Coalescing random walks and the Kingman coalescent model

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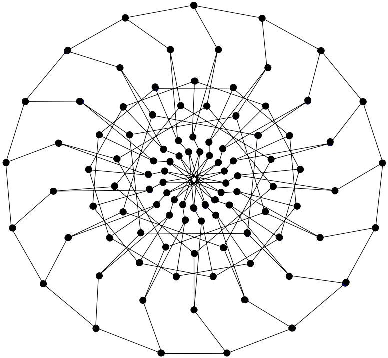
Joint with: Enrique Chávez, Claudio Landim

Relaxation time

For a finite simple connected graph

$$G = (V, E)$$



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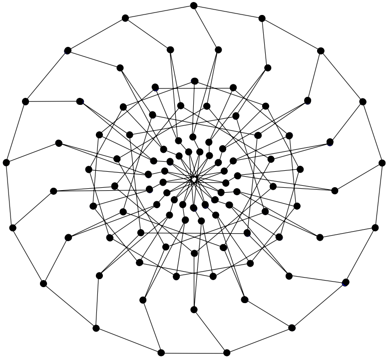

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$$\Delta f(u) = -\sum_{v} \big(f(v) - f(u)\big) \mathbf{1}_{v \sim u}, \quad u \in V.$$

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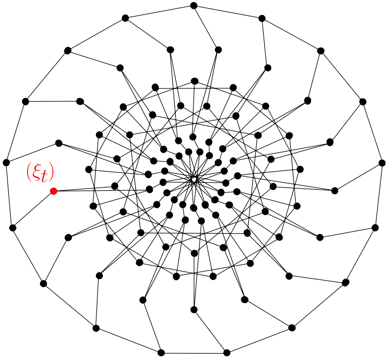

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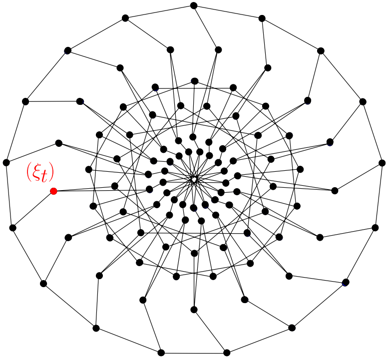

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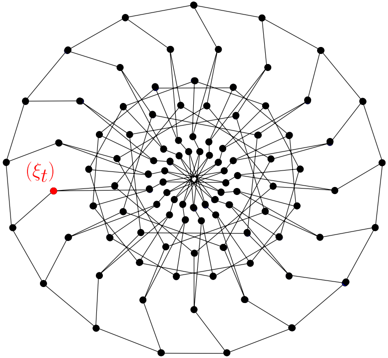

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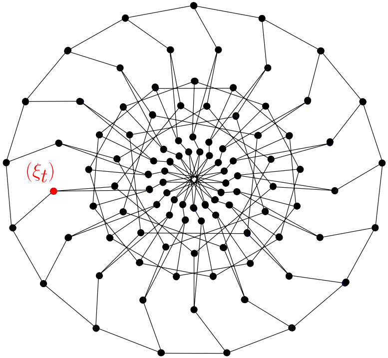

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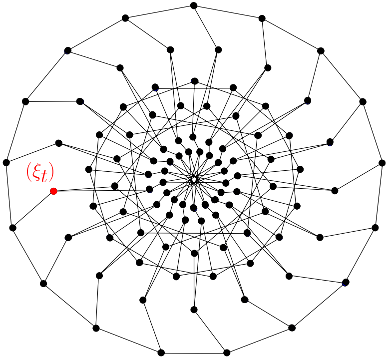

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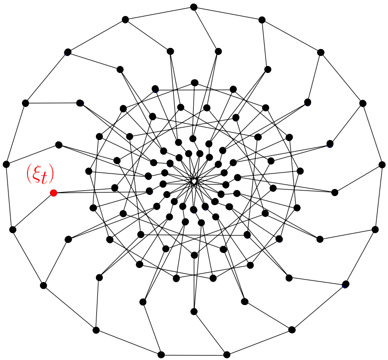

