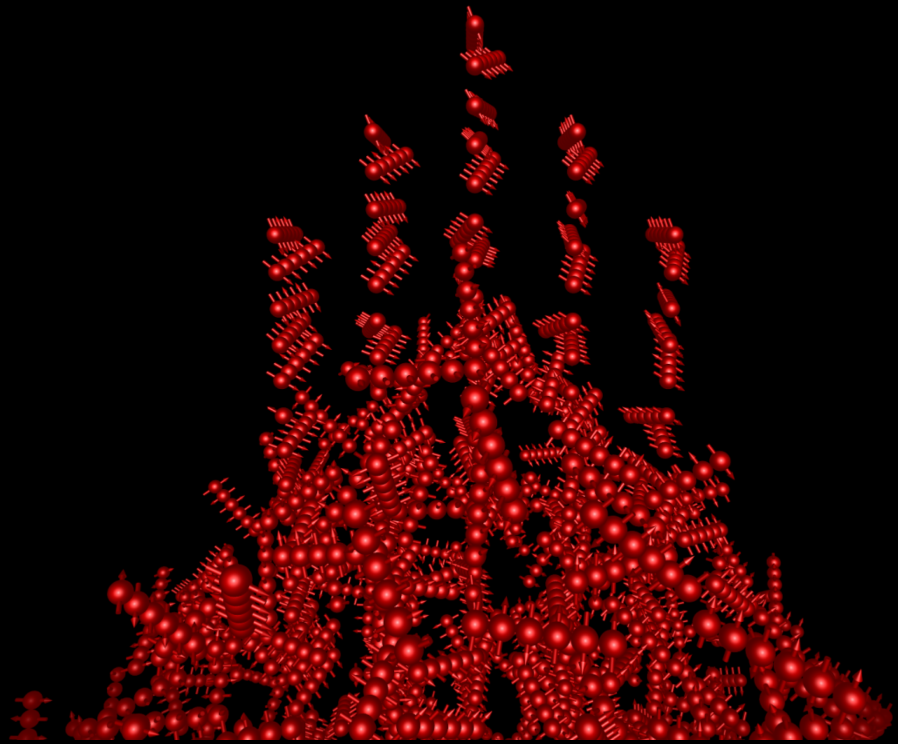
Quantum many-body scars:
a new form of weak ergodicity breaking in constrained quantum systems

Zlatko Papic



C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics 14, 745 (2018);
Phys. Rev. B 98, 155134 (2018)
(see also viewpoints by V. Dunjko & M. Olshanii, *Escape the thermal fate*
& N. Robinson, *Cold Atoms Bear a Quantum Scar*)

Quantum Matter Meets Math, Zoom seminar, 10/06/2020

UNIVERSITY OF LEEDS

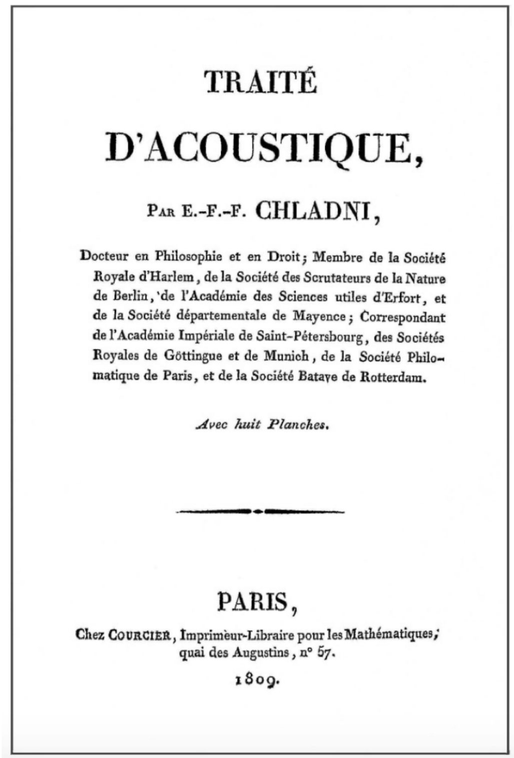
UKRI Engineering and Physical Sciences Research Council

What is a quantum scar?

Ernst Chladni

Chladni received 6000 francs from Napoleon, with the request to publish the *Akustik* in French

Napoleon



TRAITÉ

D'ACOUSTIQUE,

PAR E.-F.-F. CHLADNI,

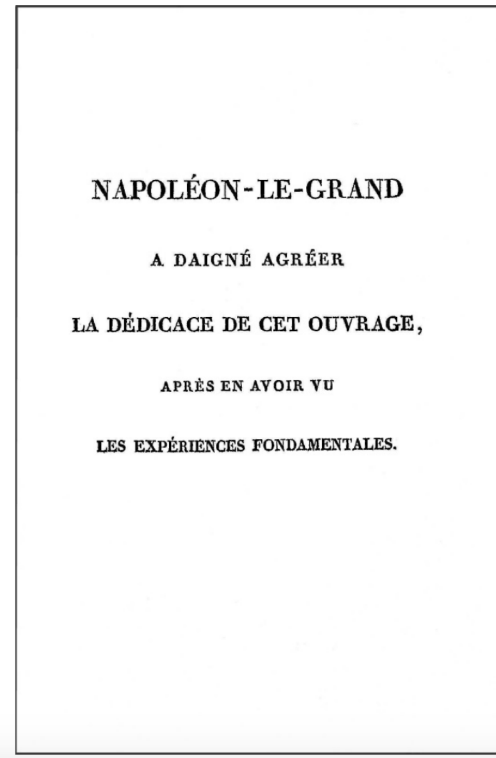
Docteur en Philosophie et en Droit; Membre de la Société Royale d'Harlem, de la Société des Scrutateurs de la Nature de Berlin, de l'Académie des Sciences utiles d'Erfort, et de la Société départementale de Mayence; Correspondant de l'Académie Impériale de Saint-Pétersbourg, des Sociétés Royales de Göttingue et de Munich, de la Société Philomatique de Paris, et de la Société Bataye de Rotterdam.

Avec huit Planches.

PARIS,

Chez COURCIER, Imprimèur-Libraire pour les Mathématiques; quai des Augustins, n° 57.

1809.



NAPOLÉON-LE-GRAND

A DAIGNÉ AGRÉER

LA DÉDICACE DE CET OUVRAGE,

APRÈS EN AVOIR VU

LES EXPÉRIENCES FONDAMENTALES.

Napoleon also gave a prize of 3000 francs for the theory of the sound patterns.

In 1816 this prize was awarded to Sophie Germain

What is a quantum scar?

WIKIPEDIA
The Free Encyclopedia

Scar (physics)

From Wikipedia, the free encyclopedia

In physics, and especially quantum chaos, a wavefunction **scar** is an enhancement (i.e. increased norm squared) of an eigenfunction along unstable classical periodic orbits in classically chaotic systems .They were discovered and explained in 1984 by E.J. Heller[1] and are part of the large field of quantum chaos. Scars are unexpected in the sense that stationary classical distributions at the same energy are completely uniform in space with no special concentrations along periodic orbits, and quantum chaos theory of energy spectra gave no hint of their existence. Scars stand out to the eye in some eigenstates of classically chaotic systems, but are quantified by projection of the eigenstates onto certain test states, often Gaussians, having both average position and average momentum along the periodic orbit. These test states give a provably structured spectrum that reveals the necessity of scars, especially for the shorter and least unstable periodic orbits.[2][3]

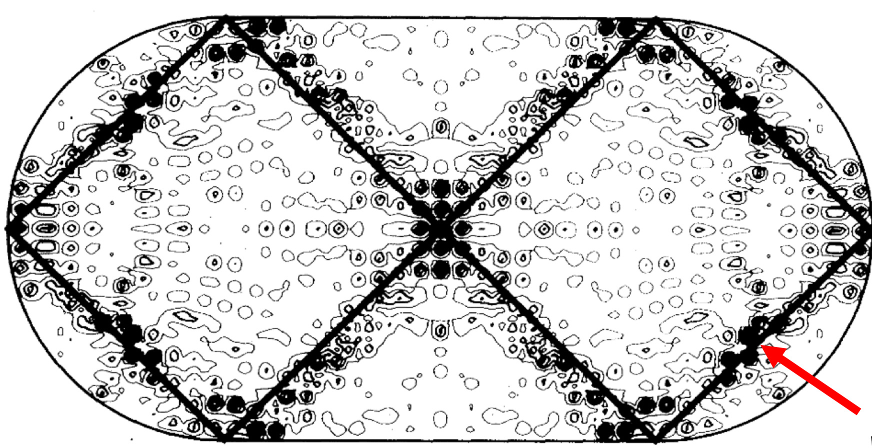
Scars have been found and are important in membranes[4], wave mechanics, optics, microwave systems, water waves, and electronic motion in microstructures.

