New materials for holographic hydrodynamics

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Motivation and Overview

- Collaboration between string theorists and condensed matter theorists
- Proposing new materials to test predictions from gauge/gravity duality
- Outline:
  - Hydrodynamics for electrons in solids
  - Brief intro to gauge/gravity duality
  - Predictions of gauge/gravity duality for electron hydrodynamics

Motivation and Overview

- New paper:

  Turbulent hydrodynamics in strongly correlated Kagome metals

  Domenico Di Sante, J. E., Martin Greiter, Ioannis Matthaiakakis, René Meyer, David Rodriguez Fernandez, Ronny Thomale, Erik van Loon, Tim Wehling

  arXiv:cond-mat/1911.06810

- Proposal for a new Dirac material with stronger electronic coupling than in graphene: Scandium-Herbertsmithite

- in view of enhanced hydrodynamic behaviour of the electrons

  Reaching smaller η/s (ratio of shear viscosity over entropy density)

Motivation and Overview

- Strongly coupled electron fluids in the Poiseuille regime

  J.E., I. Matthaiakakis, R. Meyer, D. Rodriguez, Phys. Rev. B 98 (2018) 195143

- Functional dependence of the Hall viscosity-induced transverse voltage in two-dimensional Fermi liquids

  J.E., E. Hankiewicz, I. Matthaiakakis, R. Meyer, D. Rodriguez, C. Tutschku,
  Phys. Rev. B 101 (2020) 045423

Hydrodynamics for electrons in solids

When phonon and impurity interactions are suppressed,

Electron-electron interactions may lead to a hydrodynamic electron flow

(Small parameter window)

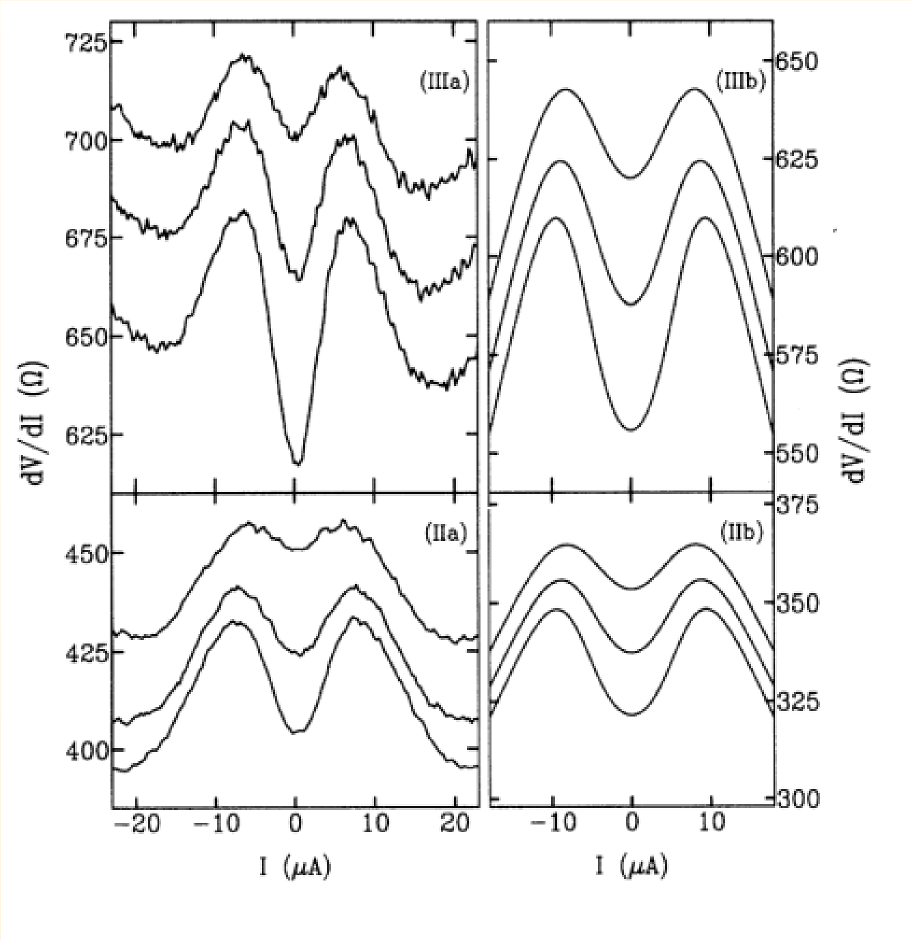
Some Implications:

- Decrease of differential resistance dV/dI with increasing current I

Weak coupling: High mobility wires

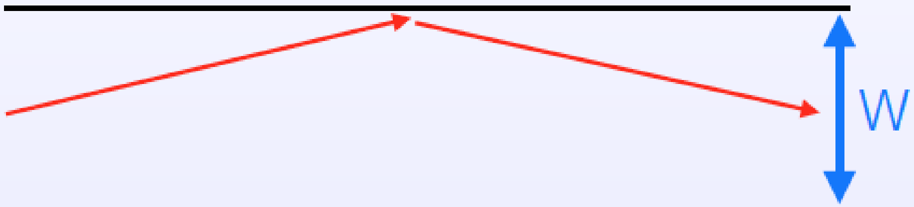
Transition: Knudsen flow $\Rightarrow$ Poiseuille flow   Gurzhi effect

Molenkamp, de Jong Phys. Rev. B 51 (1995) 13389 for GaAs in 2+1 dimensions



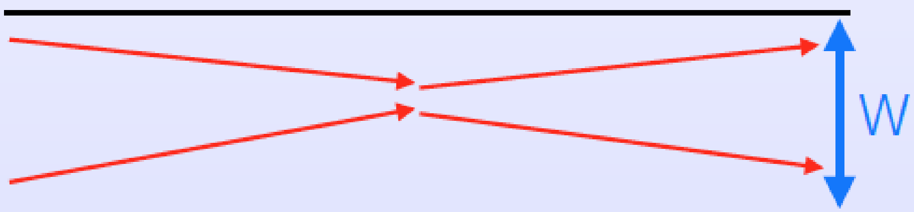
- 2D Electrons in (Al)GaAs Heterostructures

Knudsen flow regime

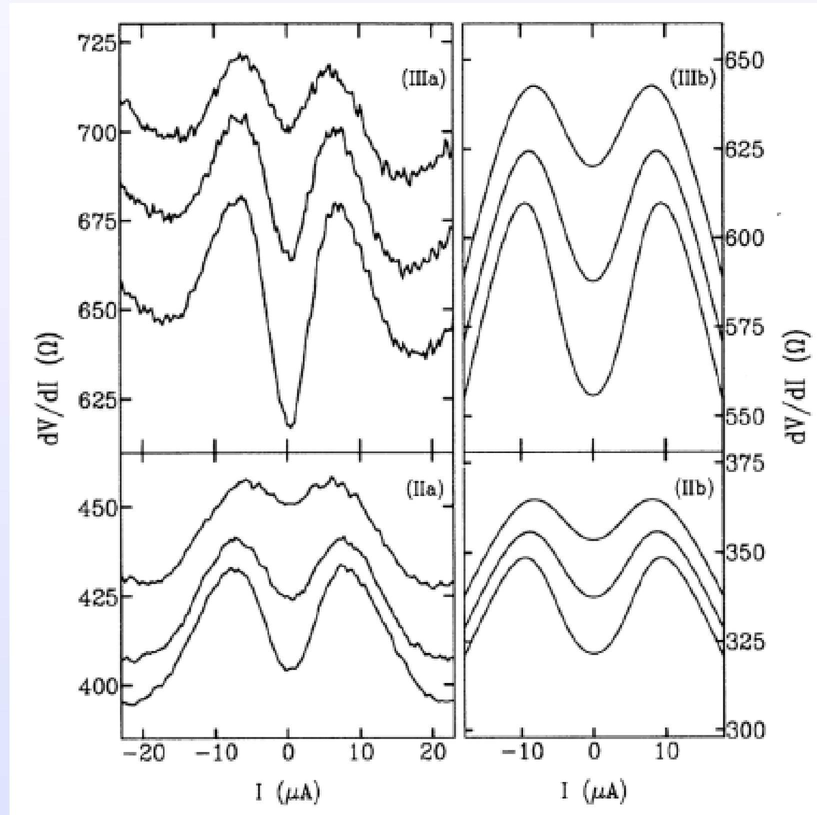


$$\ell_{ee}(T) > W$$

Poiseuille flow regime



$$\ell_{ee}(T) < W$$



[Gurzhi 1968]