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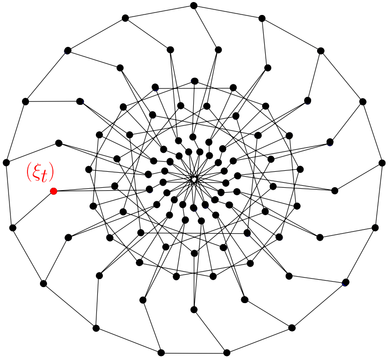

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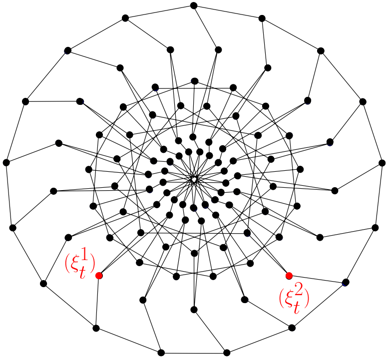

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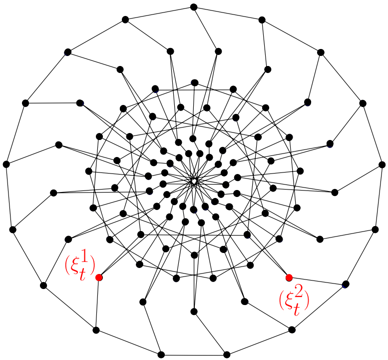

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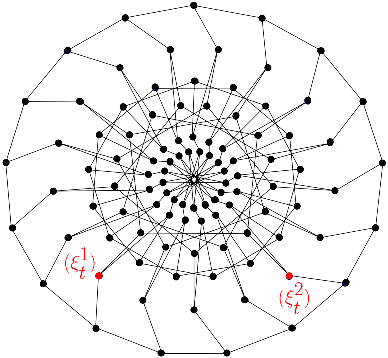

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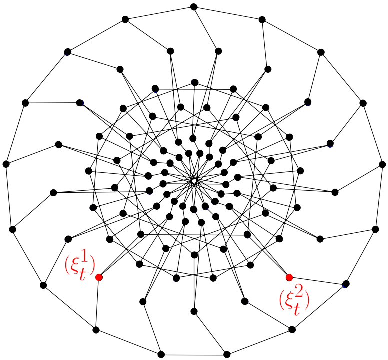

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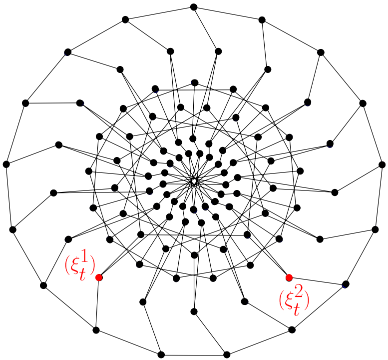

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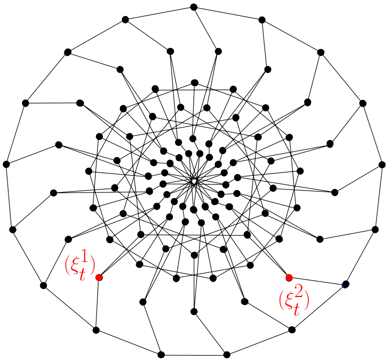

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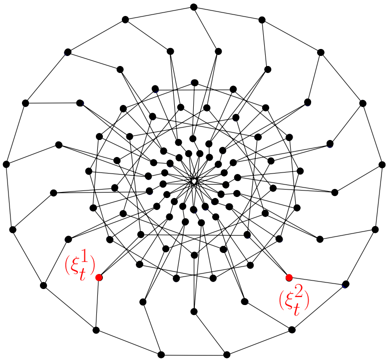