References [edit]

1. ^ Heller, Eric J. (15 October 1984). "Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits". *Physical Review Letters*. **53** (16): 1515–1518. Bibcode:1984PhRvL..53.1515H. doi:10.1103/PhysRevLett.53.1515.

Eric Heller

$|\psi|^2$



[Gerard, Leichtman, Shnirelman, Zeldich, Colin de Verdiere, Rudnick, …]

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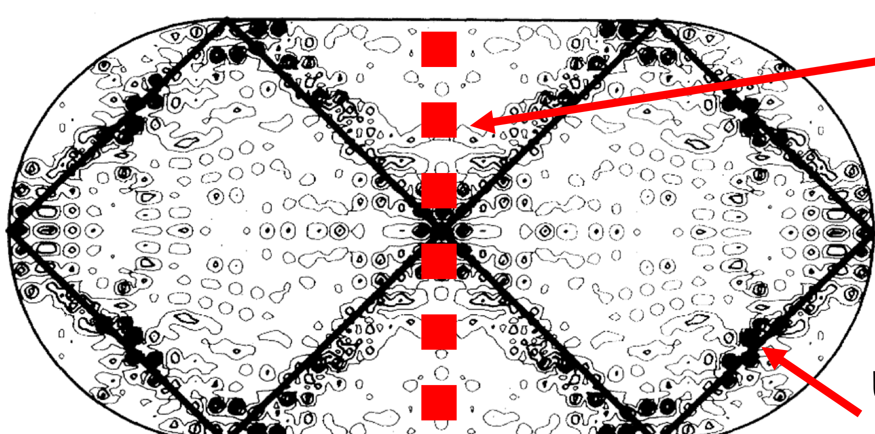

"Quasimodes"

$$\tilde{\psi}(x, y) = \phi(x)\sin(\pi n y/L)$$

Not an eigenstate but has high overlap with O(1) eigenstates

Unstable classical periodic orbit

[Gerard, Leichtman, Shnirelman, Zeldich, Colin de Verdiere, Rudnick, …]

What is a quantum scar?

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Does this have any analogue in a quantum many-body system?

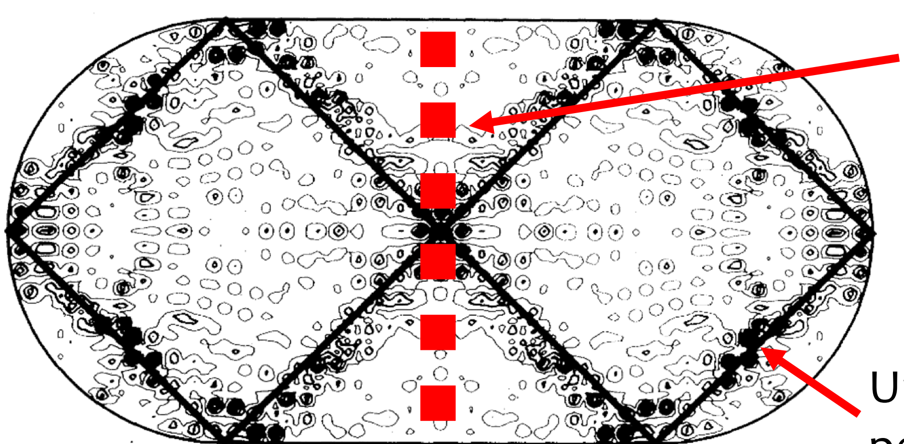
waves, and electronic motion in microstructures.

References [edit]

1. ^ Heller, Eric J. (15 October 1984). "Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits". *Physical Review Letters*. **53** (16): 1515–1518. Bibcode:1984PhRvL..53.1515H. doi:10.1103/PhysRevLett.53.1515.

Eric Heller

$|\psi|^2$



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[Gerard, Leichtman, Shnirelman, Zeldich, Colin de Verdiere, Rudnick, …]

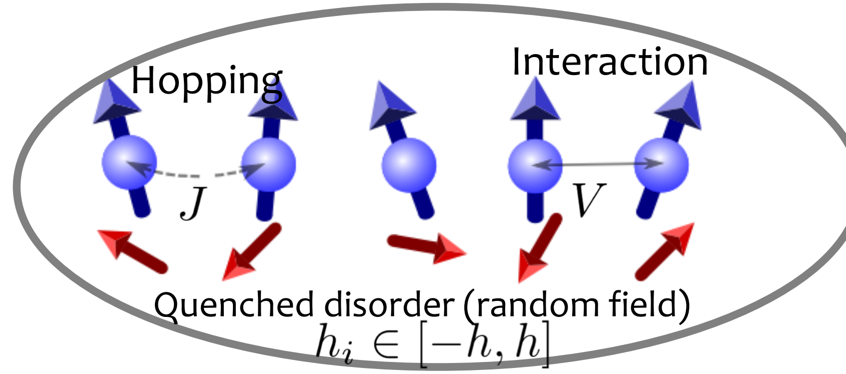
Outline

- Motivation: many-body dynamics in quantum simulators
- The model: “Fibonacci chain”
- Many-body scars in the Fibonacci chain:
  quasimodes and periodic orbits
- Future directions

Motivation #1: Understand generic behavior of closed quantum systems

What is the generic behavior of isolated quantum many-body systems at arbitrary energy density?
(open problem even in 1D)

Example: (isolated) disordered spin chain



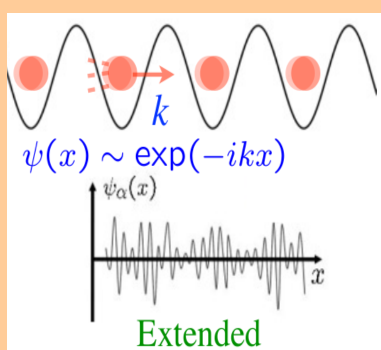
Experimental probe: global quench

1. Prepare a simple initial state $\psi_0 = |\ldots\uparrow\downarrow\uparrow\downarrow\ldots\rangle$
2. Evolve with a known Hamiltonian $\psi(t) = e^{-\frac{i}{\hbar}tH}\psi_0$

One body

Complexity increases linearly with the number of lattice sites $\sim L$

Wavefunction has direct real space interpretation



vs.

Many body

Complexity increases exponentially with the number of lattice sites $\sim 2^L$

Wavefunction lives in Fock space; no direct real space interpretation

$|\uparrow\downarrow\uparrow\ldots\rangle$

$|\uparrow\uparrow\downarrow\ldots\rangle$

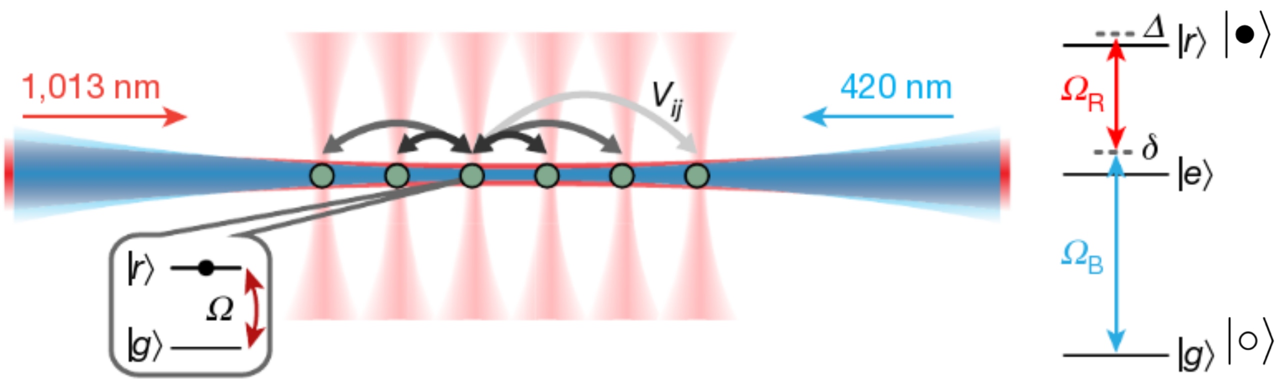
$|\downarrow\downarrow\uparrow\ldots\rangle$

