Black Hole Entropy in Loop Quantum Gravity

Daniele Pranzetti

A. Ghosh and DP, Nucl. Phys. B 889, 1 (2014); e–print: gr-qc/1405.7056

DP and H. Sahlmann, Phys. Lett. B (in print), e–print: gr-qc/1412.7435

and w.i.p. in collaboration with Jose Mourao

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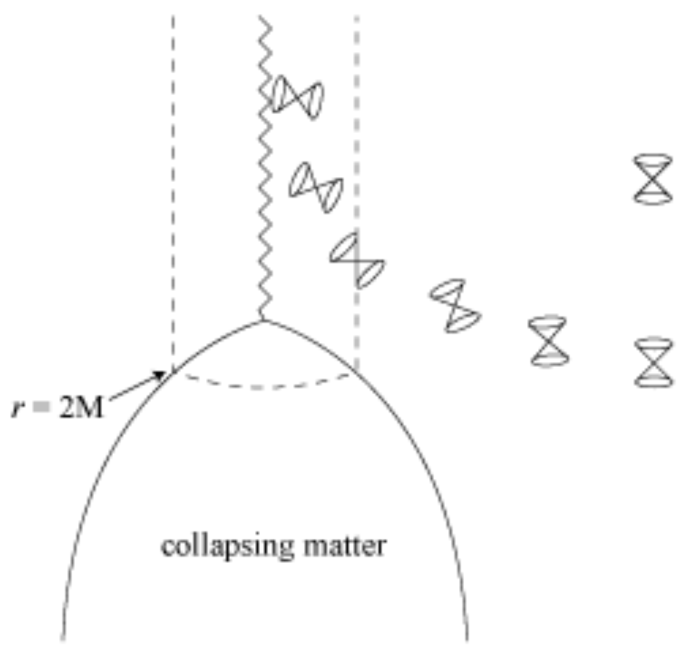
Black hole thermodynamics



[Bekenstein 72; Bardeen, Carter, Hawking 73; Hawking 74]

Black holes in their stationary phase behaves as thermodynamical systems:

$$S \longleftrightarrow A/(8\pi\alpha) \qquad T \longleftrightarrow \alpha\kappa$$

But, in classical GR: $T = 0$

Hawking radiation:
thermal emission of particles from a BH at $T = \dfrac{\kappa\hbar}{2\pi}$ $\longrightarrow$ $\boxed{S_{BH} = \dfrac{Ak_B}{4G\hbar}}$

Semiclassical result

☞ Questions:

Statistical physics: entropy of any system is given by $S = \ln N$

N = number of states of the system for the given macroscopic parameters

for a solar mass black hole

$$N = e^S \sim 10^{10^{77}}$$

1) Microscopic origin of the entropy?
2) Where do all these d.o.f. live?

☞ Call for a quantum treatment of the gravitational dof

Weak holographic principle:

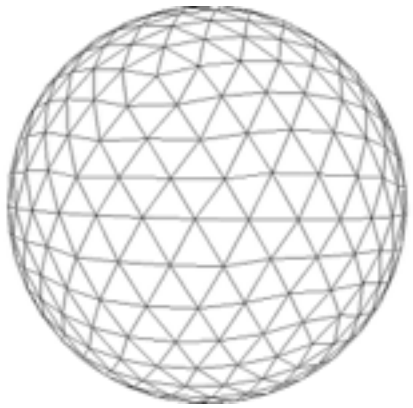
The entropy in the 1st law is the log of the number of states of the black hole that can affect the exterior

[Bekenstein; Sorkin; Smolin; Jacobson; Rovelli…]

➥ The horizon carries some kind of information with a density of approximately 1 bit per unit area

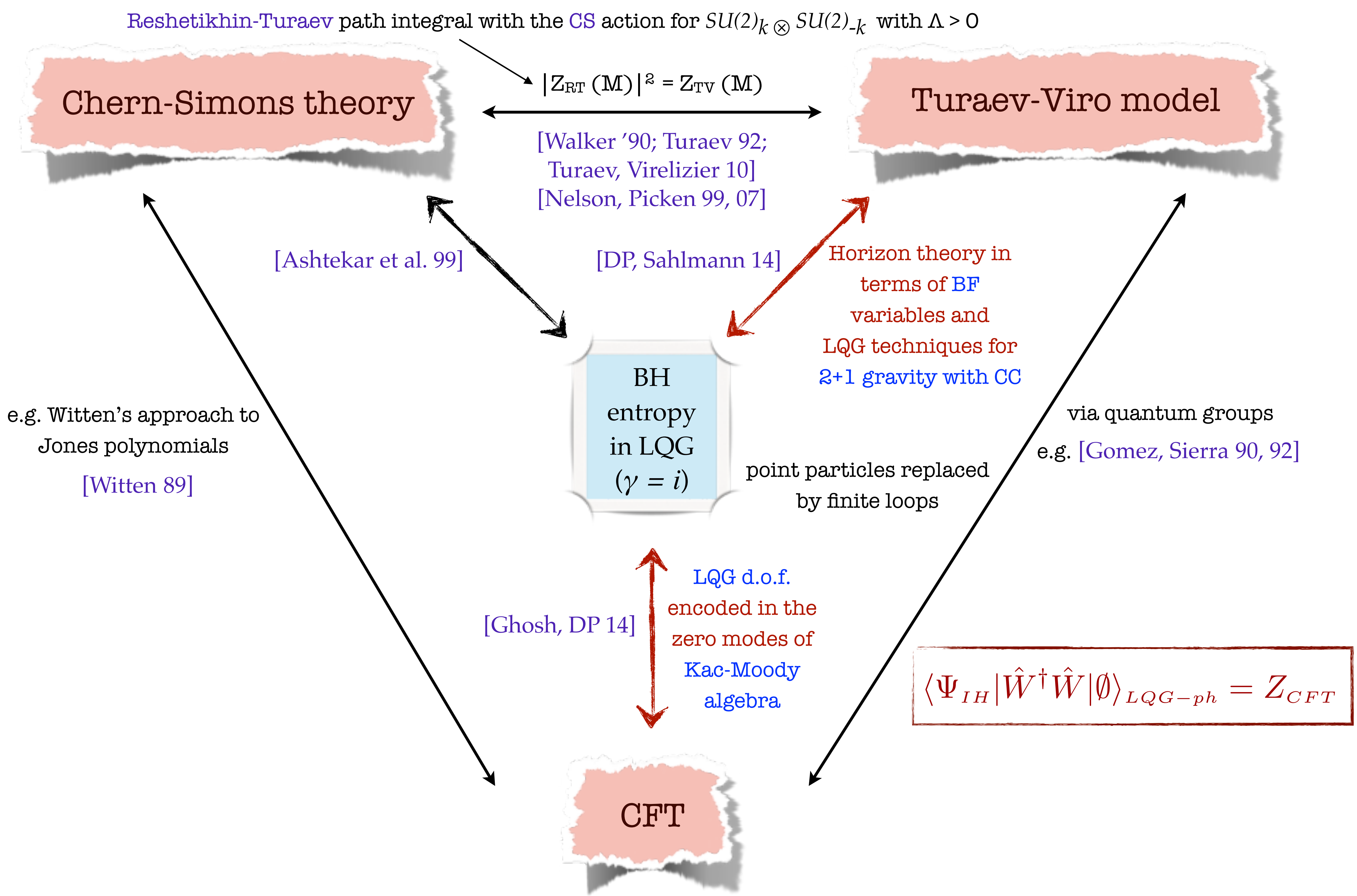
“It from Bit”

