Quantum Order at Infinite Temperature, "Time Crystals", Dissipation, and Constraints

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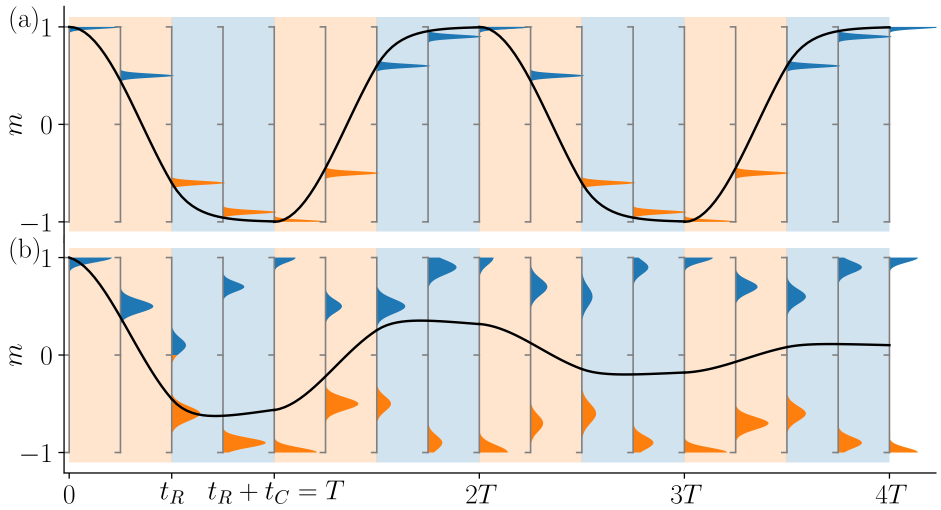

Main References: PRL 116, 250401 (2016), PRE 100, 060105 (2019), PR Research 2, 022002 (2020), PR Research 2, 023159 (2020)

Collaborators

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This talk

- Will be mostly on Floquet-type systems, ie, periodically driven
- Describe the main problem with Floquet systems: **Heating** and the need to prevent it
- Introduce and describe a **closed/unitary** prototypical disordered model
  - Breaking ergodicity via disorder: MBL
  - Observe “time-crystalline” order
- ~~Introduce an **open** (markovian) model~~
  - ~~prevent heating via contact with environment~~
  - ~~Observe “time-crystalline” order~~
- A different direction:
  - Breaking ergodicity via **dynamical constraints**

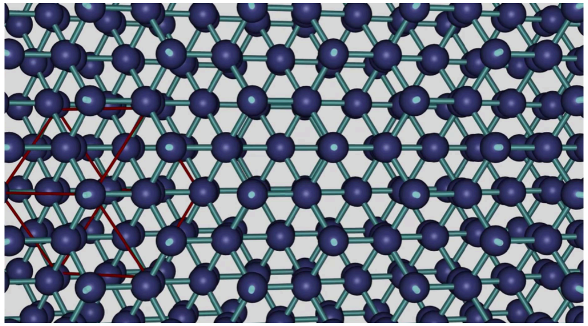
Thermodynamics and Thermalisation

Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

What is our overall goal?

Interesting phases of matter
far from equilibrium
(starting with time crystals)

Phases of matter in or out of equilibrium



- A **crystal** is an example of an **equilibrium** symmetry-broken phase
- Many-body system breaking **spatial** translational invariance (of its description, ie of its Hamiltonian)

- The analogous symmetry in time can be **continuous** (static systems) or **discrete** (periodically-driven, or **Floquet**, systems)
- If broken, the resulting phase might be called a **time crystal**
- Breaks **temporal** translational invariance

- Name was used in 2012 for breaking a **continuous** symmetry, which was promptly proven impossible in 2013 and again in 2015
- For **discrete** symmetry, it is possible as shown in 2016 for **Floquet** systems

- Two main theoretical directions:
  - **Closed** quantum systems
  - **Open** systems (environment)
- Experiments

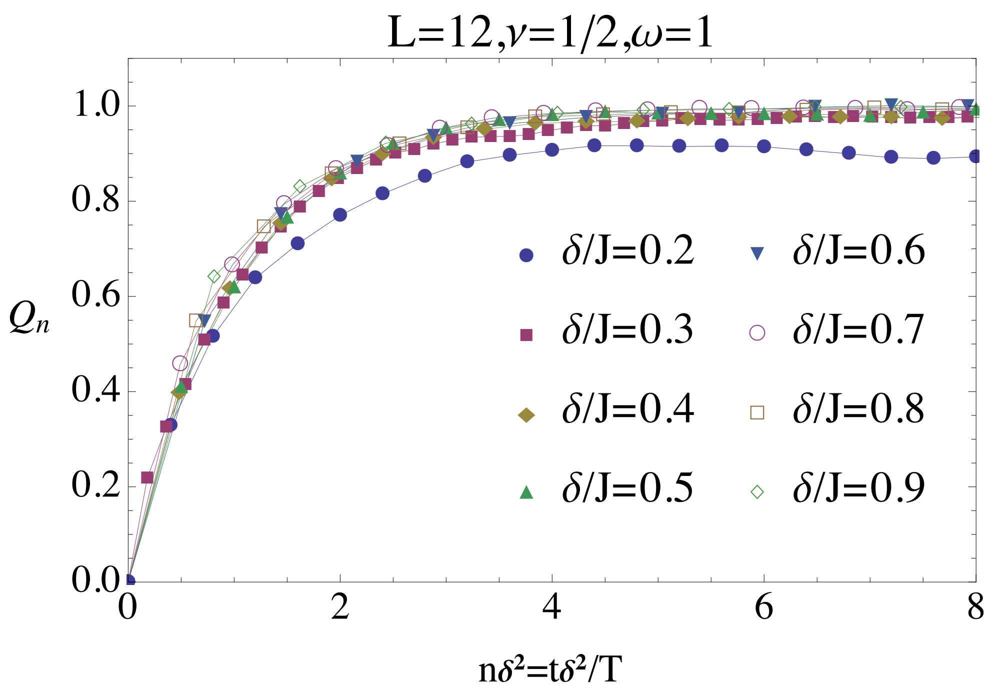
Thermodynamics and Thermalisation in Floquet systems

Thermalisation in *periodically-driven* quantum systems

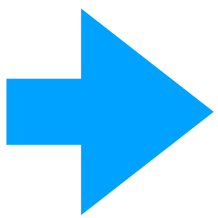
- Hamiltonian varies periodically in time $H(t) = H(t+T)$
  Purely unitary time evolution, example Hamiltonian (hard-core bosons in 1d)

$$H(t) = -\frac{1}{2}\left(J + \delta\cos(\omega t)\right)\sum_{i=1}^{L-1} b_i^\dagger b_{i+1} + V\sum_{i=1}^{L-1} n_i n_{i+1}$$

