Twisted eleven-dimensional supergravity & ∞-dimensional exceptional simple super-Lie algebras

Surya Raghavendran

Based on 2111.03049
2210.07910 with I. Saberi and B. Williams
...

- 11D SUGRA is the unique SUSY theory in 11D.

  It's supposed to be the low energy limit of M-THEORY

- [Cremmer - Julia - Scherk ; 80s]:

  - Bosonic Fields include metric, 3-Form, ...

  - EXCEPTIONAL LIE ALGEBRAS $E_6 \cdot E_7 \cdot E_8, E_9, E_{10}, E_{11}$ as enhanced symmetries of toroidal compactifications on $T^6, \ldots, T^{11}$

INFINITE-DIM'L EXCEPTIONAL SIMPLE SUPER-LIE ALGEBRAS as enhanced symmetries of TWISTS

$E(5|10)$ on $\mathbb{R} \times \mathbb{C}^5$

$E(3|6)$ $AdS_7 \times S^4$

$E(1|6)$ $AdS_4 \times S^7$

Perturbative classical field theory

Work in the Batalin-Vilkovisky formalism

Definition

A *perturbative classical field theory* on M is

- a graded vector bundle $E \to M$ with sheaf of sections $\mathcal{E}$
- a collection of polydifferential operators $\{\ell_n : \mathcal{E}^{\otimes n} \to \mathcal{E}[2-n]\}$ equipping $\mathcal{E}$ with the structure of an L_∞-algebra
- a nondegenerate invariant pairing $\langle -, - \rangle : E^{\otimes 2} \to \mathrm{Dens}_M$ of degree -3.

Example

① M a 3-mfld ⇝ CS theory

$\mathcal{E} = \Omega^\bullet_M \otimes \mathfrak{g}$

$\ell_1 = d$, $\ell_2 = (- \wedge -) \otimes [-,-]_{\mathfrak{g}}$

$\langle \alpha, \beta \rangle = \int_M \mathrm{Tr}(\alpha \wedge \beta)$

② (X, Ω) CY3 ⇝ Type I minimal BCOV [Costello-Li]

$\mathcal{E} = PV^{1,\bullet}_X \xrightarrow{\partial_\Omega} PV^{0,\bullet}_X$

$\ell_1 = \bar\partial + \partial_\Omega$, $\ell_2 = [-,-]_{SN}$

Has a degenerate copairing

supersymmetric theories — TWISTING → **holomorphic topological theories**

- let $\mathcal{E}$ be a SUSY field theory ⇝ $\mathfrak{siso} \curvearrowright \mathcal{E}$ (homotopically)

  $Q \in \mathfrak{siso}$ odd, $Q^2 = 0$ ⇝ (derived) invts $\mathcal{E}^Q$

  - UV finite and anomaly-free at 1-loop [Li, Williams; Gwilliam-Williams-Rabinovich]
  - ∞-DIM'L SYMMETRY ENHANCEMENTS generalizing appearance of Virasoro in 2d chiral CFT

  - Resolves sections of bundles/cxns flat along leaves of $\mathrm{Im}[Q,-] \subseteq T_M^{\mathbb{C}}$
  - local observables described by eg $\mathbb{E}_n$-factorization algs

- let $\mathcal{E}$ be a SCFT

  $(Q,S) \in \mathfrak{sconf}$, $Q^2 = S^2 = 0$, $[Q,S] = V$ ⇝ $\mathfrak{sconf} \curvearrowright \mathcal{E}$, $\mathcal{E}^S$

  - lower dim'l field theory supported at fixed pts of $\mathcal{L}_V$

Examples

- rank 1 ABJM at level 1 (3d $\mathcal{N}=8$ SCFT)

$$\mathcal{E} = \Omega^{\bullet}_{\mathbb{R}} \otimes \Omega^{0,\bullet}_{\mathbb{C}} \left(K^{1/4} \otimes \mathbb{C}^4 \oplus \Pi K^{3/4} \otimes \mathbb{C}^{4*} \right)$$

$$= \Omega^{\bullet}_{\mathbb{R}} \otimes \Omega^{0,\bullet}_{\mathbb{C}} [\varepsilon_1, \varepsilon_2, \varepsilon_3]$$

$\circlearrowleft E(1|6)$

Deform by $z \partial_{\varepsilon_3}$:

$\mathcal{E}^S = \Omega^{\bullet}_{\mathbb{R}} [\varepsilon_1, \varepsilon_2] \rightsquigarrow$ top'l mechanics

$\circlearrowleft \mathfrak{sl}_2$

- 6d $\mathcal{N}=(2,0)$ abelian tensor multiplet [Saberi - Williams]

$$\mathcal{E} = \Omega^{2,\bullet}_{\mathbb{C}^3} \longrightarrow \Omega^{3,\bullet}_{\mathbb{C}^3}$$

$$\Pi \mathbb{C}^2 \otimes \Omega^{0,\bullet}_{\mathbb{C}^3} (K^{1/2})$$

$\circlearrowleft E(3|6)$

$\circlearrowleft \quad \supset \mathfrak{sp}(6|2) \ni S^2 = 0$

Similar deformation:

$\mathcal{E}^S = \Omega^{\leq 1, \bullet}_{\mathbb{C}} \rightsquigarrow$ free chiral boson

local ops are Heisenberg VOA

$\circlearrowleft$ Virasoro

Twisted SUGRA

In SUGRA, supersymmetry is gauged. The BV complex includes:

$$\Omega^0(M;\mathrm{ad}(FM)) \qquad \Omega^1(M;\mathrm{ad}(FM))_\omega$$

$$\Omega^0(M;TM)_V \qquad \Omega^1(M;TM)_e \qquad + \text{ antifields } \ldots$$

$$\Omega^0(M;\Pi S)_q \qquad \Omega^1(M;\Pi S)_\Psi$$

Definition (Costello-Li [CL16])

A *twisted SUGRA background* is SUGRA in perturbation theory around a point where the bosonic ghost q takes a nonzero value Q.

Properties:

- twists of supergravity theories govern deformations of a THF
- twisting a nongravitational SUSY field theory $\Longleftrightarrow$ coupling to a twisted SUGRA background

The minimal twist

Conjecture (R.-Saberi-Williams [RSW21])

The minimal twist of 11d SUGRA on $\mathbb{R}\times\mathbb{C}^5$ is given by the following $\mathbb{Z}/2$-graded BV theory

- The fields and linearized BRST differentials are:

$$\mathcal{E}_{11d}=\Omega^\bullet(\mathbb{R})\otimes\begin{pmatrix} \mathrm{PV}^{1,\bullet}(\mathbb{C}^5)_\mu & \xrightarrow{\partial_\Omega} & \mathrm{PV}^{0,\bullet}(\mathbb{C}^5)_\nu \\ \Omega^{0,\bullet}(\mathbb{C}^5)_\beta & \xrightarrow{\partial} & \Omega^{1,\bullet}(\mathbb{C}^5)_\gamma \end{pmatrix}$$

- The BV pairing $\omega:\mathcal{E}\otimes\mathcal{E}\to\mathbb{C}$ is given by $\int(\gamma\vee\mu+\beta\nu)\wedge\Omega$.
- The interaction is:

$$I(\mu,\nu,\beta,\gamma)=\int\frac{\Omega}{1-\nu}\mu^2\vee\partial\gamma+\int\gamma\partial\gamma\partial\gamma.$$

Modular interpretation is nebulous, but lots of evidence for this conjecture!

Evidence

- The character of local operators recovers the graviton contribution to the M-theory index computed by [Nek09]
- The free limit matches with the minimal twist of the 11d SUGRA multiplet computed by [SW21].
- The theory admits a deformation to the nonminimal twist of 11d SUGRA studied by [Cos16], [EH21].
- Dimensional reductions recover known descriptions of twists of type IIA and type I SUGRA in 10d [CL16]

Infinite dimensional symmetries

The minimal twist is intimately related to a series of infinite dimensional exceptional simple super Lie algebras.

Theorem (R.-Saberi-Williams [RSW21])

There is an L_∞ equivalence

$$H^\bullet\left(\Pi\mathcal{E}_{11d}(\mathbb{R}\times\mathbb{C}^5); d+\bar{\partial}+\partial+\partial_\Omega\right)\cong\widehat{E(5|10)}$$

where $\widehat{E(5|10)}$ denotes a Lie-2 extension of $E(5|10)$

Definition

$E(5|10)$ is the super-Lie algebra $\mathrm{Vect}_0(\mathbb{C}^5)_\mu \ltimes \Pi\Omega^2_{cl}(\mathbb{C}^5)_\alpha$ with odd bracket given by

$$[\alpha,\alpha'] = (\alpha\wedge\alpha')\vee\Omega^{-1}.$$

The extension is determined by a cocycle given by

$$(\mu,\mu',\alpha)\mapsto\langle\mu\wedge\mu',\alpha\rangle|_{z=0}.$$

INFINITE DIMENSIONAL SYMMETRIES

Local operators are given by the Lie algebra cochains $C^\bullet\left(\widehat{E(5|10)}\right)$

Proposition (R.-Saberi-Williams [RSW21])

$$\chi_{SU(5)}\left(C^\bullet\left(\widehat{E(5|10)}\right)\right) = \prod_{i=1}^{5} \prod_{(m_i)\in\mathbb{Z}^5_{\geq 0}} \frac{1 - q_1^{m_1+1}\cdots q_i^{m_i}\cdots q_5^{m_5+1}}{1 - q_1^{m_1}\cdots q_i^{m_i+1}\cdots q_5^{m_5}}$$

where q_i are such that $\prod_{i=1}^{5} q_i = 1$.

