(0, t_f; \mathcal{V}')$$

Theorem (strong solutions)

i) For $n = 2$, f and v_0 given;

$$f \in L^\infty(0, t_f; \mathcal{H}), \;\; v_0 \in \mathcal{V}, \qquad (*)$$

there exists a unique strong solution u to the N-S equations satisfying

$$u \in L^2(0, t_f; \mathcal{H}^2(\Omega)), \;\; \partial_t u \in L^2(0, t_f; \mathcal{H}), \quad u \in \mathcal{C}([0, t_f]; \mathcal{V}).$$

ii) For $n = 3$, f and v_0 given, satisfying (*), there exists $t_f^* = t_f^*(v_0) = \min(t_f, t_{f_1}(\|v_0\|))$, $t_{f_1}(\|v_0\|)$ given by

$$t \leq t_{f_1}(\|v_0\|) = \frac{\mathcal{K}}{(1 + \|v_0\|^2)^2}$$

and, on $[0, t_f^*]$, there exists a unique strong solution u to the N-S equations.

Ref. *J. Leray, E. Hopf, O. A. Ladyzhenskaya, J.L. Lions, W. Layton, and J. Serrin.*

What is nanofluid?

- Nanofluid are a relatively new class of fluids which consist of a base fluid with nano-sized particles ($1 - 100nm$) suspended within them.
- Introduced by Choi (Argonne National Laboratory) in 1995.

Nanoparticle materials include:

- Oxide ceramics –Al_2O_3, CuO, SiO_2
- Metal carbides – SiC
- Nitrides–AlN, SiN
- Metals –Al, Cu
- Nonmetals–Graphite, carbon nanotubes
- Layered–Al + Al_2O_3, Cu + C
- PCM – S/S
- Functionalized nanoparticles

Base fluids include:

- Water
- Ethylene- or tri-ethylene-glycols
- Oil and other lubricants
- Bio-fluids
- Polymer solutions
- Other common fluids

Nanofluid Models

- Nanofluid can be assumed to be
  - Single phase fluids.
    Physical properties of nanofluid are taken as a function of properties of both structures and their concentrations.

Model	Shape of nanoparticles	Thermal conductivity	Dynamic viscosity
I	Spherical	$k_{nf}=k_f\left\{\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right\}$	$\mu_{nf}=\mu_f(1-\phi)^{-2.5}$
II	Spherical	$k_{nf}=k_f\left\{\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right\}$	$\mu_{nf}=\mu_f(1+7.3\phi+123\phi^2)$
III	Cylindrical (nanotubes)	$k_{nf}=k_f\left\{\frac{k_s+0.5k_f-0.5\phi(k_f-k_s)}{k_s+0.5k_f+\phi(k_f-k_s)}\right\}$	$\mu_{nf}=\mu_f(1-\phi)^{-2.5}$
IV	Cylindrical (nanotubes)	$k_{nf}=k_f\left\{\frac{k_s+0.5k_f-0.5\phi(k_f-k_s)}{k_s+0.5k_f+\phi(k_f-k_s)}\right\}$	$\mu_{nf}=\mu_f(1+7.3\phi+123\phi^2)$

  - Two-phase approach

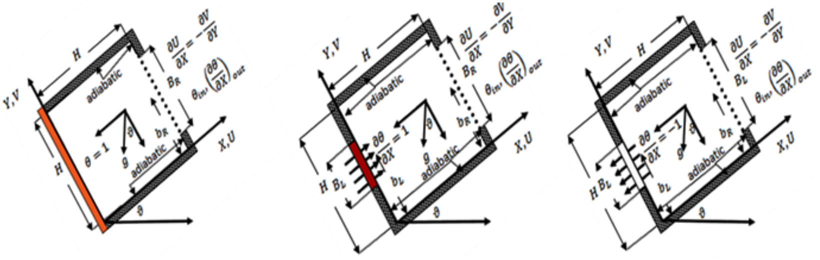
Nanofluid Application

- Nanofluids have novel properties that make them potentially useful in many applications in heat transfer.
  - Transportation (Engine cooling/vehicle thermal management)
  - Nuclear systems cooling
  - Electronics cooling
  - Solar water heating
  - Heat exchanger
  - Biomedicine
  - Heat pipes
  - Fuel cell , etc...

Natural Convection of a Nanofluid in Inclined, Partially Open Cavities: Thermal Effects [3]

Problem definition and Mathematical formulation

The schematic diagram of the two-dimensional system considered in this study is displayed in Figure 1



Case 1 Case 2 Case 3

Figure 1: The physical model of the problem

The following assumptions have been made:

I. In the cavity, the bottom, along x-axis, and top (parallel to x-axis) are kept to be adiabatic in all the three cases.

II. The right wall, parallel to y-axis, is considered to be partially opened where the remaining parts are considered to be adiabatic for all the cases.