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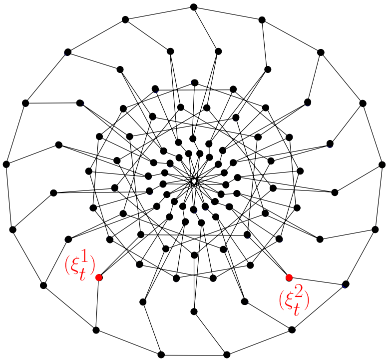

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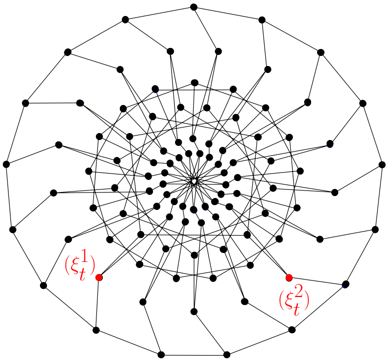

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Sequence of finite connect simp graphs

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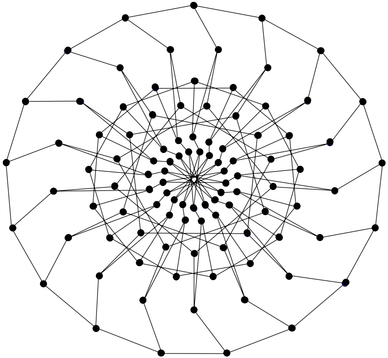

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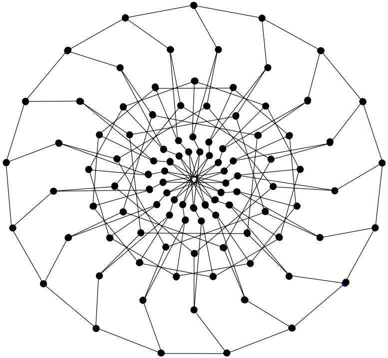

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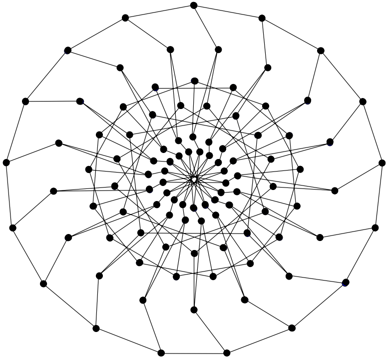

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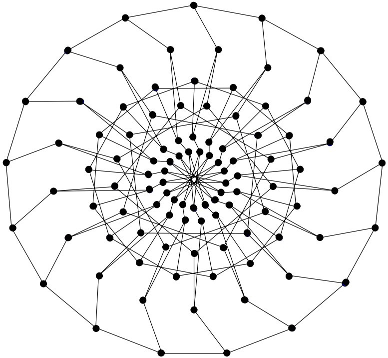

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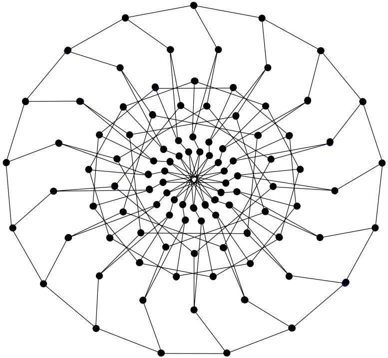

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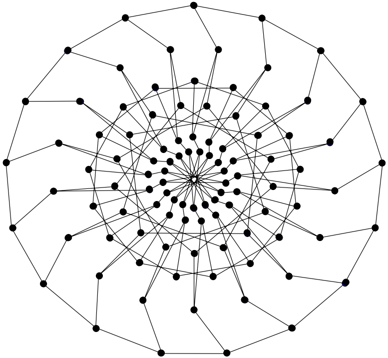