Heating up:



- δ=driving amplitude
- T=period
- $Q_n = 2\frac{\langle H\rangle - E_{min}}{E_{max} - E_{min}}$



Heats up to a “maximum entropy” or “infinite temperature” state; Want to stop it

Lazarides *et al.* (2014), D’Alessio *et al.* (2014), Ponte *et al.* (2015)

Closed Floquet "time crystals": How and what

Key idea: Prevent heating by ergodicity breaking (due to localisation)

1. Does localisation prevent heating to infinite temperatures ?

2. Do any "interesting" phases, *exclusive to Floquet systems* appear?

   ie, not mimicking some static model or system but "genuinely out of equilibrium"

3. What is the phenomenology and mechanism of emergence of these phases?

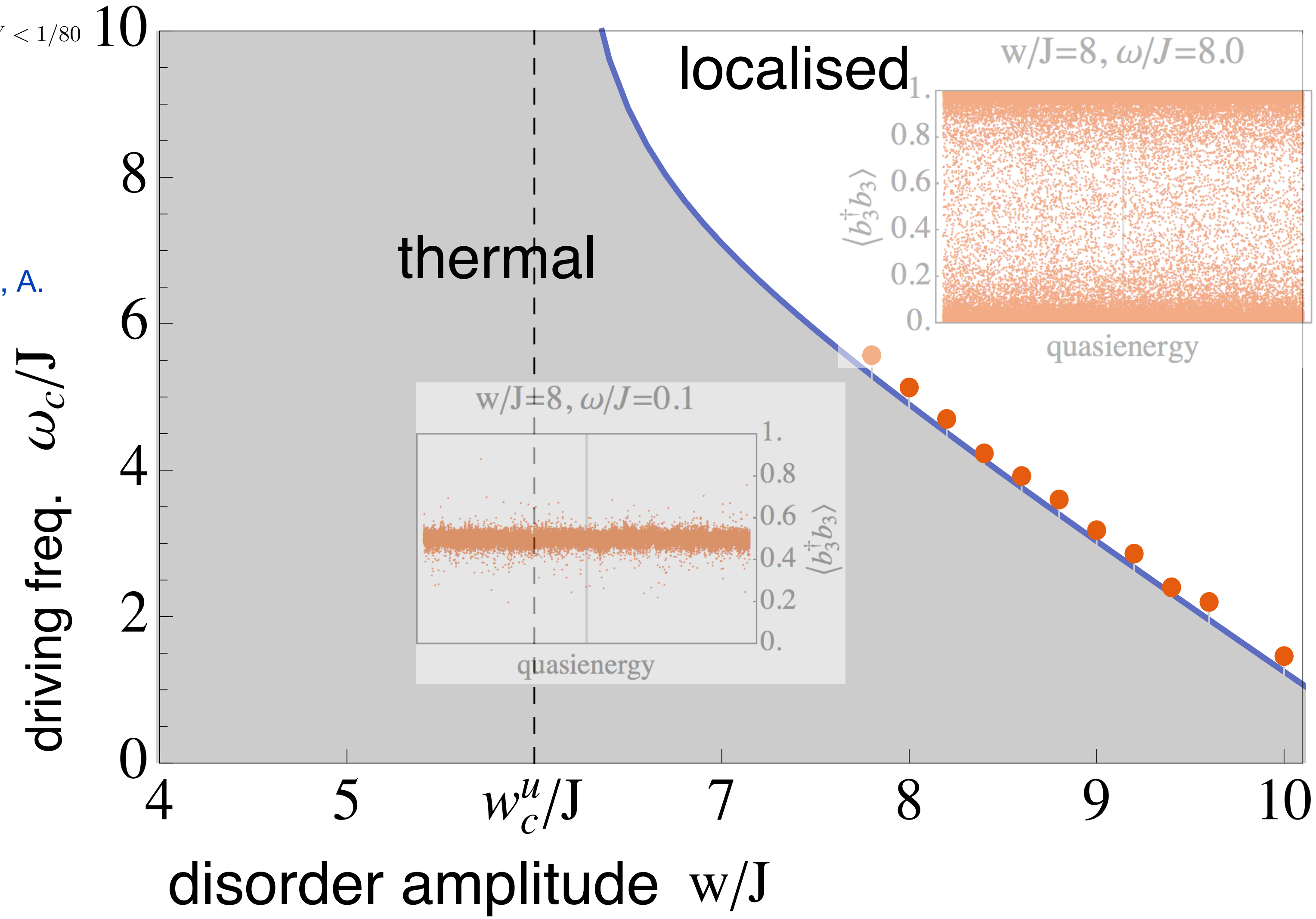
Does localisation prevent heating to infinite temperatures?

$$H_0 = -\frac{1}{2}\left(J + \delta J(t)\right)\sum_{i=1}^{L-1} b_i^\dagger b_{i+1} + \sum_{r=1}^{2} V_r \sum_{i=1}^{L-1} n_i n_{i+r} + \sum_{i=1}^{L} U_i n_i$$ hard-core bosons

$U_i \in [-W, W]$; weak driving: $\delta J(t)/W < 1/80$

$$\delta J(t) = \begin{cases} \delta J, & \mod(t,T) < T/2 \\ -\delta J, & \mod(t,T) > T/2 \end{cases}$$

AL, A. Das, R. Moessner (2015)
P. Ponte, Z. Papic, F. Huveneers, A. Abanin (2015)



Experiment: Bordia, Luschen, Schneider, Knap, Bloch, Nat. Phys. 2017

Does localisation prevent heating to infinite temperatures?