This agrees with the graviton contribution to the M-theory index and specializes to the DT series of $\mathbb{C}^3$ [Nek09], [NO14].

TWISTED BACKREACTIONS

In the physical theory, the *AdS* backgrounds arise from *backreacting* M2 and M5 branes on flat space respectively. We can mimic this at the twisted level.

- Branes determine certain curved deformation of the L_∞-algebra underlying the theory:

$$I_{M2} = \int_{\mathbb{R}\times\mathbb{C}} \gamma, \quad I_{M5} = \int_{\mathbb{C}^3} \partial^{-1}\mu \vee \Omega$$

- Backreaction amounts to deforming the theory on the complement of the brane by a solution to the resulting curved Maurer-Cartan equation

$$\bar\partial\mu + \frac{1}{2}[\mu,\mu] + \partial\gamma\partial\gamma = \Omega^{-1}\delta_{\mathbb{R}\times\mathbb{C}} \qquad \bar\partial\partial\gamma + \partial_\Omega\left(\frac{\mu}{1-\nu}\right)\wedge\partial\gamma = \delta_{\mathbb{C}^3}$$

$$\partial_\Omega\mu = 0 \qquad (\bar\partial + d)\mu + \partial\gamma\partial\gamma = 0$$

Solutions are described by variants of the *Bochner-Martinelli kernel*:

$$\mu = F_{M2} = \frac{6}{(2\pi i)^4} \frac{\sum_{a=1}^4 \bar{w}_a \mathrm{d}\bar{w}_1 \cdots \widehat{\mathrm{d}\bar{w}_a} \cdots \mathrm{d}\bar{w}_4}{\|w\|^8} \partial_z,$$

$$\partial\gamma = F_{M5} = \frac{1}{(2\pi i)^3} \frac{\bar{w}_1 \mathrm{d}\bar{w}_2 \wedge \mathrm{d}t - \bar{w}_2 \mathrm{d}\bar{w}_1 \wedge \mathrm{d}t + t\mathrm{d}\bar{w}_1 \wedge \mathrm{d}\bar{w}_2}{(\|w\|^2 + t^2)^{5/2}} \wedge \mathrm{d}w_1 \wedge \mathrm{d}w_2$$

Conjecture (R.-Saberi-Williams [RSW21])

- The minimal twist of 11d SUGRA on $AdS_4 \times S^7$ is the classical BV theory on $\mathrm{Tot}(K_{\mathbb{C}}^{1/4} \otimes \mathbb{C}^4 \to \mathbb{R} \times \mathbb{C}) \setminus 0(\mathbb{R} \times \mathbb{C})$ given by

$$\mathcal{E}_{AdS_4 \times S^7} = \left(\mathcal{E}_{11d}|_{(\mathbb{R}\times\mathbb{C})\times(\mathbb{C}^4\setminus 0)},\ S(\mu + NF_{M2}, \nu, \beta, \gamma)\right)$$

- The minimal twist of 11d SUGRA on $AdS_7 \times S^4$ is the classical BV theory on $\mathrm{Tot}(\mathbb{R} \oplus K_{\mathbb{C}^3}^{1/2} \otimes \mathbb{C}^2 \to \mathbb{C}^3) \setminus 0(\mathbb{C}^3)$ given by

$$\mathcal{E}_{AdS_7 \times S^4} = \left(\mathcal{E}_{11d}|_{\mathbb{C}^3\times(\mathbb{R}\times\mathbb{C}^2)\setminus 0},\ S(\mu, \nu, \beta, \gamma + N\partial^{-1}F_{M5})\right)$$

COUNTING GRAVITONS

We can numerically test this conjecture by counting *supergravity states* - field configurations localized at the origin in the boundary of AdS:

- Write the complement of the brane as a sphere bundle:

$$\begin{array}{ccc} S^7 & \longrightarrow & \mathrm{Tot}(K_{\mathbb{C}}^{1/4} \otimes \mathbb{C}^4 \to \mathbb{R} \times \mathbb{C}) \setminus 0(\mathbb{R} \times \mathbb{C}) \\ & & \downarrow p \\ & & \mathbb{R}_{>0} \times \mathbb{R} \times \mathbb{C} \end{array} \qquad \begin{array}{ccc} S^4 & \longrightarrow & \mathrm{Tot}(\mathbb{R} \oplus K^{1/2} \otimes \mathbb{C}^2 \to \mathbb{C}^3) \setminus 0(\mathbb{C}^3) \\ & & \downarrow p \\ & & \mathbb{R}_{>0} \times \mathbb{C}^3 \end{array}$$

