MMAC Day 2024

Infinite-valued logics and applications

Master's research topics in
Logic and Computation

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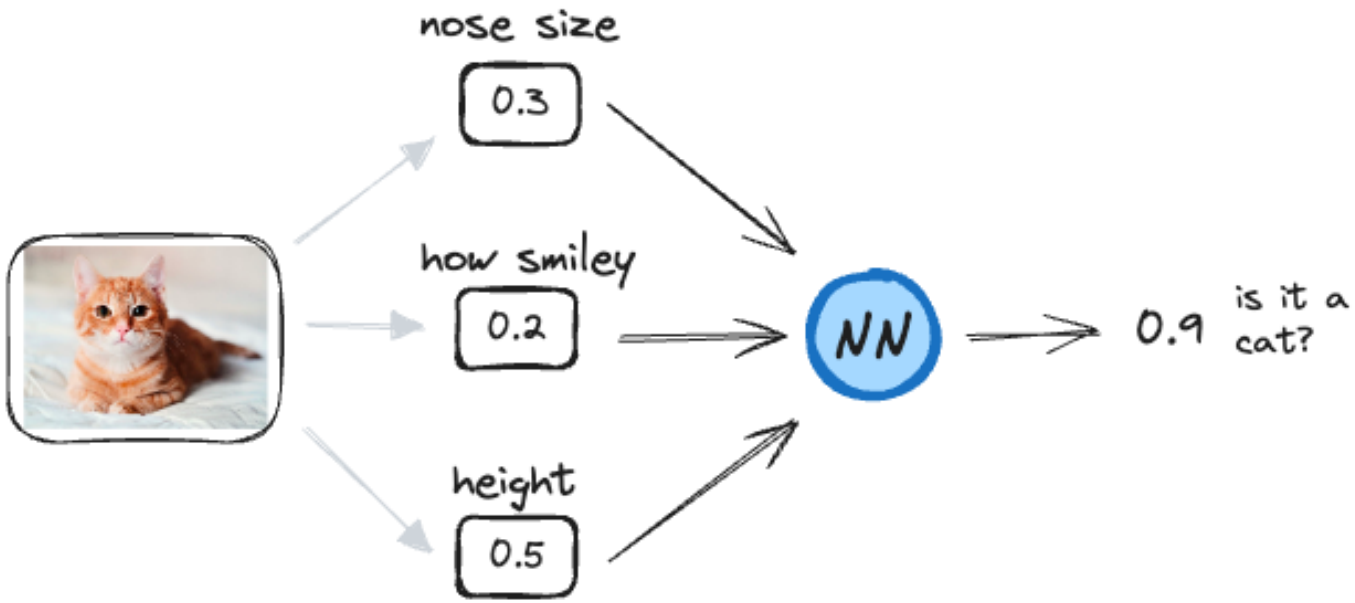
Logic and applications...

goals:

- automated symbolic reasoning
- find trade-offs between expressivity and computational feasibility

going beyond classical logic

- 2-valued : classical logic, dealing with **sharp** concepts
- infinite-valued : fuzzy logics, dealing with **vague** concepts
- motivation/applications : neuro-symbolic approaches to AI



Fuzzy logics: connectives are interpreted as functions $f : [0,1]^n \to [0,1]$

Semantics for Łukasiewicz logic

falsum	implication	negation
$\bot = 0$	$x \to y = \min\{0, 1-(x-y)\}$	$\neg x = 1 - x$

Deductive system for Łukasiewicz logic (Hilbert style axiomatization)

$$p \to (q \to p) \qquad (p \to q) \to ((q \to r) \to (p \to r))$$

$$((p \to q) \to q) \to ((q \to p) \to p) \qquad (\neg q \to \neg p) \to (p \to q)$$

$$\bot \to p \qquad \frac{p \quad p \to q}{q}$$

FACTS:

- Same computational complexity as propositional classical logic
- Piecewise linear NNs can be represented by formulas of Łukasiewicz logic
- Many other fuzzy logics have been considered, as Gödel or Product logics