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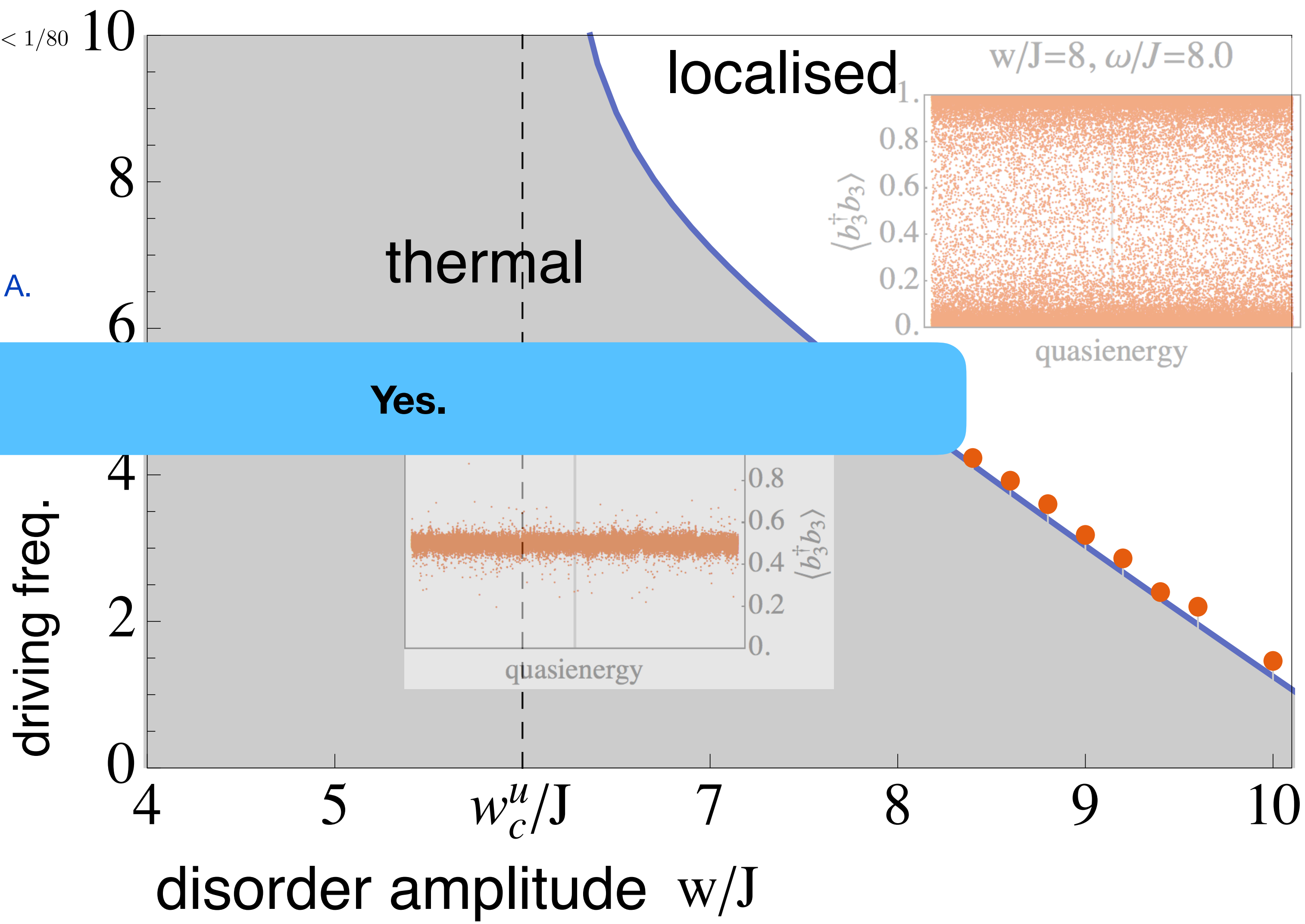


Yes.

Experiment: Bordia, Luschen, Schneider, Knap, Bloch, Nat. Phys. 2017

Interesting phases?

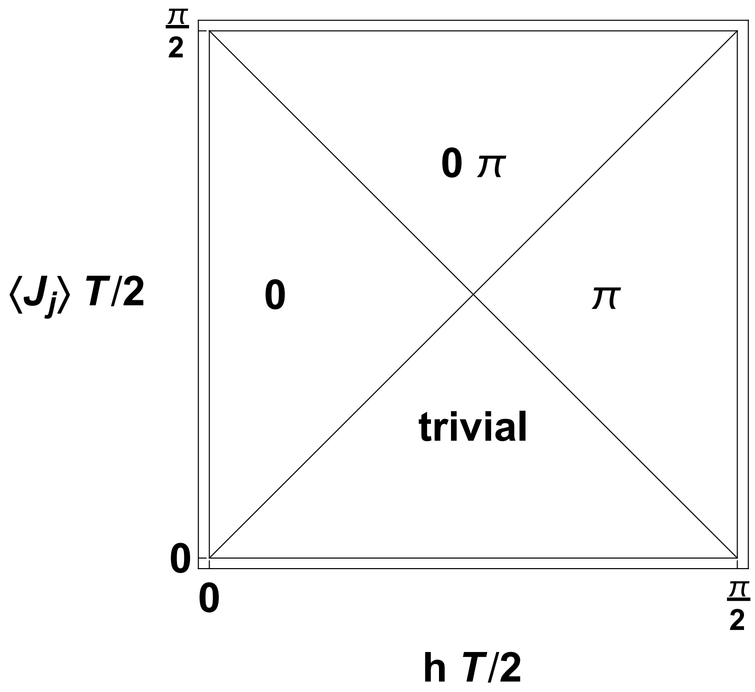
Interesting phases? Yes.

Minimal model: Driven disordered TFIM

$$H(t) = \begin{cases} H_1 = -h\sum_j \sigma_j^x, & 0 < t \le T/2 \\ H_2 = -\sum_j J_j \sigma_j^z \sigma_{j+1}^z, & T/2 < t \le T \end{cases}$$

Unitary time-evolution operator has Z_2 Ising symmetry

$$U_F = \mathcal{T} \exp\left[-i \int_0^T dt H(t)\right]$$
$$= \exp(-iH_2T/2)\exp(-iH_1T/2)$$



Region in parameter space (ie, a **phase**, labelled π) with paired but **not** degenerate eigenstates (cf static Ising model in FM phase), cartoon:

$$|\omega, \pm\rangle = (|\uparrow\downarrow\uparrow\uparrow \ldots \downarrow\uparrow\rangle \pm |\downarrow\uparrow\downarrow\downarrow \ldots \uparrow\downarrow\rangle)/\sqrt{2}$$ → $$\langle\omega, \pm| \sigma_i^z \sigma_j^z |\omega, \pm\rangle \neq 0 \quad |i-j| \to \infty$$

with, instead of degeneracy (as in static TFIM), the following relation

$$U_F |\omega, +\rangle = \exp(-i\omega T)|\omega, +\rangle$$
$$U_F |\omega, -\rangle = -\exp(-i\omega T)|\omega, -\rangle$$