- Compute the pushforward and consider certain natural boundary conditions at $\infty \in \mathbb{R}_{>0}$:

$$\mathcal{L}^N_{\mathbb{R}\times\mathbb{C}} \to (p_* \mathcal{E}_{AdS_4 \times S^7})|_{\infty \times \mathbb{R} \times \mathbb{C}}, \quad \mathcal{L}^N_{\mathbb{C}^3} \to (p_* \mathcal{E}_{AdS_7 \times S^4})|_{\infty \times \mathbb{C}^3}$$

- Supergravity states are the costalk at 0 of compactly supported sections:

$$\mathcal{H}_{AdS_4 \times S^7} = \mathcal{L}^N_{\mathbb{R}\times\mathbb{C},c}(0), \quad \mathcal{H}_{AdS_7 \times S^4} = \mathcal{L}^N_{\mathbb{C}^3,c}(0)$$

Counting gravitons

Theorem (R.-Williams)

- $\chi_{\mathfrak{sl}(4)\oplus\mathfrak{sl}(2)}(\mathcal{H}_{AdS_4\times S^7}) =$

$$\frac{q\begin{pmatrix} q^{1/4}(t_1+t_2+t_3+t_1^{-1}t_2^{-1}t_3^{-1})+q^{-1} \\ -q^{-1/4}(t_1^{-1}+t_2^{-1}+t_3^{-1}+t_1t_2t_3)-q \end{pmatrix}}{(1-q)(1-q^{1/4}t_1)(1-q^{1/4}t_2)(1-q^{1/4}t_3)(1-q^{1/4}t_1^{-1}t_2^{-1}t_3^{-1})}$$

- $\chi_{\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathfrak{gl}(1)}(\mathcal{H}_{AdS_7\times S^4}) =$

$$\frac{q^4(t_1^{-1}+t_1t_2^{-1}+t_2)-q^2(t_1+t_1^{-1}t_2+t_2^{-1})+(q^{3/2}-q^{9/2})(r+r^{-1})}{(1-t_1^{-1}q)(1-t_2q)(1-t_1t_2^{-1}q)(1-rq^{3/2})(1-r^{-1}q^{3/2})}$$

- These characters agree with single particle indices enumerating gravitons on $AdS_4 \times S^7$ and $AdS_7 \times S^4$.
- $\chi_{\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathfrak{gl}(1)}(\mathcal{H}_{AdS_7\times S^4})(q, t_1, t_2 = 1, r = q^{1/2}) = \frac{q}{(1-q)^2}$ - the plethystic exponential of this is the MacMahon function.

Global symmetries of twisted AdS

The $\begin{matrix} 3d\ \mathcal{N}=8 \\ 6d\ \mathcal{N}=(2,0) \end{matrix}$ superconformal algebras act on $\begin{matrix} AdS_4 \times S^7 \\ AdS_7 \times S^4 \end{matrix}$.

Their complexifications are both $\mathfrak{osp}(8|4)$.

Proposition (Saberi-Williams [SW21])

The minimal twists of $\mathfrak{osp}(8|4)$ are both isomorphic to $\mathfrak{osp}(6|2)$.

Proposition (R.-Saberi-Williams [RSW21])

There are Lie maps

$$\mathfrak{osp}(6|2) \to H^\bullet\left(\Pi\mathcal{E}_{AdS_4\times S^7}\left(\mathbb{R}\times\mathbb{C}\times(\mathbb{C}^4\setminus 0)\right)\right),$$
$$\mathfrak{osp}(6|2) \to H^\bullet\left(\Pi\mathcal{E}_{AdS_7\times S^4}\left(\mathbb{C}^3\times(\mathbb{R}\times\mathbb{C}^2)\setminus 0\right)\right)$$

The specialization in the previous slide is induced by deforming by a nilpotent superconformal element $S \in \mathfrak{osp}(6|2)$

Twisted Kaluza-Klein spectrometry

Theorem (R.-Williams [RW22])

There are $\mathbb{C}^\times$ actions on $\mathcal{L}^N_{\mathbb{R}\times\mathbb{C}}$ and $\mathcal{L}^N_{\mathbb{C}^3}$ compatible with the L_∞ structures. The induced decompositions

$$\mathcal{L}^N_{\mathbb{R}\times\mathbb{C}} = \prod_{j\geq -2} \mathcal{F}^{(j)}_{\mathbb{R}\times\mathbb{C}}, \quad \mathcal{L}^N_{\mathbb{C}^3} = \prod_{j\geq -1} \mathcal{G}^{(j)}_{\mathbb{C}^3}$$

are such that there are L_∞-equivalences

$$J^\infty_0 \mathcal{F}^{(0)}_{\mathbb{R}\times\mathbb{C}} \cong E(1|6), \quad J^\infty_0 \mathcal{G}^{(0)}_{\mathbb{C}^3} \cong E(3|6).$$

Moreover, $E(1|6)$ and $E(3|6)$ contain copies of $\mathfrak{osp}(6|2)$.