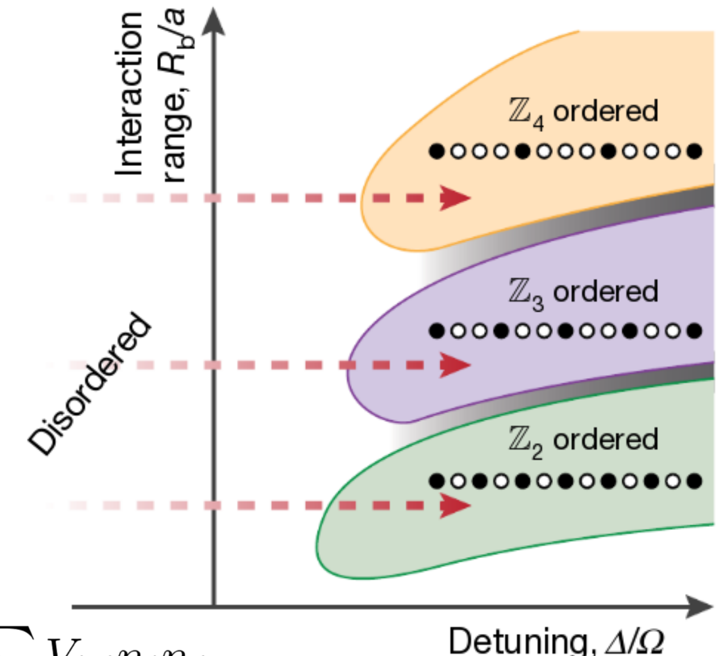
Motivation #2: Many-Body Dynamics in Quantum Simulators

doi:10.1038/nature24622

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien[1], Sylvain Schwartz[1,2], Alexander Keesling[1], Harry Levine[1], Ahmed Omran[1], Hannes Pichler[1,3], Soonwon Choi[1], Alexander S. Zibrov[1], Manuel Endres[4], Markus Greiner[1], Vladan Vuletić[2] & Mikhail D. Lukin[1]





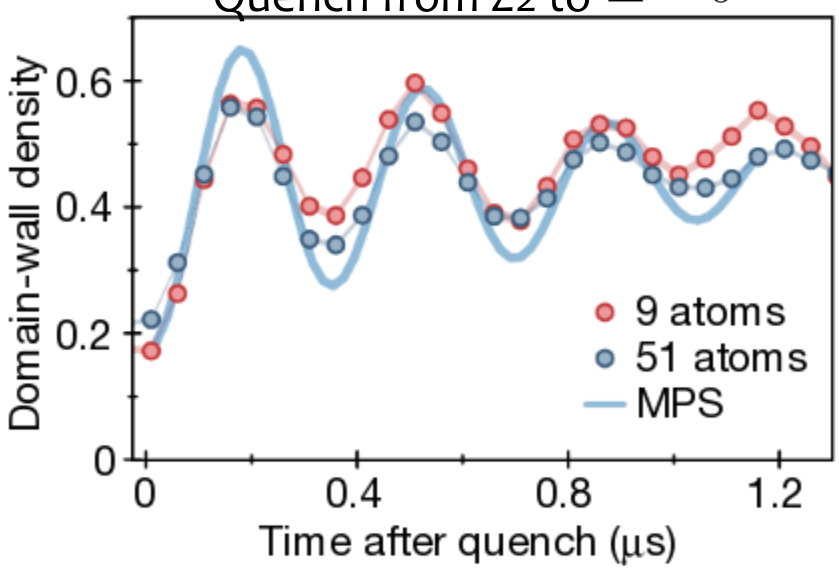
[also, the 53-qubit simulator with trapped ions: J. Zhang *et al.*, Nature **551**, 601 (2017)]

$$H = \sum_j \left(\frac{\Omega}{2} X_j - \Delta \cdot n_i\right) + \sum_{i<j} V_{i,j} n_i n_j$$

$$|\circ_i\rangle \rightarrow n_i = 0$$
$$|\bullet_i\rangle \rightarrow n_i = 1$$

$$V_{i,j} \sim 1/r_{i,j}^6$$

Quench from Z2 to $\Delta = 0$



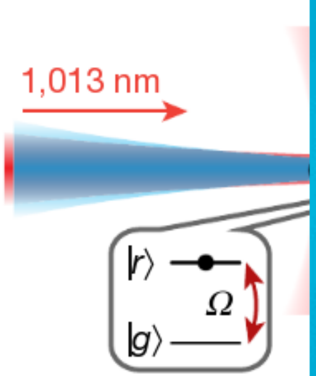
Even more striking is the coherent and persistent oscillation of the crystalline order after the quantum quench. With respect to the quenched Hamiltonian ($\Delta = 0$), the energy density of our $\mathbb{Z}_2$-ordered state corresponds to that of an infinite-temperature ensemble within the manifold constrained by Rydberg blockade. Also, our Hamiltonian does not have any explicitly conserved quantities other than total energy. Nevertheless, the oscillations persist well beyond the natural timescale of local relaxation ($1/\Omega$) and the fastest timescale ($1/V_{i,i+1}$).

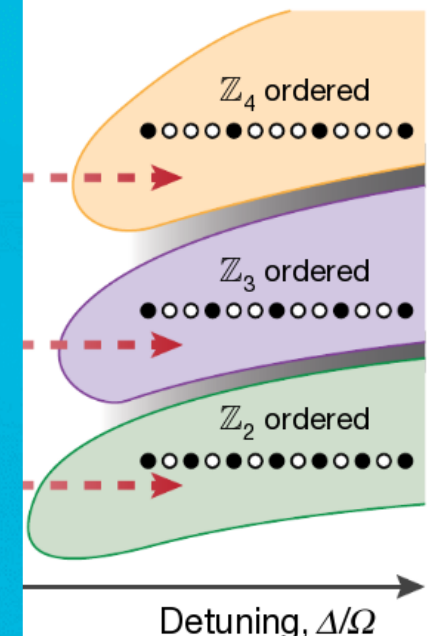
Motivation #2: Many-Body Dynamics in Quantum Simulators

doi:10.1038/nature24622

Probing m
51-atom q

Hannes Bernien[1], Sylvain Schw
Alexander S. Zibrov[1], Manuel E





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J. Zhang *et al.*, Natu

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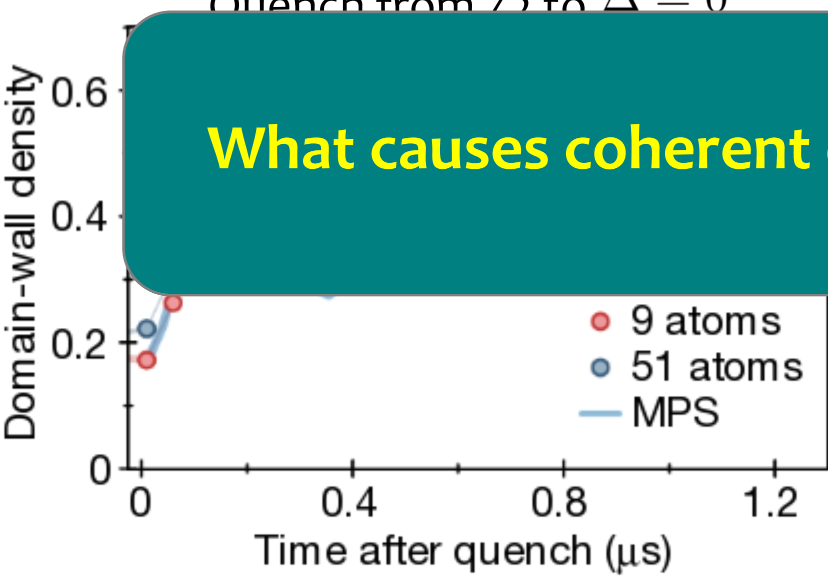

What causes coherent oscillations in a thermalizing system?

...lation ...espect ...of our $\mathbb{Z}_2$-ordered state corresponds to that of an infinite-temperature ensemble within the manifold constrained by Rydberg blockade. Also, our Hamiltonian does not have any explicitly conserved quantities other than total energy. Nevertheless, the oscillations persist well beyond the natural timescale of local relaxation ($1/\Omega$) and the fastest timescale ($1/V_{i,i+1}$).

