Resurgence, Random Matrices, and Strings

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THINK OF A **LARGE** NUMBER

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$$10^{120}$$

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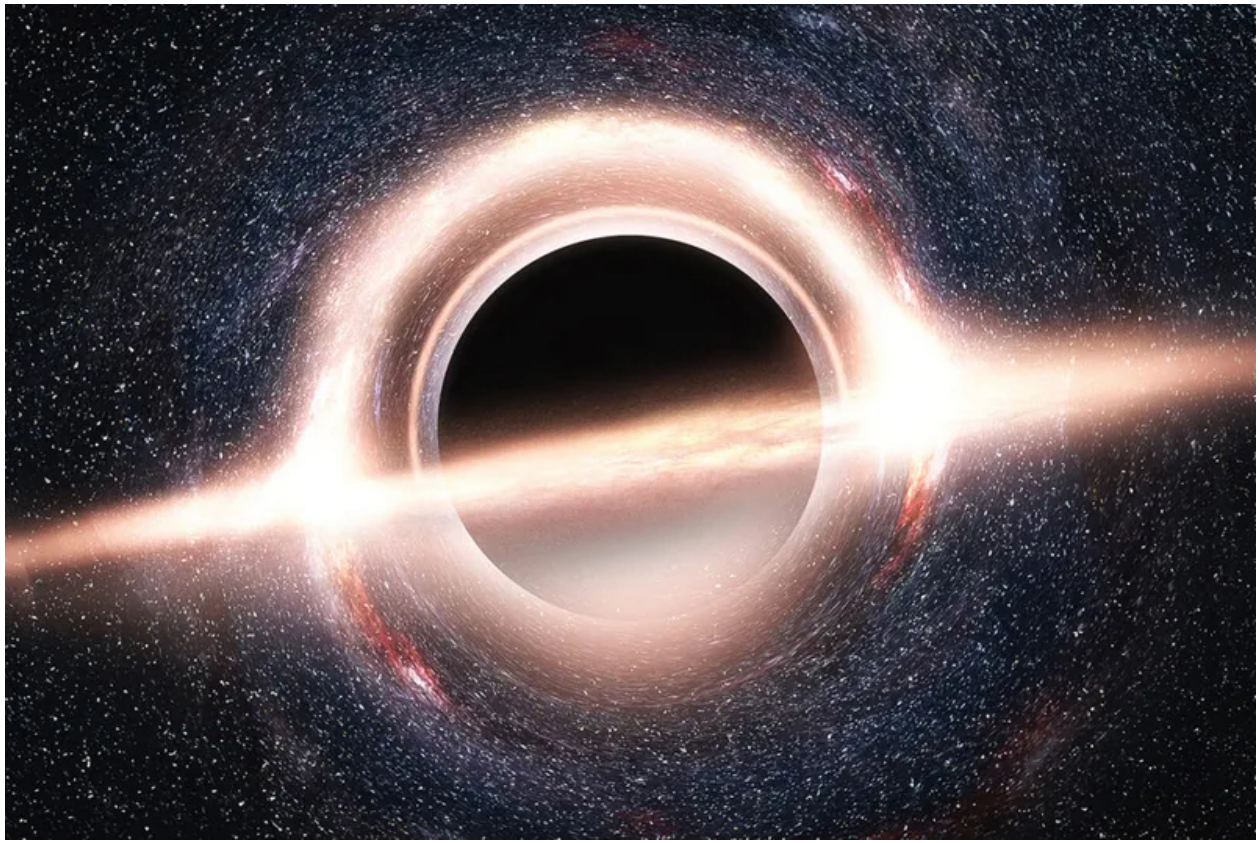
What is Inside a Black Hole?



[keanu2/GettyImages]

- According to General Relativity ... $\mathbf{1}$ state...
- According to Quantum Mechanics ... $\mathbf{10^{120}}$ states...

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How Much Does the Vacuum Weight?

[KyotoU/Takayanagi]

- Quantum field theory yields $\approx 10^{120}$ what is measured...

