How to Give Chiral Fermions a Mass

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Question

What symmetries are broken when fermions get a mass?

Simplest Example

$$\mathcal{L}_{\text{mass}} = m\psi_L^\dagger \psi_R$$

Vector symmetry survives, axial symmetry broken

Another Example

In QCD, masses are dynamically generated. But so are condensates

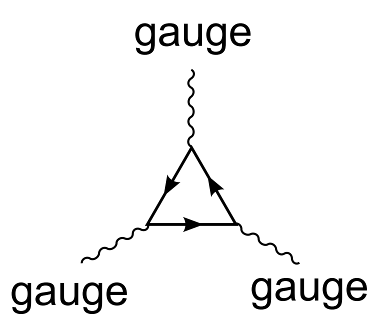
$$\langle \psi^\dagger_{L\,i} \psi_{R\,j} \rangle \sim \Lambda^3 \, \delta_{ij}$$

This breaks chiral symmetry

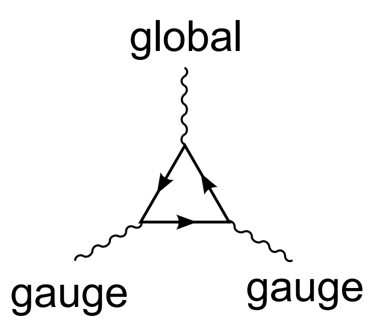
$$SU(N)_L \times SU(N)_R \longrightarrow SU(N)_{\rm diag}$$

't Hooft Anomaly

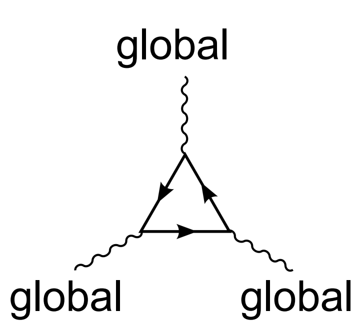
The real obstacle is the *’t Hooft* anomaly.



Gauge anomaly = inconsistency



Classical symmetry doesn’t survive quantisation



‘t Hooft anomaly = obstruction to gauging

The ‘t Hooft anomaly characterises the symmetry and does not change under deformations or RG.

't Hooft Anomaly

A global symmetry *G* has a ‘t Hooft anomaly. If the anomaly is non-vanishing then either

- The symmetry *G* is spontaneously broken
- There exist massless fermions to saturate the anomaly

An Example: QCD

SU(3) with *N* massless quarks

Global symmetry: $G = SU(N)_L \times SU(N)_R \times U(1)_B$

't Hooft anomalies: $\mathcal{A}[SU(N)_L^3] = \mathcal{A}[SU(N)_L^2 \cdot U(1)_B] = 3$

Suppose that the theory confines into *weakly interacting* particles.

- If *G* is unbroken, there must be massless baryons to saturate the ‘t Hooft anomaly
- It turns out that no such massless baryons quantitatively work
- Therefore *G* is broken

In QCD, confinement $\Longrightarrow$ chiral symmetry breaking

What if the ‘t Hooft Anomaly Vanishes?

Can we give chiral fermions a mass without breaking *G*?

For example:

$$G = SU(3) \times SU(2) \times U(1)$$

Can we give a mass to the 15 fermions carrying quantum numbers of quarks and leptons in the Standard Model without breaking *G?*

How to Gap Chiral Fermions

The Rules of the Game

- Start from free massless fermions realising a non-anomalous chiral symmetry *G*

Add extra degrees of freedom and flow to the IR. The goal is to gap everything while preserving *G.*

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- Fermions.
  - If these are charged under *G,* they must come in left/right conjugate pairs.
- Gauge Fields.
  - These gauge a different symmetry *H* providing
    - $[H,G] = 0$
    - There are no mixed anomalies with G.
    - There are scalars that allow a phase in which *H* is Higgsed.