[Wheeler]



What these bits of information represent depends on the deep structure of space-time

✧ The finiteness of the BH entropy hints at discreteness of space-time at the Planck scale


Reshetikhin-Turaev path integral with the CS action for $SU(2)_k \otimes SU(2)_{-k}$ with $\Lambda > 0$
$|Z_{RT}(M)|^2 = Z_{TV}(M)$
Chern-Simons theory
Turaev-Viro model
[Walker '90; Turaev 92; Turaev, Virelizier 10] [Nelson, Picken 99, 07]
[Ashtekar et al. 99]
[DP, Sahlmann 14]
Horizon theory in terms of BF variables and LQG techniques for 2+1 gravity with CC
BH entropy in LQG ($\gamma = i$)
e.g. Witten's approach to Jones polynomials
[Witten 89]
via quantum groups
e.g. [Gomez, Sierra 90, 92]
point particles replaced by finite loops
[Ghosh, DP 14]
LQG d.o.f. encoded in the zero modes of Kac-Moody algebra
$\langle \Psi_{IH}|\hat{W}^{\dagger}\hat{W}|\emptyset\rangle_{LQG-ph} = Z_{CFT}$
CFT

Outline

- ➢ Basic ingredients of LQG
- ➢ Quantization of an Isolated Horizon
- ➢ Entropy counting from Chern-Simons theory:
  - Old results and open issues
- ➢ New persepctives:
  - $\gamma = i$
  - Entropy from LQG methods
  - CFT/gravity correspondence

The LQG approach

Metric variables

Einstein-Hilbert action

$$I[g_{ab}] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R$$

$$\kappa = 8\pi G$$

upon foliation of spacetime in terms of space-like three dimensional surfaces Σ

$$q_{ab},\ \pi^{ab} = \frac{1}{\sqrt{q}}(K^{ab} - Kq^{ab})$$

extrinsic curvature of Σ

symplectic structure

$$\{\pi^{ab}(x), q_{cd}(y)\} = 2\kappa\delta^a_{(c}\delta^b_{d)}\delta(x,y)$$

Hamiltonian

$$H(q_{ab},\pi^{ab},N_a,N) = N_aV^a(q_{ab},\pi^{ab}) + NS(q_{ab},\pi^{ab})$$

vanishes identically on solutions of the e.o.m.

$\Longrightarrow$

Connection variables

Triad $e^i_a,\ i = 1,2,3\ \ su(2)$ indices

set of three 1-forms defining a frame at each point in Σ $\quad q_{ab} = e^i_a e^j_b \delta_{ij}$

densitized triad

$$E^a_i \equiv \frac{1}{2}\epsilon^{abc}\epsilon_{ijk}e^j_b e^k_c \qquad K^i_a \equiv \frac{1}{\sqrt{det(E)}}K_{ab}E^b_j\delta^{ij}$$

symplectic structure

$$\{E^a_j(x), K^i_b(y)\} = \kappa\delta^a_b\delta^i_j\delta(x,y)$$

spin connection

$$\partial_{[a}e^i_{b]} + \epsilon^i{}_{jk}\Gamma^j_{[a}e^k_{b]} = 0$$

Ashtekar-Barbero connection

$$A^i_a = \Gamma^i_a + \gamma K^i_a \qquad \{E^a_j(x), A^i_b(y)\} = \kappa\gamma\delta^a_b\delta^i_j\delta(x,y)$$

Hamiltonian

$$H = N_aV^a(E^a_j, A^j_a) + NS(E^a_j, A^j_a) + N^iG_i(E^a_j, A^j_a)$$

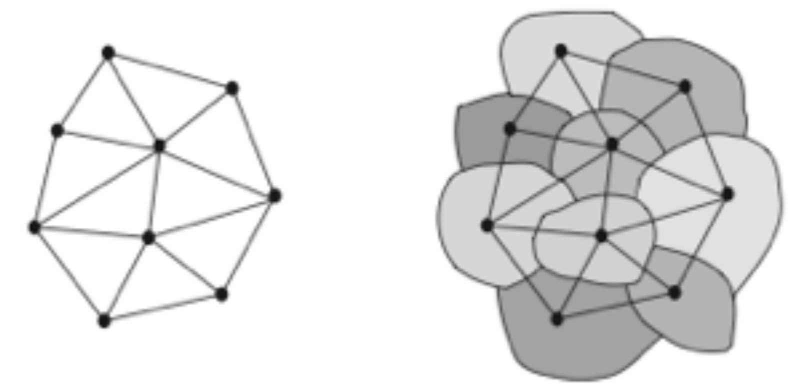
GR = background independent SU(2) gauge theory

(partly analogous to SU(2) Yang-Mills theory)

➢ Kinematical structure: holonomy along a path γ $$h_\gamma[A] = P\exp - \int_\gamma A$$

Cylindrical functionals $\Psi_{\Gamma,f}[A] = f(h_{\gamma_1}[A], \dots, h_{\gamma_{N_\ell^\Gamma}}[A])$

$$\begin{aligned}\langle \Psi_{\Gamma_1,f}, \Psi_{\Gamma_2,g}\rangle &\equiv \mu_{AL}(\overline{\Psi_{\Gamma_1,f}[A]}\Psi_{\Gamma_2,g}[A]) \\ &= \int \prod_{i=1}^{N_\ell^{\tilde\Gamma}} dh_i \overline{\tilde f(h_{\gamma_1}, \cdots, h_{\gamma_{N_\ell^{\tilde\Gamma}}})}\tilde g(h_{\gamma_1}, \cdots, h_{\gamma_{N_\ell^{\tilde\Gamma}}})\end{aligned}$$