III. For the left wall, along y-axis, the following three cases are considered:

a. *The left wall is considered to be uniform heat source.*

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II. The right wall, parallel to y-axis, is considered to be partially opened where the remaining parts are considered to be adiabatic for all the cases.
III. For the left wall, along y-axis, the following three cases are considered:
    a. *The left wall is considered to be uniform heat source.*
    b. *A heat source is located on a part of the left wall while the residual parts are adiabatically reserved.*

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II. The right wall, parallel to y-axis, is considered to be partially opened where the remaining parts are considered to be adiabatic for all the cases.
III. For the left wall, along y-axis, the following three cases are considered:
   a. *The left wall is considered to be uniform heat source.*
   b. *A heat source is located on a part of the left wall while the residual parts are adiabatically reserved.*
   c. *A heat sink is located on a part of the left wall and the other parts are thermally insulated.*

IV. The gravity acts in the vertical direction and there is no viscous dissipation.

V. The base fluid (water) and the solid spherical nanoparticles (Al_2O_3) are in thermal equilibrium.

VI. Boussinesq approximation is used to determined the variation of density in the buoyancy term where the other thermo-physical properties of the nanofluid are assumed constant.

Under the above assumptions, the continuity, momentum and energy equations can be written in the dimensional form:

$$\begin{cases} \nabla.\mathbf{v} = 0 \\ \partial_t \mathbf{v} + (\mathbf{v}.\nabla)\,\mathbf{v} = \frac{1}{\rho_{nf}}\left(-\nabla p + \mu_{nf}\ \Delta \mathbf{v} + \mathbf{f}\right) \\ \partial_t T + \mathbf{v}.\nabla T = \alpha_{nf}\ \Delta T \\ f_x = g(\rho\beta)_{nf}(T - T_c)\sin\vartheta,\ f_y = g(\rho\beta)_{nf}(T - T_c)\cos\vartheta \end{cases}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \frac{1}{\rho_{nf}}\left(-\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right. \\ &\quad \left. + g(\rho\beta)_{nf}(T - T_c)\sin\vartheta\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= \frac{1}{\rho_{nf}}\left(-\frac{\partial p}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right. \\ &\quad \left. + g(\rho\beta)_{nf}(T - T_c)\cos\vartheta\right) \end{aligned}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

where

- ρ_{nf}: the effective density of the nanofluid is given by

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_{sp}$$

  ϕ is the solid volume fraction of nanoparticles.

- α_{nf}: the thermal diffusivity of the nanofluid is given by

$$\alpha_{nf} = k_{nf} \;/(\rho_{C_{sp}})_{nf}$$

  where, $(\rho_{C_{sp}})_{nf}$ is the heat capacitance of the nanofluid and it can be expressed as:

$$(\rho_{C_{sp}})_{nf} = (1-\phi)(\rho C_{sp})_f + \phi(\rho C_{sp})_{sp}$$

- $(\rho\beta)_{nf}$: the thermal expansion coefficient of the nanofluid can be determined by

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_{sp}$$

- μ_{nf}; the effective dynamic viscosity of the nanofluid which is given by Brinkman [4]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

- Following the first model used by Mahdy and Elshehabey [5] and Oztop and Abu-nada [6], the thermal conductivity of the nanofluid k_{nf} can be expressed as:

$$k_{nf} = k_f \left[\frac{(k_{sp} + 2k_f) - 2\phi(k_f - 2k_{sp})}{(k_{sp} + 2k_f) + \phi(k_f - 2k_{sp})} \right]$$

k_{sp} is the thermal conductivity of dispersed nanoparticles,
k_f is the thermal conductivity of pure fluid.

Introducing the following dimensionless parameters:

$$X=\frac{x}{H}, Y=\frac{y}{H}, \tau=\frac{\alpha_f\ t}{H^2}, U=\frac{uH}{\alpha_f}, V=\frac{vH}{\alpha_f}, \theta=\frac{T-T_C}{\delta T}, P=\frac{pH^2}{\alpha_{nf}\ \alpha_f^2},$$
$$Ra=\frac{buoyancy}{viscosity}=\frac{g\beta_f\ H^3\Delta T}{\nu_f\ \alpha_f}, Pr=\frac{viscous\ diffusion}{thermal\ diffusion,}=\frac{\nu_f}{\alpha_f}.$$

Then we have the following dimensionless form

$$\begin{cases} \nabla.\mathbf{V}=0 \\ \partial_\tau\mathbf{V}+(\mathbf{V}.\nabla)\,\mathbf{V}=-\nabla P+\frac{\mu_{nf}}{\rho_{nf}\ \alpha_f}\ \Delta\mathbf{V}+\mathbf{F} \\ \partial_\tau\theta+\mathbf{V}.\nabla\theta=\frac{\alpha_{nf}}{\alpha_f}\Delta\theta \end{cases}$$

with

$$\begin{cases} f_X=\frac{(\rho\beta)_{nf}}{\rho_{nf}\ \beta_f}Ra\ \theta\,\mathrm{Pr}\sin\vartheta \\ f_Y=\frac{(\rho\beta)_{nf}}{\rho_{nf}\ \beta_f}Ra\ \theta\,\mathrm{Pr}\cos\vartheta \end{cases}$$