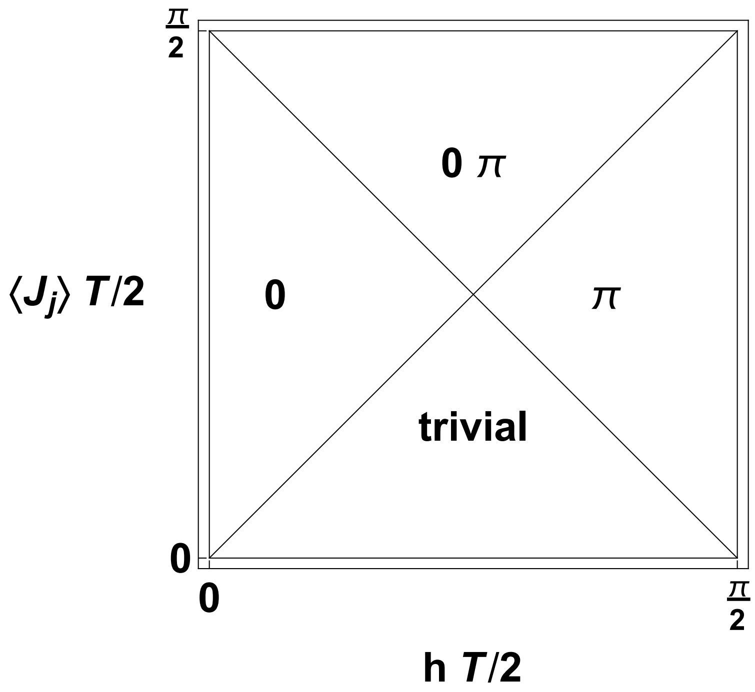
Khemani, AL, Moessner, Sondhi PRL 2016

AL, Moessner, PRB 2017

Implications of eigenstate structure

Minimal model: Driven disordered TFIM in π phase.

$$U_F\,|\omega,\pm\rangle = \pm\exp(-i\omega T)\,|\omega,\pm\rangle$$
$$\pm 1 = \exp\left(-i\frac{\pi}{T}T\right)$$



$$|\omega,\pm\rangle = \left(|\uparrow\downarrow\uparrow\uparrow\ldots\downarrow\uparrow\rangle \pm |\downarrow\uparrow\downarrow\downarrow\ldots\uparrow\downarrow\rangle\right)/\sqrt{2}$$

Pick locally correlated initial state:

$$|\psi\rangle = \left(|\omega,-\rangle + |\omega,+\rangle\right)/\sqrt{2} = |\uparrow\downarrow\uparrow\uparrow\ldots\downarrow\uparrow\rangle$$

Stroboscopic time evolution, $|\psi_n\rangle = U_F^n\,|\psi\rangle$; pick some operator O:

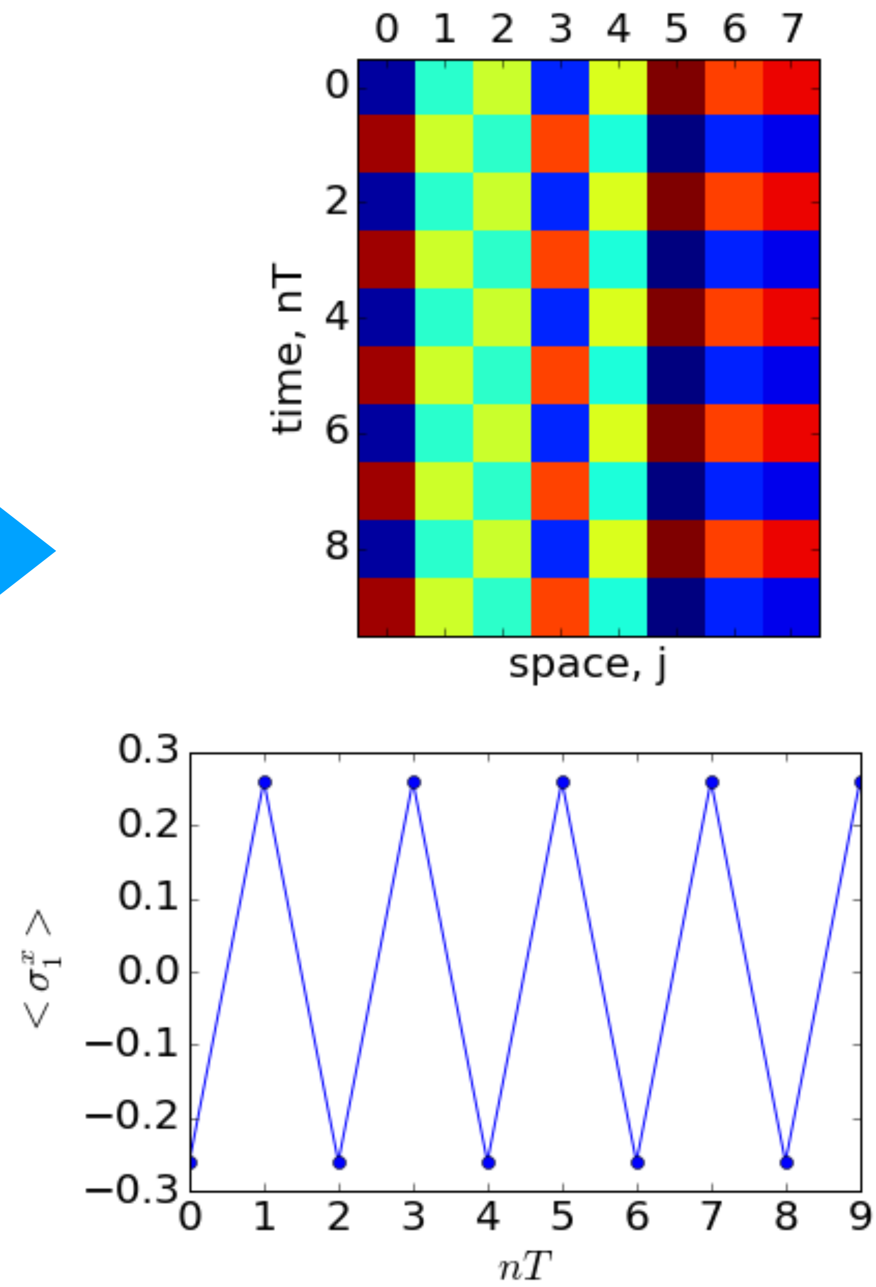
$$\langle\psi_n|\,\hat{O}\,|\psi_n\rangle = \langle\omega_\alpha +|\,\hat{O}\,|\omega_\alpha +\rangle + \langle\omega_\alpha -|\,\hat{O}\,|\omega_\alpha -\rangle + (-1)^n\left(\langle\omega_\alpha -|\,\hat{O}\,|\omega_\alpha +\rangle - \langle\omega_\alpha +|\,\hat{O}\,|\omega_\alpha -\rangle\right)$$