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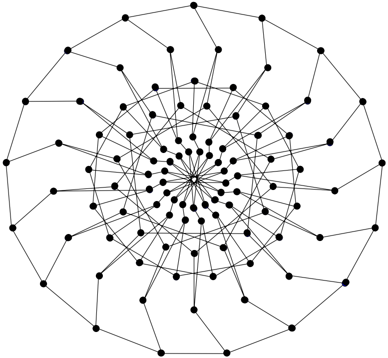

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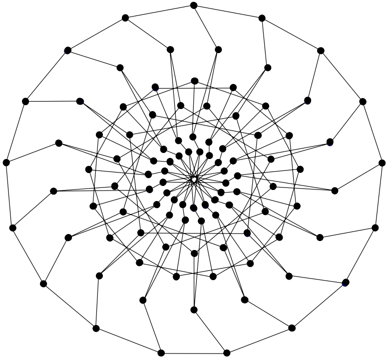

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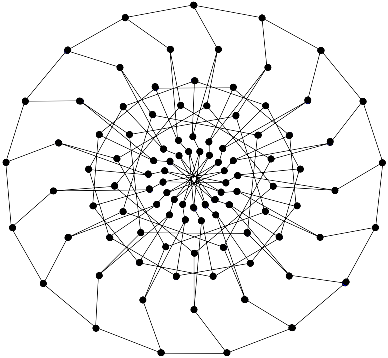

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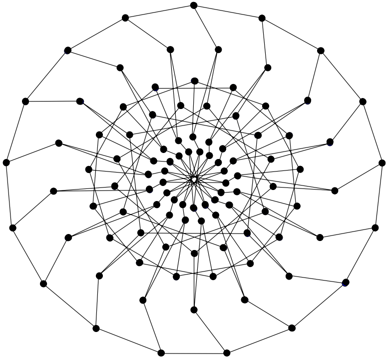

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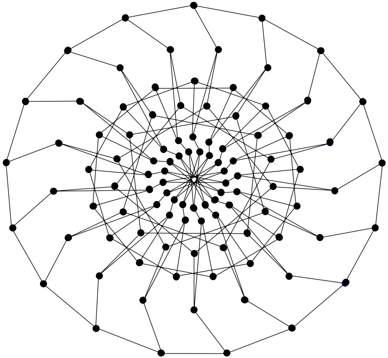

Aldous Conditions

- Each G_n is vertex-transitive
- $\dfrac{\mathbf{t}_{rel}}{\boldsymbol{\theta}} \to 0$

Exmp **Torus:** $G_n = (\mathbb{Z}/n\mathbb{Z})^d,\ d \geq 2$

$$\mathbf{t}_{\mathrm{rel}} \asymp n^2 \quad \text{and} \quad \boldsymbol{\theta} \asymp \begin{cases} n^d, & \text{for } d \geq 3, \\ n^2 \log n, & \text{for } d = 2. \end{cases}$$

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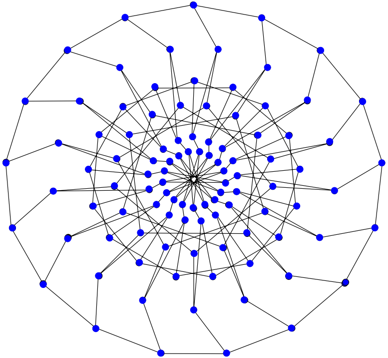

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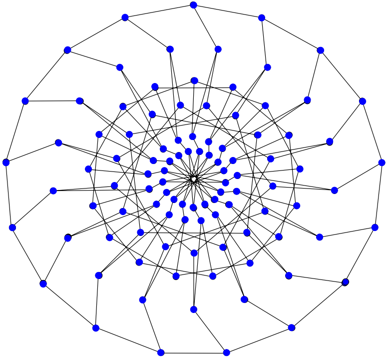