The Basic Idea

Find *H* such that:

Gauge dynamics of *H* with global symmetry *G*

$$\Big\downarrow \text{RG}$$

Confinement *without* chiral symmetry breaking

Example 1

G = *SU(N)* with $\square\!\square$ and *N+4* $\overline{\square}$

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Gauge *H* = *SU(N+4).* Must also add:

- Additional fermion in $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ of *H.*
- Scalars that can Higgs *H.*

- Scalars condense $\Longrightarrow$ auxiliary fields heavy and decouple
- Scalars heavy $\Longrightarrow$ have to understand dynamics of strongly coupled *H* gauge theory

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Under *G* x *H,* we have $\overset{\lambda}{\downarrow}$ $\overset{\psi}{\downarrow}$ $\overset{\tilde{\chi}}{\downarrow}$

$$(\square\square,\mathbf{1}) + \underbrace{(\overline{\square},\square) + \left(\mathbf{1},\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right)}_{\text{These two charged under } H\, =\, SU(N+4)}$$

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Gauge $H = SU(N+4)$. Must also add:

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H is expected to confine *without* breaking chiral symmetry G.
The low-energy spectrum us believed to be a massless composite fermion

$$\tilde{\lambda} = \psi\tilde{\chi}\psi \quad \text{in } (\overline{\square\square}, \mathbf{1})$$

Georgi ’79; Dimopoulos, Raby and Susskind ’80; Eichten, Peccei, Preskill and Zeppenfeld, ‘85

Example 1

$G = SU(N)$ with $\square\!\square$ and $\underbrace{N{+}4\ \overline{\square}}$

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$$\tilde{\lambda} = \psi\tilde{\chi}\psi \quad \text{in } (\overline{\square\!\square}, \mathbf{1})$$

Add in the UV

$$\mathcal{L}_{UV} \sim \lambda\psi\tilde{\chi}\psi \xrightarrow{\text{RG}} \mathcal{L}_{IR} \sim \lambda\tilde{\lambda}$$

This gaps the fermions, preserving *G.*

Example 2: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

(left-handed)^c		right-handed		
<u>leptons</u>	<u>quarks</u>	<u>electron</u>	<u>up quark</u>	<u>down quark</u>
$(\mathbf{1}, \mathbf{2})_{-3}$	$(\bar{\mathbf{3}}, \mathbf{2})_{+1}$	$(\mathbf{1}, \mathbf{1})_{+6}$	$(\mathbf{3}, \mathbf{1})_{-4}$	$(\mathbf{3}, \mathbf{1})_{+2}$

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- Add three further pairs of fermions

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- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry

Example 2: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

(left-handed)^c — leptons, quarks

right-handed — electron, up quark, down quark

leptons	quarks	electron	up quark	down quark	neutrino
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- Add three further pairs of fermions
- Gauge the *H* = SU(2) symmetry
- Supersymmetrize.
  - Add scalar superpartners for all fermions, and a *H* = SU(2) gaugino

Example 2: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$L' \longrightarrow (\mathbf{1},\mathbf{2})_{+3}$				$D' \longrightarrow (\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- The H = SU(2) gauge theory is coupled to six doublets.
- This confines *without* breaking the global symmetry.

Seiberg '94

- The low-energy physics consists of 15 free mesons:

$$\epsilon_{ab}L^aL^b \qquad \epsilon_{ijk}D^iD^j \qquad L^aD^i \qquad L^aN \qquad D^iN$$

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If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q^b_i + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

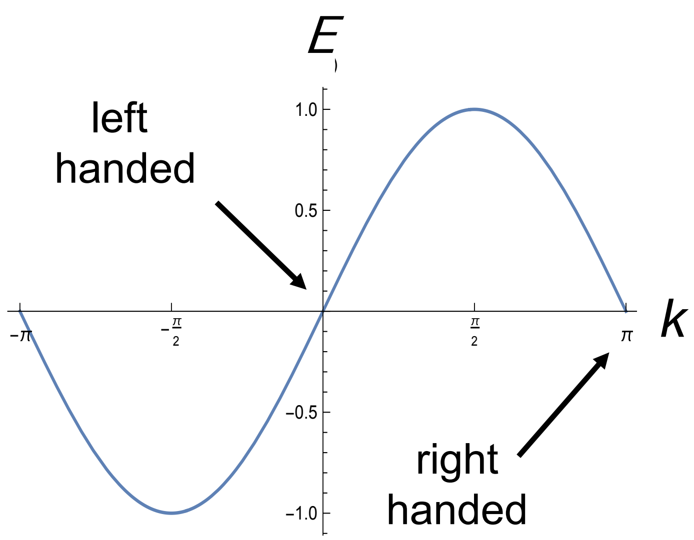
$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}^i_b Q^b_i + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$