The above implies that the costalks at the origin of the cosheaves

$$\mathcal{F}^{(j)}_{\mathbb{R}\times\mathbb{C},c}(0), \qquad \mathcal{G}^{(j)}_{\mathbb{C}^3,c}(0)$$

are $E(1|6)$ and $E(3|6)$-modules respectively. The spectra of gravitons on twisted AdS geometries carry actions of infinite dimensional exceptional simple super-Lie algebras!

Abelian superconformal field theories

The lowest pieces of this decomposition are related to minimal twists of free SCFTs in dimensions 3 and 6.

Proposition (R.-Williams)

There are equivalences of abelian local L_∞-algebras

$$\mathcal{F}^{(-1)}_{\mathbb{R}\times\mathbb{C}} \cong \Omega^\bullet_{\mathbb{R}} \otimes \Omega^{0,\bullet}_{\mathbb{C}} \left(K^{1/4} \otimes (\mathbb{C}^4)^* \oplus \Pi K^{3/4} \otimes \mathbb{C}^4\right)$$

$$\mathcal{G}^{(-1)}_{\mathbb{C}^3} \cong \begin{array}{c} \mathbb{C}^2 \otimes \Omega^{0,\bullet}_{\mathbb{C}^3}(K^{1/2}) \\ \Omega^{0,\bullet}_{\mathbb{C}^3} \xrightarrow{\partial} \Omega^{1,\bullet}_{\mathbb{C}^3} \end{array}$$

$\mathcal{F}^{(-1)}_{\mathbb{R}\times\mathbb{C},c}(0)$ and $\mathcal{G}^{(-1)}_{\mathbb{C}^3,c}(0)$ recover spaces of linear local operators for minimal twists of the theories on a single M2 and M5 brane probing flat space

Characters

We can compute the characters of the modules $\mathcal{F}^{(j)}_{\mathbb{R}\times\mathbb{C},c}(0)$ and $\mathcal{G}^{(j)}_{\mathbb{C}^3,c}(0)$.

Proposition (R.-Williams [RW22])

- $\chi_{\mathfrak{sl}(4)\oplus\mathfrak{sl}(2)}(\mathcal{F}^{(j)}_{\mathbb{R}\times\mathbb{C},c}(0))$ is given by

$$\frac{q}{(1-q)}\left(q^{1/2}\chi^{\mathfrak{sl}(4)}_{[0,1,0]}(y_i) + q^{-1/2}\chi^{\mathfrak{sl}(4)}_{[2,0,0]}(y_i) - q - \chi^{\mathfrak{sl}(4)}_{[1,0,1]}(y_i)\right)$$

- $\chi_{\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathfrak{gl}(1)}(\mathcal{G}^{(j)}_{\mathbb{C}^3,c}(0))$ is given by

$$\frac{q^3\begin{pmatrix} q^{1+3j/2}\chi^{\mathfrak{sl}(2)}_{j}(q^{-1/2}y)\chi^{\mathfrak{sl}(3)}_{[1,0]}(y_i) + q^{3j/2}\chi^{\mathfrak{sl}(2)}_{j+2}(q^{-1/2}y) \\ -q^{3(j+1)/2}\chi^{\mathfrak{sl}(2)}_{j-1}(q^{-1/2}y) - q^{-1+3(j+1)/2}\chi^{\mathfrak{sl}(2)}_{j+1}(q^{-1/2}y)\chi^{\mathfrak{sl}(3)}_{[0,1]}(y_i)\end{pmatrix}}{(1-y_1q)(1-y_2q)(1-y_3q)}$$

Under the specialization $y = y_3 = 1$, $\sum_{j=0}^{N}\chi_{\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathfrak{gl}(1)}(\mathcal{G}^{(j)}_{\mathbb{C}^3,c}(0))$ recovers the vacuum character of the $\mathcal{W}_{N+2}$-algebra

Physicists speak of the $6d\ \mathcal{N}=(2,0)$ SCFT of type A_{N-1}

They say it's

UBIQUITOUS:

mother of SCFTs ⇝ unifies contemporary programs in geometry, topology, representation theory
[instanton counting, GL program, homological knot invts, ...]

but

NEBULOUS:

- no free parameters that can be used to develop an expansion
- no known field realization outside of abelian theory

Nevertheless, much is known through HOLOGRAPHY + COMPACTIFICATION TO LOWER DIMS

Twisted AdS/CFT

M-theory on $AdS_7 \times S^4$	6d $\mathcal{N} = (2,0)$ SCFT of type A_{N-1} $\xrightarrow{\text{dim red}}$	5d $\mathcal{N} = 2$ SU(N) SYM
$\downarrow$ *twist*	$\downarrow$ *twist*	$\downarrow$ *twist*
Obs_{grav}	Obs^{N}_{brane}	BPS invariants

Conjecture (Costello-Li [CP21], [CG21a],[Cos16],[Cos17])

There is a system of compatible maps $(\mathrm{Obs}_{grav}|_{\partial(AdS_7)})^! \to \mathrm{Obs}^{N}_{brane}$ that become an isomorphism in the $N \to \infty$ limit.