Therefore this class of initial states oscillates with twice the period of the driving

Khemani, AL, Moessner, Sondhi PRL 2016
AL, Moessner, PRB 2017

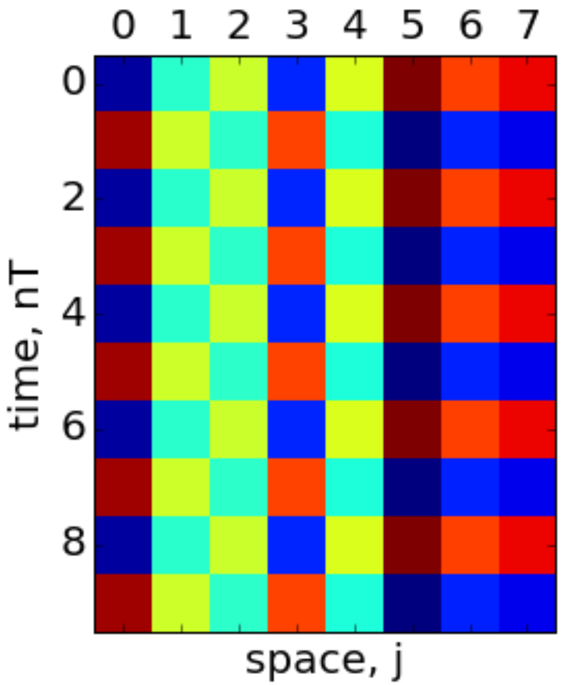
Eigenstate structure

Start in locally correlated state, display order in space, oscillate in time for all time: a purely dynamical phenomenon



Summary

- Glassy order in all eigenstates (“infinite-temperature order”)
- Disorder plays two roles:
  - prevents heating by breaking ergodicity
  - allows for structure in eigenstates
- **Initial** state breaks symmetry **spatially**, **temporal** symmetry also broken (“forever”)
- Very **delicate** effect: sensitive to exposure to external environment (AL, Moessner, PRB 2017)
- related **experiments**:
  - Choi et al (Lukin group) and Zhang et al (trapped ions, Monroe group), both Nature 2017
  - Rovny et al (Barrett group) and Pal, Nishad, Mahesh, Sreejith (NMR, both 2018)



Why is this not just a pendulum?

- A pendulum only oscillates forever if it is a single degree of freedom–motion lost to entropy otherwise: **CoM motion degenerates into internal degrees of freedom (heat)**
- Here, as long as finite imbalance initially→**perpetual** oscillations

Why is this not just a pendulum?

- A pendulum only oscillates forever if it is a single degree of freedom–motion lost to entropy otherwise
- Here, as long as finite imbalance initially→persistent oscillations
- More sophisticated viewpoint:

Usual spontaneous symmetry breaking (eg, TFIM):

$$\langle \sigma_i^z \rangle = 0$$
$$\lim_{|i-j|\to\infty} \left\langle \sigma_i^z \sigma_j^z \right\rangle = C \neq 0$$

in eigenstates

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in eigenstates

Discrete Time Crystal:

in floquet states

$$\langle \sigma_i^z(t) \rangle = 0$$

$$\lim_{t \to \infty} \langle \sigma_i^z(0) \sigma_i^z(t) \rangle = f(t) = f(t+T) \neq 0$$

Constrained Systems

Common thread in Floquet systems: preventing heating by ergodicity breaking

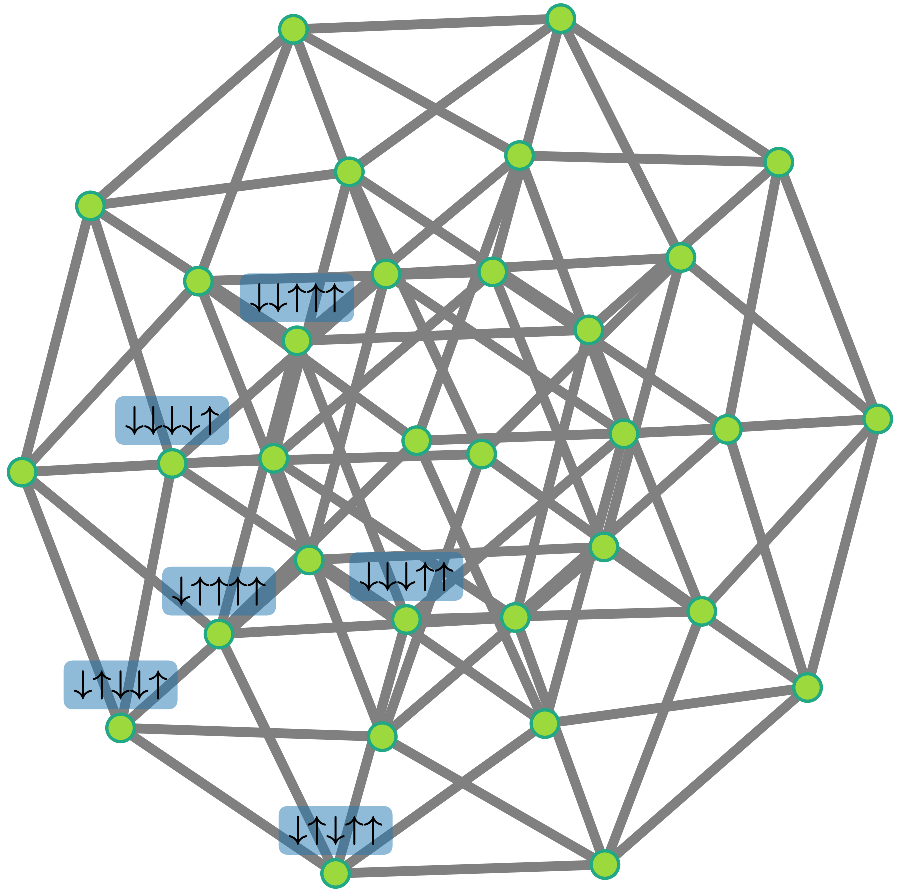
1. How else can we break ergodicity? Interesting problem in itself
2. Draw inspiration from classical glassy models
3. Focus on dynamically constrained models

Fock space

- Generic many-body spin Hamiltonian = tight binding Hamiltonian on the Fock space graph

$$H = \sum_I \mathscr{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathscr{T}_{IK} |I\rangle\langle K|$$