A Potential Application:
Chiral Gauge Theories on the Lattice

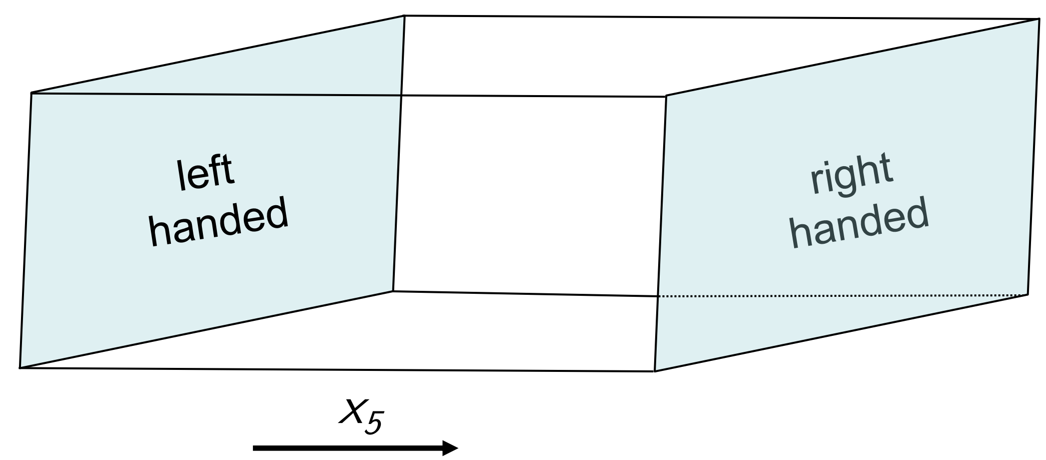
Lattice Fermions

Naïve attempts to put chiral fermions on the lattice result in doublers

Either separated in momentum space…



…or in an extra spatial dimension



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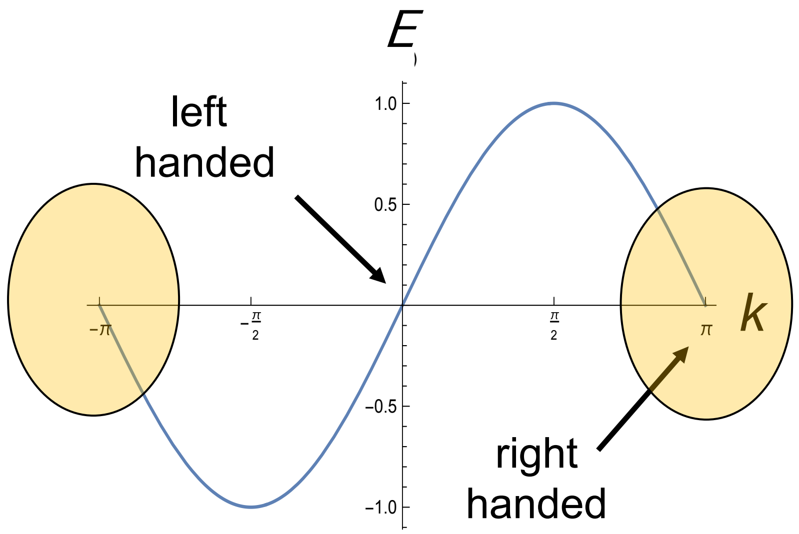

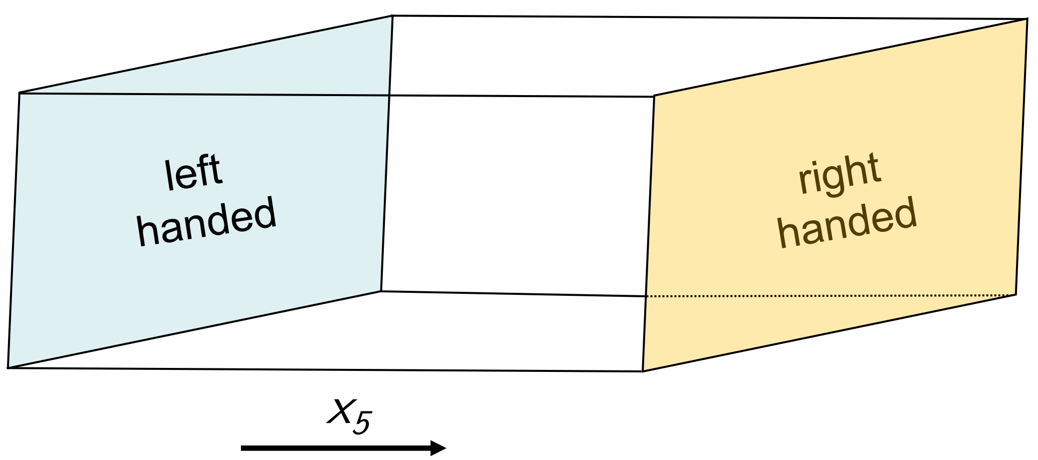

An Old Idea: Find a way to gap the doublers, leaving the original fermions untouched

