Topological Twists of Superconformal Field Theory

Chris Elliott

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Build vertex algebra from 4d field theory

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I'm going to discuss joint work with Owen Gwilliam and Matteo Lotito.

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Definition
A prefactorization algebra Obs on M is a "multiplicative precosheaf" of complex cochain complexes on M.

open $U \subseteq M \rightsquigarrow \mathrm{Obs}(U)$

vector space / cochain complex

operator product

$\mathrm{Obs}(U_1) \otimes \mathrm{Obs}(U_2)$
$\downarrow$
$\mathrm{Obs}(V)$

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and given disjoint $U_1, \ldots, U_n \subseteq V$ we have

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We say Obs is a factorization algebra if it satisfies a descent condition (won't be important for the results today).

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Definition

Lie superalgebra

For $\mathfrak{g}$ a $\mathbb{Z}/2\mathbb{Z}$-graded Lie algebra, we say Obs is $\mathfrak{g}$-supersymmetric if we're given a Lie algebra map

derived

$$\rho\colon \mathfrak{g} \to \mathrm{Der}(\mathrm{Obs}).$$

derivations of obs for operator product

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- *Supertranslation algebra* — even part is an abelian Lie algebra $\mathbb{R}^n$.

$\mathfrak{g} = \mathbb{R}^n \oplus \Pi\Sigma$ ← spinorial rep of $so(n)$

↑ odd degree

bracket $\Gamma : \mathrm{Sym}^2 \Sigma \to \mathbb{R}^n$

$so(n)$-equivariant

What sorts of $\mathfrak{g}$ are we interested in?

- *Supertranslation algebra* — even part is an abelian Lie algebra $\mathbb{R}^n$.
- *Super Poincaré algebra* — even part is the Lie algebra $\mathfrak{iso}(n)$ of infinitesimal isometries of $\mathbb{R}^n$ (or more generally $\mathbb{R}^{p,q}$, with indefinite signature).

$$iso(n) = so(n) \ltimes \mathbb{R}^n$$

$$iso(p,q) = so(p,q) \ltimes \mathbb{R}^{p+q}$$

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$$\mathrm{Conf}(p,q) \cong \mathrm{so}(p+1, q+1)$$

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In practice we ask for actions that are are "geometric", meaning the even part acts as the derivative of an action of a Lie group of transformations of $M = \mathbb{R}^n$.

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odd degree, inhomogeneous

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Let Obs *be a prefactorization algebra on* $\mathbb{R}^n$ *with an action of a supertranslation algebra. Let* Q *be a* *topological supercharge**, meaning all translations are* Q*-exact.*

$[Q, Q] = 0$

$[Q, -] : \mathfrak{g}_1 \longrightarrow \mathbb{R}^n$

surjective

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$\mathrm{Obs}^Q(U)$ only depends on homotopy type of U

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$$\mathrm{Obs}^Q(B_r(0)) \to \mathrm{Obs}^Q(B_R(0))$$

is a quasi-isomorphism for all $r < R$, which we can typically verify directly in examples.

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$$\left[\mathrm{Nilp}_{\mathfrak{g}}/(G_{\mathbb{R}})_0\right].$$

$\mathbb{R}^\wedge$

ISO(n)

Conf(n)

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In work with Safronov and Williams we have a complete description for supertranslation / super Poincaré algebras in dimensions ≤ 10.

Superconformal Theories

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Superconformal Lie algebras in dimension d include both the Lie algebra of conformal transformations of $\mathbb{R}^d$ and a super Poincaré algebra extending the Lie algebra of isometries of $\mathbb{R}^d$.

$$\mathrm{conf}_{p,q}(d|\Sigma)$$

$$\mathrm{iso}_{p,q}(d|\Sigma) \subseteq \mathrm{conf}_{p,q}(d|\Sigma) \supseteq \mathrm{conf}_{p,q}(d)$$

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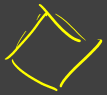
$$C(\mathbb{R}^{p,q}) \cong (S^p \times S^q)/(\mathbb{Z}/2\mathbb{Z}).$$



Example: Dimension 4

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$\mathfrak{conf}(p, 4-p) \cong \mathfrak{so}(p+1, 5-p)$

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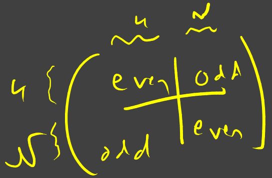
← traceless endomorphisms of $\mathbb{C}^{4|\mathcal{N}}$

for a choice of natural number $\mathcal{N}$.



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for a choice of natural number $\mathcal{N}$. We can study the nilpotence variety concretely, it consists of pairs

$$\mathcal{Q}_+ \in \mathrm{Hom}(\mathbb{C}^4,\mathbb{C}^{\mathcal{N}}), \mathcal{Q}_- \in \mathrm{Hom}(\mathbb{C}^{\mathcal{N}},\mathbb{C}^4)$$

such that

$$\mathcal{Q}_+ \circ \mathcal{Q}_- = \mathcal{Q}_- \circ \mathcal{Q}_+ = 0.$$

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Idea: to generalize Witten's construction of topological field theories, restrict to affine subspaces $\mathbb{R}^k \subseteq C(\mathbb{R}^{p,q})$ where some or all of the isometries are Q-exact.

↑ topological theory on $\mathbb{R}^k$

↑ e.g. k even

$k = 2\ell$

$\mathbb{R}^{2\ell} \cong \mathbb{C}^\ell$

antiholomorphic isometries Q-exact

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so(3,1)
∩|
Conf(2)

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- *The complexified isometry $\frac{\partial}{\partial \bar{z}}$ is $\mathcal{Q}$-exact.*

Then $\iota^*(\mathrm{Obs}^{\mathcal{Q}})$ naturally has the structure of a vertex algebra.

Williams – Saberi

holomorphic factorization algebra

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Future Work

- We have a description of the nilpotence variety for each superconformal algebra, and there are finitely many orbits for the even part of the complex superconformal group (classified by Duflo and Serganova). What does the orbit structure look like for real forms, which will look different depending on the signature?
- How does this look like for actual concrete field theories? Can we extend the construction of super Poincaré twists of super Yang–Mills theory to the superconformal nilpotence variety in some examples?

Thanks for listening!