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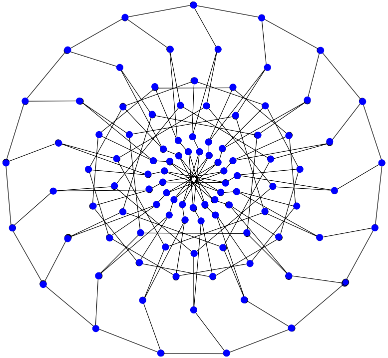

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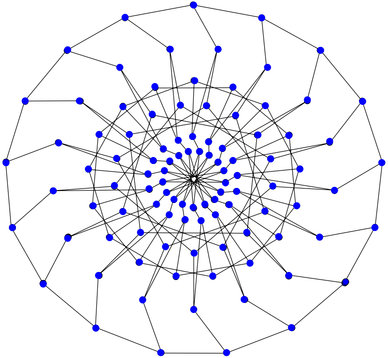

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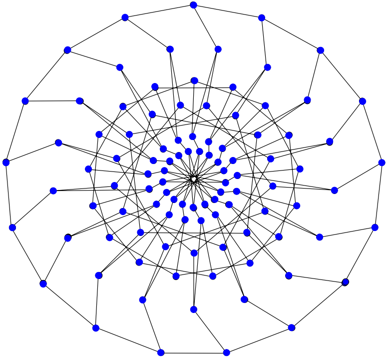

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For the **complete graph:**

$$\boxed{\frac{T_{\text{full}}}{\theta} \sim \sum_{k=2}^{|V_n|} Z_k}$$

where $Z_k \sim \exp\binom{k}{2}$, $k \geq 1$, are independent.

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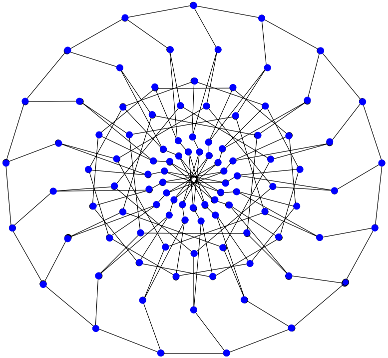

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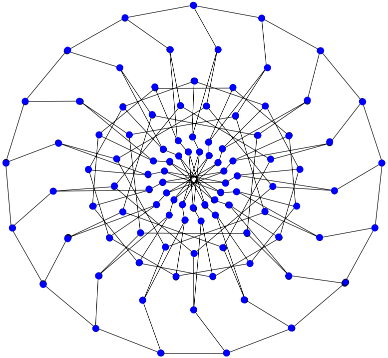

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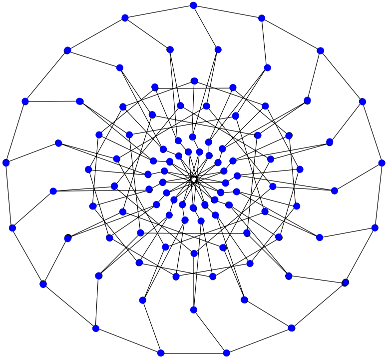

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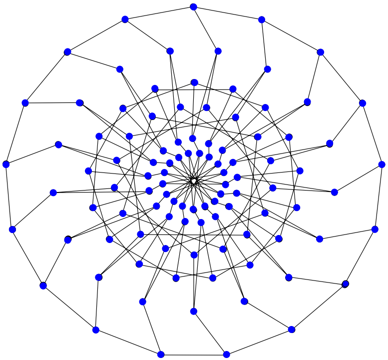

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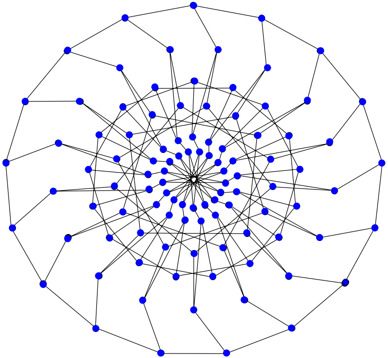