By considering the following definition of non-dimensional stream function and vorticity

$$U = \partial_Y \ \psi, V = -\partial_X \ \psi, \Omega = -\Delta\psi,$$

We have the vorticity as

$$\partial_\tau \Omega + \mathbf{V}.\nabla\Omega = \frac{\mu_{nf}}{\rho_{nf} \ \alpha_f}\Delta\Omega + \frac{(\rho\beta)_{nf}}{\rho_{nf} \ \beta_f} Ra \ \Pr \left(\partial_X \ \theta \cos\vartheta - \partial_Y \ \theta \sin\vartheta\right)$$

The dimensionless boundary conditions are as follows:

$$\textbf{B.C.}\begin{cases}\textbf{Bottom and Top Walls}\{Y=0 \ or \ Y=H, 0\le X\le H: U=V=0, \partial_Y\theta=0 \\[2ex] \begin{matrix}\textbf{Right Wall}\\ (X=H)\end{matrix} \begin{cases} 0\le Y\le (b_R-0.5B_R): \ U=V=0, \partial_X\theta=0\\ (b_R-0.5B_R)\le Y\le (b_R+0.5B_R): (\partial_X V)_{out}=0, \ \theta_{in}=0\\ (b_R+0.5B_R)\le Y\le H: U=V=0, \partial_X\theta=0\end{cases} \\[2ex] \begin{matrix}\textbf{Left Wall}\\ (X=0)\end{matrix} \begin{cases} Case"1" \ \{X=0, 0\le Y\le H: U=V=0, \theta=1 \\[1ex] Case"2" \begin{cases} 0\le Y\le (b_L-0.5B_L): U=V=0, \partial_X\theta=0,\\ (b_L-0.5B_L)\le Y\le (b_L+0.5B_L): \partial_X\theta=k_f \ /k_{nf}\\ (b_L-0.5B_L)\le Y\le (b_L+0.5B_L): U=V=0, \partial_X\theta=0\end{cases} \\[1ex] Case"3" \begin{cases} 0\le Y\le (b_L-0.5B_L): U=V=0, \partial_X\theta=0\\ (b_L-0.5B_L)\le Y\le (b_L+0.5B_L): \partial_X\theta=-k_f \ /k_{nf}\\ (b_L-0.5B_L)\le Y\le (b_L+0.5B_L): U=V=0, \partial_X\theta=0\end{cases}\end{cases}\end{cases}$$

The local Nusselt number along the heated wall:

$$Nu = \frac{Convection\ heat\ transfer}{Conduction\ heat\ transfer} = \frac{hH}{k_f},$$

- For case 1 (uniform heat source) the heat transfer coefficient and the thermal conductivity of the nanofluid are computed from:

$$h = \frac{q_w}{\delta T}, \quad k_{nf} = -\frac{q_w}{\partial_X T}$$

Then, the local Nusselt number along the heated wall is:

$$Nu(Y) = -(k_{nf}/k_f)\ (\partial_X T)_{X=0}$$

- For cases 2,3 the convection heat transfer coefficient $h = \frac{q''}{\Delta T}$ then using the dimensionless parameters, we can get

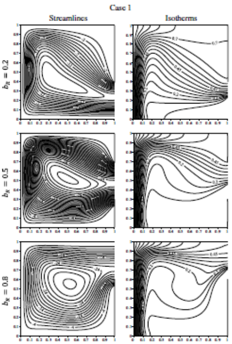
$$Nu(Y) = \left| \frac{1}{\theta_S(Y)} \right|$$

Numerical method and validation

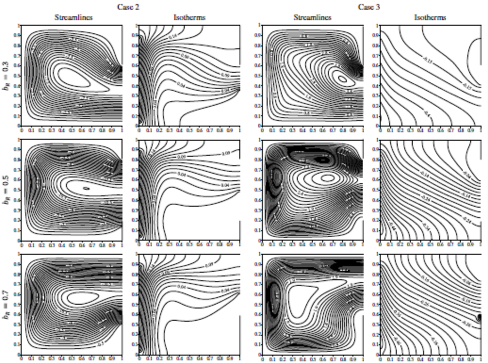
- A fully implicit finite difference method.
- Central difference approaches were used to approximate the first and second derivative ($O(h^2)$).
- Success Under Relaxation(SUR) algorithm was used to solve the obtained algebraic system.
- During each axial step, the numerical evaluation is iterated until the relative errors of U, V and T at sequential iterations are less or equal 10^{-6}.
- The numerical method was implemented in a FORTRAN software.
- The obtained results are plotted in 2D graphs ORIGIN 9 and contour maps SURFER 11.
- This method is found to be suitable and gives results that are very close to previous published results.

Results

Does the location of the aperture affect the flow and heat transfer?



Streamline and isotherm contours for different values of the open part position at $B_R = 0.4$, $Ra = 10^5$, $\theta = \frac{\pi}{6}$, and $\phi = 5\%$.