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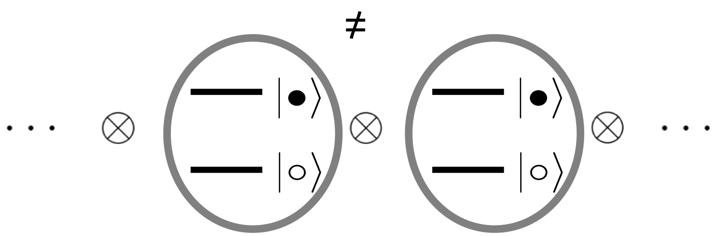
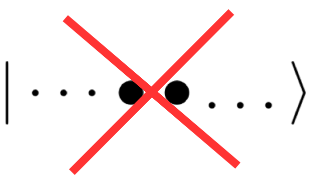
Effective model: Fibonacci chain

For homogeneous couplings
in the limit $V_{i,i+1} \gg \Omega \gg \Delta$:

Hilbert space





Remove such states because they cost ∞ energy

"Fibonacci chain" $\dim \sim \phi^L$

[Lesanovsky and Katsura, PRA **86**, 041601 (2012)]

$$H = \sum_j P_{j-1} X_j P_{j+1}$$

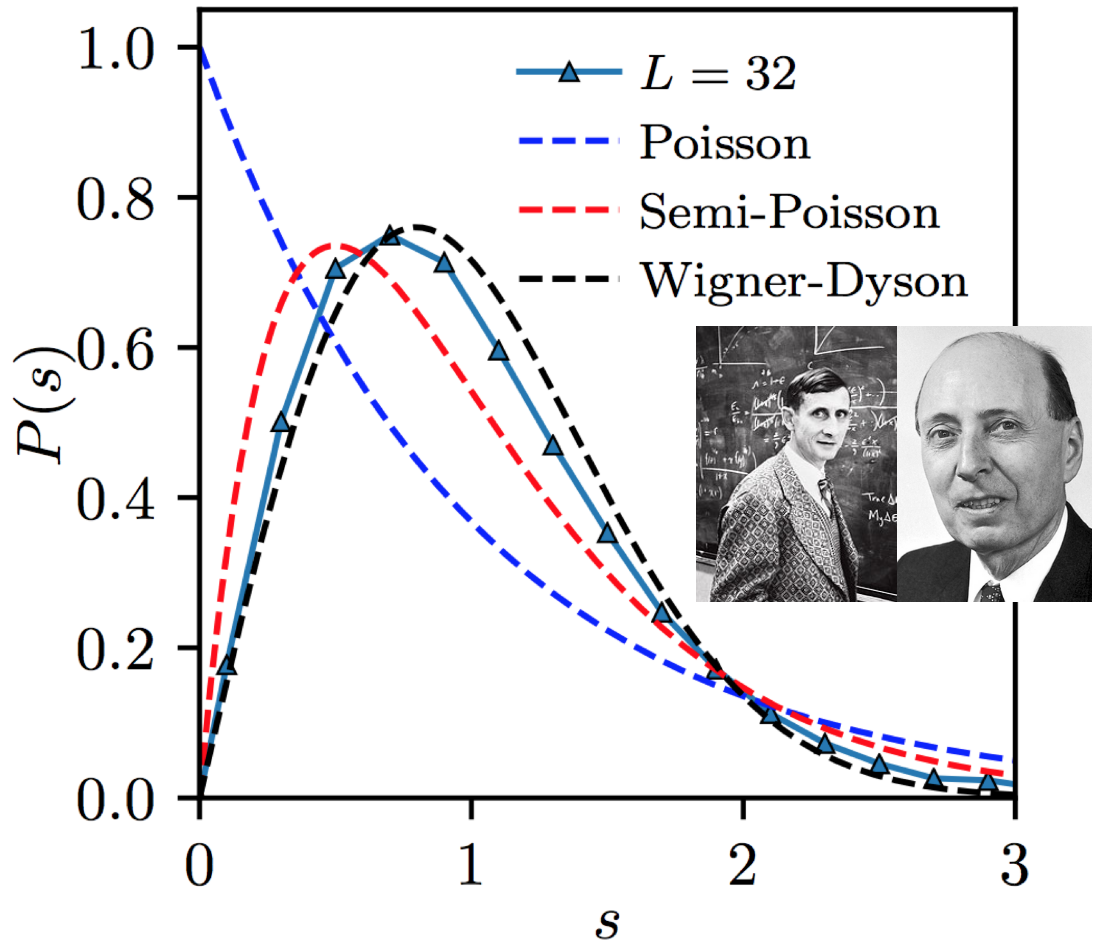
"**PXP model**"

Projector
$P = |\circ\rangle\langle\circ|$

$X = |\circ\rangle\langle\bullet| + |\bullet\rangle\langle\circ|$

$$|\cdots \circ\circ\circ \ldots\rangle \leftrightarrow |\cdots \circ\bullet\circ \ldots\rangle$$

The PXP model is **chaotic**:



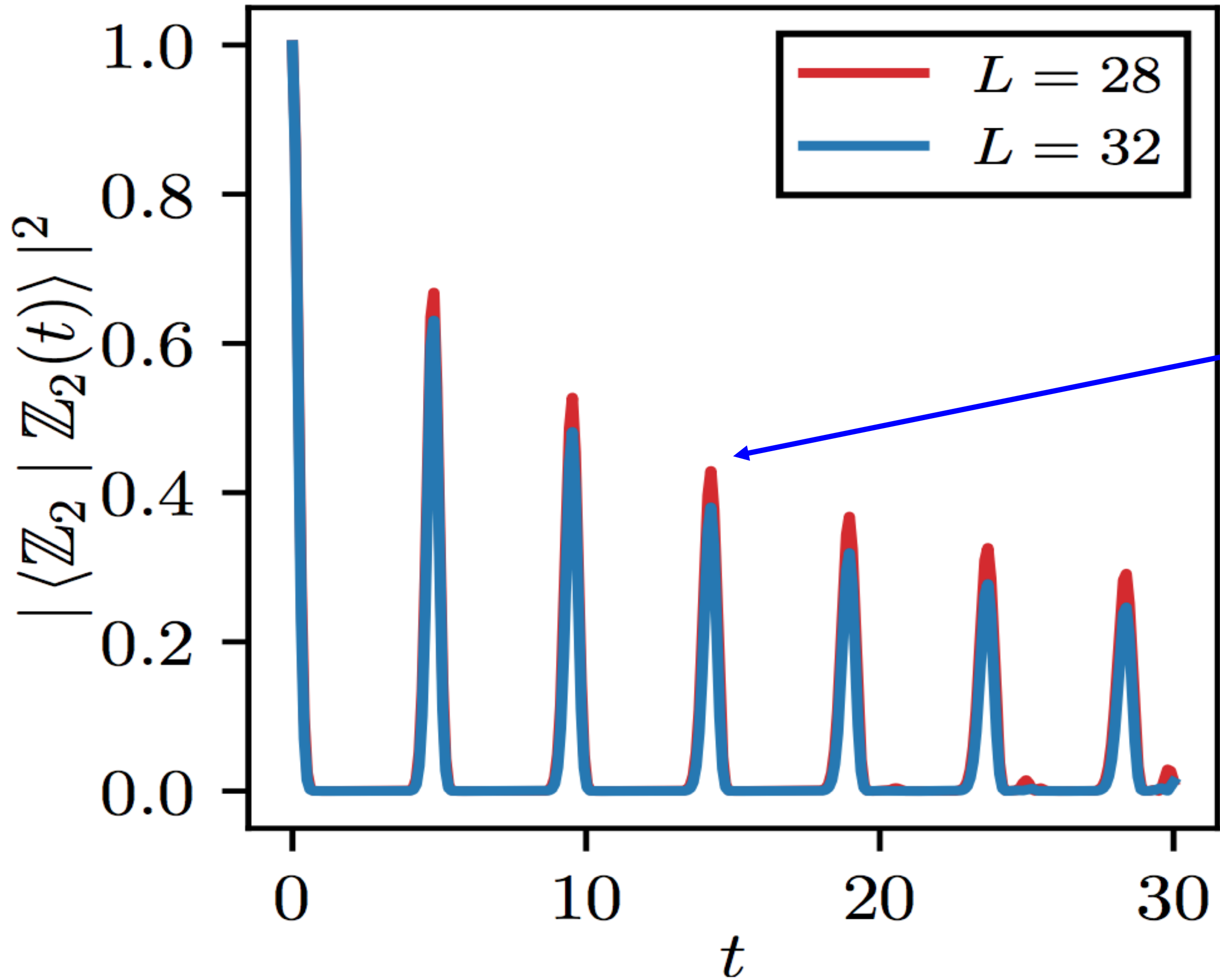
[C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics **14**, 745 (2018)]

Model has a rich ground state phase diagram under perturbations:

[Fendley, Sengupta, Sachdev, PRB **69**, 075106 (2004)]

