Universes as Bigdata: Geometry, Strings, & AI

YANG-HUI HE

London Institute of Mathematical Sciences, Royal Institution
Merton College, University of Oxford
Dept of Mathematics, City, University of London
School of Physics, NanKai University

Colloquium: Técnico Lisboa, Jan, 2023
A Classic Problem in Mathematics: Since 1736

- Trichotomy classification of (connected compact orientable) surfaces $\Sigma$

  **Euler:** topological classification of $\dim_{\mathbb{R}} = 2$

  **Gauss:** relates topology to metric geometry

  **Riemann:** complexify $\leadsto$ Riemann surfaces or complex curves: $\dim_{\mathbb{C}} = 1$

<table>
<thead>
<tr>
<th>$g(\Sigma)$</th>
<th>$\chi(\Sigma)$</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(\Sigma) = 0$</td>
<td>$\chi(\Sigma) = 2$</td>
<td>Spherical, Ricci-Flat, + curvature</td>
</tr>
<tr>
<td>$g(\Sigma) = 1$</td>
<td>$\chi(\Sigma) = 0$</td>
<td>Hyperbolic, − curvature</td>
</tr>
<tr>
<td>$g(\Sigma) &gt; 1$</td>
<td>$\chi(\Sigma) &lt; 0$</td>
<td>−</td>
</tr>
</tbody>
</table>

Euler number $\chi(\Sigma)$, genus $g(\Sigma)$
### Classical Results for Riemann Surface $\Sigma$

\[
\chi(\Sigma) = 2 - 2g(\Sigma) = [c_1(\Sigma)] \cdot [\Sigma] = \frac{1}{2\pi} \int_{\Sigma} R = \sum_{i=0}^{2} (-1)^i h^i(\Sigma)
\]

<table>
<thead>
<tr>
<th>Topology</th>
<th>Algebraic Geometry</th>
<th>Differential Geometry</th>
<th>Index Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invariants</td>
<td>Characteristic classes</td>
<td>Curvature</td>
<td>(co-)Homology</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Betti Numbers</td>
<td></td>
</tr>
</tbody>
</table>
Going up in Complex Dimension

- $\dim_{\mathbb{R}} > 2$ manifolds extremely complicated

- Luckily, for a special class of complex manifolds called **Kähler**

  \[ g_{\mu\bar{\nu}} = \partial_{\mu} \partial_{\bar{\nu}} K(z, \bar{z}) \]

  all $\Sigma$ in $\dim_{\mathbb{C}} = 1$ automatically Kähler

- **CONJECTURE** [E. Calabi, 1954, 1957]: $M$ compact Kähler manifold $(g, \omega)$ and $([R] = [c_1(M)])_{H^{1,1}(M)}$. Then $\exists!(\tilde{g}, \tilde{\omega})$ such that $([\omega] = [\tilde{\omega}])_{H^2(M;\mathbb{R})}$ and $\text{Ricci}(\tilde{\omega}) = R$.

  **Rmk:** $c_1(M) = 0 \Leftrightarrow$ Ricci-flat (rmk: Ricci-flat familiar to physicists through GR)

- **THEOREM** [S-T Yau, 1977-8; Fields 1982] Existence Proof
Two Pillars of Modern Physics

1. MACROSCOPIC (General Relativity)
   - motions of galaxies and stars
   - **Gravitational Waves**: 2018, new era for cosmology, confidence 99.99994%

2. MICROSCOPIC (Quantum Field Theory)
   - $\sim SU(3)_{str} \times [SU(2) \times U(1)]_{EW}$
   - experimentally verified to 19 digits!
   - **Higgs Boson**: 2014 at LHC, CERN, last piece to SM!
The Greatest Challenge to Science

- Problem: the two are incompatible, unrenormalisability of gravity uncancellable infinities in QFT approach to GR
- Albert Einstein spent the last years of his life on this TOE [theory of everything] in vain
  “The only thing incomprehensible about our world is that it is comprehensible.”
- Should there exist a single equation unifying the Laws of Nature?
- The universe gives a hint:

\[ L_{\text{Planck}} = \sqrt{\frac{\hbar G_{\text{Newton}}}{c^3}} \sim 10^{-35} m \]
Paradigm Shift: Points $\rightsquigarrow$ Strings $\rightsquigarrow$ ToE

- Fund constituents 1-dim, NOT points (0-dim)? $\rightsquigarrow$ strings, size $L_{Planck}$

- Smear out interactions:

- PREMISE: All particles, all of space-time, all of reality are vibration modes of an OPEN or CLOSED string
  - Heterotic string [Gross-Harvey-Martinec-Rohm]: $E_8 \times E_8$ or $SO(32)$, 1984
  - $E_8$ accommodates Standard Model

$$SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_8$$

- WORKS only in 10 dimensions
Superstring Theory 9+1 d

Unified theory of quantum gravity

I. 6 Large Dim
AdS/CFT
Brane World

II. 6 small dim
Compactification

1. Reduce Dim: 10 = 6+4
2. Break SUSY

Phenomenology [Candelas-Horowitz-Strominger-Witten]: 1985

simplest solution of 6 extra dimensions: Ricci-Flat, Kähler \( \dim_{\mathbb{C}} = 3 \)
When Physics meets Maths

- Strominger was next door to Yau in 1986 at the IAS, physicists called Ricci-Flat, Kähler manifolds, CHSW called these Calabi-Yau manifolds.

- GEOMETRIZATION PROGRAMME: Historically, the right language of physics is increasingly geometrical:
  - Gravity/Space-time $\leadsto$ GR $\leadsto$ Differential geometry;
  - Particle physics/Standard Model $\leadsto$ Gauge Theory/Yang-Mills $\leadsto$ Algebraic geometry (bundles/connections) + group theory (Lie and Finite groups);
  - Condensed matter physics of topological insulators $\leadsto$ algebraic topology; . . .
  - String theory is a brain-child of this tradition.

- TAKE-HOME MESSAGE: Whenever physics and maths converge and generate new ideas, the right things are happening.
The Confluence of Maths and Physics

**Physics**

- **A. Einstein**
  - Die grundlageder allgemeinen relativitaetstheorie
  - 1916

- **P. Dirac**
  - The quantum theory of the electron
  - 1928

- **C. N. Yang, R. Mills**
  - Conversation of isotopic spin & isospin gauge inv.
  - 1954

- **E. Witten (Fields Medal)**
  - Supersymmetry and Morse theory
  - 1994

**Mathematics**

- **B. Riemann**
  - Über die hypothesen, welcher geometrie zu grundeliegen
  - 1854

- **W. Clifford**
  - On the classification of geometric algebras
  - 1876

- **H. Seifert**
  - Lehrbuch der topologie
  - 1934

- **R. Borcherds (Fields Medal)**
  - Monstrous moonshine and monstrous Lie superalgebra
  - 1990
The Confluence of Maths and Physics

1959

The Unreasonable Effectiveness of Mathematics in the Natural Sciences

Richard Courant Lecture in Mathematical Sciences delivered at New York University, May 11, 1959

EUGENE P. WIGNER
Princeton University

2010

Phil. Trans. R. Soc. A (2010) 368, 913–926

Geometry and physics

BY MICHAEL ATIYAH¹, ROBBERT DIJKGRAAF²,* AND NIGEL HITCHIN³

¹School of Mathematics, University of Edinburgh, Edinburgh EH9 3JZ, UK
²Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 Amsterdam, The Netherlands
³Mathematical Institute, University of Oxford, 24–29 St Giles, Oxford OX1 3LB, UK

"One may be tempted to invert Wigner's comment and marvel at 'the unreasonable effectiveness of physics in mathematics.'"
An interesting sequence: 1, 2, ??? …

\[
\dim_{\mathbb{C}} = 1
\]

Torus \( T^2 = S^1 \times S^1 \)

QFT in \( 10 - 2 = 8d \)

(1) 4-Torus \( T^4 = S^1 \times S^1 \times S^1 \times S^1 \)

\[
\dim_{\mathbb{C}} = 2
\]

(2) K3 surface

QFT in \( 10 - 4 = 6d \)

Unclassified ???

\[
\dim_{\mathbb{C}} = 3
\]

(Yau’s Conjecture: Finite Number)

Desired QFT in \( 10 - 6 = 4d \)
The Inevitability of Algebraic Geometry

- How to construct CY3? Realize as vanishing locus of polynomials, Algebraic Geometry e.g., \(\{(p, q)|p^2 + q^2 - 1 = 0\} \subset \mathbb{R}^2\) is a circle (1-real dimension)
- Complexify and Projectivize (Projective algebraic variety)
  - Cubic equation in \(\mathbb{C}P^2\): e.g. CY1 = \(T^2\) \(\{(x, y, z)|x^3 + y^3 + z^3 = 0\} \subset \mathbb{C}P^2\)
    (elliptic curve); \(\dim_{\mathbb{C}} = 2 - 1 = 1\)
  - TMH: Homogeneous Eq in \(\mathbb{C}P^n\), degree = \(n + 1\) is Calabi-Yau of \(\dim_{\mathbb{C}} = n - 1\)