Challenges:

- Ensure that only the mirror fermions experience the interactions
- Find interactions that gap chiral fermions

Eichten and Preskill '86

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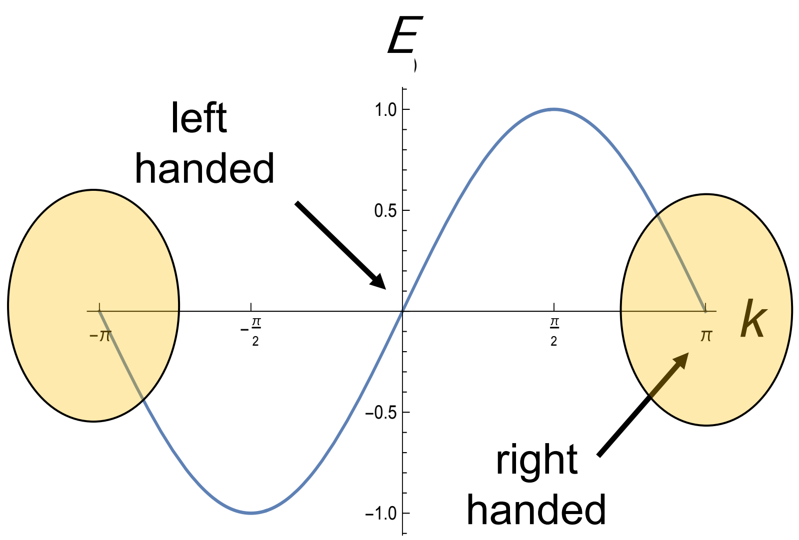

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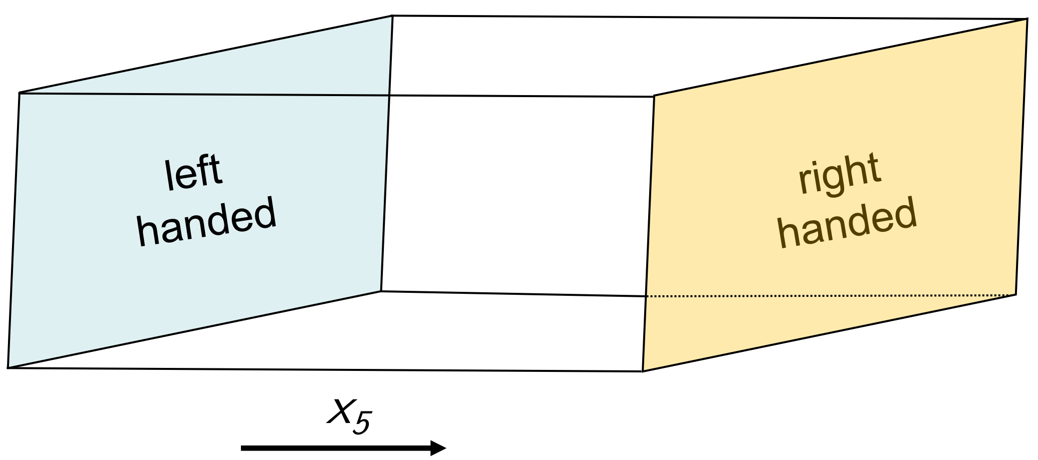


Most attempts work with irrelevant multi-fermion operators, cranked up to the lattice scale