A puzzle: oscillatory quench dynamics in a thermalizing system

$$|\mathbb{Z}_2\rangle = |\circ\bullet\circ\bullet\ldots\rangle \xrightarrow{\exp(-(i/\hbar)tH)} |\mathbb{Z}_2(t)\rangle = e^{-\frac{i}{\hbar}tH}|\mathbb{Z}_2\rangle$$



Entire many-body wavefunction (superposition of 70 000 states!) returns to itself many times

Entanglement entropy, correlation functions, local observables also revive with the same frequency

[C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics **14**, 745 (2018)]

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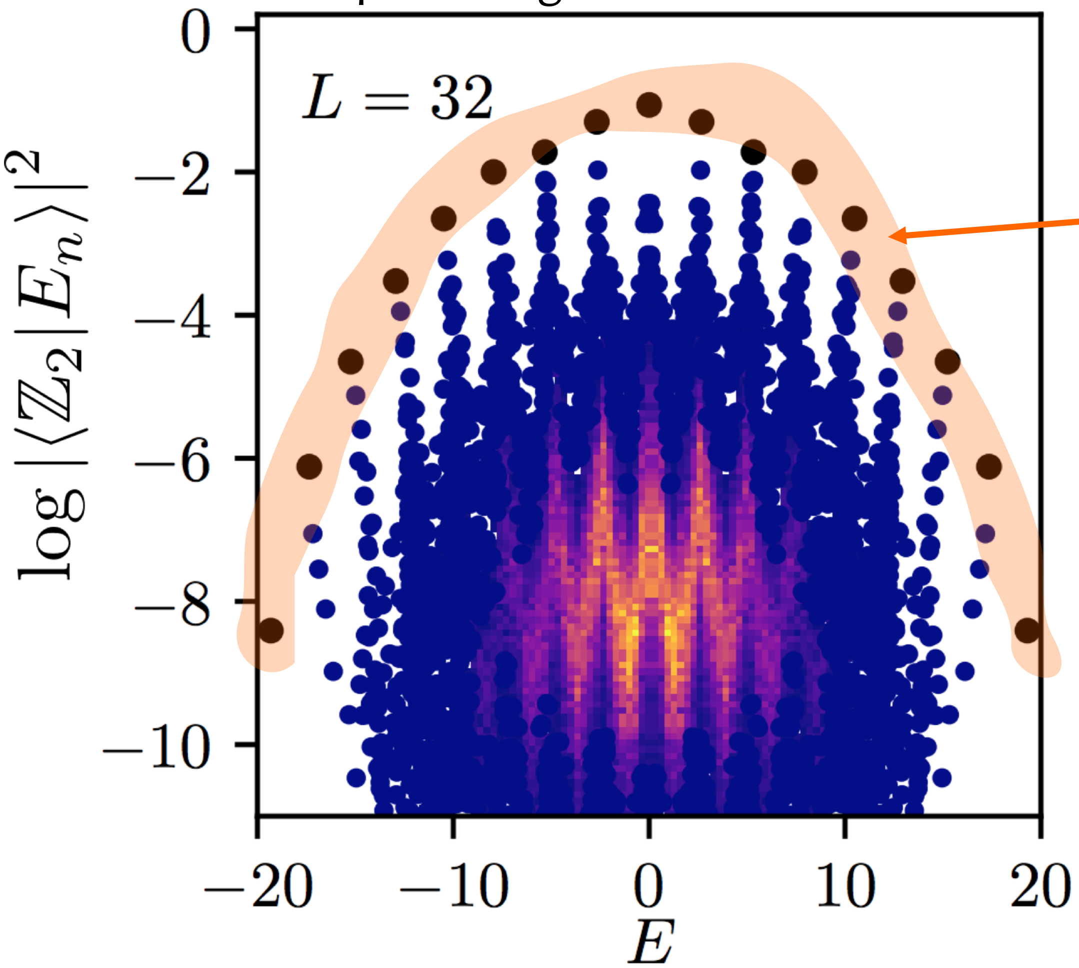
From dynamics to eigenstates

Overlap of all eigenstates with Neel state



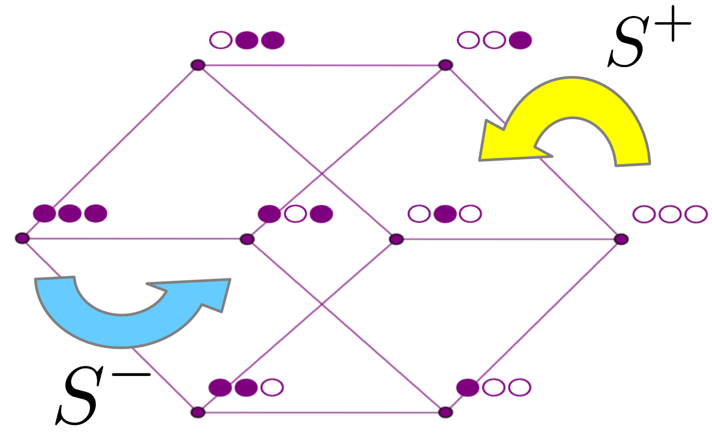
$$\langle \mathbb{Z}_2|\mathbb{Z}_2(t)\rangle = \sum_n e^{-\frac{i}{\hbar}tE_n}|\langle E_n|\mathbb{Z}_2\rangle|^2$$

- A **band of special eigenstates** with anomalously high overlap with the Neel state
- Special eigenstates are approximately **equidistant in energy**, which is the cause of oscillatory dynamics
- Apart from the special band, the rest of the spectrum is also organized in **towers**

How to construct or explain the special states?

Many-body scarred quasimodes

Inspiration: free paramagnet ➢ N-dim hypercube

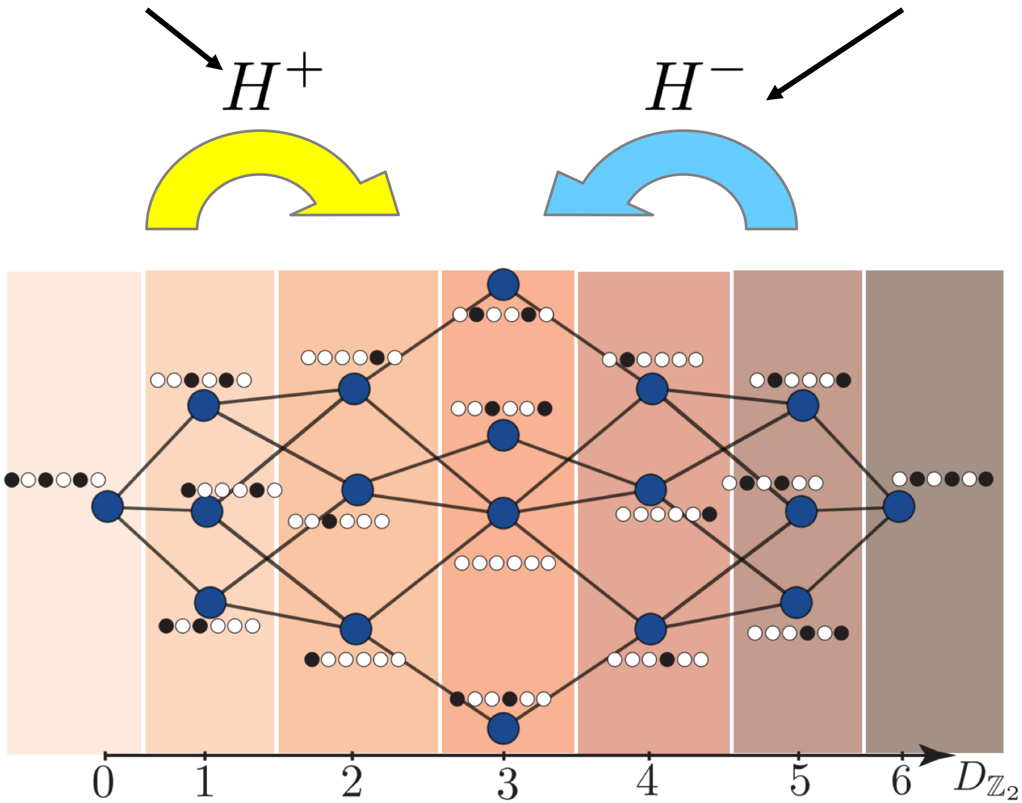


$X \sim S^+ + S^-$

PXP Hamiltonian: $H = H^+ + H^-$

Increases distance from Neel state by 1

Decreases distance from Neel state by 1



Basis of the "tight-binding" model:

