UNIVERSIDADE DE LISBOA

Topological Pressure and Dimension

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Introduction

- The theory of dynamical systems has its origins in the work of Henri Poincaré on the three-body problem of celestial mechanics
- It comprises of a set of tools and methods to study ordinary differential equations and iterated mappings

Example - Quadratic map

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = ax(1-x)$, $a > 0$
What happens to a point $x \in \mathbb{R}$ if we iterate the function f?
What is the behaviour of $\{f^n(x)\}_{n \in \mathbb{N}}$?

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One method of studying this problems is by defining new quantities over the systems, this quantities can help us analyse and classify the different dynamics

- Metric Entropy
- Topological Entropy
- Topological Pressure

Loosely speaking, these quantities give us information on how fast points are moving away from each other

- Dimension

In some systems the interesting dynamics is comprised in a complicated subset, this quantity allow us to assign a non integer dimension to this sets

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The Main Example

For the remainder of the presentation we'll consider a particular example

- $\lambda_1, \lambda_2 > 0$ and $\lambda_1 + \lambda_2 < 1$
- $\Delta_1 = [0, \lambda_1], \Delta_2 = [1 - \lambda_2, 1]$
- $\xi_1 = \{\Delta_1, \Delta_2\}$

$$\xi_n = \left\{ \bigcap_{k=0}^{n-1} f^{-k}(\Delta_{i_k}) =: \Delta_{i_0 \dots i_{n-1}} \ : \ i_0, \dots, i_{n-1} = 1, 2 \right\}$$

The set of points that remain in the interval $[0, 1]$ forever is

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Let μ be a probability measure over $[0, 1]$

We define the entropy of each collection ξ_n

$$\xi_1 = \{\Delta_1, \Delta_2\}$$

$$H_\mu(\xi_1) = -\Big(\mu(\Delta_1)\log(\mu(\Delta_1)) + \mu(\Delta_2)\log(\mu(\Delta_2))\Big)$$

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- $\operatorname{card} \xi_n = 2^n$
- $h(f) = \lim_{n\to\infty} \frac{1}{n} \log(2^n) = \log(2)$

Properties of Entropy

- Entropy is an invariant quantity over the dynamics
- $h_\mu(f^k) = kh_\mu(f)$ and $h(f^k) = kh(f)$ for all $k \in \mathbb{N}$

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- $\operatorname{card} \xi_n = 2^n$
- $h(f) = \lim_{n\to\infty} \frac{1}{n} \log(2^n) = \log(2)$

Properties of Entropy

- Entropy is an invariant quantity over the dynamics
- $h_\mu(f^k) = kh_\mu(f)$ and $h(f^k) = kh(f)$ for all $k \in \mathbb{N}$

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Variational Principle for entropy

$$h(f) = \sup_{\mu}\{h_\mu(f)\}$$

$h(f) \geq \sup_\mu\{h_\mu(f)\}$

- It can be shown that $H_\mu(\xi_n) \leq \log(\text{card}\,\xi_n)$ for all $n \in \mathbb{N}$
- $\frac{1}{n}H_\mu(\xi_n) \leq \frac{1}{n}\log(\text{card}\,\xi_n)$
- Taking $n \to \infty$ we get $h_\mu(f) \leq h(f)$

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- Consider the measure μ that assigns a measure of $\frac{1}{2^n}$ to each element of the collection ξ_n
- $H_\mu(\xi_n) = -\sum_{i_1,\dots,i_n}\mu(\Delta_{i_1\dots i_n})\log(\mu(\Delta_{i_1\dots i_n})) = -2^n\frac{1}{2^n}\log(\frac{1}{2^n})$
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Topological Pressure

Let $\varphi : [0,1] \to \mathbb{R}$ be a continuous function

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$$P(\varphi) = \sup_{\mu}\{h_{\mu}(f) + \int \varphi d\mu\}$$

- $P(0) = \sup_{\mu}\{h_{\mu}(f)\} = h(f)$
- We consider the function $\varphi(x) = \log(\lambda_i)$ if $x \in \Delta_i$
- Let d be the only solution of $\lambda_1^d + \lambda_2^d = 1$
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Computing the Topological Pressure

With this observations we can compute $P(d\varphi) = h_\mu(f) + d\int \varphi d\mu$

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$$H_\mu(\xi_n) = -\sum_{i_1,\dots,i_n} (\lambda_{i_1}\dots\lambda_{i_n})^d \log((\lambda_{i_1}\dots\lambda_{i_n})^d)$$

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With this observations we can compute $P(d\varphi) = h_\mu(f) + d\int \varphi d\mu$

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Hausdorff Dimension

Let A be any subset of $\mathbb{R}$

α-dimensional Hausdorff measuse of A

$$m(A,\alpha) = \lim_{\epsilon \to 0} \inf_{\mathcal{U}} \sum_{U \in \mathcal{U}} (\text{diam } U)^\alpha$$

Where the infimum is taken over all covers $\mathcal{U}$ of A with $|\mathcal{U}| \leq \epsilon$

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$\dim_H A = \inf\{\alpha : m(A,\alpha) = 0\}$

- We will compute the Hausdorff dimension of the set K
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THANK YOU!