An Early Physical Challenge to Algebraic Geometry

- Particle content in [CHSW] \# gens of SM particles = \(\pm \frac{1}{2}\) Euler number
- 1986 Question: Are there Calabi-Yau-3 with \(\chi = \pm 6\)?
The First Data-sets in Mathematical Physics/Geometry

  - **CICYs** (complete intersection CYs) multi-deg polys in products of $\mathbb{CP}^{n_i}$
  - Problem: classify all configuration matrices; employed the best computers at the time (**CERN supercomputer**); q.v. magnetic tape and dot-matrix printout in Philip’s office
  - 7890 matrices, 266 Hodge pairs $(h^{1,1}, h^{2,1})$, 70 Euler $\chi \in [-200, 0]$

- [Candelas-Lynker-Schimmrigk, 1990]
  - Hypersurfaces in Weighted P4
  - 7555 inequivalent 5-vectors $w_i$, 2780 Hodge pairs, $\chi \in [-960, 960]$

- [Kreuzer-Skarke, mid-1990s - 2000]
  - Hypersurfaces in (**Reflexive, Gorenstein Fano**) Toric 4-folds
  - 6-month running time on dual Pentium SGI machine
  - at least 473,800,776, with 30,108 distinct Hodge pairs, $\chi \in [-960, 960]$
The age of data science in mathematical physics/string theory not as recent as you might think.

After 40 years of research by mathematicians and physicists...

...was the first person with a tablet downloading data from the cloud.
Geometric Origin of our Universe

- Each CY3 (+ bundles, discrete symmetries) $X$ gives a 4-D universe
- The geometry (algebraic/differential geometry/topology etc.) of $X$ determines the physical properties of the 4-D world
- Particles and interactions $\sim$ cohomology theory; masses $\sim$ metric; Yukawa $\sim$ Triple intersections/integral of forms over $X$

Ubi materia, ibi geometria
– Johannes Kepler (1571-1630)

Our Universe:

1. Probabilistic/anthropic?
2. Sui generis/selection rule?
3. One of multi-verse?

Cf. Exo-planet/Habitable Zone search (so far $\sim 10^3$ in $10^{21}$)
The Calabi–Yau Landscape
From Geometry, to Physics, to Machine Learning

YH He
Springer-Nature, 2021
Lecture Notes in Maths
A playful intro for Masters to PhD students
An even more playful intro
Vacuum Degeneracy

Perhaps the biggest theoretical challenge to string theory:

- selection criterion??? metric on the landscape???
- Kachru-Kallosh-Linde-Trivedi (2003): type II/CY estimates of $10^{500}$
- Taylor-YN Wang (2015-7): F-theory estimates $10^{3000}$ to $10^{10^5}$
- Basic Reason:

*Algebraic Geometry $\sim$ Combinatorial Geometry $\sim$ Exponential Growth in dim*
Where we stand . . .

The Good  Last 10-15 years: several international groups have bitten the bullet
Oxford, London, Vienna, Blacksburg, Boston, Johannesburg, Munich, . . . computed
many geometrical/physical quantities and compiled them into
various databases Landscape Data ($10^9$~$10^{10}$ entries typically)

The Bad   Generic computation HARD: dual cone algorithm (exponential),
triangulation (exponential), Gröbner basis (double-exponential)
. . . e.g., how to construct stable bundles over the $\gg 473$ million KS
CY3? Sifting through for SM computationally impossible . . .

The ???  Borrow new techniques from “Big Data” revolution
A Wild Question

- Typical Problem in String Theory/Algebraic Geometry:

  \[ \text{INPUT} \quad \text{integer tensor} \quad \rightarrow \quad \text{OUTPUT} \quad \text{integer} \]

- Q: Can problems in computational geometry and theoretical physics be “learned” by AI? implications:
  - can we “machine-learn the landscape?”
  - can we do mathematics with ML?