$$\{|0\rangle \equiv |\mathbb{Z}_2\rangle, |1\rangle, \ldots, |L\rangle\}$$

$$|n\rangle \propto (H^+)^n |0\rangle$$

Quasimode approximations

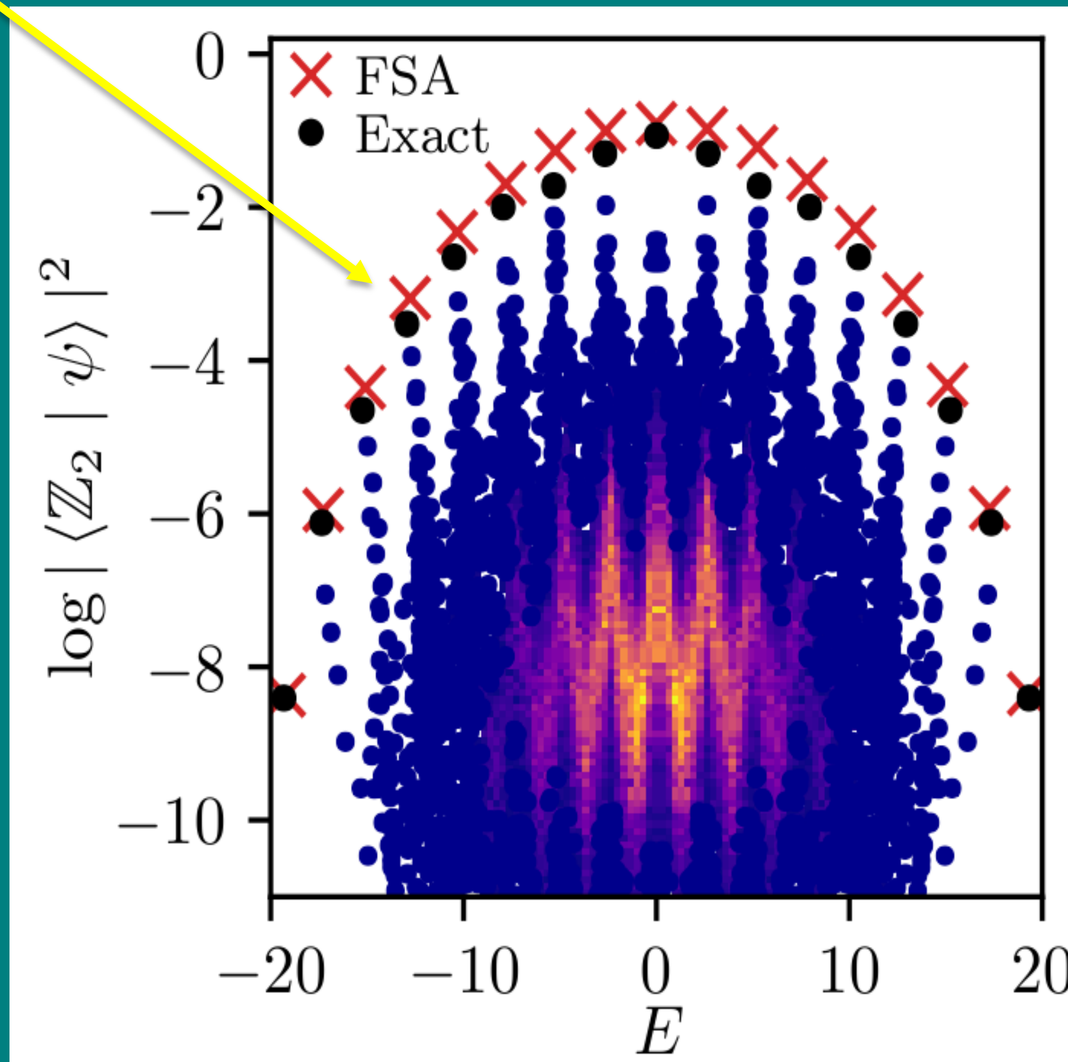
C. Lanczos

[C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics **14**, 745 (2018)]

Many-body scarred quasimodes

S^+

Quasimodes leave an imprint on eigenstates = scarring.

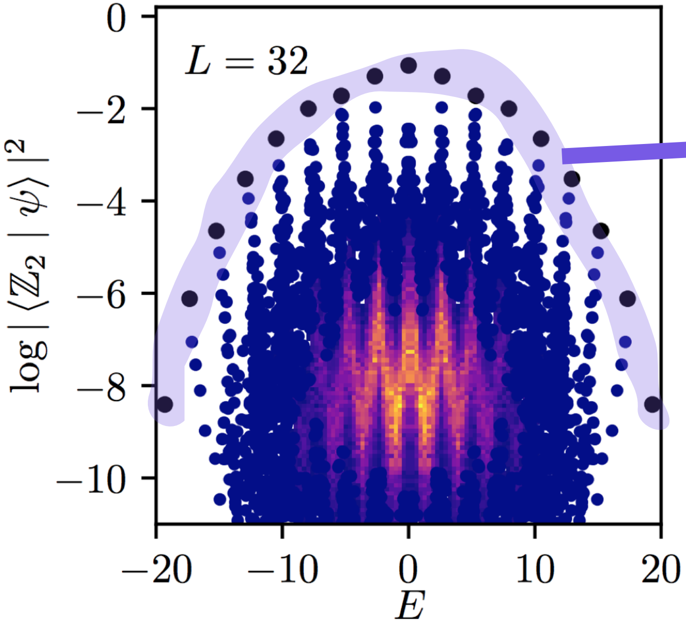


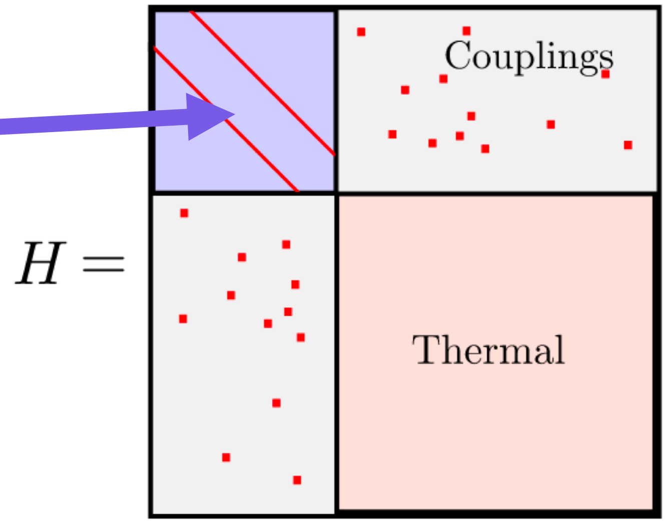
C. Lanczos

[C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics **14**, 745 (2018)]

Many-body scars as embedded algebras

[S. Choi, C. J. Turner, et al., PRL **122**, 220603 (2019); Bull, Desaules, ZP, arXiv:2001.08232]





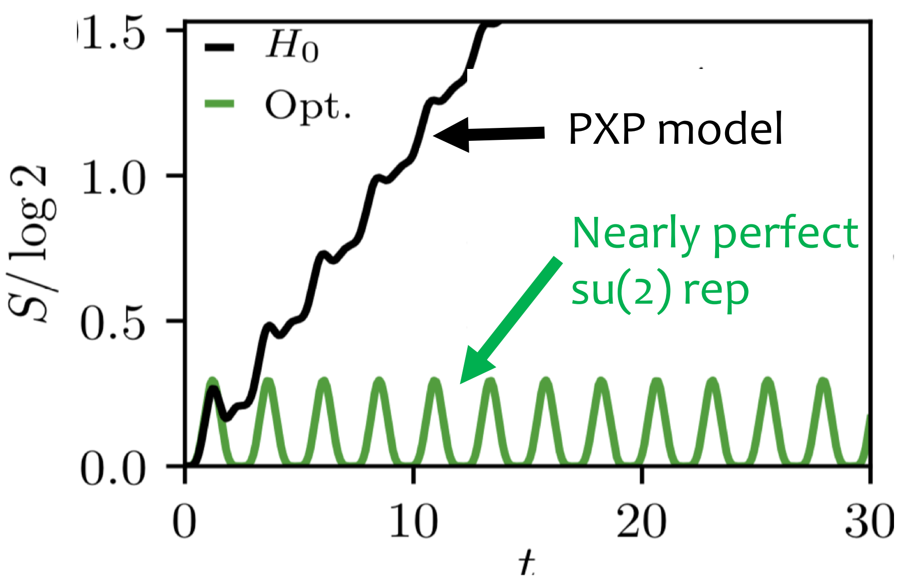
Special eigenstates form an approximate rep of su(2)