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Thm (Oliveira R.I., *Ann. Prob.* 2013)

Assuming **vertex-transitivity** and $\dfrac{\boldsymbol{t}_{mix}}{\boldsymbol{\theta}} \to 0$

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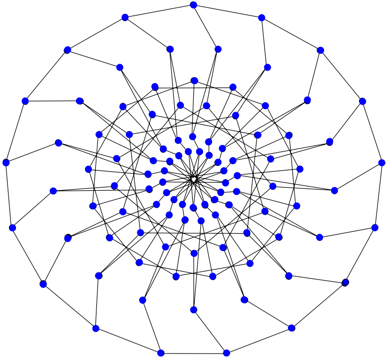

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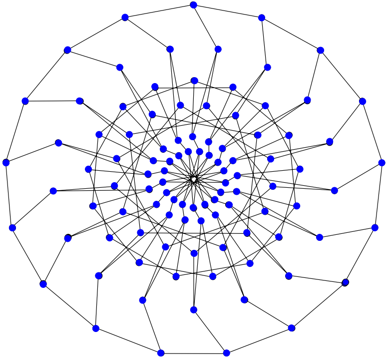

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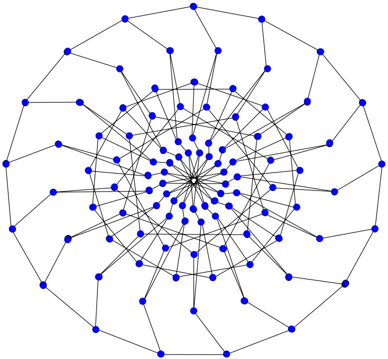

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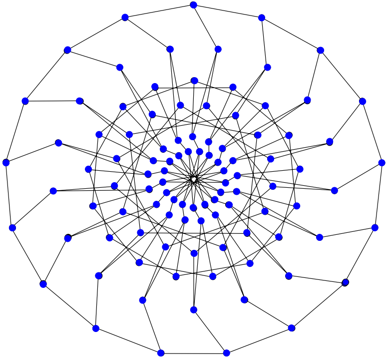

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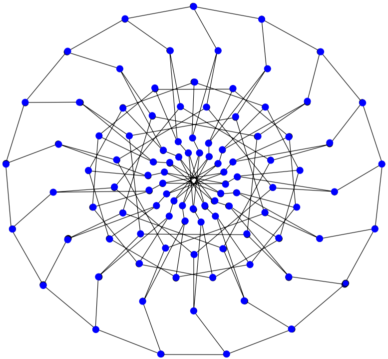

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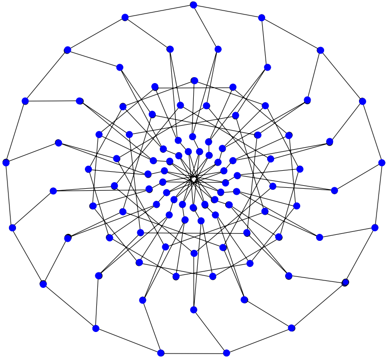

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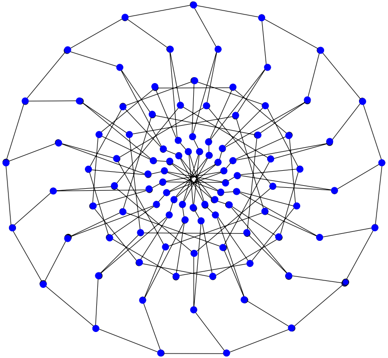

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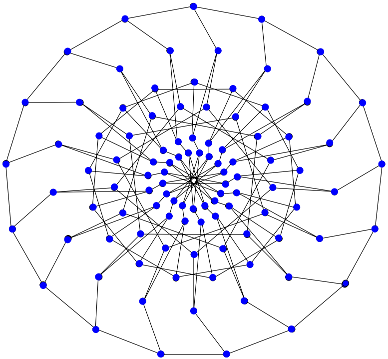