- [YHH 1706.02714] Deep-Learning the Landscape, PLB 774, 2017

  *Science* feature article, Aug, vol 365 issue 6452:

  Experimentally, it seems to be the case for many situations in geometry and beyond in a matter of seconds on ordinary laptop.
Every manifold can be represented as a numerical tensor

A typical calculation:

$$h^{2,1} \left( \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \right) = 22$$

think of as $\rightarrow 22$

NN doesn’t know/care about the maths

Proper Way

Feed to some Neural Network:

- Take samples of $X \rightarrow h$ (compiled over 40 years)
- train a NN, or SVM, etc.,
- Validation on unseen $X \rightarrow h$

can reach reaches 99.9% quickly (cf. YHH, Bull-YHH-Jejjala-Mishra, Erbin-Finotello, Constantin-Lukas, . . . )
2017: String Theory enters the ML Era

YHH (1706.02714);
(see also: Krefl-Seong (1706.03346); Ruehle
(1706.07024) Carifio-Halverson-Krioukov-Nelson
(1707.00655) )

Sophia: Hanson Robotics, HongKong

- Beginning of String_Data annual conference series
- How can ML and modern data-science help with the vacuum degeneracy problem & string phenomenology??
- Meanwhile ... Sophia becomes a “human” citizen (in Saudi Arabia)
Machine Learning Mathematics

Why stop at string/geometry?

How do different branches of mathematics respond to ML?

How does one *DO* mathematics, I?

- **Russell-Whitehead** *Principia Mathematica* [1910s] (Leibniz, Frege, ...)
  - axiomatize maths, but... **Gödel** [1931] Incompleteness; **Church-Turing** [1930s]
  - Undecidability

- **Automated Theorem Proving (ATP)** “The practicing mathematician hardly ever worries about Gödel”
  - **Newell-Simon-Shaw** [1956] Logical Theory Machine: subset of *Principia*
  - **Type Theory** [1970s] **Martin-Löf**, **Coquand**, ... **Coq**: 4-color (2005);
  - **Feit-Thompson Thm** (2012); **Lean** (2013); Univalent Foundation / Homotopy
  - **Type Theory** [2006-] **Voevodsky**

**Buzzard**: “Future of Maths” 2019, ICM 2022 **Davenport**: ICM 2018
  - “Computer Assisted Proofs” **Szegedy**: more extreme view, computers >
  - humans @ chess (1990s); @ Go (2018); @ Proving theorems (2030)

We can call this **Bottom-up Mathematics**
How does one *DO* mathematics, I?

- Russell-Whitehead *Principia Mathematica* [1910s] (Leibniz, Frege, ...)
  axiomatize maths, but ... Gödel [1931] Incompleteness ; Church-Turing [1930s]
  Undecidability

- Automated Theorem Proving (ATP) “The practicing mathematician hardly ever worries about Gödel”
  - Newell-Simon-Shaw [1956] Logical Theory Machine: subset of *Principia*
  - Type Theory [1970s] Martin-Löf, Coquand, ... Coq: 4-color (2005);
    Feit-Thompson Thm (2012); Lean (2013); Univalent Foundation / Homotopy
  - Type Theory [2006-] Voevodsky

Buzzard: “Future of Maths” 2019, ICM 2022 Davenport: ICM 2018
“Computer Assisted Proofs” Szegedy: more extreme view, computers > humans @ chess (1990s); @ Go (2018); @ Proving theorems (2030)

We can call this Bottom-up Mathematics
How does one *DO* mathematics, II?

- Historically, Maths perhaps more Top-Down: practice before foundation
  - Countless examples: calculus before analysis; algebraic geometry before Bourbaki, permutation groups / Galois theory before abstract algebra . . .
  - A lot of mathematics starts with intuition, experience, and experimentation
- The best neural network of C18-19th? brain of Gauß ; e.g., age 16
  
  \[
  \pi(x) = \# \{p \leq x\}
  \]
  
  (w/o computer and before complex analysis [50 years before Hadamard-de la Vallée-Poussin’s proof]): \( \text{PNT} \ \pi(x) \sim x / \log(x) \)

- BSD computer experiment of Birch & Swinnerton-Dyer [1960’s] on plots of rank \( r \) & \( N_p \) on elliptic curves
How does one *DO* mathematics, II?

- Historically, Maths perhaps more **Top-Down**: practice before foundation
  - Countless examples: calculus before analysis; algebraic geometry before Bourbaki, permutation groups / Galois theory before abstract algebra . . .
  - A lot of mathematics starts with **intuition, experience, and experimentation**
- The best neural network of C18-19th? **brain of Gauß**; e.g., age 16

(w/o computer and before complex analysis [50 years before Hadamard-de la Vallée-Poussin’s proof]): PNT \( \pi(x) \sim x / \log(x) \)

- **BSD computer experiment of Birch & Swinnerton-Dyer [1960’s]** on plots of rank \( r \) & \( N_p \) on elliptic curves

![Graph](image)
How does one *DO* mathematics, II?