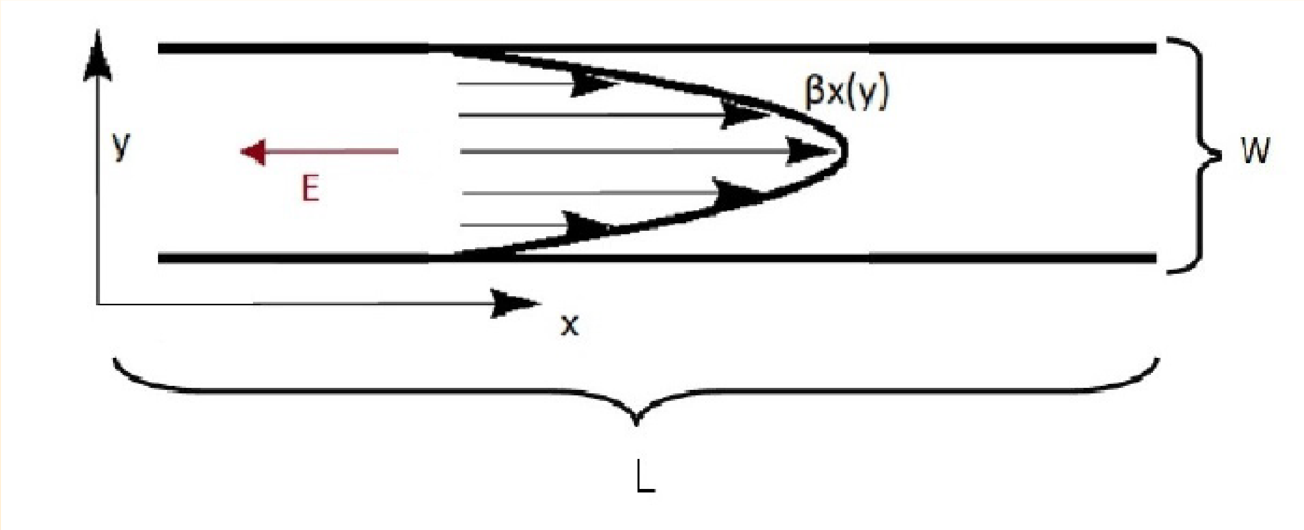
[Molenkamp+de Jong 1994,95]

Conditions for hydrodynamic behaviour of electrons in solids

$$\ell_{ee} < \ell_{\text{imp}}, \ell_{\text{phonon}}, W$$

ℓ_{ee}: Typical scale for electron-electron scattering

Flow profile in wire



Effective electron-electron coupling strength

$$\alpha_{\text{eff}} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_F}$$

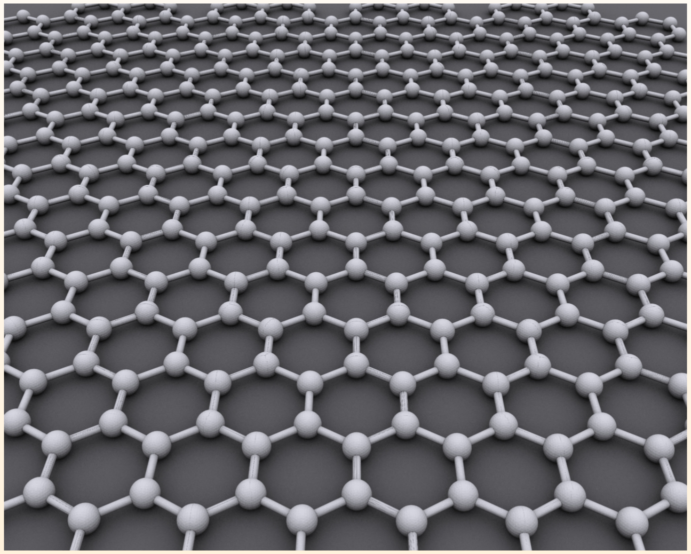
Electron-electron scattering length:

$$\ell_{\text{ee}} \propto \frac{1}{{\alpha_{\text{eff}}}^2}$$

Larger electronic coupling $\Rightarrow$ More robust hydrodynamic behaviour

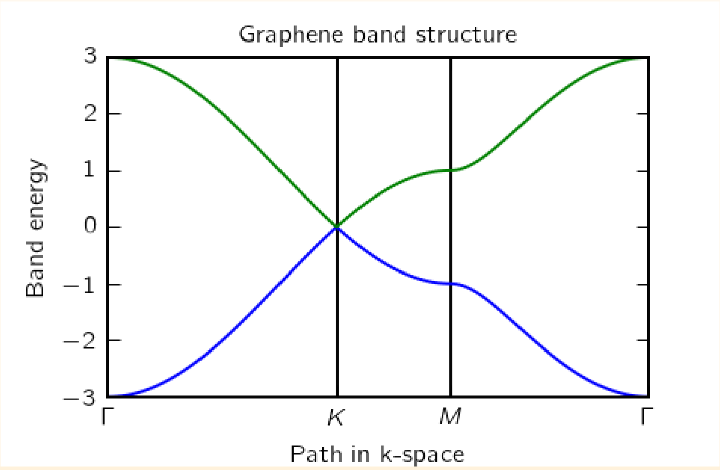
Hydrodynamics in Dirac materials: Graphene

Hexagonal carbon lattice



Source: Wikipedia

Dirac material:
Linear dispersion relation



Considerable theoretical and experimental effort for viscous fluids

Review: Polini + Geim, arXiv:1909.10615

Relativistic hydrodynamics: Expansion in four-velocity derivatives

$$T_{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + P\eta^{\mu\nu} - \sigma^{\mu\nu} + \dots$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\eta(\nabla_{\alpha}u_{\beta} + \nabla_{\beta}u_{\alpha} - \frac{2}{3}\nabla_{\gamma}u^{\gamma}\eta_{\alpha\beta}) \, + \, \zeta\nabla_{\gamma}u^{\gamma}\eta_{\alpha\beta}\right)$$

