Vortex filaments in fluid mechanics

Very thin tubes of extremely high vorticity in a small vorticity medium

video 1, video 2, video 3

Mathematical model

For each fixed time t, we consider a curve $\chi(t,x)$ parametrized by arclength. Vortex filaments correspond to solutions of the **binormal flow**

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Hasimoto transform: if $c(t,x)$ is the curvature of the curve and $\tau(t,x)$ the torsion, define the filament function

$$u(t,x) = c(t,x) \exp\left(i \int_0^x \tau(t.x')dx' \right).$$

Then u solves a nonlinear Schrödinger equation

$$iu_t + u_{xx} + |u|^2 u - A(t)u = 0$$

for some function A.

Corners and self-similar solutions

If the filament, at time $t = 0$, is made of two half-lines meeting at an angle θ, the filament becomes **self-similar** - invariant by change of scale.

The evolution of curves with corners is highly nontrivial... video 4

In the nonlinear Schrödinger side, this corresponds to initial data of the form

$$u_0(x) = a\delta_{x=0}, \quad \sin\left(\frac{\theta}{2}\right) = e^{-\frac{a^2}{2}}.$$

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Goals:

- Study the basic local existence theory for the nonlinear Schrödinger equation in 1D
- Determine the self-similar solutions and understand the behavior of the flow around these special solutions
- Transfer information back to the binormal flow