Goal:

- Use the above map to define Obs^{N}_{brane}.
- Check definition by trying to recover BPS invariants of 5d $\mathcal{N} = 2$ theories.

Distinguished subalgebras of the tree-level large N algebra

Proposition

At tree-level $(\mathrm{Obs}_{grav}|_{\partial(AdS_7)})^! = \mathcal{U}_\omega(\mathcal{L}^{N=0}_{\mathbb{C}^3})$

- The cocycle ω can be computed in terms of Witten diagrams involving the flux NF_{M5} but they quickly grow unwieldy.
- The restrictions to the lowest pieces $\mathcal{G}^{(-1)}_{\mathbb{C}^3}$ and $\mathcal{G}^{(0)}_{\mathbb{C}^3}$ are more manageable.

Conjecture (R.-Saberi-Williams)

After deforming by $S \in \mathfrak{osp}(6|2)$ the factorization algebras $\mathcal{U}_\omega(\mathcal{G}^{(-1)}_{\mathbb{C}^3})$ and $\mathcal{U}_\omega(\mathcal{G}^{(0)}_{\mathbb{C}^3})$ have no sections away from a copy of $\mathbb{C} \subset \mathbb{C}^3$.
Moreover:

- $\mathcal{U}_\omega(\mathcal{G}^{(-1)}_{\mathbb{C}^3})$ deforms to the Heisenberg vertex algebra.
- $\mathcal{U}_\omega(\mathcal{G}^{(0)}_{\mathbb{C}^3})$ deforms to the Virasoro vertex algebra.

It remains to compute the relevant Witten diagrams.

Dolbeault-AGT

Can place $\mathcal{L}^N_{\mathbb{C}^3}$ on any complex 3-fold Z with $\begin{matrix} L_1, L_2 \to Z \\ \det(L_1 \oplus L_2) = K_Z \end{matrix}$.

Suppose $Z = \mathrm{Tot}(L_0 \to S)$. Build an associative algebra and a module.

- Algebra: choose a fiberwise hermitian metric on L_0

$$Z \setminus 0(S) \xrightarrow{\pi} \mathbb{R}_{>0} \rightsquigarrow \pi_* \mathcal{U}_\omega(\mathcal{G}_Z^{(-1)})$$

- Module: Hilbert space of minimally twisted 5d $\mathcal{N} = 2$ on S in the presence of a background RR 1-form.

Conjecture (R-Williams)

There is an action of $\pi_* \mathcal{U}_\omega(\mathcal{G}_Z^{(-1)})$ on $H^\bullet(\mathrm{Higgs}_1(S, L_2), \Theta)$ where Θ is a particular line bundle on $\mathrm{Higgs}_1(S, L_2)$

For $S = \mathbb{CP}^2, L_0 = \mathcal{O}(-1), L_1 = \mathcal{O}, L_2 = K_S$, this admits a deformation to the action studied by Grojnowski-Nakajima.

Thanks!

Let us impose the divergence-free and co-closed constraints on μ, γ. The equations defining the phase space include:

$$0 = \bar{\partial}\mu^{1,1} + \frac{1}{2}\{\mu^{1,1}, \mu^{1,1}\} + c(\partial\gamma^{1,0}\partial\gamma^{1,2}) \vee \Omega \tag{1}$$

$$0 = \bar{\partial}\gamma^{1,0} + \partial\gamma^{0,1} + L_{\mu^{1,1}}\gamma^{1,0} + L_{\mu^{1,0}}\gamma^{0,1} \tag{2}$$

Fix $\beta = \partial\gamma^{1,0}$ and consider

$$F = \mathrm{Im}(\wedge^{2,0}T^{*1,0}(Z) \to \wedge^4 T^{*1,0}(Z) \to T^{1,0}(Z)) \subset T^{1,0}(Z).$$

This is a holomorphic distribution; integrability follows from $\partial\beta = 0$.

Splitting $T^{1,0}(Z) = F \oplus Q$

$$\Longrightarrow \; T^{1,0}(Z) \otimes T^{*0,1}(Z) = (F \otimes \bar{F}^*) \oplus (F \otimes \bar{Q}^*) \oplus (Q \otimes \bar{F}^*) \oplus (Q \otimes \bar{Q}^*)$$

Expanding

$$0 = \bar{\partial}\mu^{1,1} + \frac{1}{2}\{\mu^{1,1}, \mu^{1,1}\} + c(\partial\gamma^{1,0}\partial\gamma^{1,2}) \vee \Omega$$

into components:

- $\mu^{1,1}_{Q\bar{Q}}$ defines a deformation of complex structure along the leaf space of F.
- $\mu^{1,1}_{F\bar{F}}$ defines a deformation of complex structure along the leaves of F.