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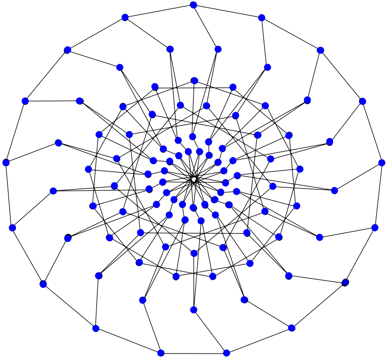

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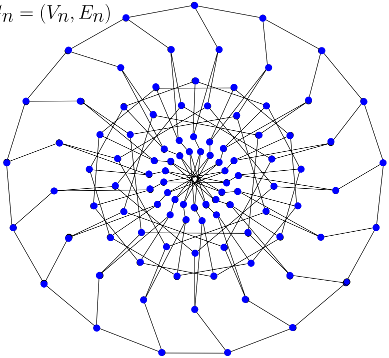
Two regimes

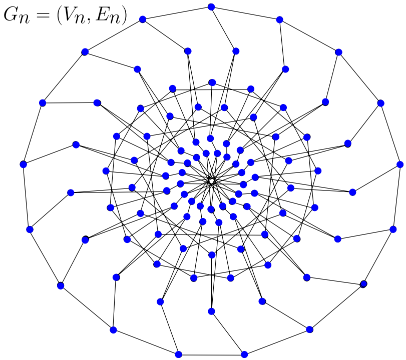
$G_n = (V_n, E_n)$



(I) The Big Bang Regime (BBR)

A term coined by Rick Durrett

Two regimes



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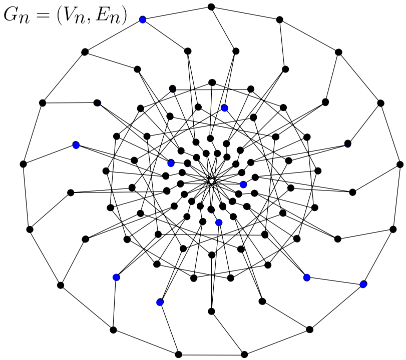

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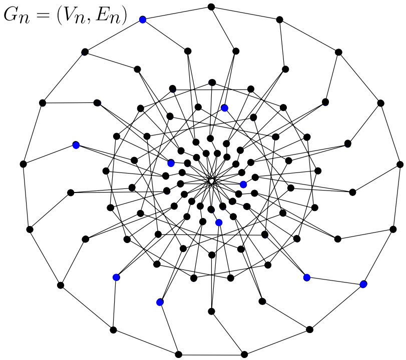
(II) The Coalescing Regime

There are typically $O(1)$ clusters

Two regimes



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A term coined by Rick Durrett

Hermon, Li, Yao, Zhang AOP (2022)

"Mean field behavior during the BBR"

(II) The Coalescing Regime

There are typically $O(1)$ clusters

(II) Let $(\eta^1, \eta^2, \ldots, \eta^k)$ be CRW, with k fixed.

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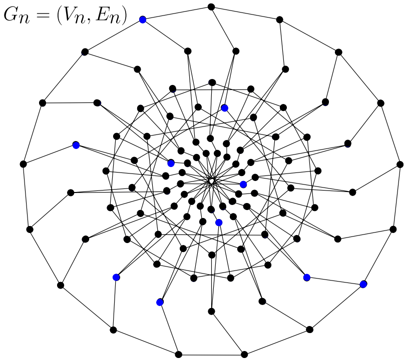

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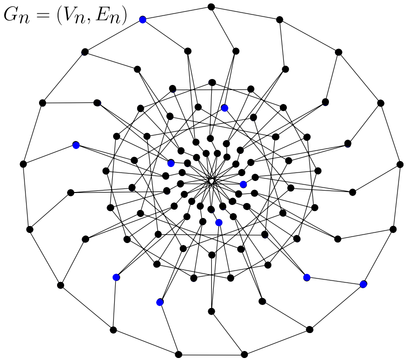