Streamline and isotherm contours for different values of the open part position at $b_L = 0.2$, $B_L = 0.8$, $B_R = 0.6$, $Ra = 10^5$, $\theta = \frac{\pi}{6}$, and $\phi = 5\%$.

For all cases, the fluid follows the geometry of the cavity by forming one clockwise circular cell inside the enclosure

As b_R increases, strong natural convection is obtained which results in strong flow inside the enclosure

Does the length of the aperture affect the flow and heat transfer?

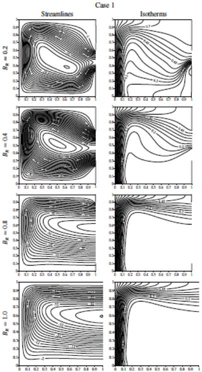


Fig. 7 Streamline and isotherm contours for different values of the open part length at $b_R = 0.5$, $Ra = 10^5$, $\vartheta = \frac{\pi}{6}$ and $\phi = 5\%$.

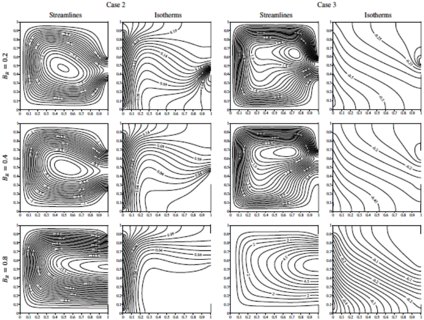


Fig. 8 Streamline and isotherm contours for different values of the open part length at $b_L = 0.5$, $B_L = 0.8$, $b_R = 0.5$, $Ra = 10^5$, $\vartheta = \frac{\pi}{6}$ and $\phi = 5\%$.

The convection is enhanced with superior aperture

The isotherms lines gather beside the left wall as B_R increases which make a thermal region in this place

What about Rayleigh number effects?

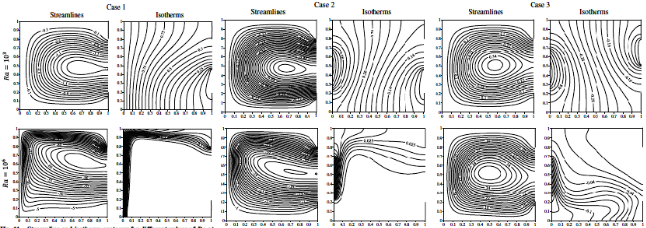


Fig. 11 Streamline and isotherm contours for different values of Ra at $b_R = 0.5$, $B_R = 0.6$, $\vartheta = \frac{\pi}{6}$, and $\phi = 5\%$.

Fig. 12 Streamline and isotherm contours for Rayleigh numbers at $b_L = 0.5$, $B_L = 0.4$, $b_R = 0.5$, $B_R = 0.6$, $\vartheta = \frac{\pi}{6}$, and $\phi = 5\%$.

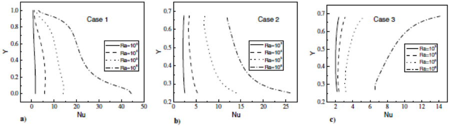


Fig. 13 Profiles of the local Nusselt number for Rayleigh numbers at $b_L = 0.5$, $B_L = 0.4$, $b_R = 0.5$, $B_R = 0.6$, $\vartheta = \frac{\pi}{6}$, and $\phi = 5\%$.

- For $Ra = 10^3$, the flow inside the cavity is so feeble and the viscous forces are dominant over the buoyancy force [weak convection, quasi-conductive heat transfer].
- The heat transfer in the cavity becomes dominated by convective regime as Ra increases.
- A clockwise contours for case 2 while for case 3 it is anticlockwise

The effects of solid volume fraction on Nusselt number

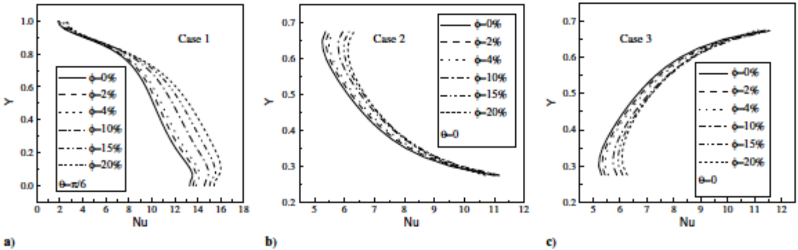
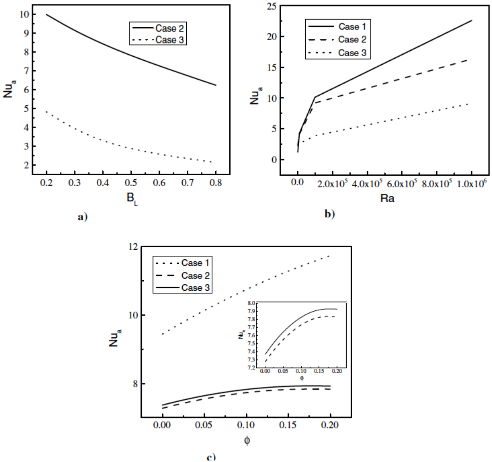


Fig. 16 Profiles of the local Nusselt number for ϕ at $b_L = 0.5$, $B_L = 0.4$, $b_R = 0.5$, $B_R = 0.6$, and $Ra = 10^5$.