Hints of a description in terms of a variant of exceptional generalized geometry

Back

One loop quantizations of holomorphic-topological field theories can be constructed by general results.

- Such theories are UV finite at 1-loop [Wil20]
- Weights attached to wheel diagrams that can contribute to the 1-loop anomaly will vanish for analytic reasons. The key step is a convenient choice of gauge afforded by holomorphicity [GRW21]

$\rightsquigarrow$ We can prove the existence of a 1-loop quantization of the 11d theory on $\mathbb{R} \times \mathbb{C}^5$.

Conjecture (Costello-Li [CL16])

Let Y be a CY4. The $SU(4)$-invariant twist of type IIA supergravity $\mathbb{R}^2 \times Y$ is given by the following $\mathbb{Z}/2$-graded BV theory:

- The fields and linearized BRST differentials are given by

$$\mathcal{E}_{IIA} = \Omega^\bullet(\mathbb{R}^2) \otimes \left(\begin{array}{ccc} & & \mathrm{PV}^{0,\bullet}(Y)_\eta \\ \mathrm{PV}^{1,\bullet}(Y)_\mu & \xrightarrow{\partial_\Omega} & \mathrm{PV}^{0,\bullet}(Y)_\nu \\ \mathrm{PV}^{4,\bullet}(Y)_\beta & \xrightarrow{\partial_\Omega} & \mathrm{PV}^{3,\bullet}(Y)_\gamma \\ \mathrm{PV}^{4,\bullet}(Y)_\theta & & \end{array} \right)$$

- the BV pairing is given by $\int (\mu\gamma + \eta\theta + \nu\beta) \vee \Omega \wedge \Omega$
- the interaction is given by

$$\int \frac{1}{1-\nu} \left(\mu\mu\partial_\Omega\gamma + \eta\mu\partial_\Omega\theta + \eta\partial_\Omega\gamma\partial_\Omega\gamma \right) \vee \Omega \wedge \Omega.$$

Conjecture (Costello-Li [CL16])

Let X be a CY5. The minimal twist of type I SUGRA on X is given by the following $\mathbb{Z}/2$-graded BV theory:

- The fields and linearized BRST differentials are given by

$$\mathrm{PV}^{1,\bullet}(X)_{\mu} \xrightarrow{\partial_\Omega} \mathrm{PV}^{0,\bullet}(X)_{\nu}$$

$$\Omega^{0,\bullet}(X)_{\tilde{\beta}} \xrightarrow{\partial} \Omega^{1,\bullet}(X)_{\tilde{\gamma}}$$

- the BV pairing is given by $\int(\mu\gamma + \eta\theta + \nu\beta) \vee \Omega \wedge \Omega$
- the interaction is given by

$$\int \frac{\Omega}{1-\nu} \left(\mu^2 \partial \tilde{\gamma}\right).$$

Usual relationships between 11d SUGRA and 10d SUGRAs hold at the level of twists:

Proposition (R.-Saberi-Williams [RSW21])

- Consider $\mathcal{E}_{11d}$ on $\mathbb{R} \times \mathbb{C}^\times \times \mathbb{C}^4$. The dimensional reduction of $\mathcal{E}_{11d}$ along $S^1 \subset \mathbb{C}^\times$ is $\mathcal{E}_{IIA}$ on $\mathbb{R}^2 \times \mathbb{C}^4$.
- Consider $\mathcal{E}_{11d}$ on $[0,1] \times \mathbb{C}^5$. The dimensional reduction of $\mathcal{E}_{11d}$ along $[0,1]$, with boundary conditions where $\gamma = 0$ at $\{0,1\} \times \mathbb{C}^5$, is $\mathcal{E}_I$ on $\mathbb{C}^5$.

We are also able to propose a conjectural description of the minimal twist of type IIA - this twist is unique because it does not seem to describe the closed string field theory of a topological string!

Back

Let X be a G2 manifold and N a holomorphic symplectic surface

Conjecture (Costello [Cos16])

The nonminimal twist of 11d SUGRA on $X \times N$ is given by the following $\mathbb{Z}/2$-graded BV theory

- The fields and linearized BRST differential are $\mathcal{E}_{nonmin} = \Omega^\bullet(X) \otimes \Omega^{0,\bullet}(N)$.
- The BV pairing is given by $\int \Omega_N(\alpha \wedge \beta)$.
- The interaction is:

$$I(\alpha) = \int \Omega_N \left(\alpha(d + \bar{\partial})\alpha + \frac{1}{2}\alpha\{\alpha, \alpha\} \right)$$

Theorem (R.-Saberi-Williams [RSW21])

Let $\tilde{\gamma} \in \Omega^{1,0}(Z)$ be such that $\partial\tilde{\gamma} = \Omega_N$ and $(d + \bar{\partial})\tilde{\gamma} = 0$. The specialization of $\Pi\mathcal{E}$ at $\gamma = \tilde{\gamma}$ is $\mathcal{E}_{nonmin}$.