$|\Gamma, j_l, v_n\rangle$

description of quantized geometries

Spin network states basis: graphs colored with SU(2) spins

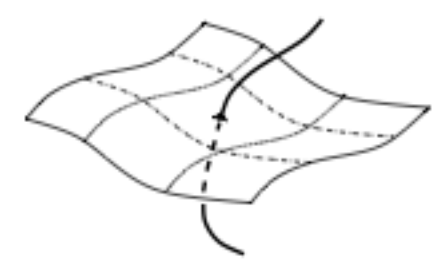
Peter-Weyl th. $f(g) = \sum_j f_j^{mm'} \Pi^j_{mm'}(g)$

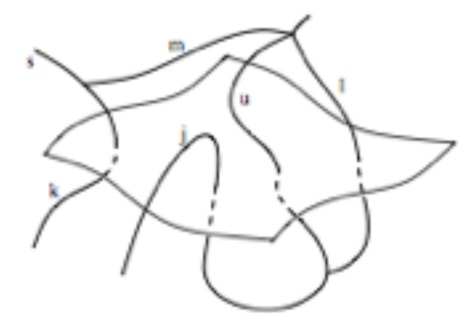
Fluxes $$\hat\Sigma^i_S(x) = \epsilon^i{}_{jk}\int_S \hat e^j(x)\wedge \hat e^k(x) = \int_S n_a \hat E^{ia}(x) = 8\pi\gamma\ell_P^2 \sum_{p\in\gamma\cap S}\delta(x,x_p)\hat J^i(p)$$

with $[\hat J^i(p), \hat J^j(p)] = \epsilon^{ij}{}_k \hat J^k(p)$

✶ Area operator:

$$\hat A_S|\Psi\rangle = \sqrt{\hat E^a_i n_a \hat E^b_j n_b \delta^{ij}}|\Psi\rangle = 8\pi\gamma\ell_P^2 \sum_{p\in\gamma\cap S}\sqrt{j_p(j_p+1)}\,|\Psi\rangle$$





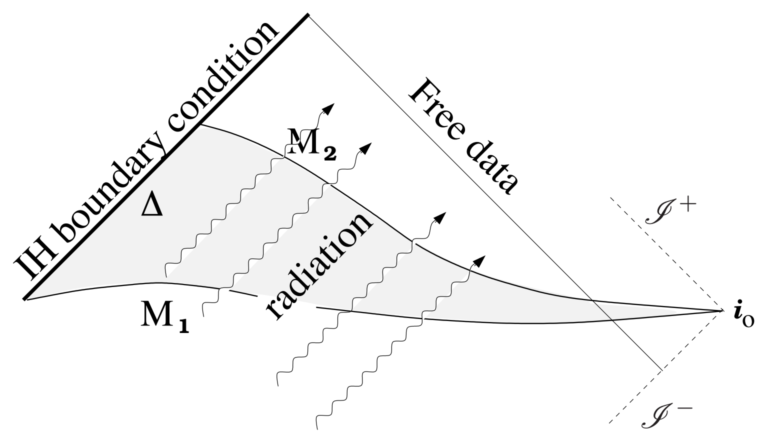
Spectral analysis of geometrical operators ➥ Planck scale discreteness

“Atoms” of quantum space = polymer-like excitations of the gravitational field

Quasi local definition of BH

Isolated Horizons

IH boundary conditions



- $\Delta = S^2 \times \mathbb{R}$ null hyper-surface with vanishing expansion
- $\ell^a =$ normal future pointing null vector field with vanishing expansion within Δ
- Einstein's field equations hold at Δ

$$F^i_{\underleftarrow{ab}}(A) = -\frac{\pi(1-\gamma^2)}{a_H}\Sigma^i_{\underleftarrow{ab}}$$

$$p = (\Sigma, A) \in \Gamma \quad \delta = (\delta\Sigma, \delta A) \in T_p(\Gamma)$$

for the pull back of fields on the horizon δ = linear combinations of SU(2) gauge transformations and diffeomorphisms preserving the preferred foliation of Δ

The presymplectic structure

$$\kappa\,\Omega_M(\delta_1, \delta_2) = \int_M 2\delta_{[1}\Sigma_i \wedge \delta_{2]}K^i$$

is preserved in the presence of an IH (no boundary term needed)

$$= \frac{1}{\gamma}\int_M 2\delta_{[1}\Sigma^i \wedge \delta_{2]}A_i \underbrace{- \frac{a_H}{\pi\gamma(1-\gamma^2)}\int_H \delta_1 A_i \wedge \delta_2 A^i}$$

boundary term given by an SU(2) Chern-Simons presymplectic structure

The single intertwiner BH model

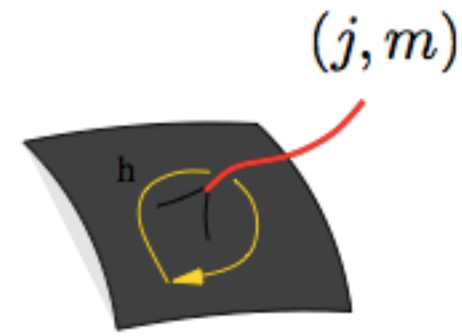
✧ Bulk theory: LQG Hilbert space associated to a fixed graph $\gamma \subset M$ with end points ps on H

$$\hat{a}_H|\{j_p, m_p\}_1^n; \cdots\rangle = 8\pi\gamma\ell_p^2 \sum_{p=1}^{n} \sqrt{j_p(j_p+1)}|\{j_p, m_p\}_1^n; \cdots\rangle$$

spin network states

boundary condition

$$-\frac{a_H}{\pi(1-\gamma^2)}\epsilon^{ab}\hat{F}^i_{ab} = 16\pi G\gamma \sum_{p\in\gamma\cap H} \delta(x, x_p)\hat{J}^i(p)$$



✧ Boundary theory: $SU(2)$ Chern-Simons with punctures

$$S_{CS} + S_{int} = \frac{k}{4\pi}\int_{D\times\mathbb{R}} \mathrm{tr}[A\wedge dA + \frac{2}{3}A\wedge A\wedge A] + \lambda_j \int_c \mathrm{tr}[\tau_3(\Lambda^{-1}d\Lambda + \Lambda^{-1}A\Lambda)]$$

$\Lambda \in SU(2)$ particle d.o.f.