For any case, the Nusselt number increases as ϕ increases. This due to the increase in the temperature gradient because of the formulation of the thermal region

Average Nusselt number for different values of the physical parameters



Numerical Investigation for Natural Convection of a Nanofluid in an Inclined L-Shaped Cavity in the Presence of an Inclined Magnetic Field [7]

Problem definition and mathematical formulation

I. In the cavity,

a. Walls A (along x-axis) and F(along y-axis) are considered to be heated walls.
b. Walls B (parallel to y-axis) and E (parallel to x-axis) are kept to be adiabatic.
c. The other two walls (C and D) are considered to be cold.

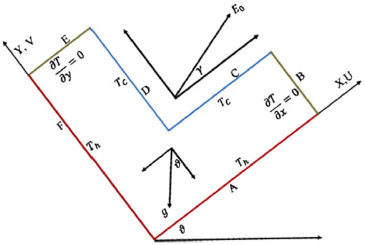


Figure 2: The physical model of the problem

II. The base fluid (water) and the solid spherical nanoparticles (Cu) are in thermal equilibrium.

III Boussinesq approximation is used to determined the variation of density in the buoyancy term.

IV. The cavity is permeated by a uniform magnetic field $\mathbf{B} = \mathbf{B}_x e_x + \mathbf{B}_y e_y$ of constant magnitude $\mathbf{B}_0 = \sqrt{\mathbf{B}_x^2 + \mathbf{B}_y^2}$

V. The direction of the magnetic field makes an angle γ with $X-$axis.

VI. According to Ohm's law, law of conservation of charge and Lorentz force, the electric current density $\mathbf{J}$ and the electromagnetic force $\mathbf{F}$ are given by the relations

$$\mathbf{J} = \sigma_l(-\nabla\varphi + \mathbf{V} \times \mathbf{B}), \ \nabla.\mathbf{J} = \mathbf{0}, \ \mathbf{F} = \mathbf{J} \times \mathbf{B} \tag{1}$$

For electrically non conducting boundaries [φ (constant)]; So

$$\mathbf{J} = \sigma_l(\mathbf{V} \times \mathbf{B}), \ \mathbf{F} = \sigma_l(\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \tag{2}$$

Under the above assumptions, the continuity, momentum and energy equations in the dimensional form are:

$$\begin{cases} \nabla.\mathbf{v} = 0 \\ \partial_t \mathbf{v} + (\mathbf{v}.\nabla)\,\mathbf{v} = \frac{1}{\rho_{nf}}\left(-\nabla p + \mu_{nf}\ \Delta \mathbf{v} + \mathbf{f}\right) \\ \partial_t T + \mathbf{v}.\nabla T = \alpha_{nf}\ \Delta T \end{cases}$$

where

$$\begin{cases} f_x = g(\rho\beta)_{nf}(T - T_c)\sin\vartheta + \sigma_{nf}\ B_0^2(v\sin\gamma\cos\gamma - u\sin^2\gamma) \\ f_y = g(\rho\beta)_{nf}(T - T_c)\cos\vartheta + \sigma_{nf}\ B_0^2(u\sin\gamma\cos\gamma - v\cos^2\gamma) \end{cases}$$

Introducing the following dimensionless parameters:

$$X = \frac{x}{H}, Y = \frac{y}{H}, \tau = \frac{\alpha_f\ t}{H^2}, U = \frac{uH}{\alpha_f}, V = \frac{vH}{\alpha_f}, \theta = \frac{T - T_C}{T_h - T_c}, P = \frac{pH^2}{\alpha_{nf}\ \alpha_f^2},$$

$$Ha = \frac{electromagnetic\ \ force}{viscous\ \ force} = B_0 H\sqrt{\frac{\sigma_f}{\mu_f}},\ Ra = \frac{g\beta_f\ H^3\Delta T}{\nu_f\ \alpha_f}, Pr = \frac{\nu_f}{\alpha_f}$$