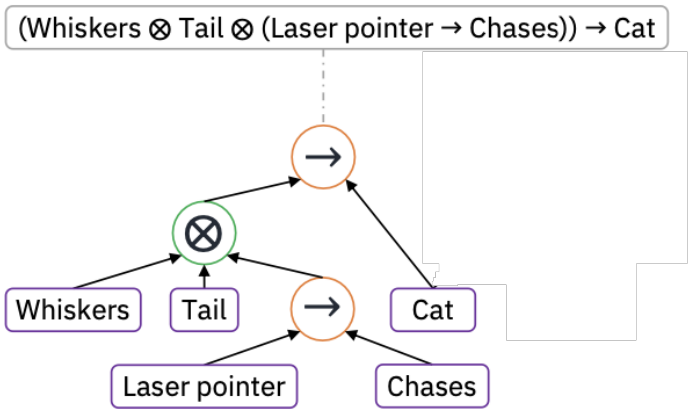
Gödel implication

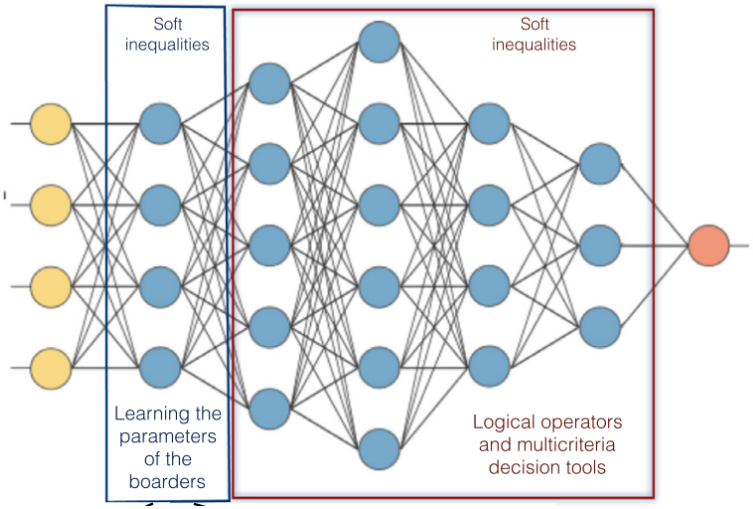
$$x \to y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases}$$

Product implication

$$x \to y = \begin{cases} 1 & \text{if } x \leq y \\ \frac{x}{y} & \text{if } x > y \end{cases}$$

Fuzzy logics in neuro-symbolic approaches to AI



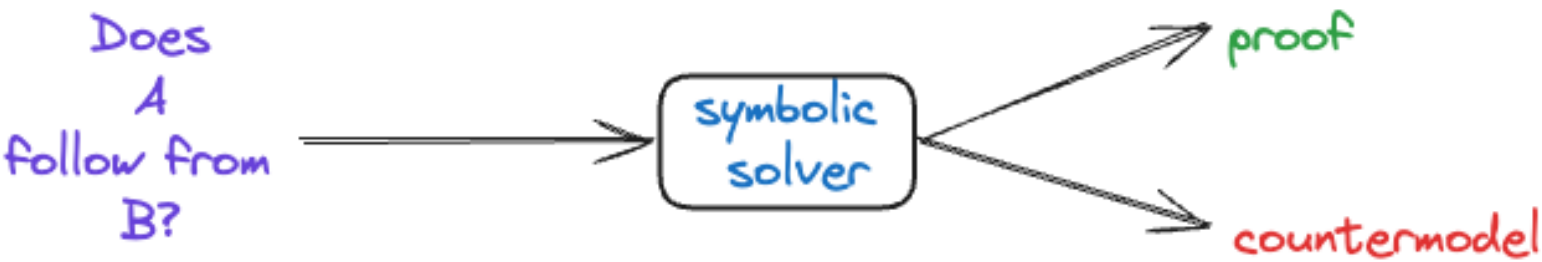


- Symbolically represent NN and their properties
  minimize NN, analyse NN behavior

- Multi-criteria layers using Łukasiewicz or other fuzzy connectives
  design inherently interpretable NN, reduce number of learning parameters

T1 deductive systems

- focus on symbolic solvers allowing effective proof-search
- extract explanations on why properties hold or not
- use, improve and compare with existing solvers



T2 approximation strategies . . .

T3 extensions . . .

T1 deductive systems . . .

T2 approximation strategies

Find sequence of more tractable logics $\mathcal{L}_1, \mathcal{L}_2, \ldots$ approximating $\mathcal{L}_{\text{target}}$

- from above $\mathcal{L}_1 \supseteq \mathcal{L}_2 \supseteq \ldots \supseteq \mathcal{L}_{\text{target}}$ focus on counter-model generation
- from bellow $\mathcal{L}_1 \subseteq \mathcal{L}_2 \subseteq \ldots \subseteq \mathcal{L}_{\text{target}}$ focus on proofs generation

E.g. finitization techniques

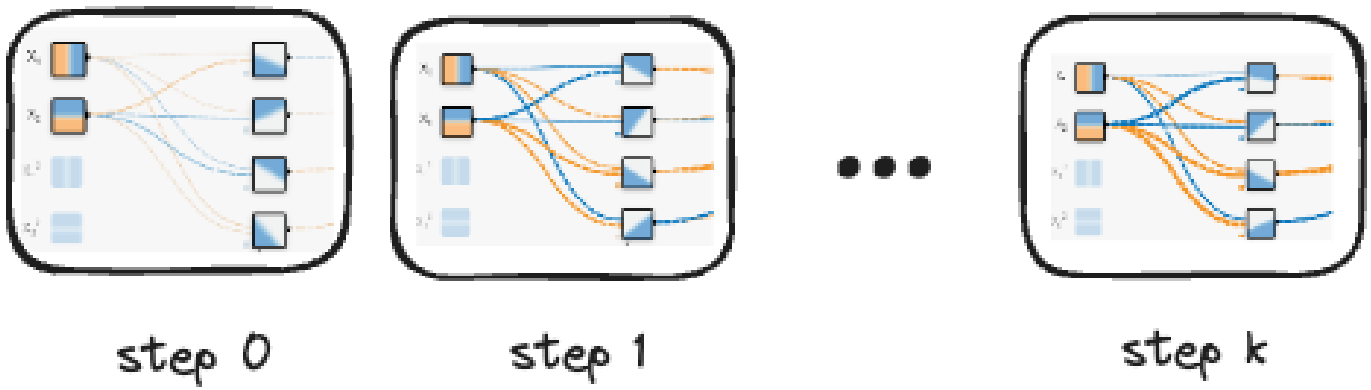
T3 extensions . . .

T1 deductive systems . . .

T2 approximation strategies . . .

T3 extensions

- Combine infinite-valued logics with other logics
- Control interaction to find tractable fragments



E.g. combine with temporal logic to study the evolution of NN and its properties during the learning process

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