$S^i \in \mathfrak{su}(2)$ momentum conjugate to Λ

Poisson brackets:

$$\{A^i_a(x), A^j_b(y)\} = \delta_{ij}\epsilon_{ab}\frac{2\pi}{k}\delta^2(x-y), \quad a,b = 1,2;\; x^0 = y^0$$

$$\{S^i, \Lambda\} = -\tau^i\Lambda\,, \quad \{S^i, S^j\} = i\epsilon^{ij}{}_k S^k$$

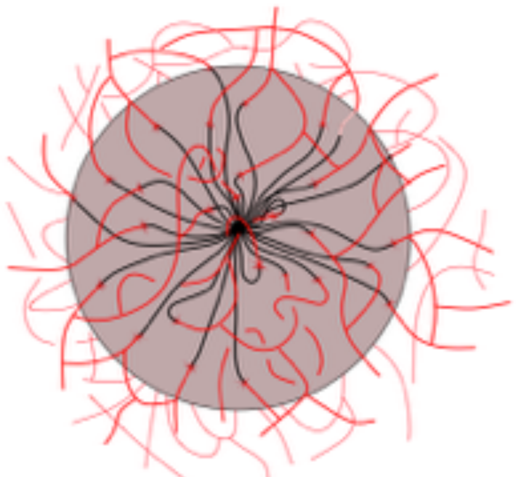
E.O.M. $\quad \epsilon^{ab}F^i_{ab}(A(x)) = -\frac{2\pi}{k}S^i\delta^2(x-p)$

➢ Combinatorial quantization:

➥ $\quad k \leftrightarrow a_H/(4\pi\ell_P^2\gamma(1-\gamma^2)), \quad S^i \leftrightarrow J^i\,, \quad \mathscr{H}^{CS}_{kin}(j_1\cdots j_n) \leftrightarrow \mathrm{Inv}(\otimes_p j_p)$

Quantum BH dof described by a Chern-Simons theory on a punctured 2-sphere H

[Ashtekar, Baez, Corichi, Krasnov 99]
[Engle, Noui, Perez, DP 11]

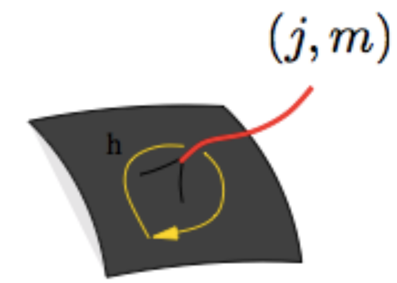


$\Rightarrow$

$$\dim[\mathscr{H}^{\mathrm{CS}}(\mathrm{j}_1 \ldots \mathrm{j}_\mathrm{n})] = \dim[\mathrm{Inv}(\mathrm{j}_1 \otimes \cdots \otimes \mathrm{j}_\mathrm{n})]$$

we can model the IH by a single SU(2) intertwiner

BH entropy d.o.f. = polymer-like excitations of the gravitational field



$$S = \ln \sum_{j_1,\ldots,j_n} \dim[\mathscr{H}^{CS}(j_1 \cdots j_n)] = \frac{a_H}{4\ell_P^2}\frac{\gamma_0}{\gamma} - \frac{3}{2}\log a_H$$

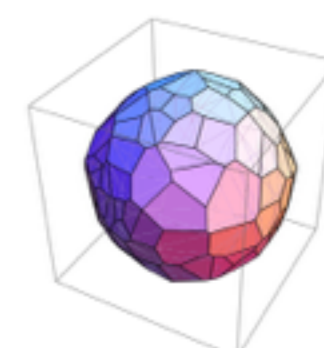
Bekenstein-Hawking formula for

$\gamma = \gamma_0\,, \quad$ with $\gamma_0 = 0.274067\ldots$

[Kaul, Majumdar 98]
[Agullo, Barbero, Diaz-Polo, Fernandez-Borja, Villasenor 08]
[Ghosh, Mitra 05]
[Livine, Terno 05]
[Engle, Noui, Perez, DP 11]

Semiclassical limit of the SU(2) intertwiner quantum geometry: tesselated surfaces

[Livine, Terno 05; Bianchi 10]



BH microstates $\Longleftrightarrow$ horizon *quantum shapes*

$$\gamma_0 = 0.274067\ldots$$ quite random number!!

No physical insight: γ not expected to play any role in the semi-classical limit

[Frodden, Geiller, Noui, Perez 12; Ben Achour, Mouchet, Nuoi 14]

However, one can make sense of the **analytic continuation** of the Verlinde formula to $\gamma = i$

and obtain an entropy which does not depend on the Immirzi parameter any more

➢ But what's so special about $\gamma = i$?

The self-dual Ashtekar connection can be derived from a manifestly covariant action (maintaining full local Lorentz invariance) [Jacobson, Smolin 88]

while the Ashtekar-Barbero connection cannot be interpreted as a space-time connection [Samuel 00; Alexandrov 01]

"The nature of BH entropy is intimately related to the nature of BH temperature. We cannot understand the one without the other." [Bill Unruh, Loops13]

* Local observer perspective + Unruh temp. by hand [Ghosh, Perez 11; Frodden, Ghosh, Perez 11]
* KMS-state of a quantum IH: $\beta_{IH} = 2\pi(1-1/k) \Leftrightarrow \gamma = i$ [DP 13]

☞ Thermality of the density matrix associated to the horizon quantum state originates from the entanglement between internal and external horizon dof

➥ $$S = \frac{A_{IH}}{4\ell_P^2} + \mu N$$

quantum hair argued to be associated to a new horizon microscopic observable

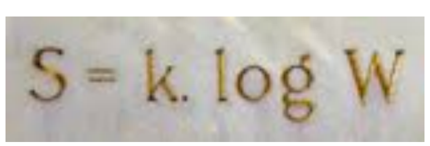
(call for a GFT description in order to make sense of it)

☞ $$S_{Bol} = -\beta^2 \frac{\partial}{\partial\beta}\left(\frac{1}{\beta}\ln Z\right)$$

Boltzmann ent. = Entanglement ent.

$$S_{ent} = -\mathrm{tr}(\hat{\rho}\ln\hat{\rho})$$

[Sorkin 86]



W = number of horizon \`quantum shapes'

$\Leftarrow$ **Intertwiner structure** encoding $\Rightarrow$

[DP 13]

Correlations of quantum geometry dof across the horizon

Carlip's proposal

- ➢ 2+1 gravity acquires new degrees of freedom in presence of a boundary (broken gauge invariance)
- ➢ In the Chern-Simons formulation, these are described by WZW theory
- ➢ new, dynamical "would-be gauge" d.o.f. can account for the BH entropy

Attempt to describe the microphysics of BH in terms of a "dual" 2-dim Conformal Field Theory

Powerful method

Cardy formula:

$$S = 2\pi\sqrt{\frac{cL_0}{6}}$$

However, several open questions:

- ✻ what is the microscopic nature of the d.o.f.?
- ✻ where do the d.o.f. live?
- ✻ extension to higher dimensions?