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Eugene Paul Wigner (Nobel Prize 1963)

[AIP/VisualArchives]

"The Unreasonable Effectiveness of Mathematics in the Natural Sciences" **(1960)** *"...enormous usefulness of mathematics in the natural sciences [...] laws of nature must already be formulated in mathematical language [...] great importance which mathematical concepts possess in the formulation of the laws of physics [...] theory of relativity — macroscopic bodies — mathematical concepts of four-dimensional Riemannian spaces [...] quantum theory — microscopic world — mathematical concepts of infinite-dimensional Hilbert space..."*

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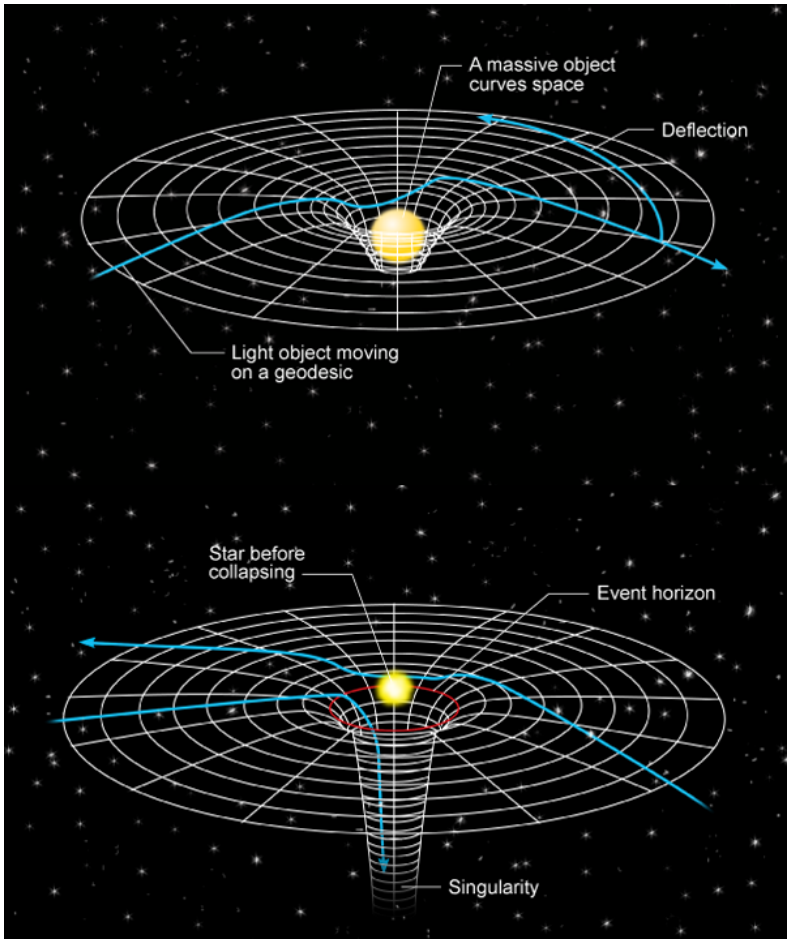
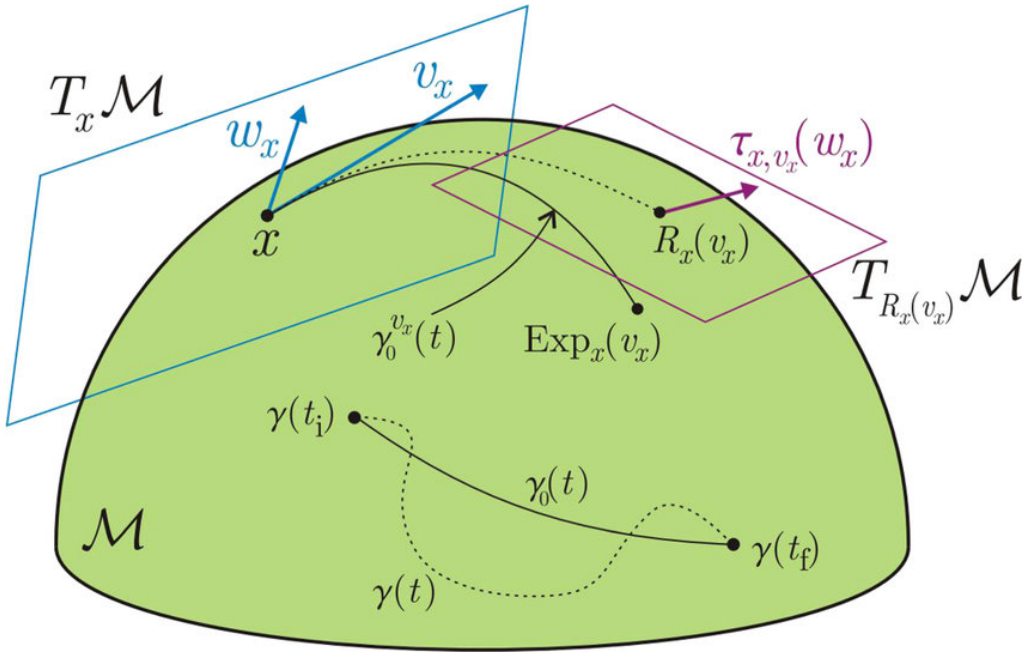
General Relativity ($\sim$ 1915) and Riemannian Geometry ($\sim$ 1854)