Shear viscosity η, bulk viscosity ζ

$P^{\mu\nu} = \eta^{\mu\nu} + u^{\mu}u^{\nu}$

Relativistic hydrodynamics: Expansion in four-velocity derivatives

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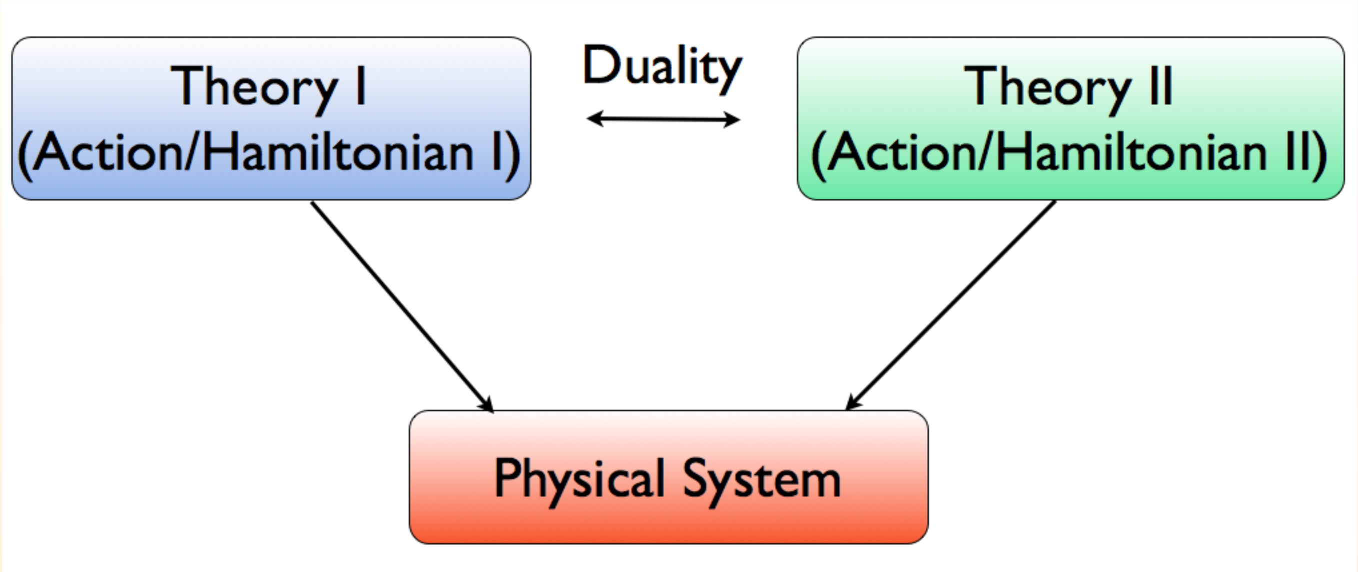
Shear viscosity η, bulk viscosity ζ

$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$

Shear viscosity for strongly correlated systems may be calculated from gauge/gravity duality!

Duality:


Theory I
(Action/Hamiltonian I)
Duality
Theory II
(Action/Hamiltonian II)
Physical System

Gauge/Gravity Duality

- Conjecture which follows from a low-energy limit of string theory

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

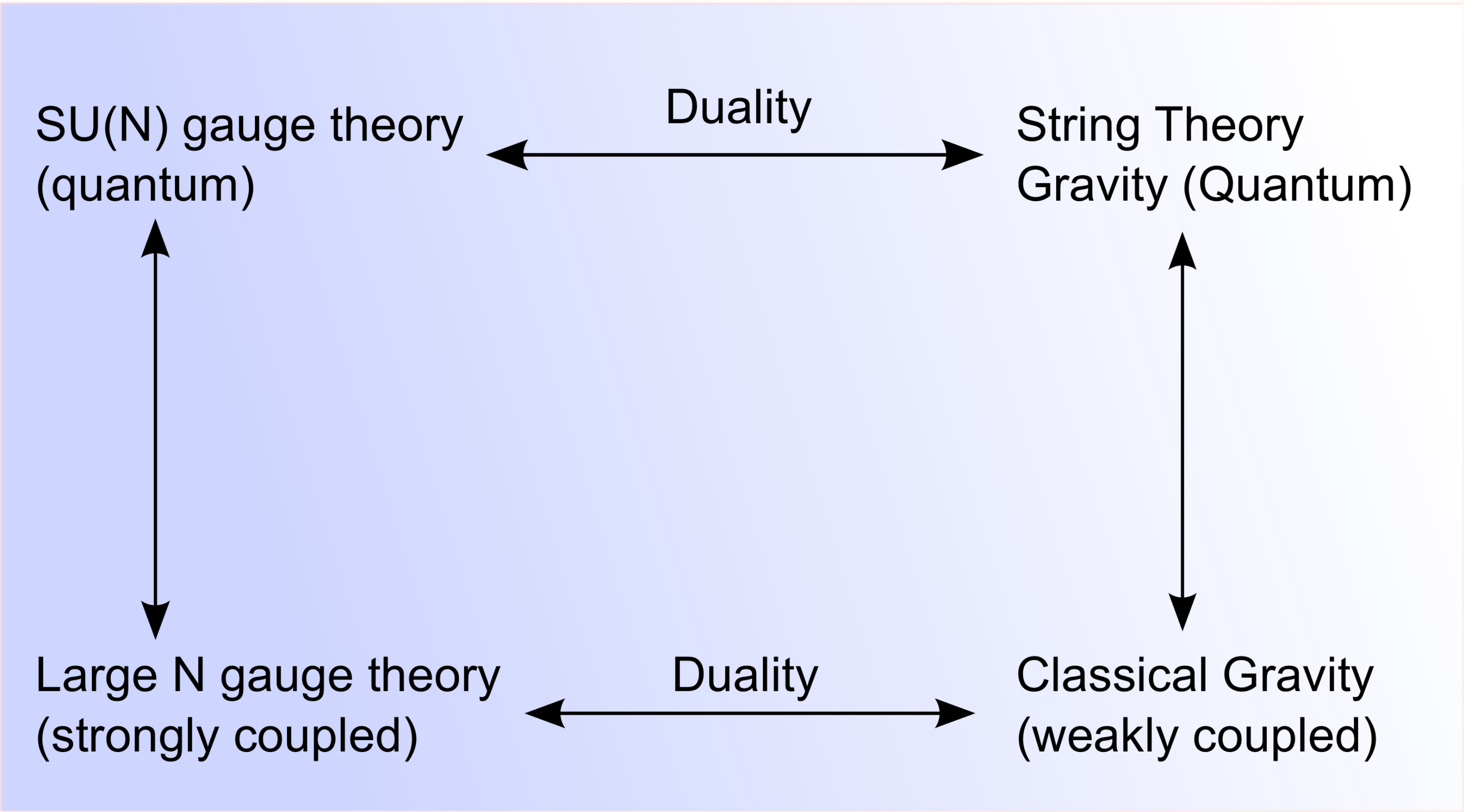
- Holography:

  Quantum field theory in d dimensions

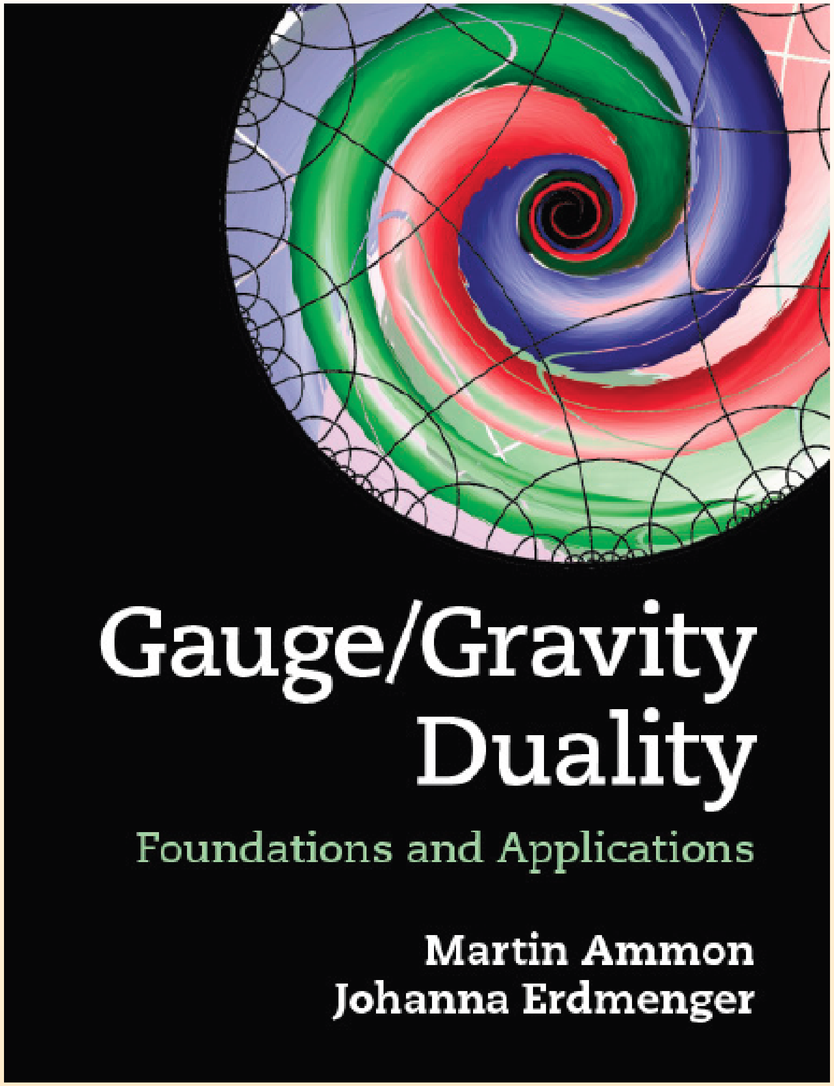
  $\Leftrightarrow$ Gravitational theory in $d+1$ dimensions