☞ Universality problem:

(hidden) CFT symmetry underlying different microscopic approaches to BH entropy?

BH Entropy in LQG

$$S_{LQG} = \frac{A}{4\ell_p^2} + \mu N$$

Main open questions:

- Can inclusion of new (matter) d.o.f. on the IH give the Bekenstein-Hawking formula?
  (see e.g. proposal of [Ghosh, Noui, Perez 13])

- Is there a unified treatment to quantize bulk and boundary d.o.f.?

- Are there CFT d.o.f. lurking somewhere?
  (does LQG belongs to Carlip's `universality class'?)

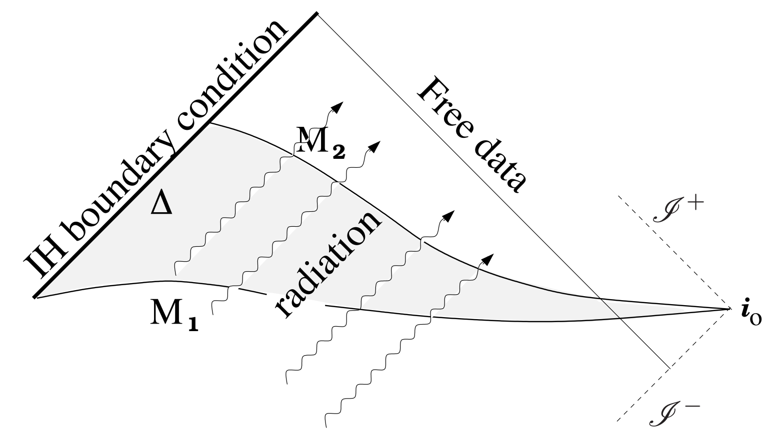
- Can we learn something about the full theory?
  (see the example of AdS/CFT)

Turaev-Viro/LQG

SU(2) BF variables for IH

[DP, Sahlmann 14]

$$\begin{aligned}\kappa\Omega(\delta_1,\delta_2) &= \frac{1}{\gamma}\int_M \delta_{[1}\Sigma^i\wedge\delta_{2]}\gamma K_i\\ &= \frac{1}{\gamma}\int_M \delta_{[1}\Sigma^i\wedge\delta_{2]}A_i - \frac{1}{\gamma}\int_{\partial M}\delta_{[1}e^i\wedge\delta_{2]}e_i\\ &= \frac{1}{\gamma}\int_M \delta_{[1}\Sigma^i\wedge\delta_{2]}A_i + \frac{1}{\gamma^2}\sqrt{\frac{a_{IH}}{2\pi}}\int_{IH}\delta_{[1}e^i\wedge\delta_{2]}A_i\end{aligned}$$



$p=(\Sigma,A)\in\Gamma \quad \delta=(\delta\Sigma,\delta A)\in T_p(\Gamma)$

for the pull back of fields on the horizon δ = linear combinations of SU(2) gauge transformations and diffeomorphisms preserving the preferred foliation of Δ

$$K_a^i = -\sqrt{\frac{2\pi}{a_{IH}}}e_a^i$$

$$\{e_a^i(x), e_b^j(y)\} = -\kappa\gamma\epsilon_{ab}\;\delta^{ij}\delta^{(2)}(x,\,y)$$

$$\Rightarrow$$

$$\{A_a^i(x)\,,\,\tilde{e}_b^j(y)\} = \kappa\gamma\epsilon_{ab}\;\delta^{ij}\delta^{(2)}(x,\,y)$$

with $\quad \tilde{e}_a^i := \frac{1}{\gamma}\sqrt{\frac{a_{IH}}{2\pi}}e_a^i$

and the Ashtekar-Barbero boundary connection becomes non-commutative

IH boundary conditions:

$$F^i(A) = -\frac{\pi}{a_{IH}}(1-\gamma^2)\,\Sigma^i$$

$$d_A\tilde{e}^i = -\Sigma^i$$

resemblance with 2+1 gravity with CC in presence of point particles

But we know how to deal with non-commutative holonomies in 2+1 LQG [Noui, Perez, DP 11]:

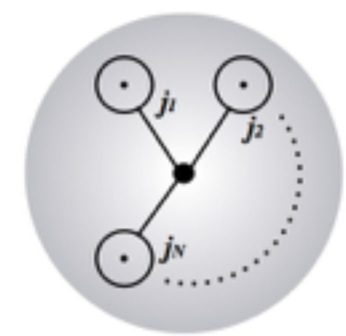
$$\Rightarrow \quad \begin{aligned} A_a^i &= \Gamma_a^i + \gamma K_a^i = \Gamma_a^i - \frac{2\pi\gamma^2}{a_{IH}}\tilde{e}_a^i \\ \tilde{A}_a^i &= A_a^i + \alpha_\pm \tilde{e}_a^i = \Gamma_a^i \pm \frac{2\pi\gamma}{a_{IH}}\tilde{e}_a^i \end{aligned} \qquad \text{with} \qquad \alpha_\pm = \gamma(\gamma \pm 1)\frac{2\pi}{a_{IH}}$$

after introducing a cellular decomposition Γ_{IH} of the horizon 2-sphere

$$\boxed{\begin{aligned} &F_p^i(A) = 0, \ \ d_A\tilde{e}^i = 0 && \forall p \notin \cup\ell_i \\ &F_p^i(\tilde{A}) = 0, \ \ d_A\tilde{e}^i = -\Sigma_p^i && \forall p \in \cup\ell_i \end{aligned}}$$

➢ Quantization: we can extend LQG techniques from the bulk to the IH

IH quantum states:



blowing up of point punctures to finite loops due to the extended nature of the phase space variables used for quantization in LQG

$\longrightarrow$ generalized spin-network states [Freidel, Louapre 04] [Noui, Perez 04]

IH Hilbert space observables: holonomies of the non-commutative connections and appropriately smeared functionals of the dyad field represented as quantum operators on $L_2(\overline{\mathcal{A}}, \mathrm{d}\mu^q_{\mathrm{AL}})$

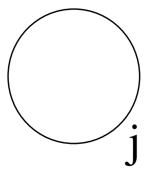
$$[\hat{\tilde{e}}(\eta), h_\gamma] = i\hbar\kappa\gamma \sum_{p\in\eta\cap\gamma} \mathrm{sign}(\epsilon_{ab}\dot{\eta}^a\dot{\gamma}^b(p))h_{\gamma_2(p)}\hat{J}_i h_{\gamma_1(p)} \quad \rightarrow \quad \overset{\text{modified Gauss law}}{\epsilon^{ab}\hat{\Sigma}^i_{ab}(x) = 2\kappa\gamma \sum_{p\in\Gamma\cap IH} \delta(x, x_p)\hat{J}^i(p)}$$

the bulk geometry induces conical singularities in the boundary torsion, which can be interpreted as point particles