[APS/Cain]



[IOP/Luchnikov-Krechetov-Filippov]

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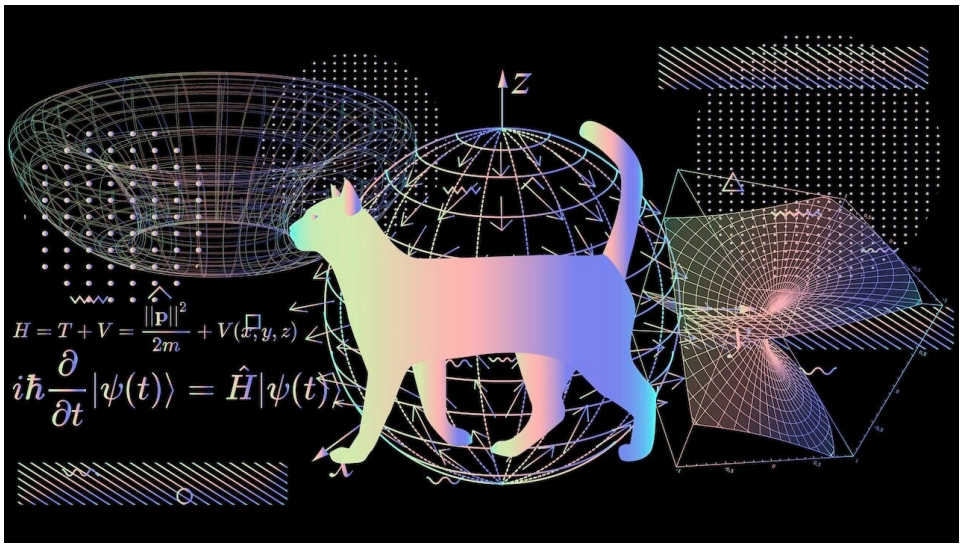
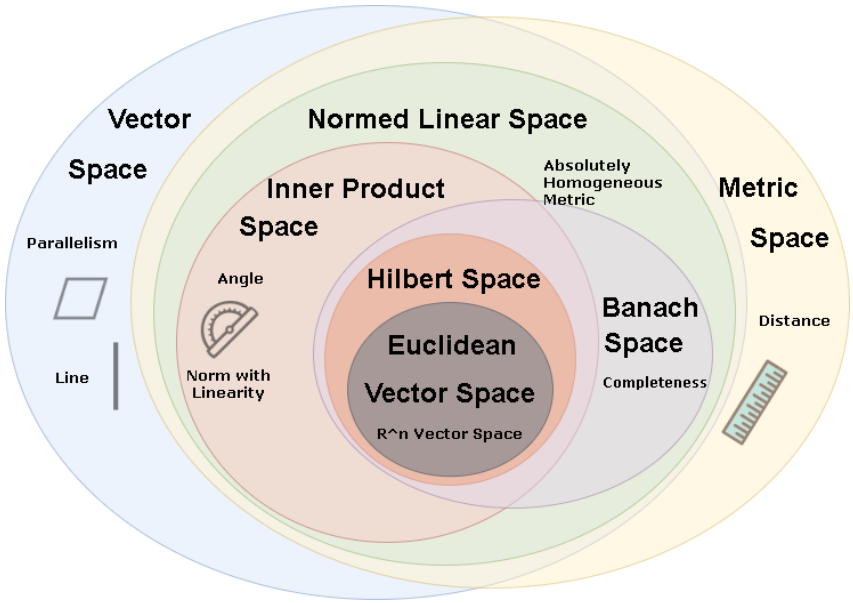
Quantum Mechanics ($\sim$ 1925) and Hilbert Spaces ($\sim$ 1909)