- Historically, Maths perhaps more Top-Down: practice before foundation
  - Countless examples: calculus before analysis; algebraic geometry before Bourbaki, permutation groups / Galois theory before abstract algebra . . .
  - A lot of mathematics starts with intuition, experience, and experimentation

- The best neural network of C18-19th? brain of Gauß ; e.g., age 16

  (w/o computer and before complex analysis [50 years before Hadamard-de la Vallée-Poussin’s proof]): PNT $\pi(x) \sim x/\log(x)$

  BSD computer experiment of Birch & Swinnerton-Dyer [1960’s] on plots of rank $r$ & $N_p$ on elliptic curves
Mathematical Data

- NOISELESS Data: different from real-world data to which ML is usually applied; If I gave you 100,000 cases of

```
5 3 4 3 5 4 2 4 4 1 2
1 0 1 0 1 0 4 4 4 5 0
2 3 2 3 2 3 1 0 0 2 1
2 5 0 3 0 4 4 4 1 5
```

or, labeled data e.g.

```
5 3 4 3 5 4 2 4 4 1 2
1 0 1 0 1 0 4 4 4 5 0
2 3 2 3 2 3 1 0 0 2 1
2 5 0 3 0 4 4 4 1 5
```

\[ \rightarrow 3 \]

- Q: Is there a pattern? Can one conjecture & then prove a formula?

- Q: What branch of mathematics does it come from?
**Mathematical Data**

- NOISELESS Data: different from real-world data to which ML is usually applied; If I gave you 100,000 cases of

  \[
  \begin{pmatrix}
  5 & 3 & 4 & 1 & 2 & 4 & 1 & 2 \\
  0 & 1 & 2 & 3 & 4 & 5 & 0 & 1 \\
  0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  \end{pmatrix},
  \quad \text{or, labeled data e.g.}\quad
  \begin{pmatrix}
  5 & 3 & 4 & 1 & 2 & 4 & 1 & 2 \\
  0 & 1 & 2 & 3 & 4 & 5 & 0 & 1 \\
  0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  \end{pmatrix} \rightarrow 3
  \]

- Q: Is there a pattern? Can one conjecture & then prove a formula?
- Q: What branch of mathematics does it come from?
Mathematical Data

- NOISELESS Data: different from real-world data to which ML is usually applied; If I gave you 100,000 cases of

\[
\begin{pmatrix}
3 & 4 & 3 & 5 & 1 & 4 & 4 & 2 & 4 \\
4 & 3 & 5 & 1 & 4 & 4 & 2 & 4 & 1 \\
5 & 0 & 1 & 2 & 0 & 4 & 0 & 5 & 0 \\
1 & 2 & 0 & 3 & 0 & 2 & 0 & 3 & 0 \\
2 & 0 & 5 & 0 & 3 & 0 & 4 & 4 & 1 \\
5 & 4 & 3 & 5 & 1 & 4 & 4 & 2 & 4 \\
4 & 3 & 5 & 1 & 4 & 4 & 2 & 4 & 1 \\
5 & 0 & 1 & 2 & 0 & 4 & 0 & 5 & 0 \\
1 & 2 & 0 & 3 & 0 & 2 & 0 & 3 & 0 \\
2 & 0 & 5 & 0 & 3 & 0 & 4 & 4 & 1 
\end{pmatrix}
\]

- e.g. \( \begin{pmatrix} 3 \end{pmatrix} \rightarrow 3 \)

- or, labeled data e.g. \( \begin{pmatrix} 3 \end{pmatrix} \rightarrow 3 \)

- Q: Is there a pattern? Can one conjecture & then prove a formula?