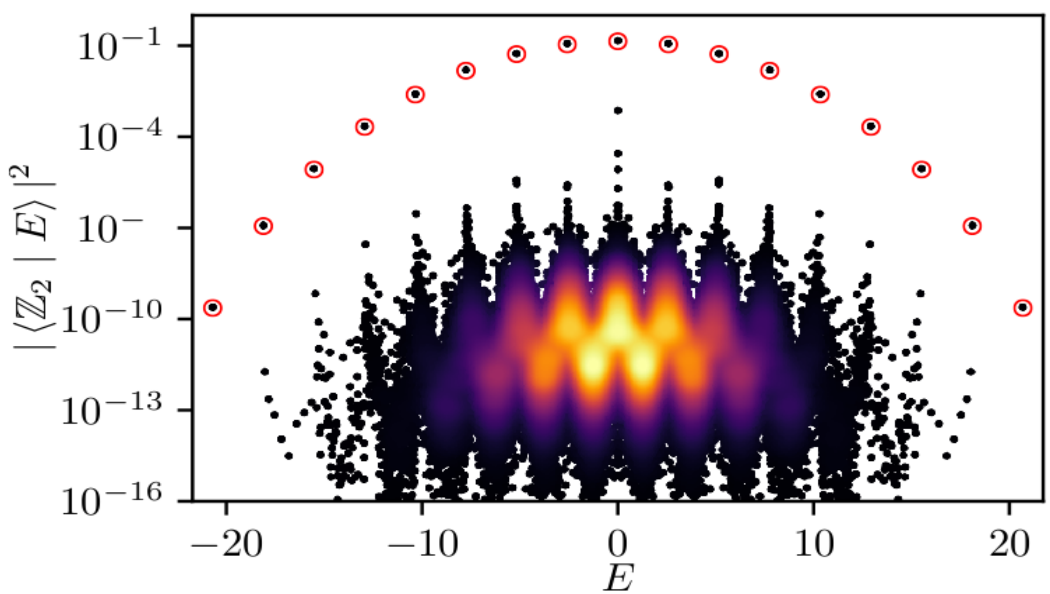
$$H^+, H^-, H^z \equiv \frac{1}{2}[H^+, H^-]$$

$|\mathbb{Z}_2\rangle$ = lowest weight state of H^z

Algebra is only approximate but can be systematically improved: $[H^z, H^\pm] = \pm H^\pm + \delta^\pm$

[see also Khemani, Laumann, Chandran, PRB **99**, 161101 (2019)]

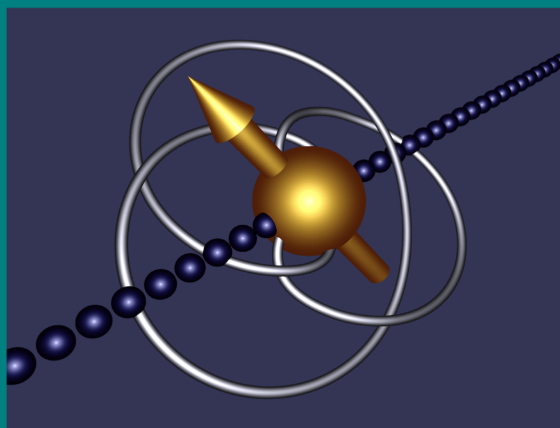


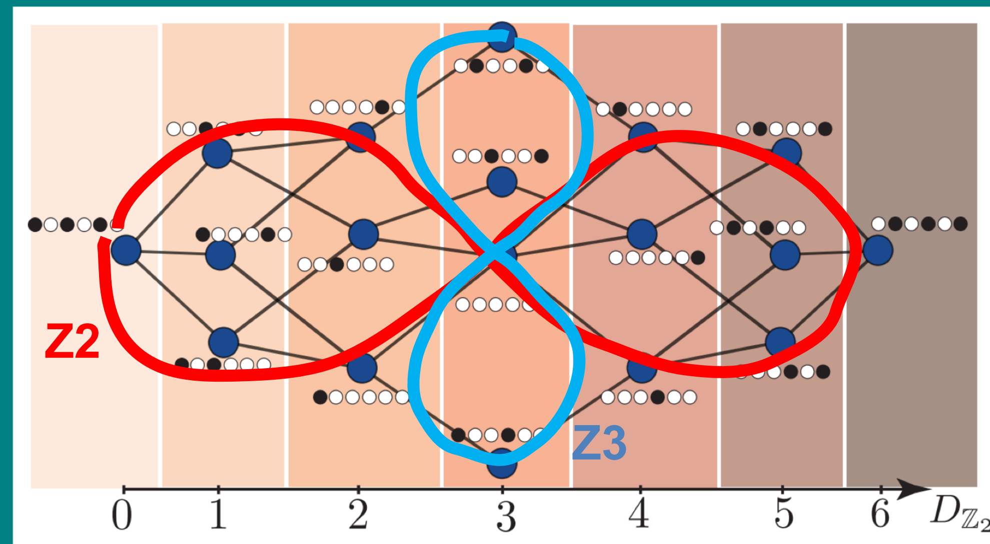


Many-body scars as embedded algebras

[S. Choi, C. J. Turner, et al., PRL **122**, 220603 (2019); Bull, Desaules, ZP, arXiv:2001.08232]

su(2) rep = embedded spin;
spin precession = periodic orbit





Multiple su(2) reps can be embedded!

Orbits have an elegant semiclassical description in terms of TDVP on a small bond-dimension MPS manifold!

[Ho *et al.*, PRL **122**, 040603(2019),
Michailidis *et al.*, arXiv:1905.08564 (to appear in PRX)]

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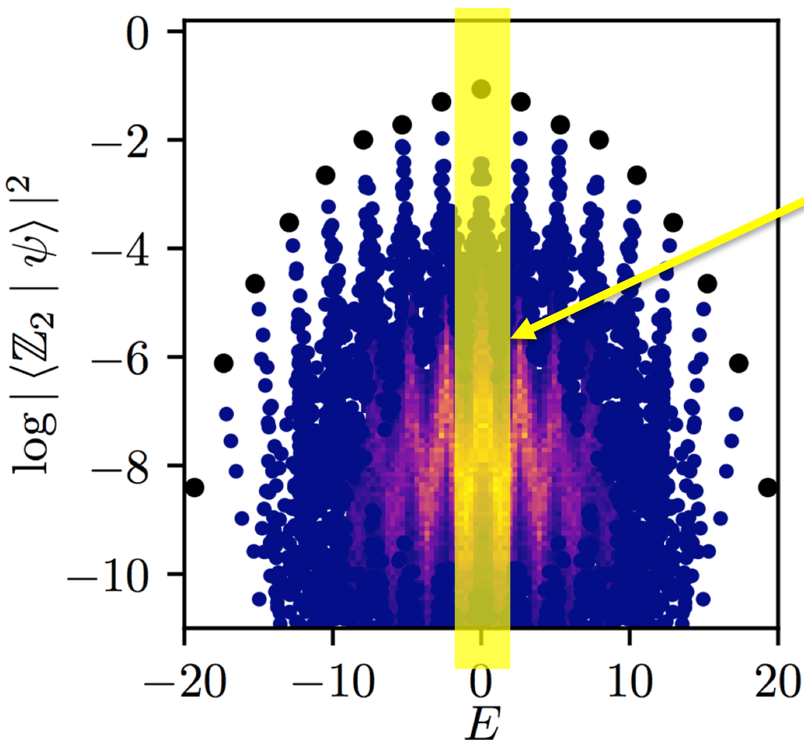
(More) rigorous results on PXP model