$|I\rangle \equiv$ Fock-basis state



- For example, a disordered spin-1/2 chain

$$H = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

$|I\rangle \equiv \sigma^z$- product state

- Or the Quantum Random Energy Model (QREM)

$$H_{\text{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Gaussian IRVs

Constrained dynamics

— switch off some links on the Fock space thereby creating bottlenecks

Example

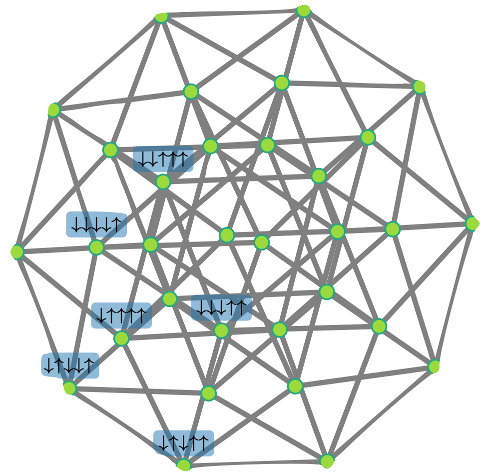
— start with the QREM as our reference *unconstrained* model (**always delocalised**)

$$H_{\text{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Gaussian IRVs

spins free to flip

Baldwin+Pal+Laumann+Scardichhio
PRB (2016)



— **There is no localised phase in the QREM: Eigenstates extended (in Fock space)**

Roy and AL, Phys Rev Research 2020

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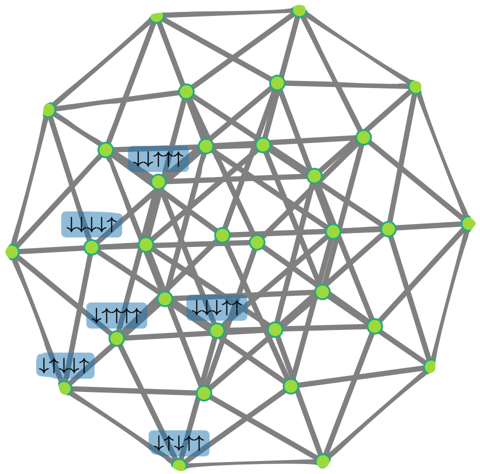

— **There is no localised phase in the QREM: Eigenstates extended (in Fock space)**

— introduce *East-glass* type constraints (“EastREM”…)

$$H_{\mathrm{EastREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \frac{\Gamma}{2} \sum_i \sigma_i^x (1 + \sigma_{i+1}^z)$$

Gaussian IRVs

a spin can flip only if the one to its right is up

Roy and AL, Phys Rev Research 2020

Constraints: What do they do in Fock space?

$$H_{\mathrm{EastREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \frac{\Gamma}{2}\sum_i \sigma_i^x(1+\sigma_{i+1}^z)$$

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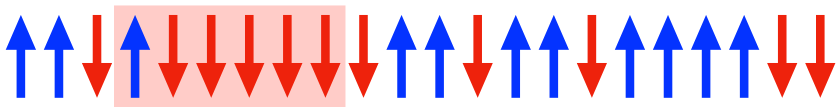
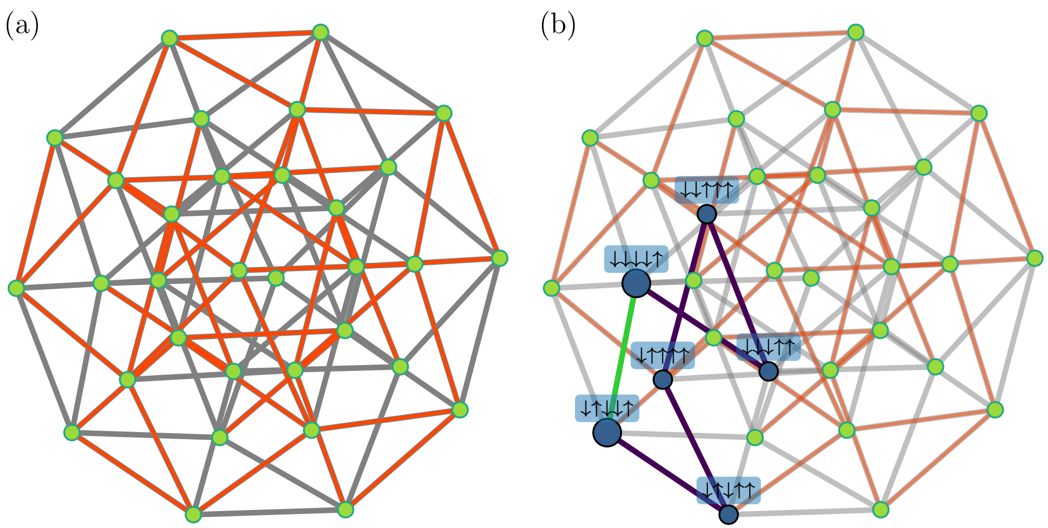


Frozen block of spins; can melt only from the right; arrested dynamics !!

On Fock space



- **Constraints switch off some of the links**
- Increase the typical distance between two nodes
- Decrease the number of paths between two nodes

