Space Bounded **Scatter Machines**

João Alves Alírio

Supervisor:
Prof. José Félix Gomes da Costa

Instituto Superior Técnico - Universidade de Lisboa

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Computability theory

Abstract digital device

Complexity theory

Resources; e.g. time or space

Decidability

P *NP*

Definition

A deterministic Turing machine $\mathcal{M}$ is said to decide a set $A \subseteq \Sigma^*$ if,
(a) $\mathcal{M}$ halts for all $w \in \Sigma^*$ and
(b) $\mathcal{M}$ accepts w if and only if $w \in A$.

Definition

A non-deterministic Turing machine $\mathcal{M}$ is said to decide a set $A \subseteq \Sigma^*$ if,
(a) if $w \in A$ $\mathcal{M}$ halts and accepts at least one possible computation of w and
(b) if $w \notin A$ $\mathcal{M}$ do not accept any computation of w.

Oracle Turing machine

An oracle is usually considered as a set.
The oracle Turing machine can consult the oracle during its computations.

Advice Turing machine (non-uniform complexity)

An advice function $f : \mathbb{N} \to \Sigma^*$ is a total function which assigns a word for each (input size) $n \in \mathbb{N}$.
The advice Turing machine receives extra input information, depending on (main) input size.

$P/\log\star$ $\qquad$ $PSPACE/poly$

Bounded error **probabilistic decidability**

BPP *BPPSPACE*

Definition

A bounded error probabilistic Turing machine $\mathcal{M}$ *is said to decide a set* $A \subseteq \Sigma^*$ *if, there is a rational number* ϵ *with* $0 < \epsilon < 1/2$ *such that, for every* $w \in \Sigma^*$ *(a) if* $w \in A$*, then* $\mathcal{M}$ *rejects* w *with probability at most* ϵ *and (b) if* $w \notin A$*, then* $\mathcal{M}$ *accepts* w *with probability at most* ϵ*.*

Non-uniform probabilistic decidability

$BPP//\log\star$ $BPPSPACE//poly$

Definition

Let F be a class of advice functions, we denote by $BPPSPACE//F$ the class of sets A for which there exists a probabilistic advice Turing machine $\mathcal{M}$ bounded in polynomial space, a constant ϵ with $0 < \epsilon < 1/2$ and an advice function $f \in F$ such that, for every $w \in \Sigma^*$,
(a) if $w \in A$, then $\mathcal{M}$ rejects $\langle w, f(|w|)\rangle$ with probability at most ϵ and
(b) if $w \notin A$, then $\mathcal{M}$ accepts $\langle w, f(|w|)\rangle$ with probability at most ϵ.

Scatter machine

Abstract analogue-digital device

Turing machine

Abstract digital device

Physical experiment

Abstract analogue device

Communication protocol

Sharp scatter experiment

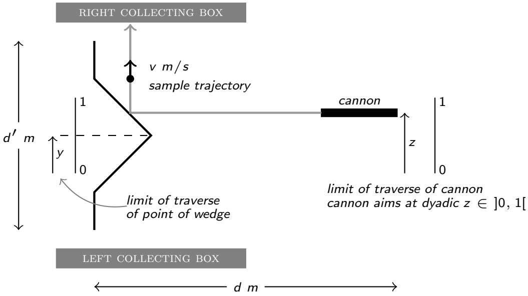

Figure: Sharp scatter experiment

Infinite precision

Arbitrary precision

Fixed precision

Smooth scatter experiment

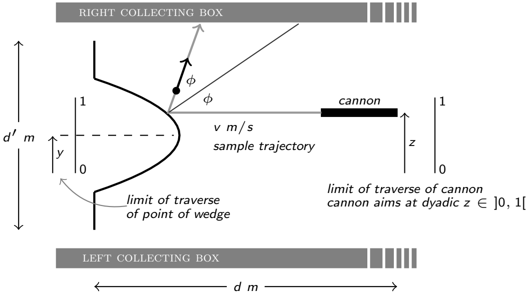

Figure: Smooth scatter experiment

Smooth scatter experiment — Physical time

$$\frac{A}{|y-z|^{n-1}} \leq t(z) \leq \frac{B}{|y-z|^{n-1}}$$

New communication protocol

Query tape not bounded

Space bounded clocks

Proposition

In polynomial space, any clock ticks at most an exponential amount of transitions.