- Q: What branch of mathematics does it come from?
Thank you! Hundreds of Experiments since 2017-

**my fantastic students** Jiakang Bao, Elli Heyes, Ed Hirst Tejas Acharya, Daatta Aggrawal, Malik Amir, Kieran Bull, Lucille Calmon, Siqi Chen, Suvajit Majumder, Maks Manko, Toby Peterken, Juan Pérez-Ipiña, Max Sharnoff, Yan Xiao

**my wonderful collaborators**


**Algebraic Geometry:** Anthony Ashmore, Challenger Mishra, Rehan Deen, Burt Ovrut

**Number Theory:** Laura Alessandretti, Andrea Baronchelli, Kyu-Hwan Lee, Tom Oliver, Alexey Pozdnyakov, Drew Sutherland, Eldar Sultanow

**Representation Theory:** Mandy Cheung, Pierre Dechant, Minhyong Kim, Jianrong Li, Gregg Musiker

**Combinatorics:** Johannes Hofscheier, Alexander Kasprzyk, Shiing-Tung Yau
conjecture formulation: e.g.

- '19 YHH-Kim: separating hyperplane - simple/non-simple groups; open
- '19 Brodie-Constantin-Lukas: exact formulae for cohomology surf.; proved.
- '20 YHH-Lee-Oliver: L-coefs and integer pt./torsion on ell; Known.
- '20 Craven-Jejjala-Par: Jones poly best-fit function; open
- '22 DeepMind-Oxford-Sydney, Nature: Volume bounds for knots; proved

speed-ups & accuracies: e.g.,

- computing/estimating (top.inv., charges, etc) MUCH FASTER
- '19 Ashmore-YHH-Ovrut: speed up Donaldson alg@CY metric 10-100
- '20 Douglas et al., Anderson et al. improves Donaldson 10-100 times
Launching in 2023

IJDSMS

Calling for Papers

Editor-in-Chief
Yang-Hui He
London Institute for Mathematical Sciences
& Merton College, University of Oxford
email: hey@maths.ox.ac.uk

More Information:
https://www.worldscientific.com/worldscinet/ijdsms
The London Institute for Mathematical Sciences

- UK’s only independent research institute for maths; modelled after IAS, Princeton
- Founded in 2011 by Dr. Thomas Fink
- Housed in the Faraday Suites of the Royal Institution of Great Britain
- 1 of 23 themes: AI for Maths Discovery
  
  https://lims.ac.uk/event/ai-assisted-maths-discovery/

- Just established:

  Arnold Fellowships
  Landau Fellowships
Obrigada!

CalabiYau the Game

《卡拉比丘》是一款宏大世界观的动作游戏。采用5V5的战斗模式，玩家需要选择自己的阵营，操控英雄探索地图，与队友密切协作，战胜敌人玩家即可获得比赛的胜利，更有上百位美少女英雄等待你的召唤！

游戏介绍 (from https://www.9k9k.com/shouyou/klbq/)

Back to Serious Geometry

YANG-HUI HE (London/Oxford/Nankai)  CY Landscape  Lisboa, Jan, 2023  33 / 35
Exact (MS)SM Particle Content from String Compactification

- [Braun-YHH-Ovrut-Pantev, Bouchard-Cvetic-Donagi 2005] first exact MSSM
- [Anderson-Gray-YHH-Lukas, 2007-] use alg./comp. algebraic geo & sift
- Anderson-Gray-Lukas-Ovrut-Palti $\sim 200$ in $10^{10}$ MSSM Stable Sum of Line Bundles over CICYs (Oxford-Penn-Virginia 2012-)

Constantin-YHH-Lukas '19: $10^{23}$ exact MSSMs (by extrapolation on above set)?

A Special Corner [New Scientist, 5/1/2008 feature]

Candelas-de la Ossa-YHH-Szendroi

“Triadophilia: A Special Corner of the Landscape” ATMP, 2008
Computing Geometrical Invariants

- Recall Hodge decomposition \( H^{p,q}(X) \cong H^q(X, \wedge^p T^* X) \sim \)

\[
H^{1,1}(X) = H^1(X, T_X^*) , \quad H^{2,1}(X) \cong H^{1,2} = H^2(X, T_X^*) \cong H^1(X, T_X)
\]

- Euler Sequence for subvariety \( X \subset A \) is short exact:

\[
0 \to T_X \to T_M|_X \to N_X \to 0
\]

- Induces long exact sequence in cohomology:

\[
\begin{array}{cccccccc}
0 & \to & H^0(X, T_X) & \to & H^0(X, T_A|_X) & \to & H^0(X, N_X) & \to \\
& \to & H^1(X, T_X) & \to & H^1(X, T_A|_X) & \to & H^1(X, N_X) & \to \\
& \to & H^2(X, T_X) & \to & \ldots \\
\end{array}
\]

- Need to compute \( R_k(d) \), cohomology and \( H^i(X, T_A|_X) \) (Cf. Hübsch)