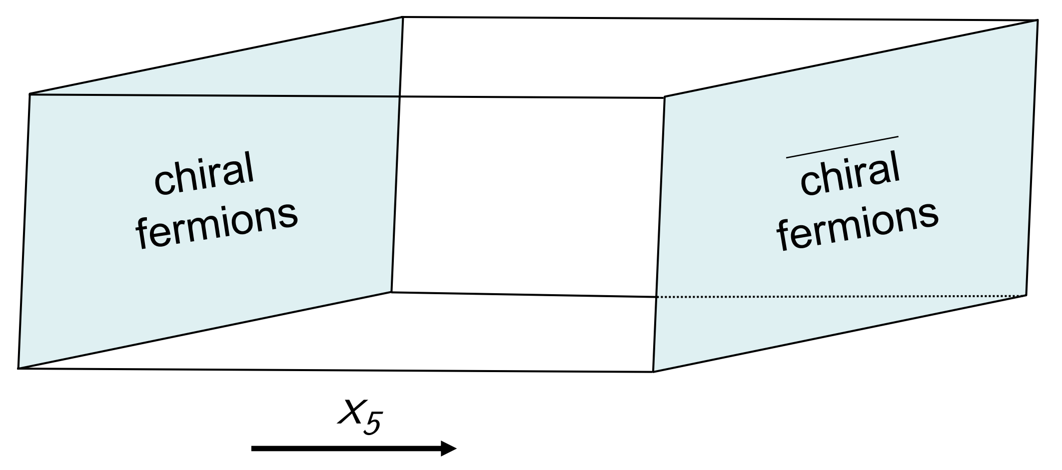
$$\mathcal{L}_{4-\mathrm{fermi}} \sim \psi\psi\psi\psi$$

Sadly, so far, to no avail.

Eichten and Preskill '86, Golterman, Petcher and Rivas '93; Creutz, Rebbi, Tytgat, Xue '96;
Poppitz and Shang '10; Chen, Giedt and Poppitz '12; Wen '13; Wang and Wen '13; Kikukawa '17'; Wang and Wen '18

Can we use continuum gapping mechanisms to help us?

Gapping Domain Wall Fermions



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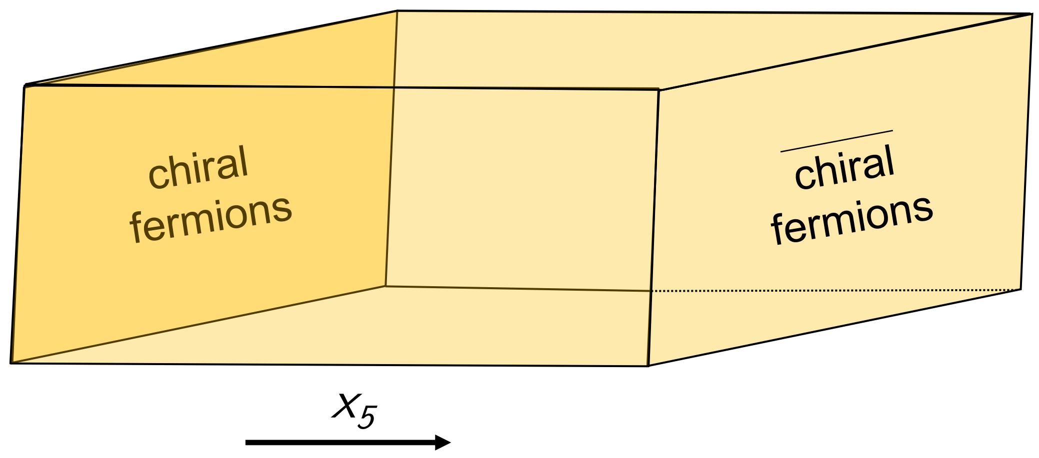
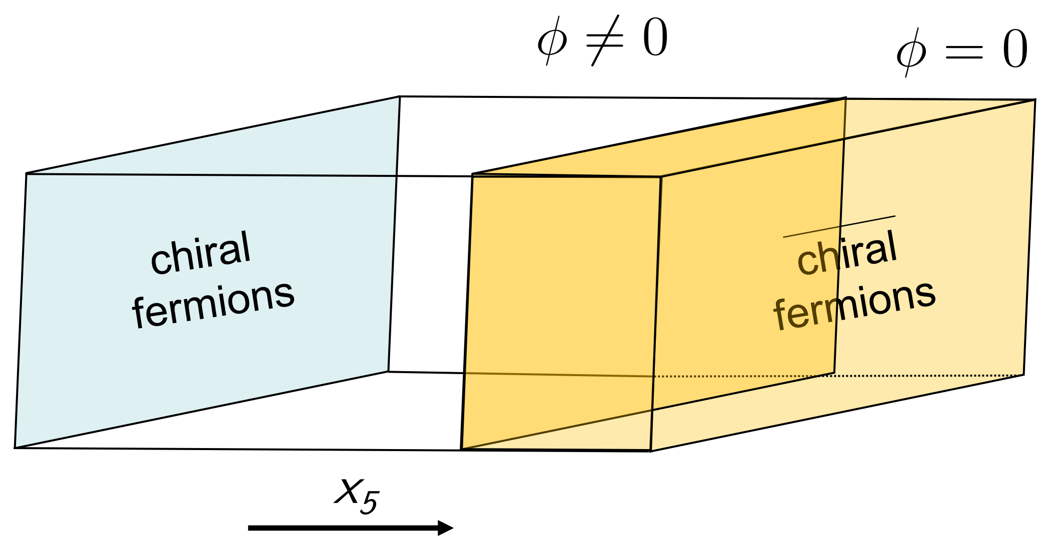