  Quantum field theory defined on the boundary of the $d+1$-dimensional space

Gauge/Gravity Duality: String Theory Origin



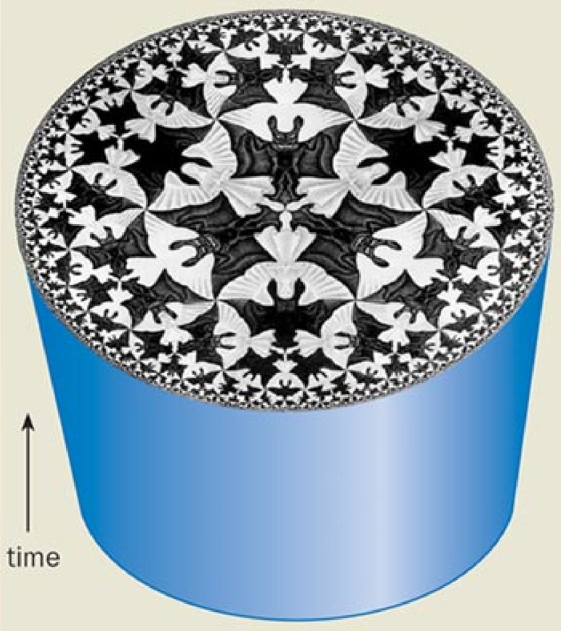
Book on gauge/gravity duality



AdS/CFT correspondence:
Example of gauge/gravity duality with huge amount of symmetry

AdS: Anti-de Sitter space:
Hyperbolic space
with constant negative curvature

CFT: Conformal field theory
Example: QFT at RG fixed point

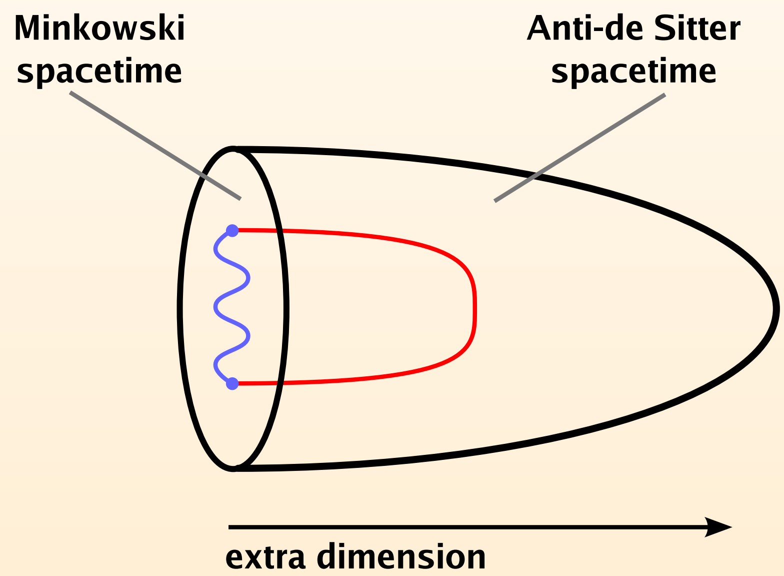


Quelle: Institute of Physics, Copyright: C. Escher

Gauge/Gravity Duality: Bulk-boundary correspondence

Quantum observables at the boundary of the curved space

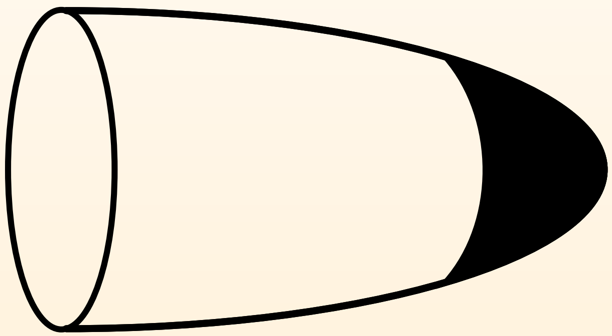
may be calculated from propagation through curved space



Gauge/Gravity Duality: Bulk-boundary correspondence

Quantum theory at finite temperature:

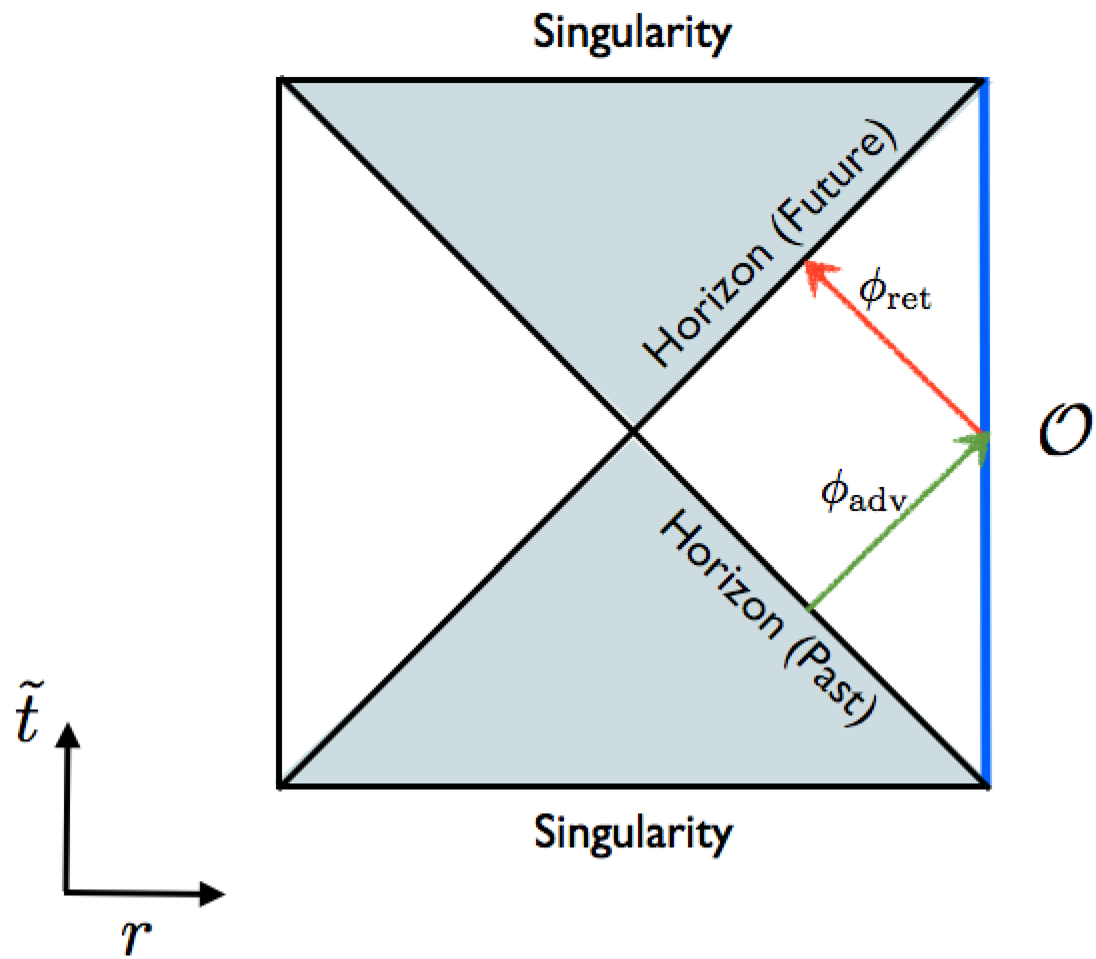
Dual to gravity theory with black hole (in Anti-de Sitter space)