Proposition

There exists a clock bounded in polynomial space such that, for an input of size n, ticks an exponential amount of transitions on n.

Random coin toss generator

Proposition

Given a biased coin with probability of heads $q \in]\delta, 1-\delta[$, for some $0 < \delta < 1/2$, and $\gamma \in]0,1[$, we can simulate, up to probability $\geq \gamma$, a sequence of independent fair coin tosses of length n by doing a linear number of biased coin tosses.

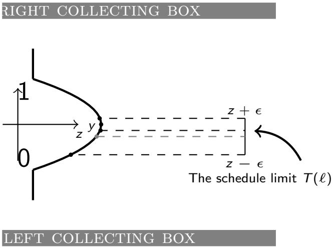

Figure: The *SmSE* with fixed precision as a coin.

We can have an exponential amount of fair coin tosses, up to probability $\geq \gamma$ and not all at once, within polynomial space.

Scatter machine

- Turing machine together with scatter experiment

Lower bounds

Simulate advice Turing machines in scatter machines

Upper bounds

Simulate scatter machines in advice Turing machines

Lower bounds

- Choose the correct scatter experiment
- Obtain the vertex position (advice)
- Perform the digital computations
- (Possibly) Obtaining fair random bits

Upper bounds

- Choose the correct advice
- Perform the digital computations
- Simulating digitally the oracle calls using the advice

Time bounded scatter machine

	Infinite	Arbitrary	Fixed
Lower Bound	$P/poly$	$P/poly$	$BPP//\log\star$
Upper Bound	$P/poly$	$P/poly$	$BPP//\log\star$
Lower Bound	$P/\log\star$	$BPP/\log\star$	$BPP/\log\star$
Upper Bound Exponential schedule	$P/\log\star$	$BPP//\log^2\star$	$BPP//\log^2\star$
Upper Bound Explicit Time	—	$BPP/\log\star$ Exponential schedule	$BPP/\log\star$ Exponential schedule

Computational power of the scatter machine bounded in polynomial time with the sharp scatter experiment (above) and the smooth scatter experiment (below)

Space bounded scatter machine

	Infinite	Arbitrary	Fixed
Lower Bound	$PSPACE/poly$	$BPPSPACE//poly$	$BPPSPACE//poly$
Upper Bound	$PSPACE/poly$	$BPPSPACE//poly$	$BPPSPACE//poly$
Lower Bound	$PSPACE/poly$	$BPPSPACE//poly$	$BPPSPACE//poly$
Upper Bound	$PSPACE/poly$	$BPPSPACE//poly$	$BPPSPACE//poly$

Computational power of the scatter machine with the standard communication protocol bounded in polynomial space with the sharp scatter experiment (above) and the smooth scatter experiment (below)

Space bounded scatter machine (generalized)

	Infinite	**Arbitrary**	**Fixed**
Lower Bound	2^{Σ^*}	2^{Σ^*}	$BPPSPACE//poly$
Upper Bound	2^{Σ^*}	2^{Σ^*}	$BPPSPACE//poly$

Lower Bound with time schedule	$PSPACE/poly$	$BPPSPACE//poly$	$BPPSPACE//poly$
Lower Bound without time schedule	2^{Σ^*}	2^{Σ^*}	—
Upper Bound	2^{Σ^*}	2^{Σ^*}	$BPPSPACE//poly$

Computational power of the scatter machine with the generalized communication protocol bounded in polynomial space with the sharp scatter experiment (above) and the smooth scatter experiment (below)

Bibliography

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