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Then the governing equation in the following dimensionless form:

$$\begin{cases} \nabla.\mathbf{V} = 0 \\ \partial_\tau \mathbf{V} + (\mathbf{V}.\nabla)\,\mathbf{V} = -\nabla P + \frac{\mu_{nf}}{\rho_{nf}\ \alpha_f}\ \Delta\mathbf{V} + \mathbf{F} \\ \partial_\tau \theta + \mathbf{V}.\nabla\theta = \frac{\alpha_{nf}}{\alpha_f}\Delta\theta \end{cases}$$

where

$$\begin{cases} f_X = \frac{(\rho\beta)_{nf}}{\rho_{nf}\ \beta_f} Ra\ \theta \Pr \sin\vartheta + Ha^2 \Pr \frac{\rho_f}{\rho_{nf}} \\ \qquad (V\sin\gamma\cos\gamma - U\sin^2\gamma)\left(1 + \frac{3\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_p}{\sigma_f}+2\right)-\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}\right) \\ f_Y = \frac{(\rho\beta)_{nf}}{\rho_{nf}\ \beta_f} Ra\ \theta \Pr \cos\vartheta + Ha^2 \Pr \frac{\rho_f}{\rho_{nf}} \\ \qquad (U\sin\gamma\cos\gamma - V\cos^2\gamma)\left(1 + \frac{3\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_p}{\sigma_f}+2\right)-\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}\right) \end{cases}$$

- Considering the stream function $U = \partial_Y \psi$, $V = -\partial_X \psi$, vorticity $\Omega = -\Delta\psi$. Momentum equation is

The dimensionless boundary conditions

$$\left.\begin{array}{ll} U = V = 0, \theta = 1, & \text{(on hot walls } A, F) \\ U = V = 0, \theta = 0\ , & \text{(on the cold walls } C, D) \\ U = V = 0, \partial_S\ \theta = 0 & \text{(on adiabatic walls } B, E) \end{array}\right\}$$

$$\left.\begin{array}{c} \partial_\tau \Omega + \mathbf{V}.\nabla\Omega = \frac{\mu_{nf}}{\rho_{nf}\ \alpha_f}\Delta\Omega + \frac{(\rho\beta)_{nf}}{\rho_{nf}\ \beta_f} Ra\ \Pr\left(\partial_X \cos\vartheta - \partial_Y \sin\vartheta\right) \\ +Ha^2 \Pr \frac{\rho_f}{\rho_{nf}} \left(1 + \frac{3\left(\frac{\sigma_p}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_p}{\sigma_f} + 2\right) - \left(\frac{\sigma_p}{\sigma_f} - 1\right)\phi}\right) \\ \left(\partial_X U \sin(2\gamma) - \partial_X V \cos^2\gamma + \partial_Y U \sin^2\gamma\right) \end{array}\right\}$$

Numerical method and validation

- Fully implicit F. D.
- SUR algorithm.
- Relative errors $< 10^{-7}$.
- Comparison.

Comparison of average Nusselt number with the previous work.

Ra	Tasnim, Mahmud [8]	Mahmoodi [9]	Present Study
10^3	3.270	3.251	3.4342
10^4	3.259	3.255	3.4372
10^5	3.855	3.903	3.8693
10^6	9.340	9.331	9.3661

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Some results

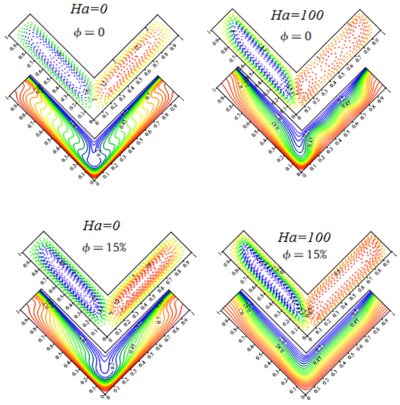
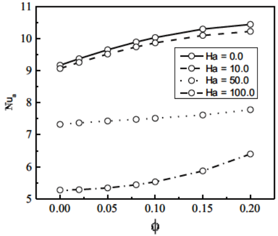


Figure 3: Streamlines (dash lines) and isotherms (solid lines) contours for different Ha and ϕ.



The average Nusselt number for different values of volume fraction ϕ with different values of Ha at $Ra = 10^6$, $\vartheta = \frac{\pi}{4}$, $\gamma = \frac{\pi}{3}$ and $AR = 0.25$.

Adding the nanoparticles to the base fluid causes an enhancement of the heat transfer rate.
An increase in solid volume fraction causes in a decrease in the intensity of buoyancy and the flow intensity, so the maximum values of stream function decreases as ϕ increases.

Explanation??

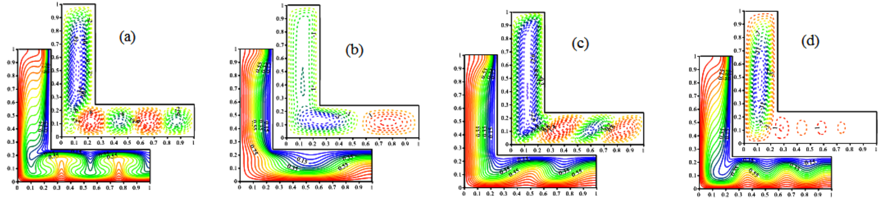


Figure 4: Streamlines (dash lines) and isotherms (solid lines) contours for (a)$Ha = 0$, (b)$Ha = 100$ and $\gamma = 0$, (c)$Ha = 100$ and $\gamma = 30^o$ and (d) $Ha = 100$ and $\gamma = 90^o$

$\Longrightarrow$The presence of the magnetic field leads to a reduction in the natural convection.