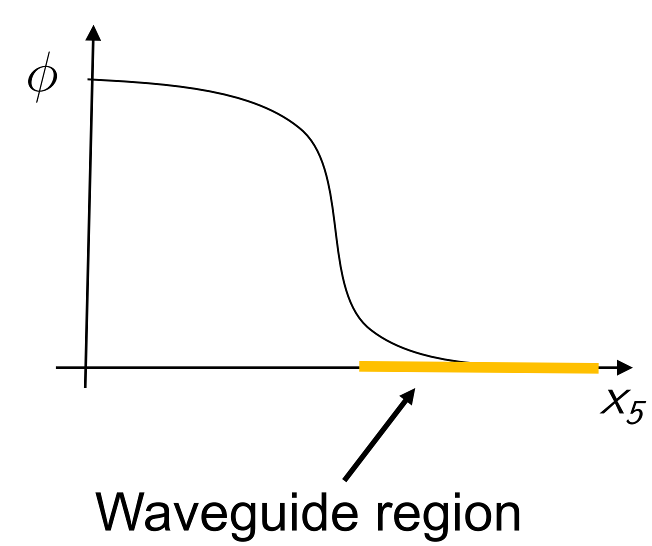
We want to introduce an auxiliary gauge field like H=SU(2) that couples to just one edge

Gapping Domain Wall Fermions



We want to introduce an auxiliary gauge field like H=SU(2) that couples to just one edge

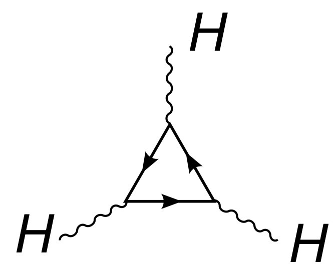
Add Higgs fields for *H* with a profile in the fifth dimension.



A Pitfall

What if chiral fermions are in anomalous representation of *H*?

$$\sum_{\text{5d Fermions}}$$



$$= \frac{k}{24\pi^2}\,\mathrm{tr} A \wedge F \wedge F + \dots$$

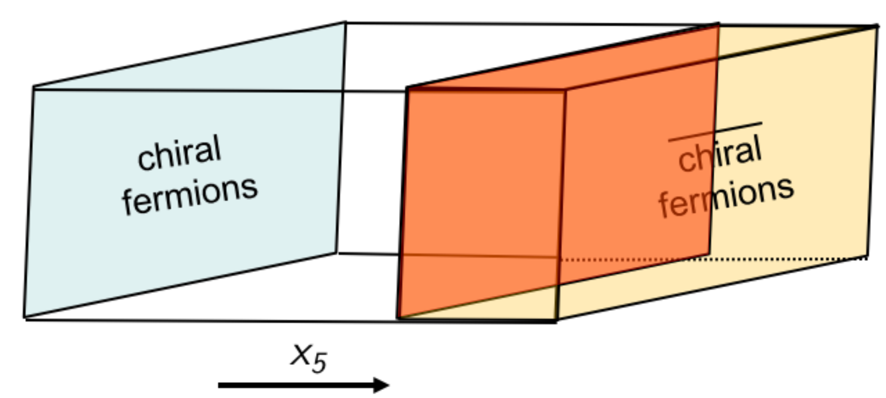
The phase of the Higgs field then fails to decouple on the interface.

