Exploring 4D Topological Physics in the Laboratory

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THE ROYAL SOCIETY

UNIVERSITY OF BIRMINGHAM

With thanks to:

Birmingham Theory

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Birmingham Experiment

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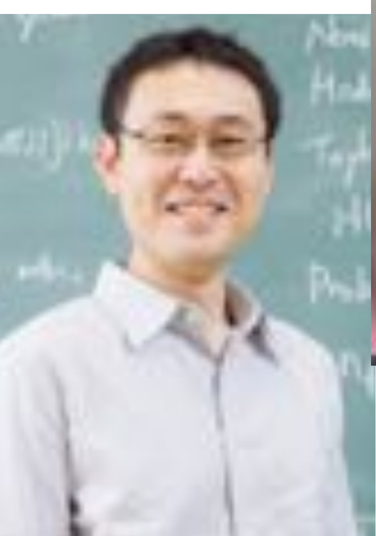

Tomoki Ozawa (Tohuku, Japan)

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Zurich:
Martin Lebrat, Samuel Hausler, Laura Corman, Tilman Esslinger

EPFL:
Jean-Philippe Brantut

Munich:
Michael Lohse, Christian Schweizer, Immanuel Bloch

Jena:
Martin Wimmer, Monika Monika, Ulf Peschel

Singapore:
Wang You, Baile Zhang, Yidong Chong

Outline

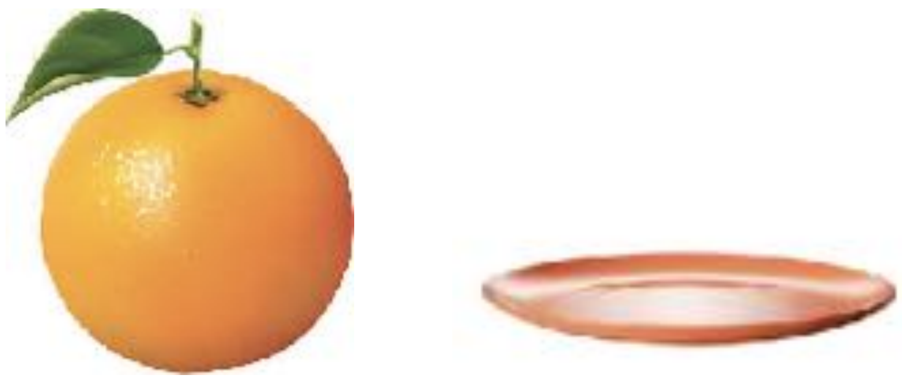
1. **Brief Introduction to 2D Quantum Hall Physics**

2. Introduction to 4D Quantum Hall Physics

3. How can we explore 4D Quantum Hall with quantum simulation?
    - *(Topological Pumping)*
    - *Connectivity*
    - *Synthetic Dimensions*

Topological Invariants

e.g. topology of surfaces

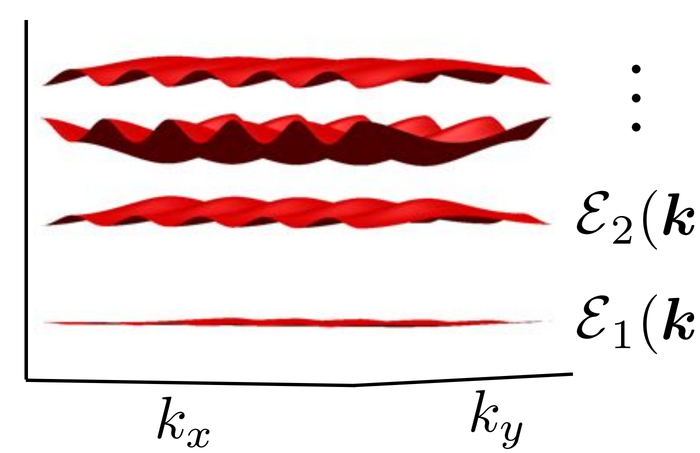


No holes: genus=0

1 hole: genus=1

- Global property
- Integer-valued
- Robust under smooth deformations

Topological band theory



Each single-particle band labelled by topological invariants

Topology from geometry

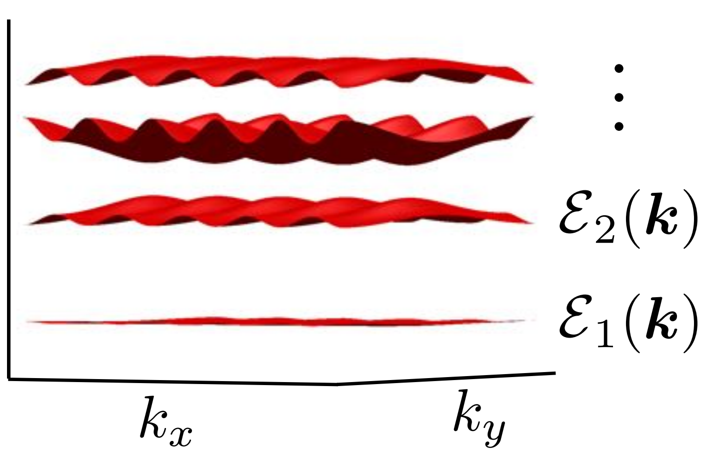
Gauss-Bonnet theorem for closed surfaces:

$$\int_{\mathcal{S}_{\rm tot}} \kappa \mathrm{d}S = 4\pi(1-g)$$

$g = 0$

$g = 1$

For energy bands:



$$\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n,k}(\mathbf{r})$$

$$\hat{H}_{\mathbf{k}} u_{n,\mathbf{k}} = \mathcal{E}_n(\mathbf{k}) u_{n,\mathbf{k}}$$

Geometrical properties: Berry curvature

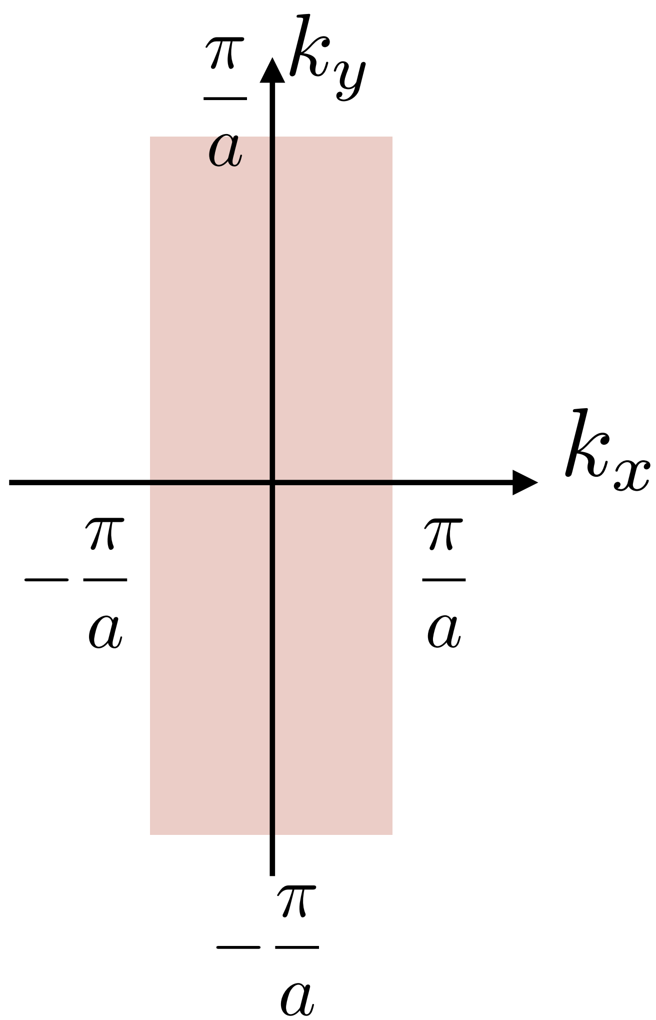
$$\Omega = \frac{1}{2}\Omega^{\mu\nu}(\mathbf{k})\mathrm{d}\mathbf{k}_\mu \wedge \mathrm{d}\mathbf{k}_\nu$$

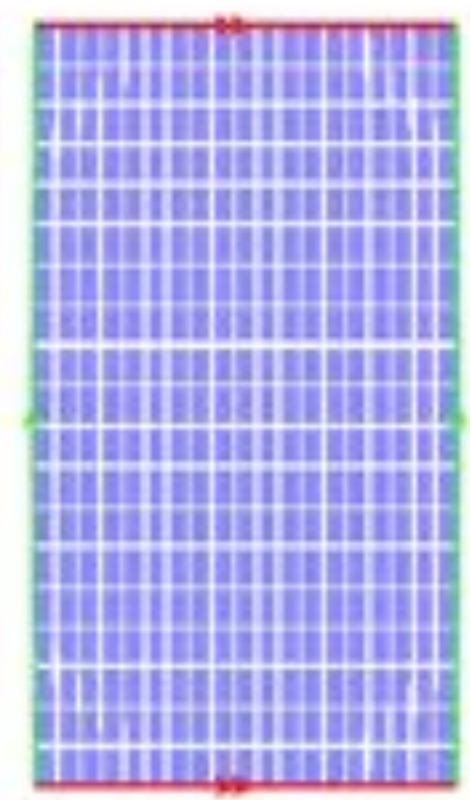
$$\Omega_n^{\mu\nu} = i\left[\langle\frac{\partial u_n}{\partial k_\mu}|\frac{\partial u_n}{\partial k_\nu}\rangle - \langle\frac{\partial u_n}{\partial k_\nu}|\frac{\partial u_n}{\partial k_\mu}\rangle\right]$$