Hawking temperature identified with temperature in the dual field theory

Retarded Green's Functions in Strongly Coupled Systems

Anti-de Sitter
black hole



Retarded Green's function: $$G^R_{\mathcal{O}_A\mathcal{O}_B} = \left.\frac{\delta\langle\mathcal{O}_A\rangle}{\delta\phi_{B(0)}}\right|_{\delta\phi=0} = \frac{\delta\phi_{A(1)}}{\delta\phi_{B(0)}}$$

subject to infalling boundary condition at horizon

Holographic calculation of shear viscosity Kovtun, Son, Starinets 2004

- Energy-momentum tensor $T_{\mu\nu}$ dual to graviton $g^{\mu\nu}$
- Calculate correlation function $\langle T_{xy}(x_1)T_{xy}(x_2)\rangle$ from propagation through black hole space
- Shear viscosity is obtained from Kubo formula:

$$\eta = -\lim \frac{1}{\omega} \operatorname{Im} G^R_{xy,xy}(\omega)$$

- Shear viscosity $\eta = \pi N^2 T^3/8$, entropy density $s = \pi^2 N^2 T^3/2$

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

(Note: Quantum critical system: $\tau = \hbar/(k_B T)$)

Holographic hydrodynamics

Holography: From propagation of graviton in dual gravity subject to

$$S_{E-H} = \int d^{d+1}x\sqrt{-g}\,(R - 2\Lambda)$$

For $SU(N)$ gauge theory at infinite coupling, $N \to \infty$, $\lambda = g^2 N \to \infty$:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

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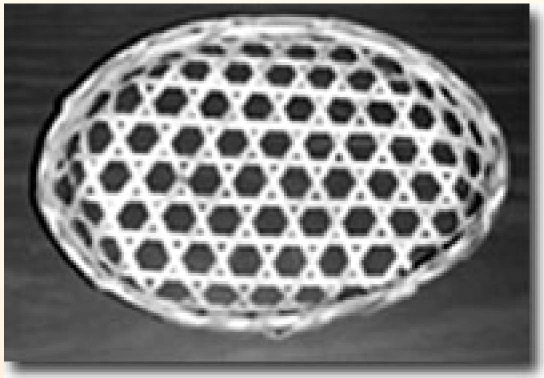
Leading correction in the inverse ’t Hooft coupling $\propto \lambda^{-3/2}$