Back

Definition

$E(1|6)$ is the super-Lie algebra with

- $E(1|6)_0 = \Gamma(\widehat{D}, T) \ltimes \left(\Gamma(\widehat{D}, \mathcal{O}) \otimes \mathfrak{sl}(4)\right)$
- $E(1|6)_1$ is the (unique) nontrivial extension of $E(1|6)_0$-modules

$$0 \to \mathrm{Sym}^2(\mathbb{C}^4) \otimes \Gamma\left(\widehat{D}, K_{\mathbb{C}}^{1/2}\right) \to E(1|6)_1 \to \wedge^2(\mathbb{C}^4) \otimes \Gamma\left(\widehat{D}, K_{\mathbb{C}}^{-1/2}\right) \to 0$$

The odd bracket is given by

$$[A \otimes f\mathrm{d}z^{1/2}, B \otimes g\mathrm{d}z^{-1/2}] = (A * B) \otimes fg$$

$$[A \otimes f\mathrm{d}z^{-1/2}, B \otimes g\mathrm{d}z^{-1/2}] = \mathrm{Tr}(A * B) \otimes fg\partial_z + \frac{1}{2}(A * B)_0 \otimes \left(\partial(f\mathrm{d}z^{-1/2})g\mathrm{d}z^{-1/2} + f\mathrm{d}z^{-1/2}\partial(g\mathrm{d}z^{-1/2})\right)$$

Definition

$E(3|6)$ is the super-Lie algebra with

- $E(3|6)_0 = \Gamma(\widehat{D}^3, T) \ltimes \left(\Gamma(\widehat{D}^3, \mathcal{O}) \otimes \mathfrak{sl}(2)\right)$
- $E(3|6)_1 = \mathbb{C}^2 \otimes \Gamma\left(\widehat{D}^3, \Omega^1(K^{-1/2})\right)$ carries the obvious $E(3|6)_0$-module structure.

The odd bracket is given by

$$\begin{aligned}
&[v_1 \otimes f_i \mathrm{d}z_i \otimes (\partial_{z_1}\partial_{z_2}\partial_{z_3})^{1/2}, v_2 \otimes g_j \mathrm{d}z_j \otimes (\partial_{z_1}\partial_{z_2}\partial_{z_3})^{1/2}] \\
&\quad = \omega(v_1, v_2)\varepsilon^{ijk} f_i g_j \partial_{z_k} \\
&\quad + (v_1 \odot v_2)\left(\partial(f_i \mathrm{d}z_i) g_j \mathrm{d}z_j - f_i \mathrm{d}z_i \partial(g_j \mathrm{d}z_j)\right) \vee (\partial_{z_1}\partial_{z_2}\partial_{z_3})
\end{aligned}$$

Back

References

[ABJM08] Ofer Aharony, Oren Bergman, Daniel Louis Jafferis, and Juan Maldacena.
N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals.
JHEP, 10:091, 2008.

[ACMV17] Mina Aganagic, Kevin Costello, Jacob McNamara, and Cumrun Vafa.
Topological Chern-Simons/Matter Theories.
6 2017.

[AFI+20] Reona Arai, Shota Fujiwara, Yosuke Imamura, Tatsuya Mori, and Daisuke Yokoyama.
Finite-n corrections to the m-brane indices.
Journal of High Energy Physics, 2020(11), nov 2020.

[AGT10] Luis F. Alday, Davide Gaiotto, and Yuji Tachikawa.
Liouville correlation functions from four-dimensional gauge theories.
Lett. Math. Phys., 91(2):167–197, 2010.

[BBMR08] Jyotirmoy Bhattacharya, Sayantani Bhattacharyya, Shiraz Minwalla, and Suvrat Raju.
Indices for Superconformal Field Theories in 3,5 and 6 Dimensions.
JHEP, 02:064, 2008.

[BBRT91] Danny Birmingham, Matthias Blau, Mark Rakowski, and George Thompson.
Topological field theory.
Physics Reports, 209(4-5):129–340, 1991.

[BCOV94] M. Bershadsky, S. Cecotti, H. Ooguri, and C. Vafa.
Kodaira–Spencer theory of gravity and exact results for quantum string amplitudes.
Commun. Math. Phys., 165:311–428, 1994.

[BG21] Kasia Budzik and Davide Gaiotto.
Giant gravitons in twisted holography, 2021.

[BH05] Anirban Basu and Jeffrey A. Harvey.
The m2-m5 brane system and a generalized nahm's equation.
Nuclear Physics B, 713(1-3):136–150, 5 2005.

[BH11] John C. Baez and John Huerta.
Division algebras and supersymmetry II.
Adv. Theor. Math. Phys., 15(5):1373–1410, 2011.

[BL07] Jonathan Bagger and Neil Lambert.
Modeling multiple m2-branes.
Physical Review D, 75(4), 2 2007.