[Medium/Schot]

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$$10^{120}$$

Quantum Gravity and ... ?

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... Resurgence, Random Matrices, and Strings?

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Resurgence: Perturbation Theory *Diverges*

- In quantum mechanics, in quantum field theory, in gauge theory, in string theory, exact solutions are *rare*...
- Most interesting problems only tackled via perturbation theory...
- Perturbative series are asymptotic:

$$E(\hbar) \simeq \sum_{n=0}^{+\infty} E_n \hbar^n$$

  Asymptotic $\Rightarrow$ Coefficients grow as $E_n \sim n!$
- Zero radius of convergence! Now what?!?

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Resurgence: Making Sense of Divergent Series

- Niels Henrik Abel (1802–1829):

  "...Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever..." (1826)

- Johann Peter Gustav Lejeune Dirichlet (1805–1859):

  Sum of all natural numbers...a *negative rational* number! ($\sim$ 1837)

$$1+2+3+4+5+6+7+8+9+\cdots=-\frac{1}{12}$$

- Félix Édouard Justin Émile Borel (1871–1956):

  Factorial growth $(-1)^n\, n!$ is Borel summable to irrational number!

$$1-1+2-6+24-120+720-5040+40320-\cdots\simeq 0.596347\ldots$$

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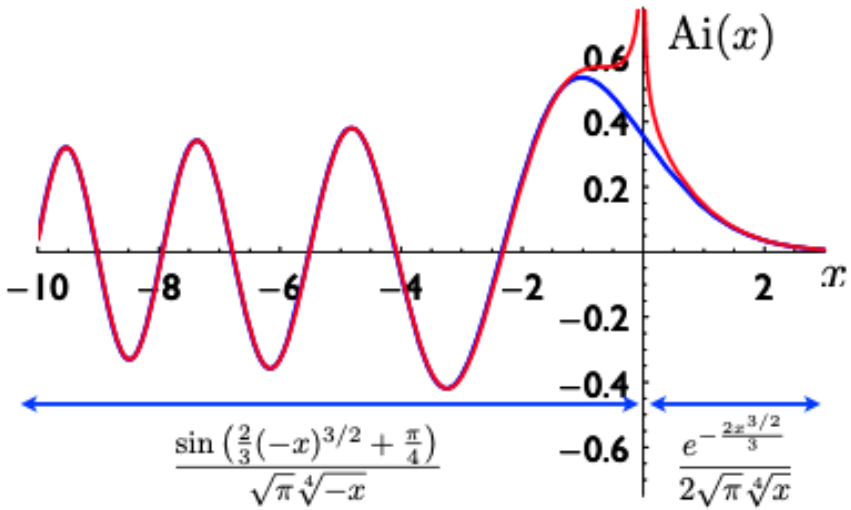
Resurgence: Unravelling the Rainbow!

- Divergence of series encodes missing nonperturbative information!
- Digging deep into divergence, find resurgence of all such information!
- George Gabriel Stokes (1819–1903): Learns to describe the rainbow...