From R^4 terms contributing to the gravity action

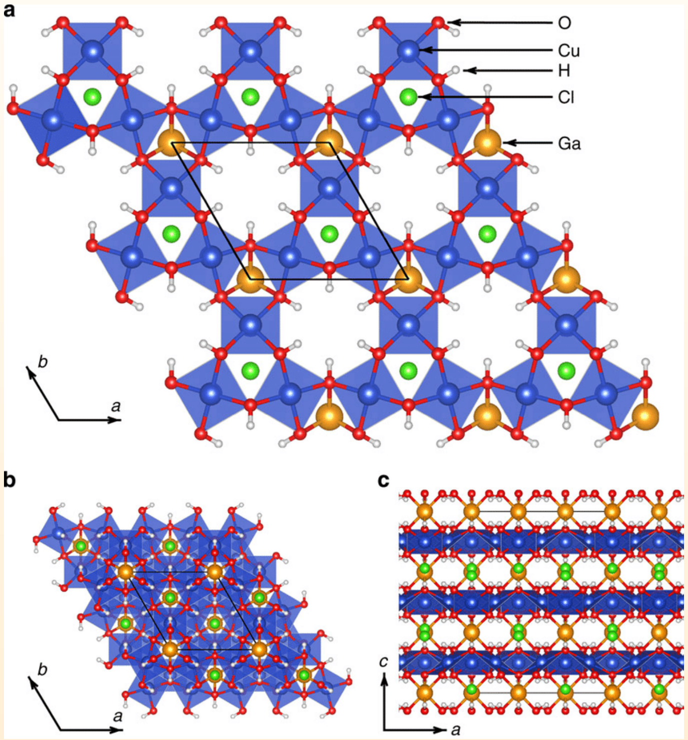
Kagome materials

Kagome: Japanese basket weaving pattern



Source: Wikipedia

Hexagonal lattice



Source: Nature

Herbertsmithite: $ZnCu_3(OH)_6Cl_2$

Source: Wikipedia

Scandium-Herbertsmithite

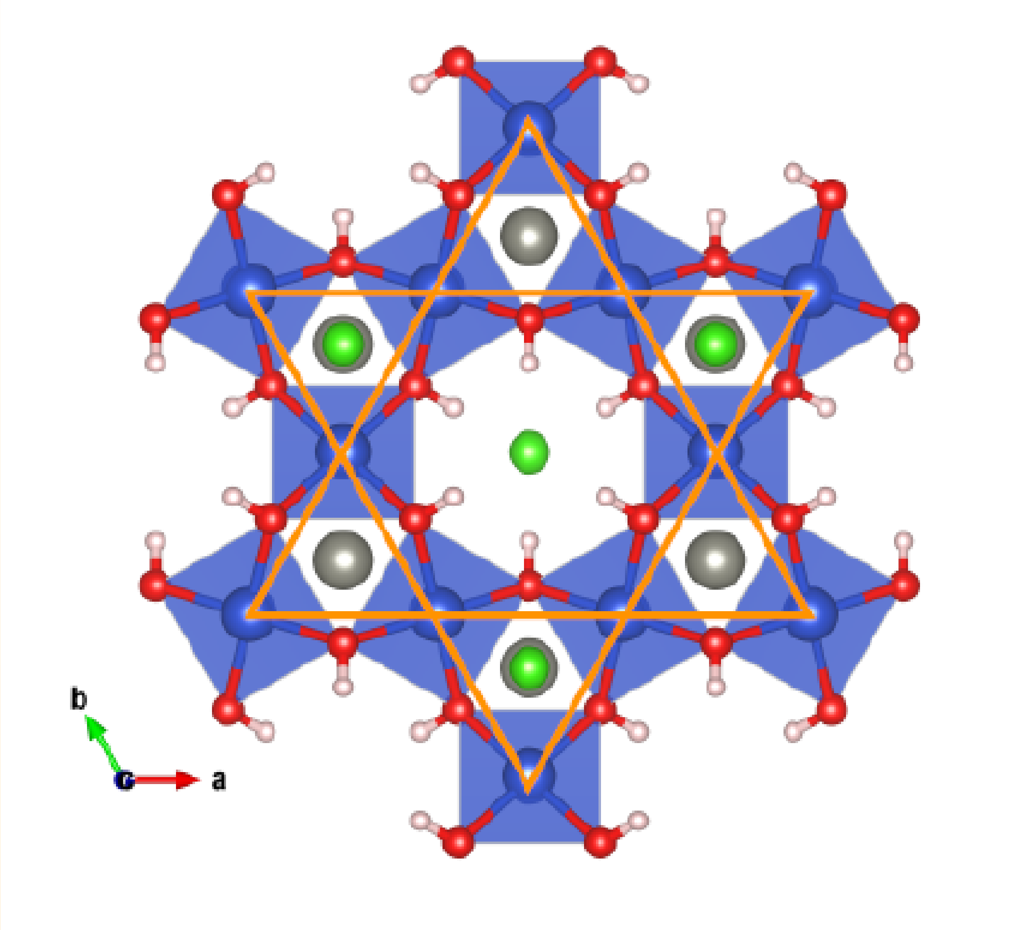
Original Herbertsmithite has Zn^{2+}

Fermi surface below Dirac point

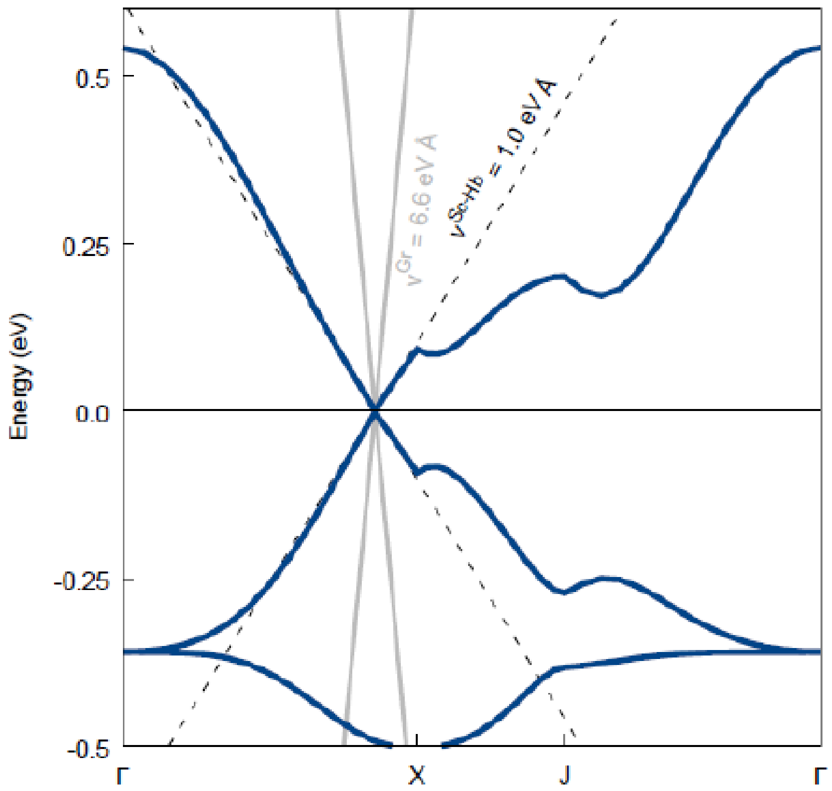
Idea: Replace Zinc by Scandium, Sc^{3+}

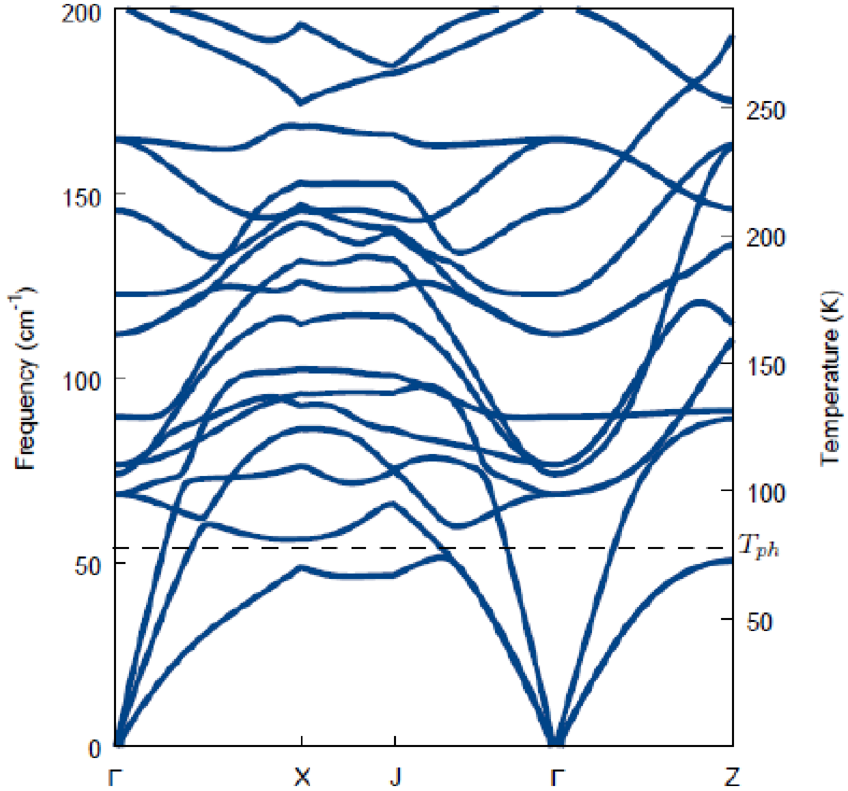
Places Fermi surface exactly at Dirac point

Scandium-Herbertsmithite



Scandium-Herbertsmithite





Band structure

Phonon dispersion

Scandium-Herbertsmithite

- CuO_4 plaquettes form Kagome lattice
- Low-energy physics captured by $d_{x^2-y^2}$ orbital at each Cu site
- Fermi level is at Dirac point (filling fraction $n = 4/3$)
- Orbital hybridization allows for larger Coulomb interaction (confirmed by cRPA calculation)
- Prediction: $\alpha^{\mathrm{Sc-Hb}} = 2.9$ versus $\alpha^{\mathrm{Graphene}} = 0.9$
- Optical phonons are thermally activated only for temperatures above $T = 80$K
- Enhanced hydrodynamic behaviour: $\ell_{ee}^{\mathrm{Sc-Hb}} = \frac{1}{6}\ell_{ee}^{\mathrm{graphene}}$
- Candidate to test universal predictions from holography

Estimate of the Shear viscosity

Weak coupling : Kinetic theory

$$\frac{\eta}{s} \propto \frac{1}{\alpha^2}$$

Strong coupling: Holography

Take correction

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left(1 + \frac{\mathcal{C}}{\alpha^{3/2}}\right)$$