We get a Wess-Zumino term living at the interface.

$$\mathcal{L}_{\text{eff}} \sim k\,\mathrm{tr}\;\Omega F \wedge F$$

$$\sim k\bar{\psi}\gamma^5\partial_5\Omega\psi \qquad \Omega \in H$$

by anomaly

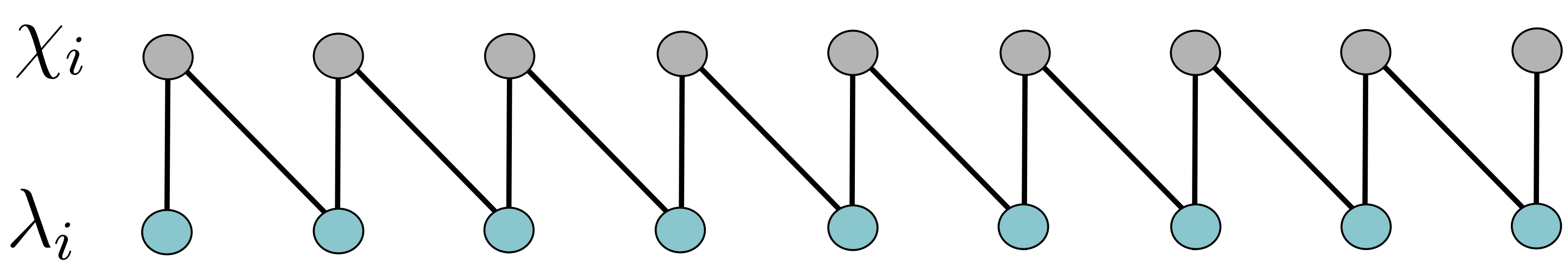


k=0 only for *non-anomalous* theories.

Lattice Domain Wall Fermions

A 5d Dirac fermion $\Psi = \begin{pmatrix} \chi \\ \lambda \end{pmatrix}$. We discretize it in the 5th dimension with Wilson parameter *r = 1*

$$S = \int d^4x \sum_{i=1}^{N} a \left\{ \overbrace{i\chi_i^\dagger \bar{\sigma}^\mu \partial_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \partial_\mu \lambda_i}^{\text{4d kinetic terms}} + \frac{1}{a}\left[\overbrace{\chi_i^\dagger(\lambda_i - \lambda_{i-1}) + \lambda_i^\dagger(\chi_i - \chi_{i+1})}^{\text{5d hopping terms}} - \overbrace{ma(\chi_i^\dagger \lambda_i + \lambda_i^\dagger \chi_i)}^{\text{masses}} \right] \right\}$$



$ma > 0$

Left-handed zero mode localized here

Right-handed zero mode localized here

Kaplan ’92; Shamir ‘93

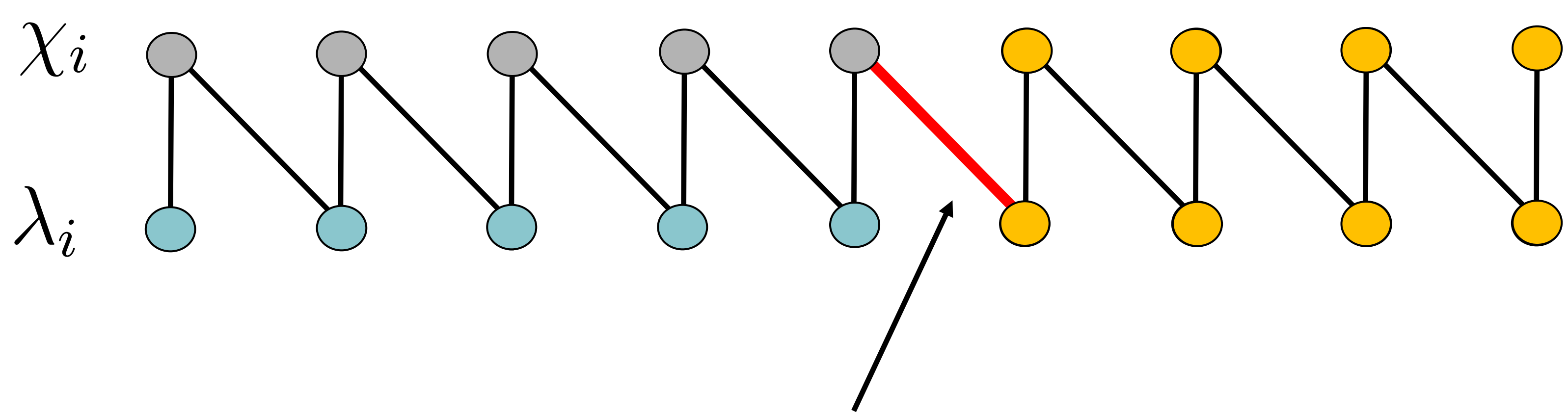
Lattice Domain Wall Fermions

Add a 5d gauge field in waveguide region. At low-energies, only 4d gauge field survives