[Bahrdt/CC-BY-SA-4]



[+Plus/Freiberger]

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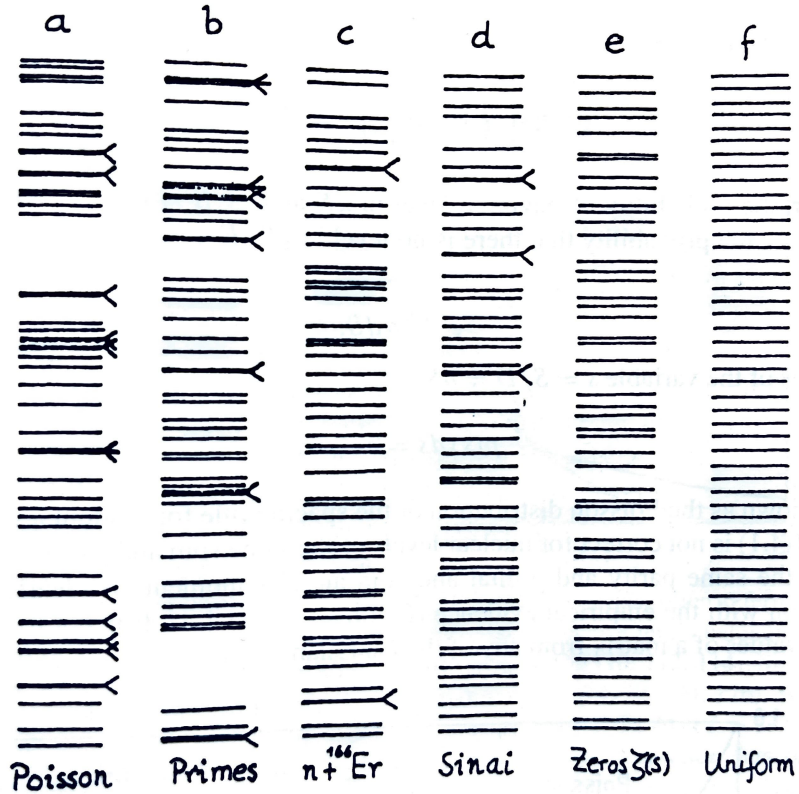
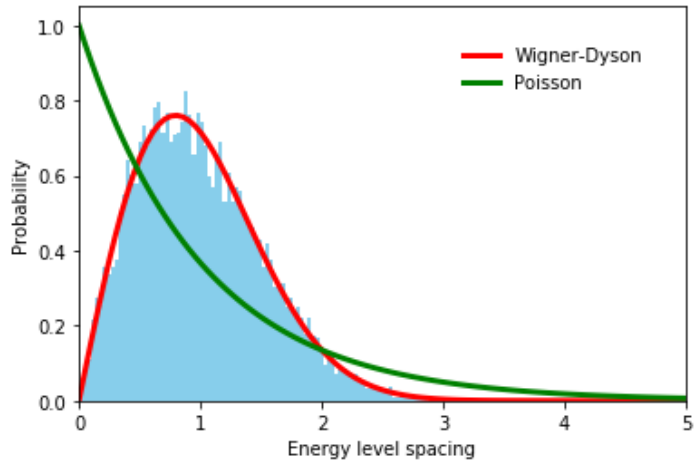
Random Matrices: Spectra with(out) Eigenvalue Repulsion

- Describe quantum behavior of chaotic, many-body systems?
- Chaotic systems $\Rightarrow$ Neighboring levels "repel"...
- Quantum chaotic spectra obey random matrix statistics! [Wigner–Dyson]