Vary $\mathcal{C}$ from 0.0005 to 2

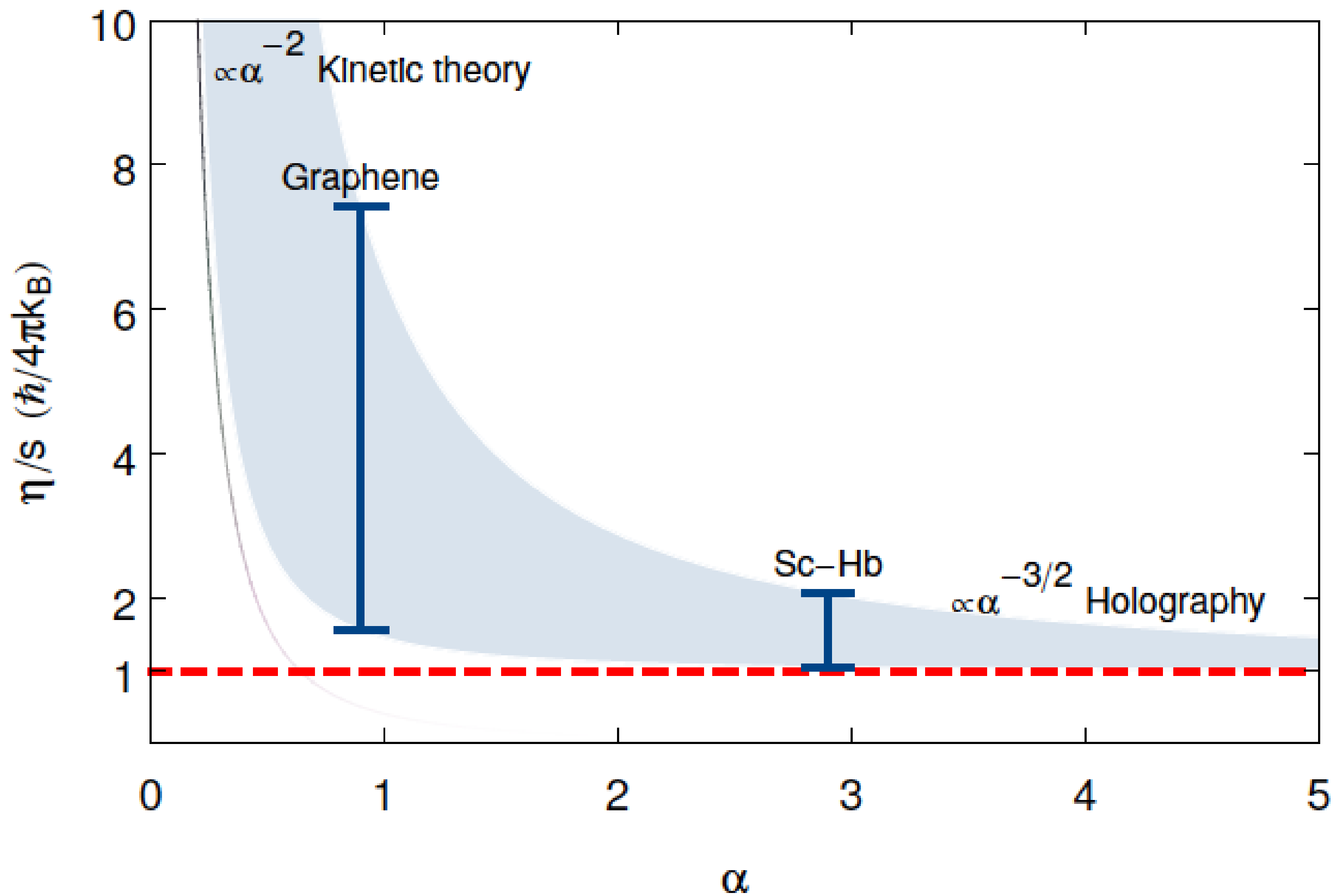
Estimate of the Shear viscosity

AdS gravity computation: Corrections of higher order in the curvature

$$S = S_{E-H} + \int \sqrt{-g}\left(\gamma_2 R^2 + \gamma_3 R^3 + \gamma_4 R^4 + \ldots\right)$$

- R^2 term is topological for bulk theory in $d = 4$
- R^3 terms absent in type II supergravity parent theories
- R^4 term: Coefficient $\mathcal{O}(\lambda^{-3/2})$
- $$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left(1 + \frac{\mathcal{C}}{\alpha^{3/2}}\right)$$
- R^4 correction is model-dependent.
  We parametrize this by varying the coefficient $\mathcal{C}$

Estimate of the Shear viscosity



Estimate of the Reynolds number

$$\mathrm{Re} = \left(\frac{\eta}{s}\frac{k_B}{\hbar}\right)^{-1} \frac{k_B T}{\hbar v_F} \frac{u_{\rm typ}(\eta/s)}{v_F} W$$

$u_{\rm typ}$ typical velocity, enhanced at strong coupling

Navier-Stokes equation:

$$\frac{d\bar{v}}{dt} = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \bar{v} + f$$

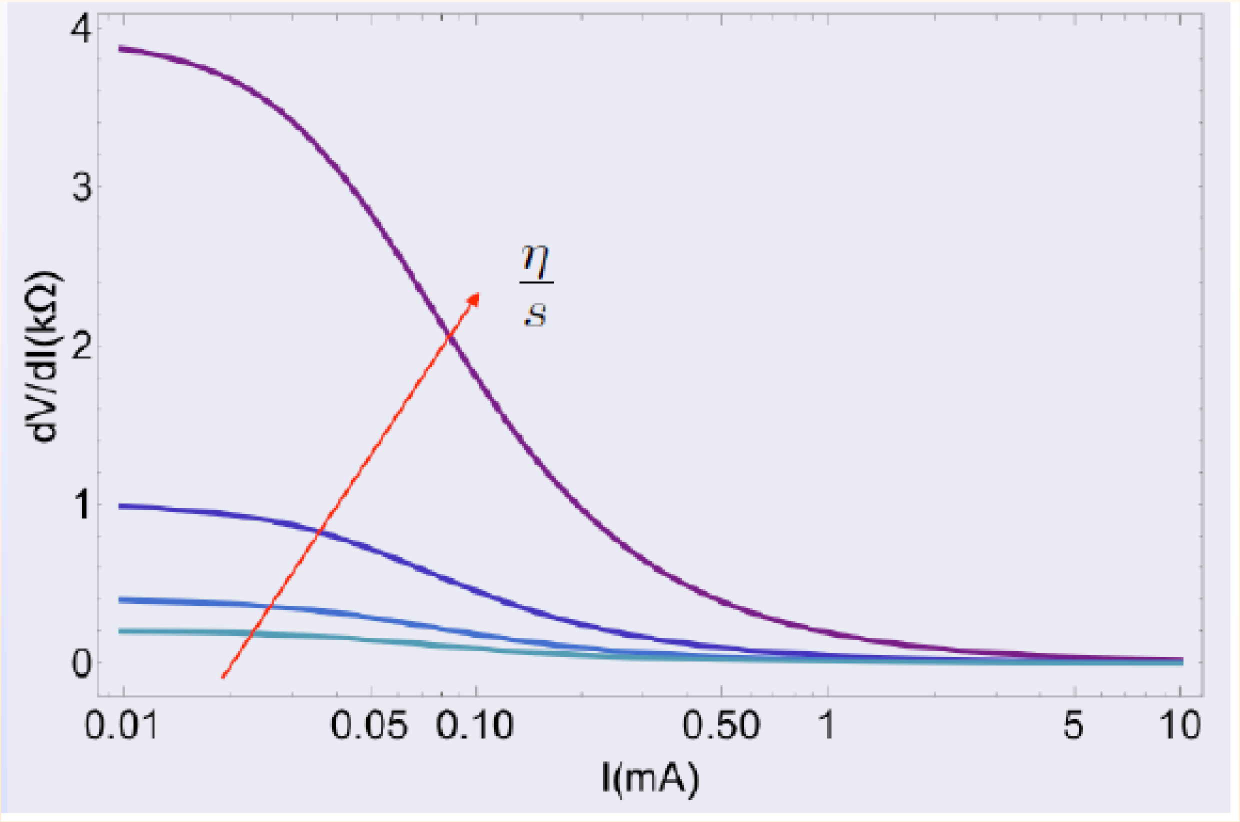
Turbulence: Reynolds number must be $\mathcal{O}(1000)$