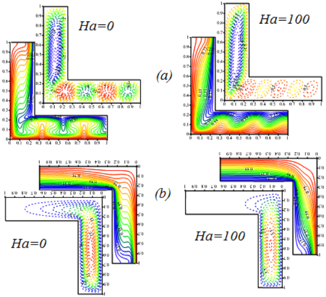


Figure 5: Streamlines (dash lines) and isotherms (solid lines) contours $\vartheta = 0, 180^o$

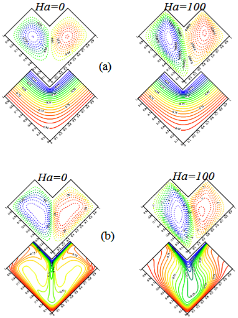


Figure 6: Streamlines (dash lines) and isotherms (solid lines) contours $Ra = 10^3, 10^6$

$\Longrightarrow$ For ($Ra = 10^3$), a weak convection - the isotherm lines parallel to the two channels. For ($Ra = 10^6$) significant convection effects which formed the boundary thermal region along the two hot walls.

MHD Mixed Convection in a Lid-driven Cavity Filled by a Nanofluid with Sinusoidal Temperature Distribution on the both Vertical Walls using Buongiorno's Nanofluid Model [10]

A 2D steady flow inside a square cavity filled with nanofluid.

Assumptions:

I. Sinusoidal temperature and nanoparticles distributions are imposed on the left and right walls.
II. The top and bottom walls are insulated walls.
III. The top wall of the cavity is moving with constant speed U_p - all other walls have no speed.

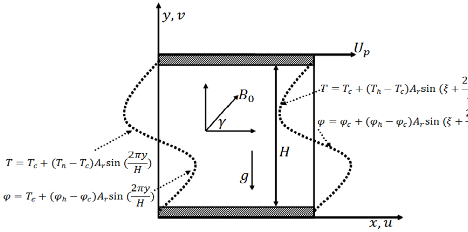


Figure 7: The physical model

IV. Thermophoresis and Brownian motion effects are included in our study in the absence of chemical reaction [11]

V. The base fluid (water) and the solid nanoparticles are in thermal equilibrium.

VI. Boussinesq approximation is used to determined the variation of density in the buoyancy term where the other thermo-physical properties of the nanofluid are assumed constant.

VII. The cavity is permeated by a uniform magnetic field $\mathbf{B} = \mathbf{B}_x e_x + \mathbf{B}_y e_y$ of constant magnitude $\mathbf{B}_0 = \sqrt{\mathbf{B}_x^2 + \mathbf{B}_y^2}$, ($e_x$, e_y the unit vectors).

VIII. The direction of the magnetic field makes an angle γ with $X-$axis.

Under the above assumptions we have

Continuity

$$\nabla.\mathbf{v} = 0$$

Momentum equation

$$(\mathbf{v}.\nabla)\,\mathbf{v} = -\frac{1}{\rho_f}\left(\nabla p + \mu_f \ \Delta\mathbf{v} + \mathbf{f}\right);$$

$$f_x = \sigma_f \ B_0^2(v\sin\gamma\cos\gamma - u\sin^2\gamma),$$

$$f_y = \sigma_f \ B_0^2(u\sin\gamma\cos\gamma - v\cos^2\gamma)$$

$$+(1-\varphi_c)\rho_{f_0}(T - T_c)\beta_f \ g - (\varphi - \varphi_c)\,(\rho_p - \rho_{f_0})\,g$$

Energy equation

$$\mathbf{v}.\nabla T = \ \alpha_f \ \Delta T + \frac{(\rho c)_p}{(\rho c)f}\left[D_B\left(\nabla\varphi \ .\nabla T\right) + \frac{D_T}{T_c}\left|\nabla T\right|^2\right]$$

Conservation equation for the nanoparticles

$$\mathbf{v}.\nabla\varphi = \ D_B\Delta\varphi + \frac{D_T}{T_c}\Delta T$$

Introducing the following dimensionless parameters:

$$X = \frac{x}{H},\ \ Y = \frac{y}{H},\ \ U = \frac{u}{U_p},\ \ V = \frac{v}{U_p},$$
$$\theta = \frac{T - T_C}{T_h - T_c}, \phi = \frac{\varphi - \varphi_C}{\varphi_h - \varphi_c},\ \ P = \frac{p}{\rho_f U_p^2}.$$