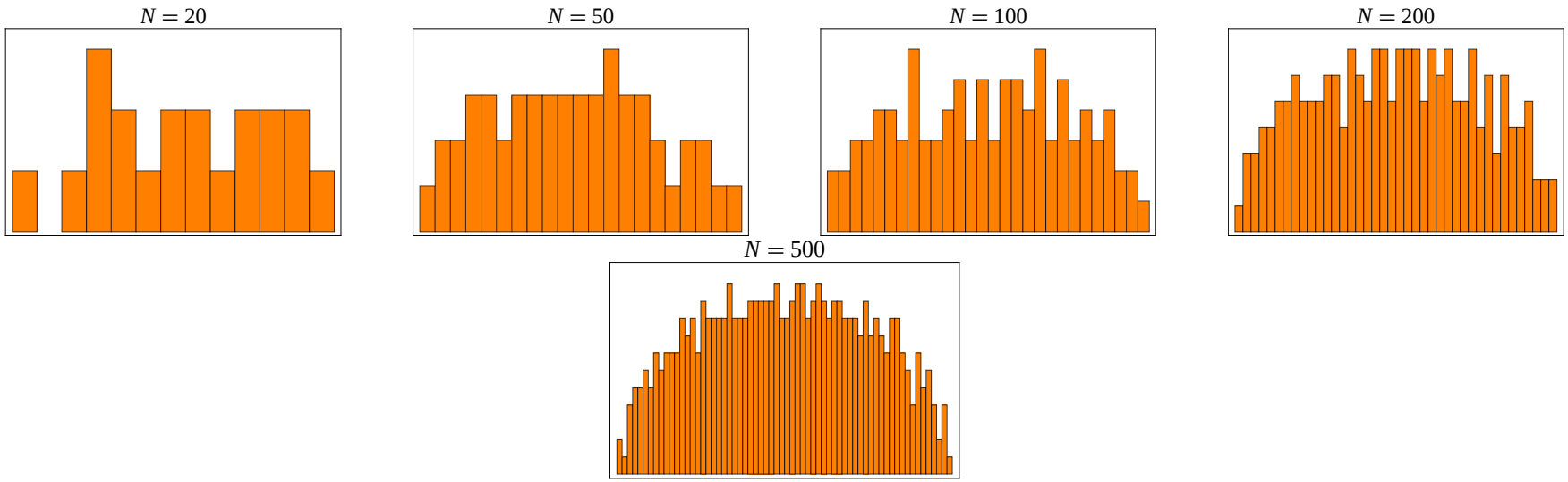
[Elsevier/Mehta]



[Stanford/Nakayama]

Random Matrices: Universality at Large N

- $N \times N$ hermitian matrix (whose entries have normal distribution with zero mean and unit standard deviation) $\Rightarrow$ Eigenvalue histogram:



- Use for quantum black holes and quantum gravity? YES!
- Black holes are chaotic systems $\Rightarrow$ Their quantum spectra should be well described by very large random matrices!
- Model quantum gravity with random matrix models!