We can use techniques developed for the quantization of 2+1 gravity with CC [DP 14]:

$$C[N] = \lim_{\epsilon\to 0} \sum_{p\notin \cup \ell_i} \mathrm{tr}\left[N_p W_p(A)\right] + \lim_{\epsilon\to 0} \sum_{p\in \cup \ell_i} \mathrm{tr}\left[N_p W_p\left(\tilde{A}\right)\right] = 0$$

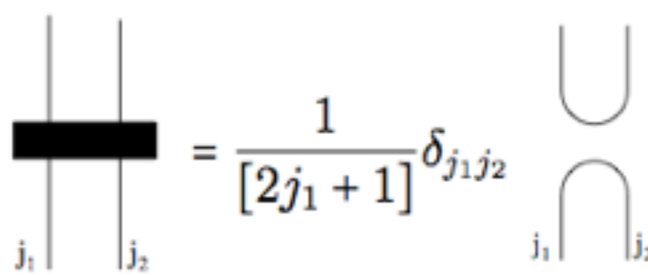
and in order to have an anomaly-free (first class) constraint algebra



$$= (-)^{2j}[2j+1]_q = (-)^{2j}\frac{q^{2j+1}-q^{-(2j+1)}}{q-q^{-1}} \qquad \text{where} \qquad q = \begin{cases} e^{\frac{\pi i \hbar \kappa \gamma^3}{a_{IH}}}, & \text{for } p \notin \cup \ell_i \\ e^{\frac{\pi i \hbar \kappa \gamma^2}{a_{IH}}}, & \text{for } p \in \cup \ell_i \end{cases}$$

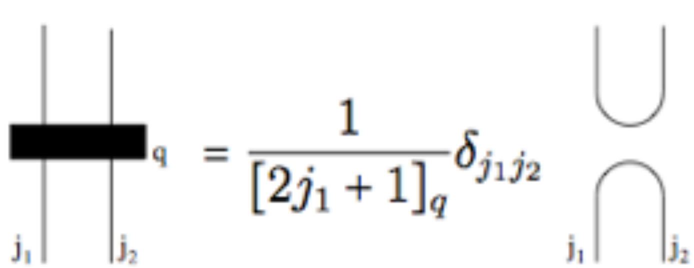
☞ at each plaquette, the recoupling theory of the classical SU(2) group has to be replaced with the one of the quantum group UqSL(2)

the skein relation

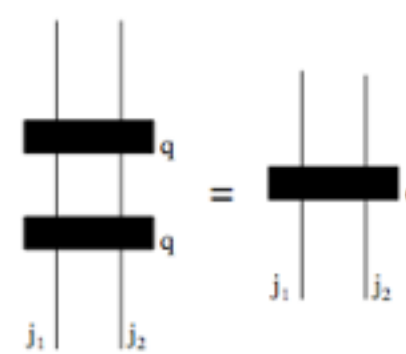


has to be modified, since it does not satisfy the projector property anymore

in order to preserve the properties of the Ashtekar-Lewandowski measure:



$\Rightarrow$



✦ Physical scalar product for the IH boundary theory : $\langle s,\ s' \rangle_{phys} = \langle P[A,\tilde{A}]s,\ s' \rangle$ where

projector operator into the IH physical Hilbert space (same form of the physical projector of 2+1 gravity with CC)

$$
\begin{aligned}
P[A,\tilde{A}] &= \lim_{\epsilon \to 0} \prod_{p \notin \cup \ell_i} \delta(W_p(A)) \prod_{p \in \cup \ell_i} \delta(W_p(\tilde{A})) \\
&= \lim_{\epsilon \to 0} \sum_{j_p} \prod_{p \notin \cup \ell_i} (-)^{2j_p} [2j_p + 1]_q\, \chi_{j_p}(W_p(A)) \prod_{p \in \cup \ell_i} (-)^{2j_p} [2j_p + 1]_q\, \chi_{j_p}(W_p(\tilde{A}))
\end{aligned}
$$

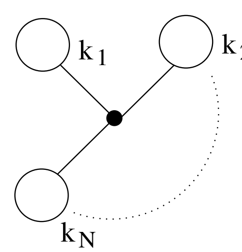
[Witten 89] argument: if M is obtained from the connected sum of two three manifolds M_1 and M_2 joined along a two sphere S^2 and containing N unlinked and unknotted circles C_i

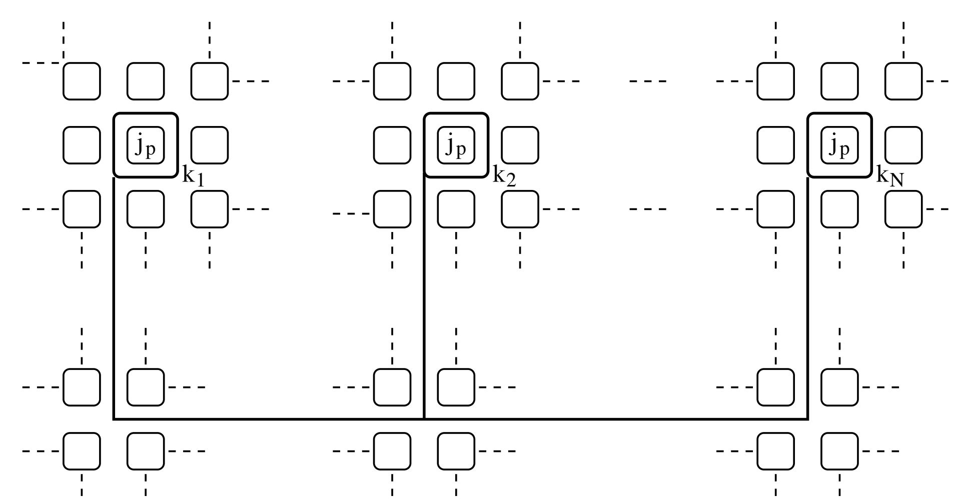
$$
\langle \Psi_2 | \Psi_1 \rangle = Z(\underbrace{M}_{M = S^2 \times S^1}; \prod_{i=1}^{N} C_i) = \dim \mathscr{H}_{S^2;\otimes_i j_i}
$$

equivalence between Chern-Simons and BF formulations

$|Z_{RT}(M)|^2 = Z_{TV}(M)$

➥ $S_{IH} = \log(\mathscr{N})$ with

$$\mathscr{N} = \langle P\,\emptyset,$$  $$\rangle$$



$$
\sim \prod_i (-)^{2k_i} [2k_i + 1]_q = \prod_i \underbrace{e^{2\pi i k_i}} [2k_i + 1]_q
$$