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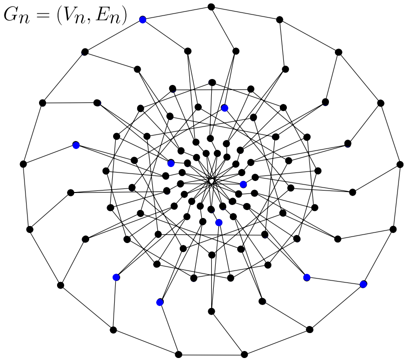

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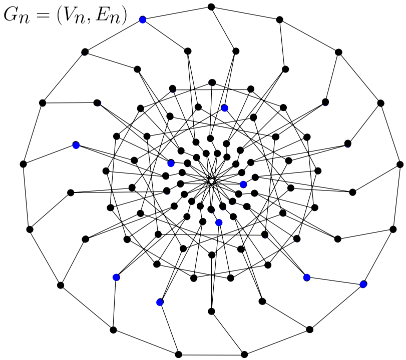

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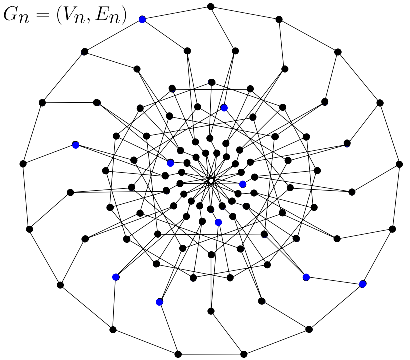

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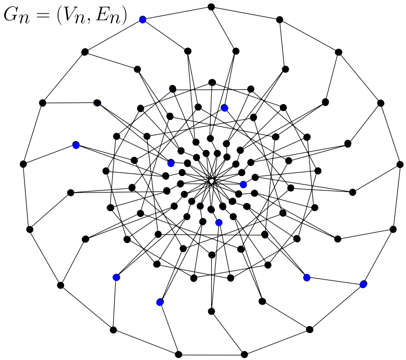
Thm (B.J., Chavez E., '25+)

Under **Aldous Cond**: **v-trans** and $\frac{\mathbf{t}_{rel}}{\boldsymbol{\theta}} \to 0$

Two regimes



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$$(\Pi_{\theta t}) \xrightarrow{\text{(law)}}$$

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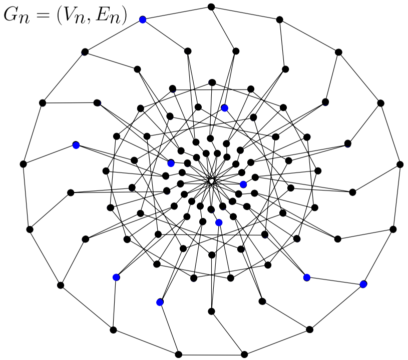

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$$(\Pi_{\boldsymbol{\theta} t}) \xrightarrow{\text{(law)}} \begin{array}{c}\text{Kingman } k\text{-coalescent} \\ \text{starting at } \{\{1\},\{2\},\ldots,\{k\}\}\end{array}$$

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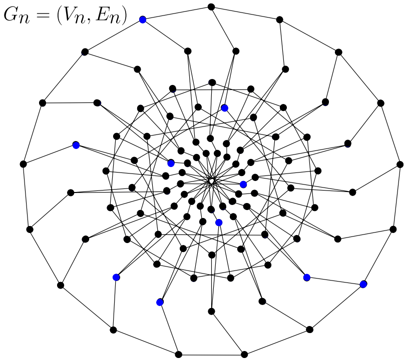

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In particular, $\frac{T_{full}^{(k)}}{\boldsymbol{\theta}} \xrightarrow{\text{(law)}} Z_2 + Z_3 + \cdots + Z_k$