Roy and AL, Phys Rev Research 2020

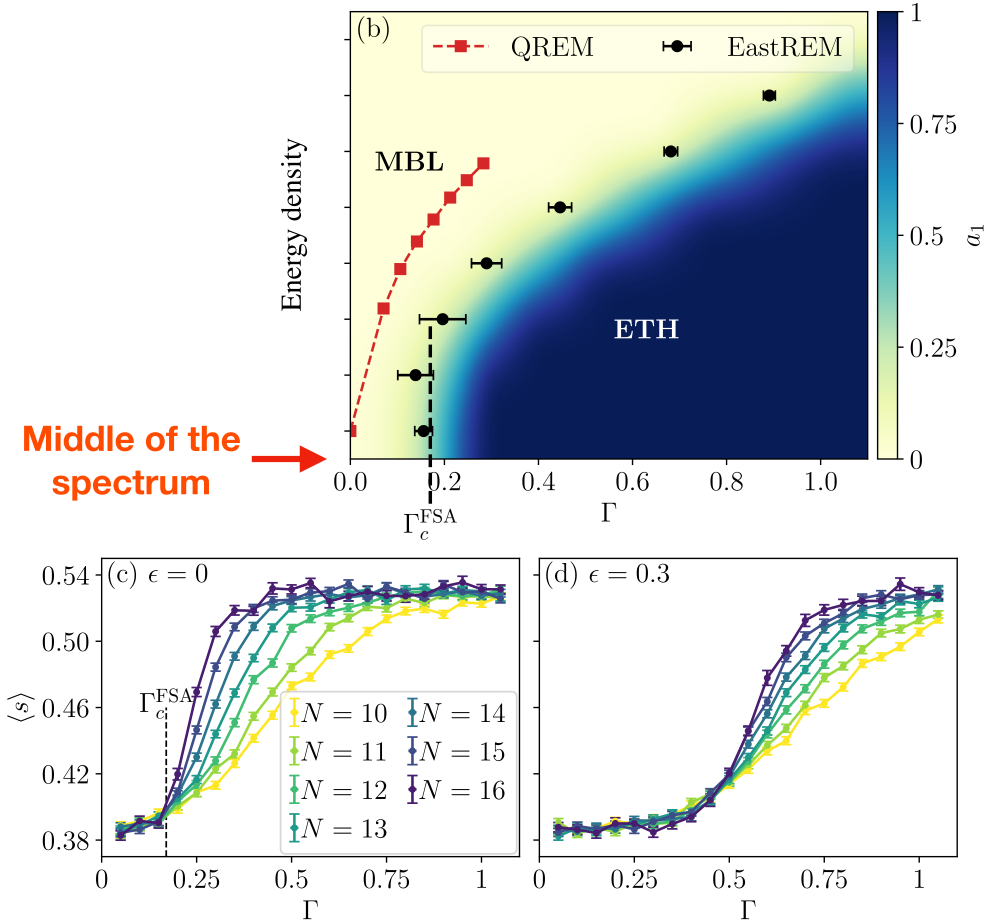
Constraints: can they stabilise a full MBL phase?

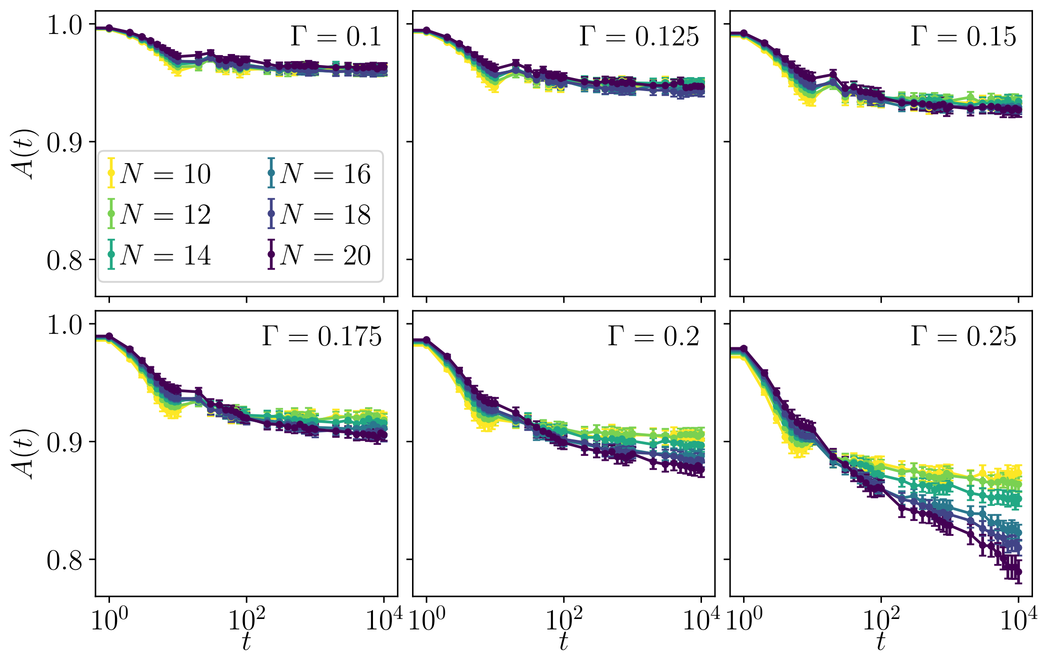
Constraints clearly disfavour delocalisation but can they stabilise localisation?

- spectral properties of the EastREM suggest an affirmative answer
- dynamical autocorrelations also do so
- analytical results agree with numerics

$$H_{\text{EastREM}} = \sum_I \mathcal{E}_I |I\rangle\langle I| + \frac{\Gamma}{2}\sum_i \sigma_i^x(1+\sigma_{i+1}^z)$$

$$A(t) = \frac{1}{N}\sum_i \langle\psi_0|\sigma_i^z(t)\sigma_i^z|\psi_0\rangle$$

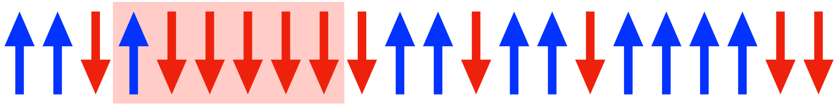




Roy and AL, Phys Rev Research 2020

Constraints: Is there a random matrix-like structure for this?

- **“Random Matrix Theory”:** Minimally-structured matrix that behaves like a Hamiltonian with the properties of interest (here, that shows EastREM phenomenology)
- What is the “structure” we wish to preserve here? Clustering:



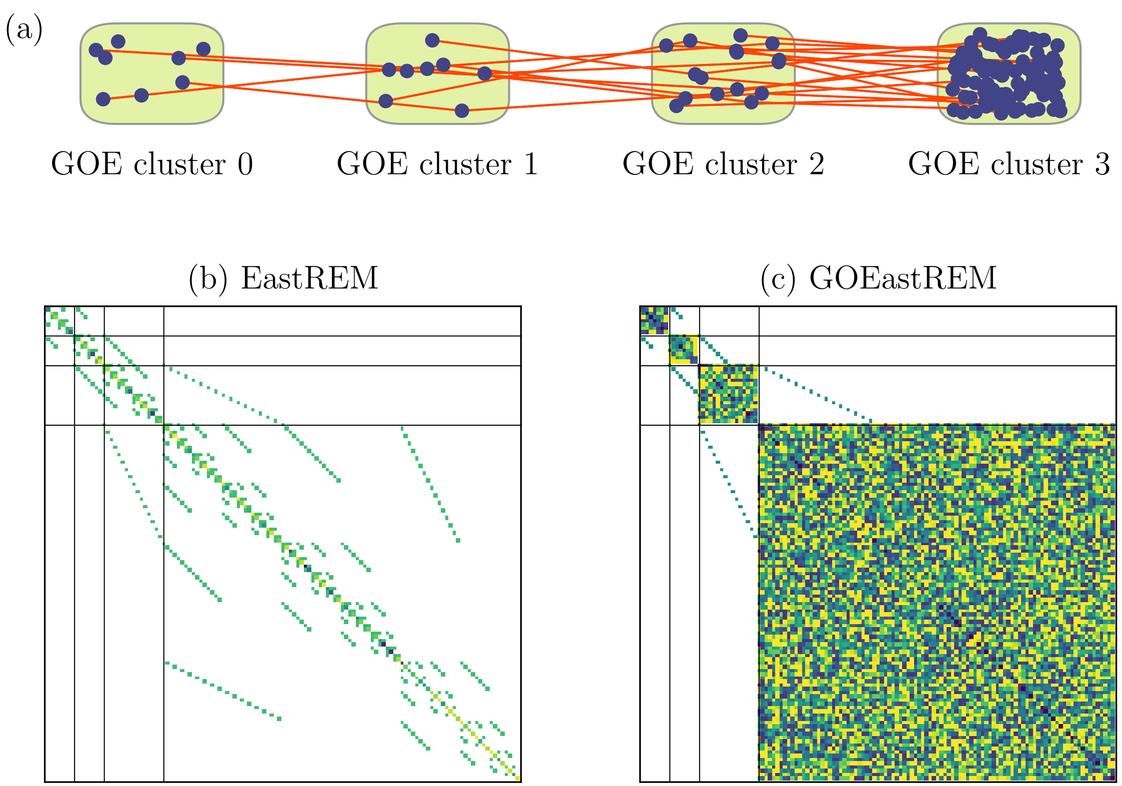
$$H_{\text{EastREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \frac{\Gamma}{2}\sum_i \sigma_i^x(1+\sigma_{i+1}^z)$$

$$\text{Cluster\# } 0 \; : \; \overbrace{\downarrow\downarrow \cdots\cdots \downarrow\downarrow}^{N/2}\overbrace{\circ\circ\cdots\cdots\circ\uparrow}^{N/2},$$

$$\text{Cluster\# } 1 \; : \; \overbrace{\downarrow\downarrow \cdots\cdots \downarrow\uparrow}^{N/2-1}\overbrace{\circ\circ\cdots\cdots\circ\uparrow}^{N/2},$$

$$\vdots$$

$$\text{Cluster\# } i \; : \; \overbrace{\downarrow\downarrow \cdots \downarrow\uparrow}^{N/2-i}\overbrace{\circ\circ\cdots\cdots\circ\uparrow}^{N/2+i-1},$$



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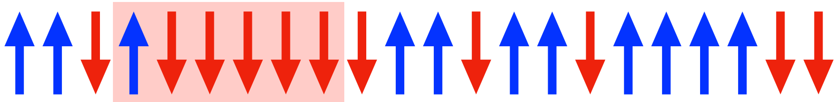

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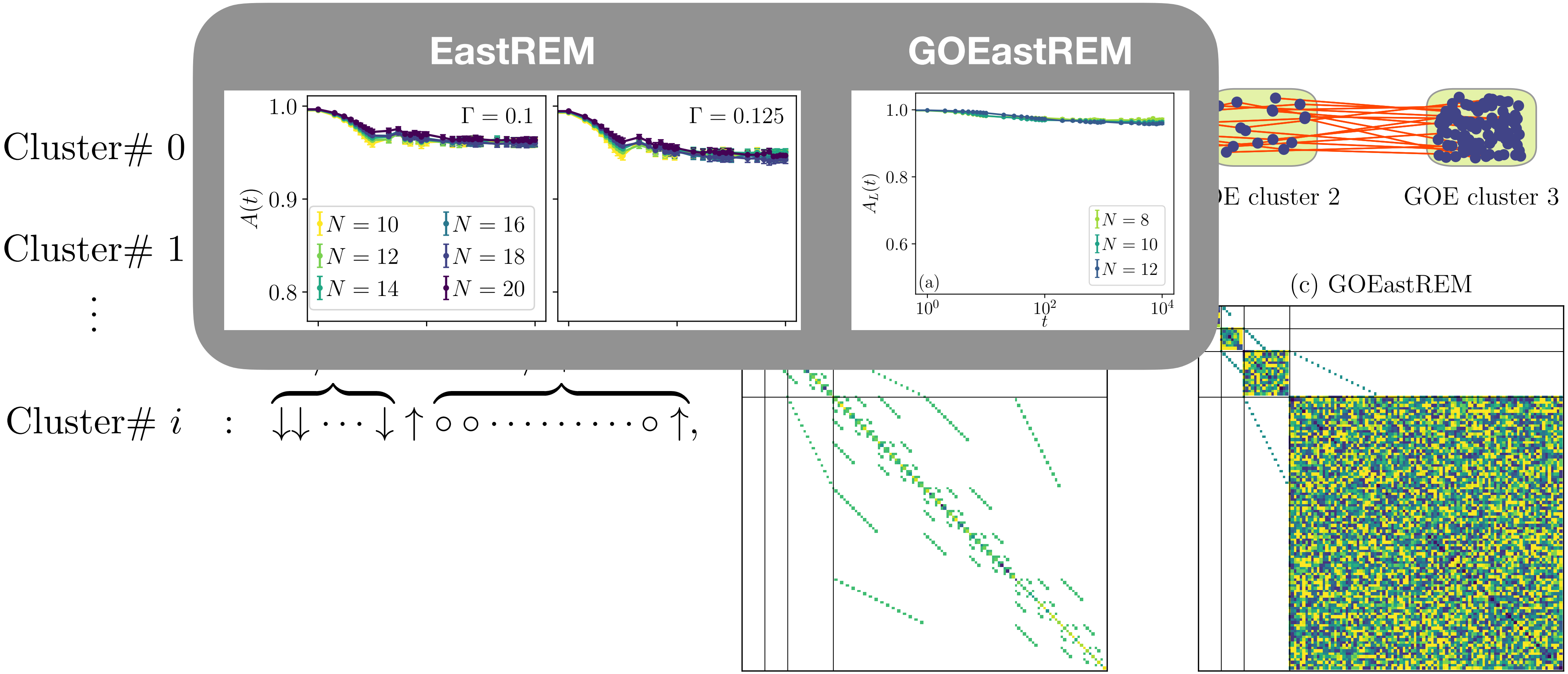

Overall conclusions

- **Unitary** system: relies on structure of eigenstates and eigenvalues of propagator
- Requires disorder
- “Fundamentally quantum” (relies on coherences)

- **Constrained** system: ergodicity breaking due to constraints
- **Does it support interesting phases? Future work…**

Collaborators

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