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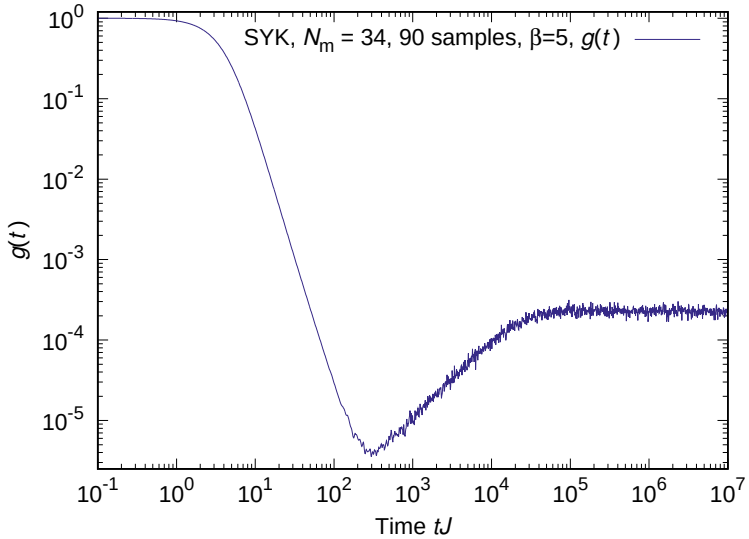
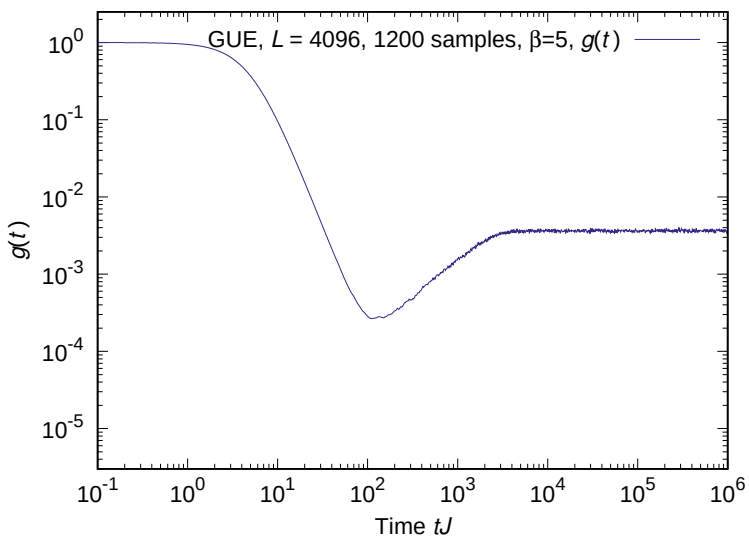
Random Matrices: Quantum Black Holes & Universes

- Fully understanding quantum-black-hole random-matrix-model requires going beyond perturbation theory $\Rightarrow$ Requires resurgence!
- SYK black hole *versus* random matrix spectral-form-factor:





[arXiv:1611.04650[hep-th]]

- Bound on chaos $\equiv$ bound on Lyapunov exponent $\lambda_L \leq 2\pi k_B T/\hbar$ $\Rightarrow$ Saturated by black holes! [Maldacena–Shenker–Stanford]
- Construction of random matrices for de Sitter spacetime ongoing!

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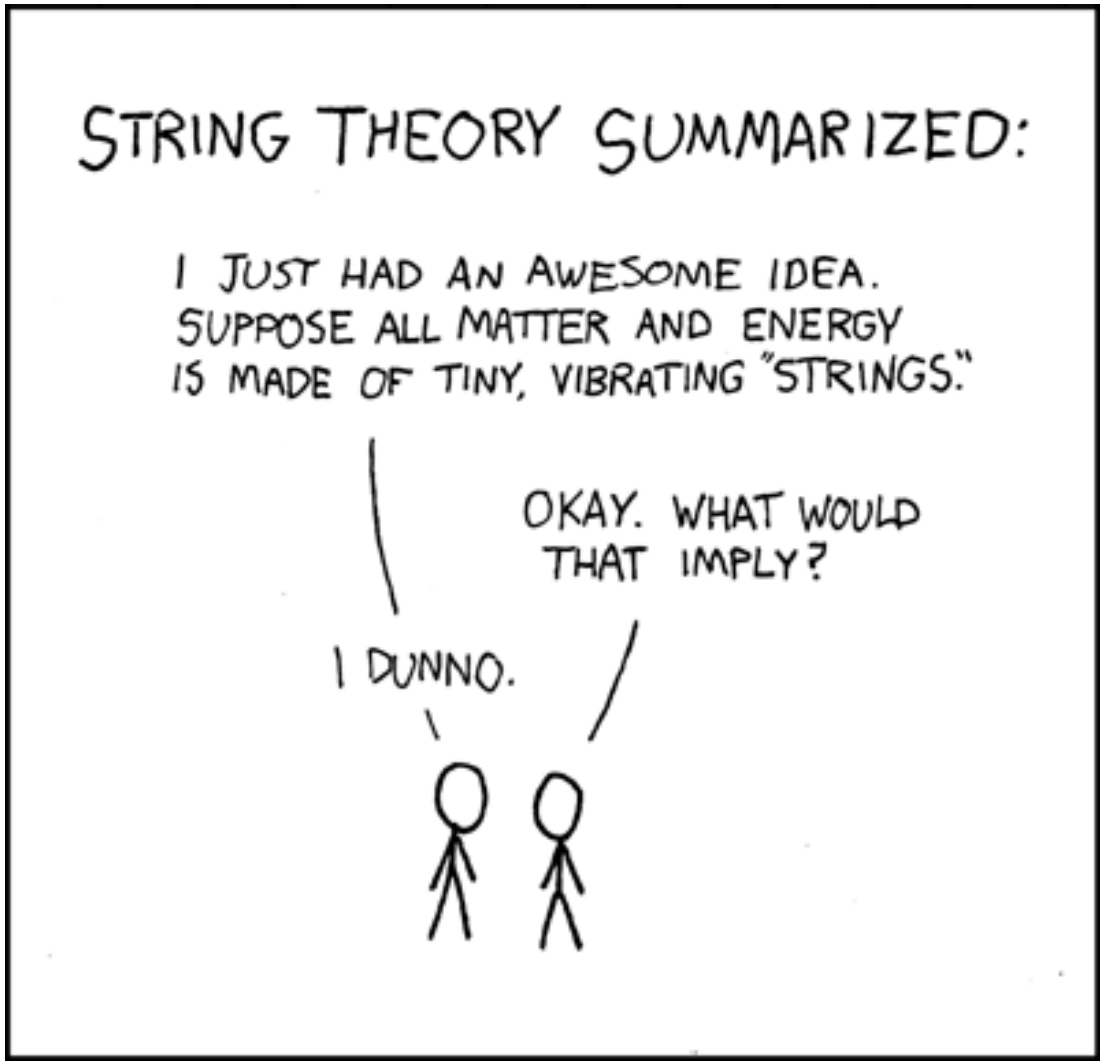
String Theory: What is It?