$$S = \int d^4x \; a \sum_{i \notin \mathrm{WG}} \left[i\chi_i^\dagger \bar{\sigma}^\mu \partial_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \partial_\mu \lambda_i \right] + a \sum_{i \in \mathrm{WG}} \overbrace{\left[i\chi_i^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \mathcal{D}_\mu \lambda_i \right]}^{\text{gauged}}$$

$$+ \sum_i{}' \frac{1}{a} \left[\chi_i^\dagger (\lambda_i - \lambda_{i-1}) + \lambda_i^\dagger (\chi_i - \chi_{i+1}) - ma(\chi_i^\dagger \lambda_i + \lambda_i^\dagger \chi_i) \right]$$

$$+ \, y \left(\frac{1}{a} \chi_\star^\dagger \Omega \lambda_{\star-1} - \chi_{\star-1}^\dagger \Omega^\dagger \lambda_\star \right)$$

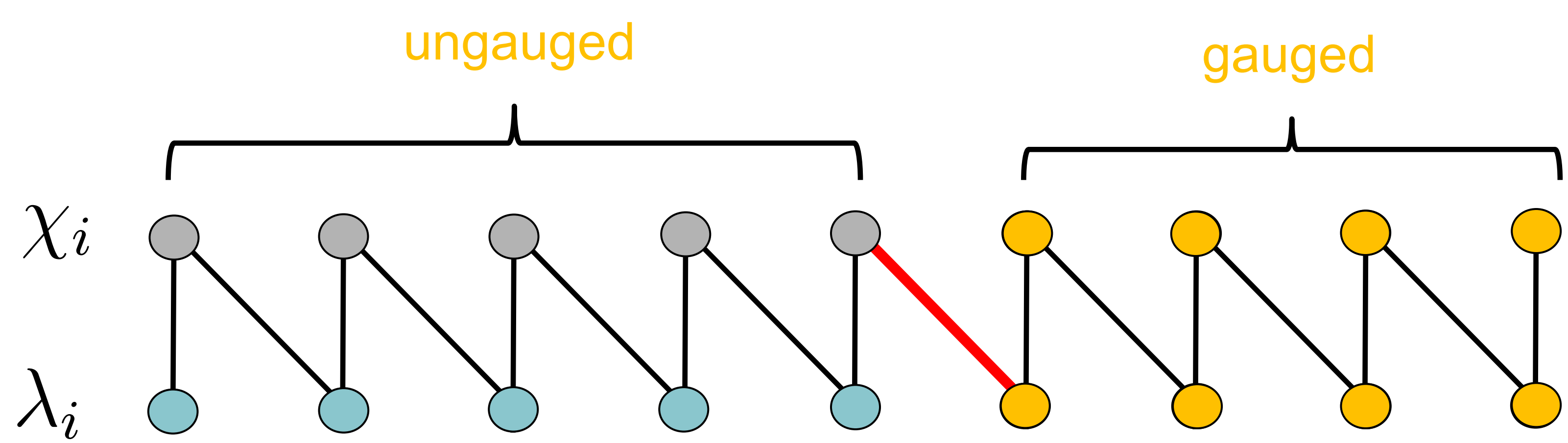


New dynamical field needed here: $\Omega \in H$. This is the Wess-Zumino term on the interface

Golterman, Jansen, Petcher, Vink ’93; Golterman and Shamir ‘94

Lattice Domain Wall Fermions

The same pitfall is seen in the lattice



Left-handed zero mode localized here

Right-handed zero mode localized here

Result: Neither Ω nor the two neighbouring fermions are gapped in <u>*quenched*</u> simulations.

But this is the expected behavior! These fields should only be gapped by fermion loops.

$\Rightarrow$ Sign problem!

Thank you for your attention