Holographic bound: $\exp\left(\frac{a_i}{4\ell_P^2}\right)$ [DP, Sahlmann 14]

with $\gamma = i$, $(a_i = 8\pi \ell_P^2 \gamma k_i)$

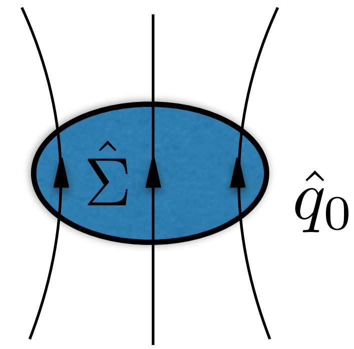
CFT/Turaev-Viro correspondence

(see also [Freidel, Krasnov 02])

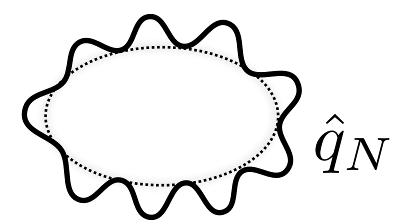
CFT/LQG

- Regularization procedure introduces a new boundary at each puncture
- Infinite set of charges satisfying a Kac-Moody algebra (diffeos on the circle)
- Due to central extension would-be-gauge d.o.f. become physical
- IH boundary conditions ➞ CFT/gravity correspondence

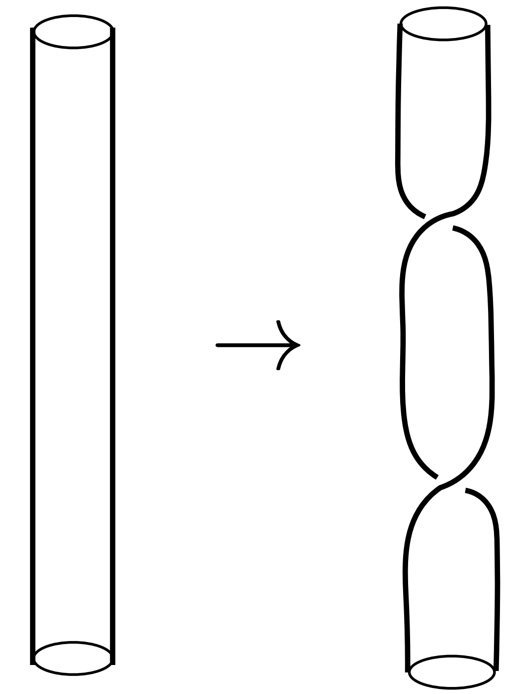
$$q_0^{(B)i} = -\frac{k}{2\pi}\oint_{\partial B} A^i = J^i$$ ➞ gravitational d.o.f.



$$q_N^{(B)i} = -\frac{k}{2\pi}\oint_{\partial B} e^{iN\theta} A^i\,, \quad N > 0$$ ➞ new matter d.o.f. (bosonic modes)



dynamics induced by L_0
=
particles self-interactions



\`Affinization' of the gravitational SU(2) finite Lie algebra ➥ Infinite tower of new d.o.f.

Local conformal symmetry
at each puncture on the horizon

$\gamma = i$
ultimately related to
the horizon thermality

Extra (matter) d.o.f.:

holographic degeneracy factor in Z in agreement
with Bekenstein-Hawking formula

✪ Let's exploit this CFT/Turaev-Viro duality to understand better the analytic continuation to $\gamma = i$:

Notion of phase space 'Wick rotation' via an invertible phase space make W:

$$A^{\mathbb{C}j}_a(x) = W^{-1}\cdot A^j_a(x) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\{A^j_a(x), C\}_{(n)}$$

infinitesimal generator

Complexifier

corresponding operator acting on $\mathscr{H} = L_2(\overline{\mathcal{A}}, \mathrm{d}\mu_{\mathrm{AL}})$: $\quad \hat{W} := \exp\left(-\frac{\hat{C}}{\hbar}\right), \qquad \hat{A}^{\mathbb{C}} := \hat{W}^{-1}\hat{A}\hat{W}$

Thiemann's proposal (in 2+1 for generic γ), using IH boundary conditions:

$$C = -\left(\frac{\pi}{2} - i\ln\gamma\right)\frac{1}{4\kappa G\hbar(1-\gamma^2)}\frac{1}{k}\int_{IH} d^2x\ \epsilon^{ab}\tilde{e}^i_a\tilde{e}^j_b\ \delta_{ij} \qquad \text{where} \qquad \tilde{e}^i_a := \frac{1}{\gamma}\sqrt{\frac{a_{IH}}{2\pi}}\,e^i_a$$

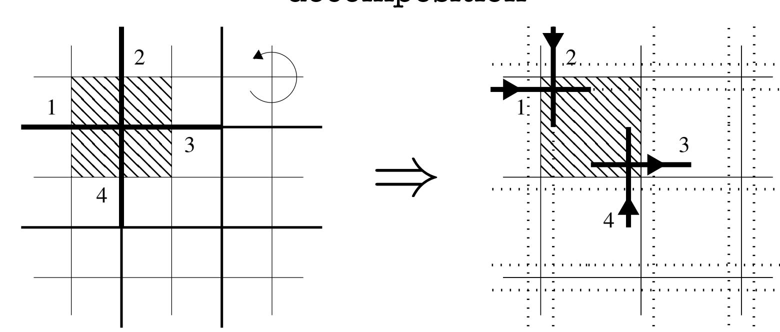
by means of the Poisson bracket of the IH theory in its BF formulation

$$\rightarrow \quad A^{\mathbb{C}}_a(x) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\{(\Gamma_a(x) + \gamma K_a(x)), C\}_{(n)} = \Gamma_a(x) - iK_a(x) \quad ✓$$

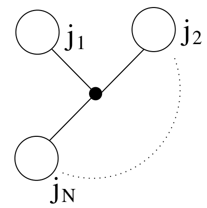
➢ Quantization: let us recall $\quad \hat{\tilde{e}}^i_a = -i8\pi G\hbar\gamma\epsilon_{ab}\,\delta^i_j\frac{\delta}{\delta A^j_b}\,, \quad \tilde{e}^i(\eta) = \int_\eta dt\ \tilde{e}^i_a\dot{\eta}^a$

$$\hat{W} = \lim_{\varepsilon\to 0}\ \exp\left(\frac{(\pi/2 - i\ln\gamma)}{32\pi(G\hbar)^2\gamma}\frac{1}{k}\sum_p\sum_{a=1}^{4}\frac{1}{4}\ \hat{\tilde{e}}^i(\eta_a)\hat{\tilde{e}}^j(\eta_{a+1})\ \delta_{ij}\right)$$