[xkcd.com]

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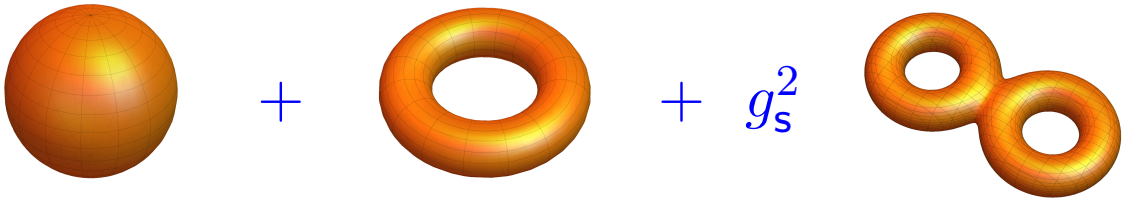
String Theory: Classical Perturbative Geometry...

- String theory generically defined *perturbatively*:

$$F = \log Z \simeq \sum_{g=0}^{+\infty} F_g(t)\, g_{\mathsf{s}}^{2g-2} =$$

$$= \frac{1}{g_{\mathsf{s}}^2} \quad + \quad + \; g_{\mathsf{s}}^2 \quad + \; \cdots .$$



- Again asymptotic series in g_{s}^2 with coefficients $F_g \sim (2g)!$...
- For A-model topological string on Calabi–Yau threefold $\mathcal{X}$,

$$F_g(t) = \sum_{d=1}^{+\infty} N_{g,d}\, \mathrm{e}^{-d\,t},$$

where $N_{g,d} \in \mathbb{Q}$ are enumerative Gromov–Witten invariants.

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String Theory: Classical Geometry Yields Quantum Theory

- Via resurgence, classical, geometric, enumerative content of string theory — Gromov–Witten invariants $N_{g,d}$ — dictate full nonperturbative quantum theory $\Rightarrow$ Build exact solutions!
- This allows for exact solutions of many random matrix models!
- Hence for exact solutions for (statistical behavior of) quantum black holes and, eventually, (statistical behavior of) quantum cosmologies.

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Quantum Gravity and ...

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