In Sc-Hb, factor 100 larger than in graphene

Differential resistance in Poiseuille flow

J.E., Matthiakakis, Meyer, Rodriguez Fernandez PRB 2018

dV/dI increases as η/s increases



More strongly coupled fluids flow faster

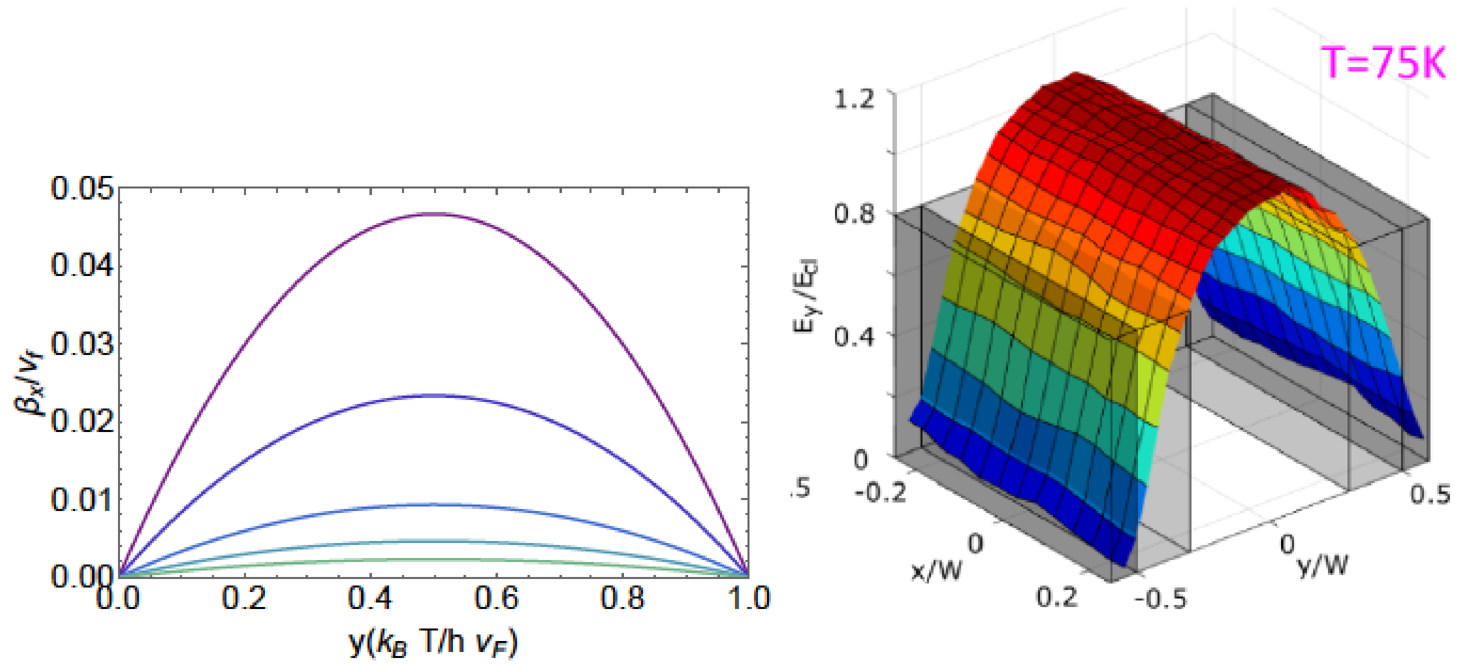


Figure: Left figure:Top curve, $\eta/s = \hbar/4\pi k_B$ (Holography). Right figure: Experimental observation of the Poiseuille flow in graphene (fig. taken from J. Sulpizio *et al* [1905.11662]

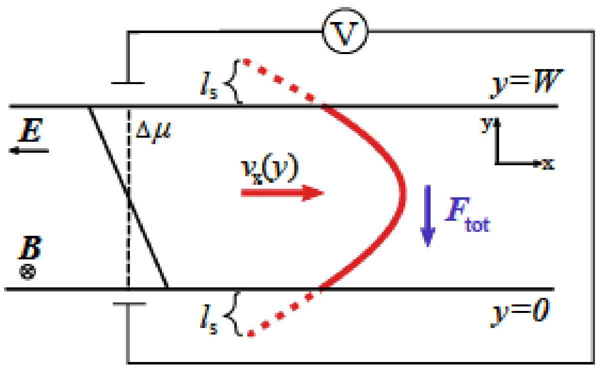
Strongly coupled fluids

- **Strongly coupled** fluids (low η/s) **flow faster**. A promising realistic material to realize this experimentally is **Sc-Hb**
- $R(I)$ highly sensitive to the Coulomb coupling strength α_{eff} (through shear viscosity) in the hydrodynamic regime
- **Stongly coupled electron** fluids show the **smallest wire resistance** and smallest **Joule heating effect** $J \sim \sigma_Q E_x^2$

Parity breaking hydrodynamics: Hall viscosity

Functional dependence of the Hall viscosity-induced transverse voltage in two-dimensional Fermi liquids

Ioannis Matthaiakakis,[1,*] David Rodríguez Fernández,[1,*] Christian Tutschku,[1,*]
Ewelina M. Hankiewicz,[1] Johanna Erdmenger,[1] and René Meyer[1]
[1]*Institute for Theoretical Physics and Astrophysics and Würzburg-Dresden Cluster of Excellence ct.qmat, Julius-Maximilians-Universität Würzburg, 97074 Würzburg, Germany*



$$(\partial_t + \mathbf{v}\cdot\nabla)\,\rho = -\rho\nabla\cdot\mathbf{v}, \qquad \text{(S1)}$$

$$m_{\text{eff}}\rho\,(\partial_t + \mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla p + \eta\nabla^2\mathbf{v} + \eta_{\mathrm{H}}\nabla^2(\mathbf{v}\times\mathbf{e}_z) + e\rho(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - \frac{\rho_0 v_{\mathrm{F}} m_{\text{eff}}}{l_{\text{imp}}}\mathbf{v}. \qquad \text{(S2)}$$

Parity breaking hydrodynamics: Hall viscosity

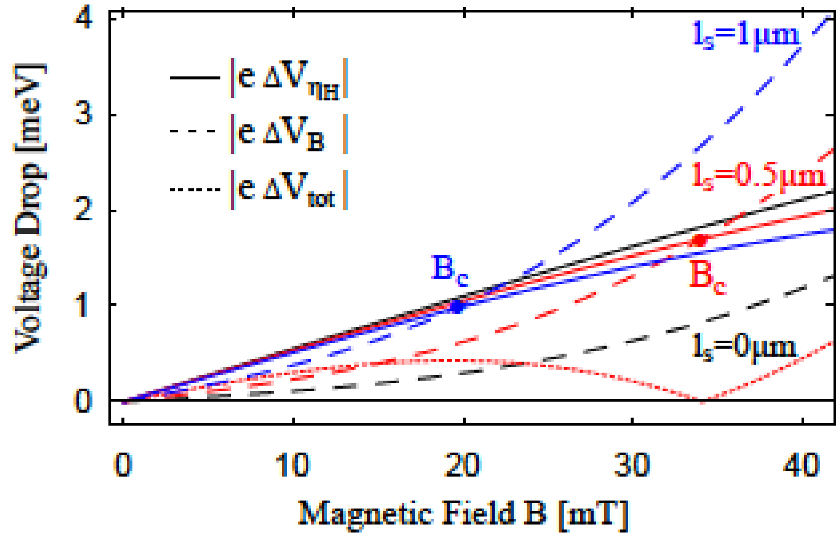


FIG. 4. Absolute values of the Lorentz ΔV_B and Hall viscous contribution $\Delta V_{\eta H}$ to the total Hall voltage ΔV_{tot} in GaAs are shown as functions of the magnetic field B for $l_s = 0, 0.5, 1.0\mu m$. Parameters for this calculation are given in the caption of Fig. 3. For $B < B_c$, we find $|\Delta V_{\eta H}|/|\Delta V_B| > 1$, whereas otherwise $|\Delta V_{\eta H}|/|\Delta V_B| < 1$. At $B = B_c$, the ratio $\Delta V_{\eta H}/\Delta V_B = -1$ implying a vanishing Hall voltage $\Delta V_{tot} = 0$.

Conclusion and outlook

- Scandium-substituted Herbertsmithite has predicted coupling $\alpha_{\text{eff}} = 2.9$
- Factor 3.2 larger than Graphene
- May reach region of robust hydrodynamics in solids
- Smaller ratio of η/s - parameter region where gauge/gravity duality applies
- Strongly coupled fluids flow faster
- Poiseuille flow
- Cancellation of Hall viscosity induced voltage with standard Hall voltage