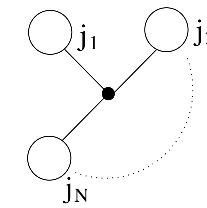
dual cellular decomposition



$$\Rightarrow \quad \hat{W} \,|$$



$$\rangle = \exp\left(2\pi \left(\frac{\pi}{2} - i \ln \gamma \right) \gamma \sum_{p=1}^{N} \frac{j_p(j_p+1)}{k} \right) |$$



$$\rangle$$

Complexifier $\hat{C}$ $\leftrightarrow$ $\hat{L}_0$ Virasoro energy generator

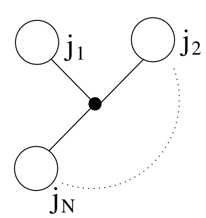
where recall

$$\hat{L}_0 |v_j\rangle = \frac{j(j+1)}{k+2} |v_j\rangle$$

[Mourao and DP w.i.p.]

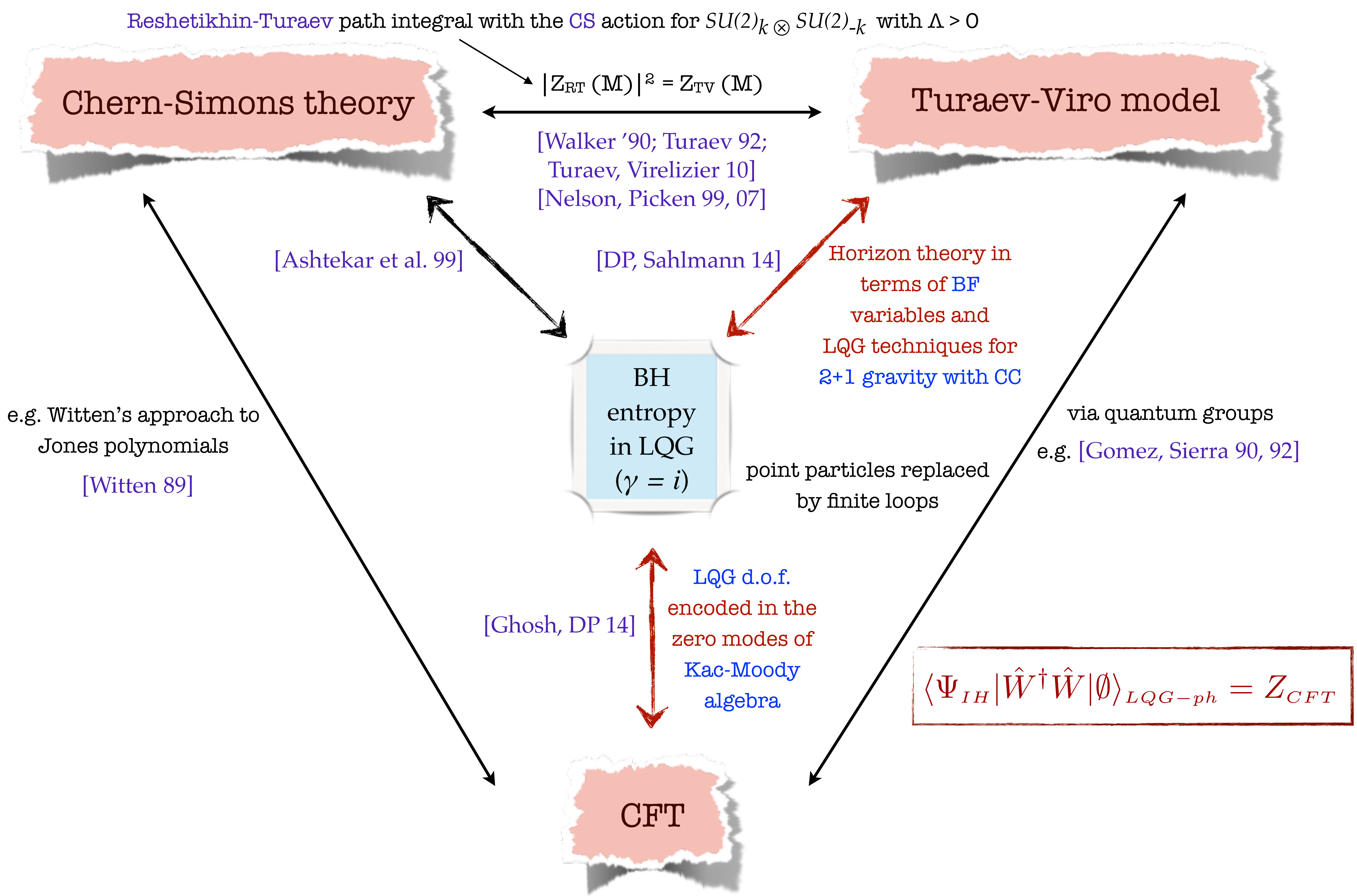
$\hat{L}_0$ is the time evolution generator in the CFT $\longrightarrow$ Entropy from the Euclidean time evolution of the physical scalar product

$$\langle$$



$$| \hat{W}^\dagger \hat{W} | \emptyset \rangle_{phys} = \prod_p Z^p_{CFT}(\tau = \pi\gamma/2)$$

☞ The CFT dual description could guide us towards the physical understanding and, at the same time, rigorous implementation of the analytic continuation to $\gamma = i$



Reshetikhin-Turaev path integral with the CS action for $SU(2)_k \otimes SU(2)_{-k}$ with $\Lambda > 0$
$|Z_{RT}(M)|^2 = Z_{TV}(M)$
Chern-Simons theory
Turaev-Viro model
[Walker '90; Turaev 92; Turaev, Virelizier 10] [Nelson, Picken 99, 07]
[Ashtekar et al. 99]
[DP, Sahlmann 14]
Horizon theory in terms of BF variables and LQG techniques for 2+1 gravity with CC
BH entropy in LQG ($\gamma = i$)
e.g. Witten's approach to Jones polynomials
[Witten 89]
via quantum groups
e.g. [Gomez, Sierra 90, 92]
point particles replaced by finite loops
[Ghosh, DP 14]
LQG d.o.f. encoded in the zero modes of Kac-Moody algebra
$\langle \Psi_{IH} | \hat{W}^\dagger \hat{W} | \emptyset \rangle_{LQG-ph} = Z_{CFT}$
CFT