- Embedded eigenstates at/close to $E \approx 0$

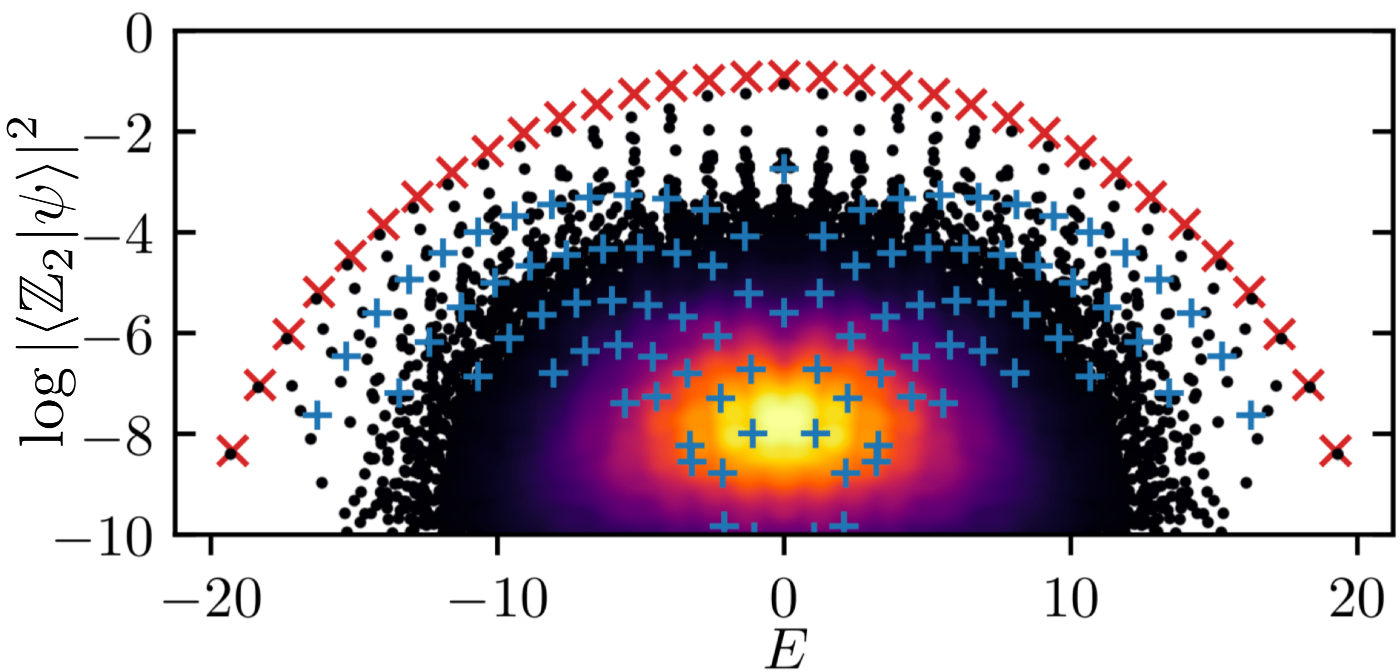
  Analogous to AKLT ground state!

  [Shiraishi and Mori, PRL **119**, 030601 (2017);
  Lin and Motrunich, PRL **122**, 173401 (2019);
  Shiraishi, J. Stat. Mech. 083103 (2019)]

- A new construction of quasimodes based on enlarged permutation symmetry

$$\mathcal{K} = \mathrm{span}\{|n_1, n_2\rangle\}$$

[C. Turner, J-Y. Desaules, K. Bull, ZP, to appear]



- Analytical expressions for matrix elements, energy variance, entanglement…

- Can be viewed as TDVP + quantum flucts at *all* orders

Many-body scars beyond 1D PXP model

- AKLT model

[Arovas, Physics Lett. A **137**, 431 (1989);
Moudgalya, Regnault, Bernevig, PRB **98**, 235156 (2018)]

- Models with U(1) symmetry

[Schecter and Iadecola, PRL **123**, 147201 (2019)]
[Chattopadhyay, Pichler, Lukin, Ho, arXiv:1910.08101]
[Lee, Melendrez, Pal, Changlani, arXiv:2002.08970]

- Periodically driven/dissipative systems

Buca, Tindall, Jaksch,
Nature Commun. **10**, 1730 (2019);
Haldar et al., arXiv:1909.04064]

- "Krylov restricted thermalization"

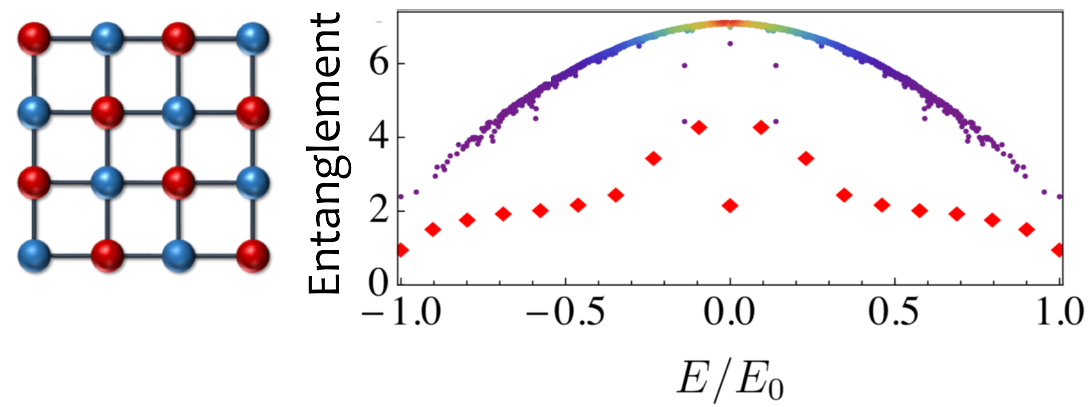
[Moudgalya *et al.*, arXiv:1910.14048]

- Miscellanea

[Seulgi Ok *et al.*, PRR **1**, 033144 (2019),
Moudgalya, Bernevig, Regnault, arXiv:1906.05292;
Iadecola and Shecter, PRB **101**, 024306 (2020);
Shibata, Yoshioka, Katsura, arXiv:1912.13399]

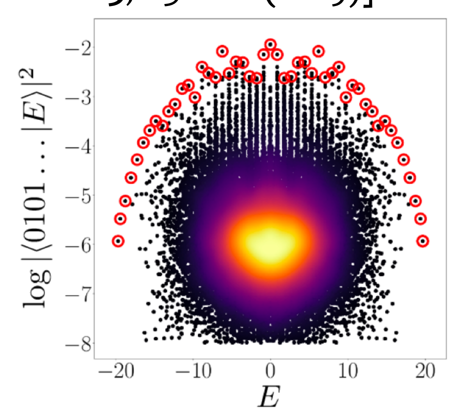
- Kinetically constrained models

2D PXP model [Michailidis *et al*, arXiv:2003.02825]



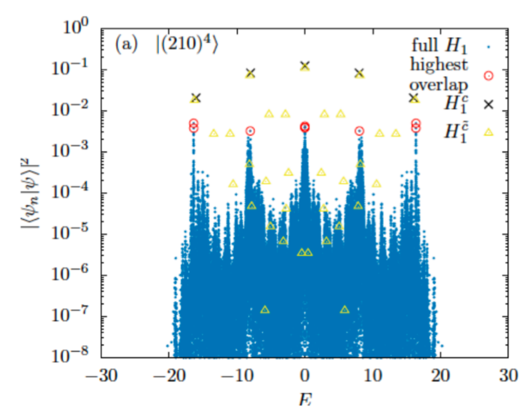
Quantum clocks

[Bull, Martin, ZP,
PRL **123**, 030601 (2019)]



Bosonic models

[Hudomal, Vasic,
Regnault, ZP, arXiv:1910.09526]



Conclusions and outlook

- A new class of **non-integrable** many-body systems which feature **atypical (non-thermal) eigenstates** throughout the spectrum and display **periodic quantum revivals** that have been observed in quench experiments
- Analogous to single-particle quantum scars (quasimodes, periodic orbits)
- Scarred eigenstates form **approximate reps of Lie algrebras** [typically su(2)]

- Stability under perturbations? Full classification of orbits and host systems?
- New experiments? Wil Kao et al., arXiv:2002.10475
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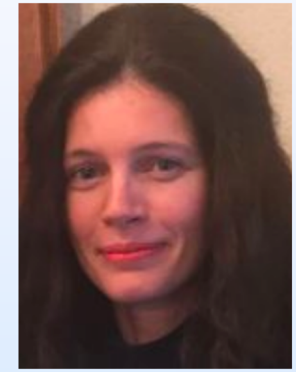

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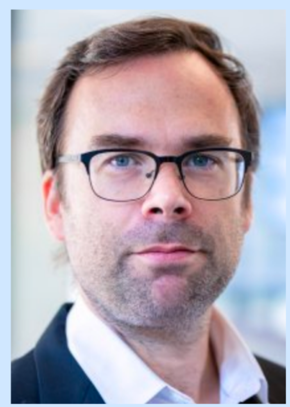

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+ Harvard/Berkeley group:

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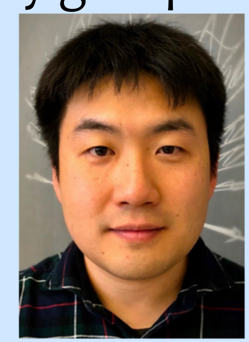

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