Then we have the governing equation in the dimensionless form as follows

$$\begin{cases} \nabla.\mathbf{V} = 0 \\ \\ (\mathbf{V}.\nabla)\,\mathbf{V} = -\nabla P + \frac{1}{Re}\ \Delta \mathbf{V} + \mathbf{F}; \\ f_X = \frac{Ha^2}{Re}(V \sin\gamma \cos\gamma - U \sin^2\gamma, \\ f_Y = \ Ri\,(\theta - Nr\phi) + \frac{Ha^2}{Re}(U \sin\gamma \cos\gamma - V \cos^2\gamma) \\ \\ \mathbf{V}.\nabla\theta = \frac{1}{\mathrm{Pr}\,.Re}\ \Delta\theta + \frac{Nb}{Re.\,\mathrm{Pr}}\,(\nabla\phi\ .\nabla\theta) + \frac{Nt}{Re.\,\mathrm{Pr}}\,|\nabla\theta|^2 \\ \\ \mathbf{V}.\nabla\phi = \frac{1}{Re.\,\mathrm{Pr}\,.Le}\ \Delta\phi + \frac{Nt}{Nb.Re.\,\mathrm{Pr}\,.Le}\ \Delta\theta \end{cases}$$

Here,

$$[Reynold\ \ Number] \rightarrow Re = \frac{inertial\ \ forces}{viscous\ \ forces} = \frac{\rho_f U_p H}{\mu_f},$$
$$[Richardson\ \ Number] \rightarrow Ri = \frac{buoyancy\ \ term}{flow\ \ gradient\ \ term}\frac{Gr}{Re^2},$$
$$[Grashof\ \ Number] \rightarrow Gr = \frac{buoyancy}{viscous} = \frac{g\beta_f\ \ H^3(1-\varphi_C)(T_h-T_c)}{\nu_f},$$
$$[Lewis\ \ Number] \rightarrow Le = \frac{thermaldiffusivity}{massdiffusivity} = \frac{\alpha_f}{D_B},$$
$$[Prandtl\ \ Number] \rightarrow \Pr = \frac{\nu_f}{\alpha_f},$$
$$[Hartmann\ \ Number] \rightarrow Ha = B_0 H\sqrt{\frac{\sigma_f}{\mu_f}},$$
$$[Thermophoresis\ \ Number] \rightarrow Nt = \frac{D_T}{T_c}\frac{(\rho c)_p}{(\rho c)f}\frac{(T_h-T_c)}{\alpha_f},$$
$$[Brownian\ \ motion\ \ parameter] \rightarrow Nb = D_B\frac{(\rho c)_p}{(\rho c)f}\left(\frac{(\varphi_h-\varphi_c)}{\alpha_f}\right),$$
$$[Buoyancy\ \ ratio] \rightarrow Nr = \frac{(\varphi_h-\varphi_c)\left(\rho_p-\rho_{f_0}\right)}{\beta(1-\phi_c)\rho_{f_0}(T_h-T_c)}.$$

- Considering the non-dimensional stream function and vorticity $U = \frac{\partial\psi}{\partial Y}, V = -\frac{\partial\psi}{\partial X}, \Omega = -\Delta\psi,$ then,

$$\mathbf{V}.\nabla\Omega - \frac{1}{Re}\Delta\Omega = Ri\left(\partial_X\theta - Nr\ \partial_X\phi\right)$$
$$+\frac{Ha^2}{Re}\left(\partial_X U\sin(2\gamma) - \partial_X V\cos^2\gamma + \partial_Y U\sin^2\gamma\right)$$

Boundary Conditions (dimensionless form):

- Top wall:

$$U = 1,\ V = \psi = 0,\ \Omega = -\partial_{YY}\ \psi,\ \partial_Y \theta = 0, \partial_Y \phi = 0$$

- Bottom wall:

$$U = V = \psi = 0,\ \Omega = -\partial_{YY}\ \psi,\ \partial_Y \theta = 0,\ \partial_Y \phi = 0$$

- Left wall:

$$U = V = \psi = 0,\ \Omega = -\partial_{XX}\ \psi,\ \theta = \sin(2\pi Y),\ \phi = \sin(2\pi Y)$$

- Right wall:

$$U = V = \psi = 0,\ \Omega = -\partial_{XX}\ \psi,\ \theta = \varepsilon \sin(2\pi Y + \xi),\ \phi = \varepsilon \sin(2\pi Y + \xi)$$

Numerical method and validation

- Finite volume method.
- The upwind differencing scheme is adopted for the convective terms
- A second-order central difference approach is used to approximate the diffusion term.$\rightarrow$ stable solution.
- The algebraic finite volume equations are written into the following implicit line tridiagonal form

$$\Gamma_N\ \Pi_N - \Gamma_P\ \Pi_P + \Gamma_S\ \Pi_S = -\Gamma_W\ \Pi_W - \Gamma_E\ \Pi_E - \delta$$

- The resulting algebraic equations are solved using the tridiagonal matrix algorithm (TDMA).
- A uniform grid has been selected in both X- and Y-directions.

- The grid sizes are tested from 21×21 to 141×141 for $Ri = 1.0$ and 100 with $\mathrm{Pr} = 0.054$ and 6.2;
- It is observed that, the 101×101 grid is sufficiently enough to obtain an independent grid size solution.
- The numerical method was implemented in a FORTRAN software.
- The obtained results are plotted in 2D graphs and contour maps by using ORIGIN 9 software and SURFER 11 software, respectively.

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Acknowledgment

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