Figure Credit: Nathan Goldman

Topology from geometry

An energy band in the Brillouin Zone is a closed surface





https://tex.stackexchange.com/questions/469051/topology-diagrams-labelled-edges

Topology from geometry

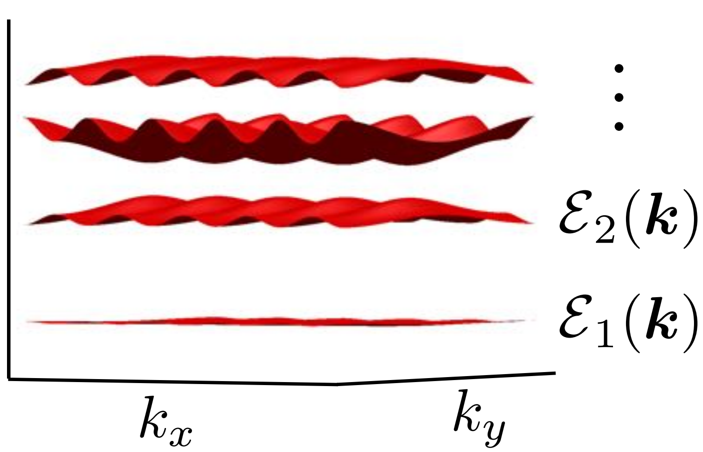
Gauss-Bonnet theorem for closed surfaces:

$$\int_{\mathcal{S}_{\rm tot}} \kappa \mathrm{d}S = 4\pi(1-g)$$

$g = 0$

$g = 1$

Analogously for energy bands:



$$\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n,k}(\mathbf{r})$$

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$$\Omega_n^{\mu\nu} = i\left[\langle\frac{\partial u_n}{\partial k_\mu}|\frac{\partial u_n}{\partial k_\nu}\rangle - \langle\frac{\partial u_n}{\partial k_\nu}|\frac{\partial u_n}{\partial k_\mu}\rangle\right]$$

Topological properties: First Chern number

$$\nu_1 = \frac{1}{2\pi}\int_{\mathbb{T}^2} \Omega$$

Figure Credit: Nathan Goldman

Analogy with Magnetic Fields

Berry connection $\mathcal{A}_n(\mathbf{k}) = i\langle u_{n,\mathbf{k}}\|\frac{\partial}{\partial \mathbf{k}}\|u_{n,\mathbf{k}}\rangle$	Magnetic vector potential $\mathbf{A}(\mathbf{r})$
Berry curvature $\Omega_n(\mathbf{k}) = \nabla \times \mathcal{A}_n(\mathbf{k})$	Magnetic field $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$
Berry phase $\gamma_n = \oint_{\mathcal{C}} d\mathbf{k} \cdot \mathcal{A}_n(\mathbf{k}) = \int_{\mathcal{S}} d\mathbf{S} \cdot \Omega_n(\mathbf{k})$	Magnetic Flux $\Phi = \int_{\mathcal{S}} \mathrm{d}\mathbf{S} \cdot \mathbf{B}(\mathbf{r})$
Chern number $\nu_1^n = \frac{1}{2\pi} \int_{BZ} \mathrm{d}^2\mathbf{k} \cdot \Omega_n(\mathbf{k})$	No/ magnetic monopoles $N = \frac{1}{\Phi_0} \int_{\mathcal{S}_{\mathrm{tot}}} d\mathbf{S} \cdot \mathbf{B}(\mathbf{r})$

Connection to Homotopy

Minimal two-band model, e.g. spinless atoms on lattice with two-site unit cell:

$$H(\mathbf{k}) = \varepsilon(\mathbf{k})\hat{I} + \mathbf{d}(\mathbf{k}) \cdot \sigma$$

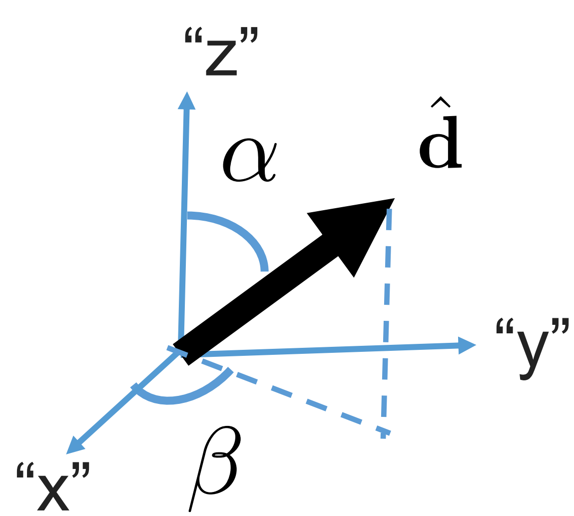
$$E_{\pm} = \varepsilon(\mathbf{k}) \pm \sqrt{\mathbf{d}(\mathbf{k}) \cdot \mathbf{d}(\mathbf{k})}$$

Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Normalized 3D “pseudo-spin” vector

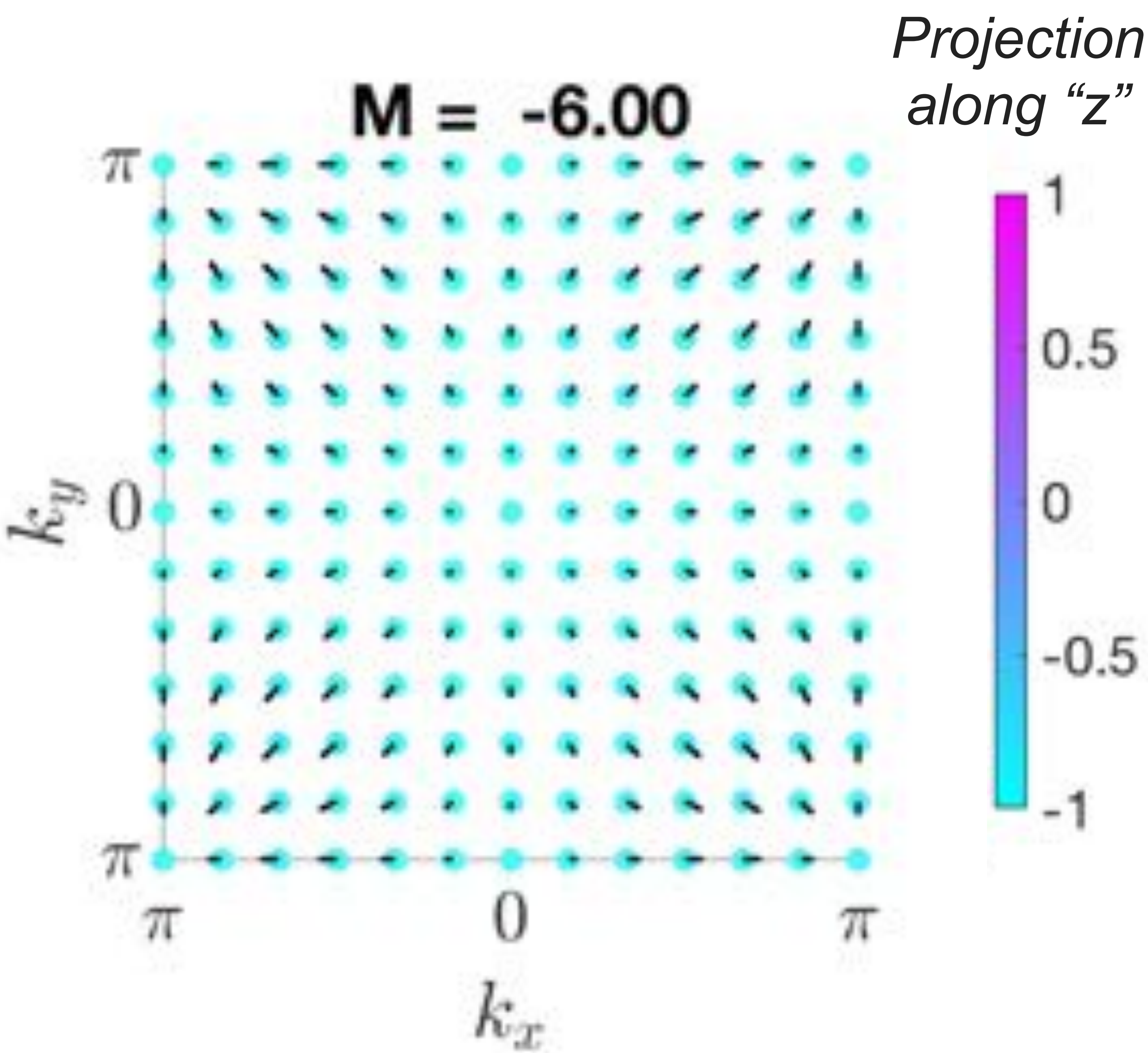
$$\hat{\mathbf{d}}(\mathbf{k}) = \frac{1}{\sqrt{\sum_i d_i^2}} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$



which is a 3D vector field over the Brillouin zone

Example: 2-Band Lattice Chern Insulator Model

$$H = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + (2 + M - \cos(k_x) - \cos(k_y))\sigma_z$$

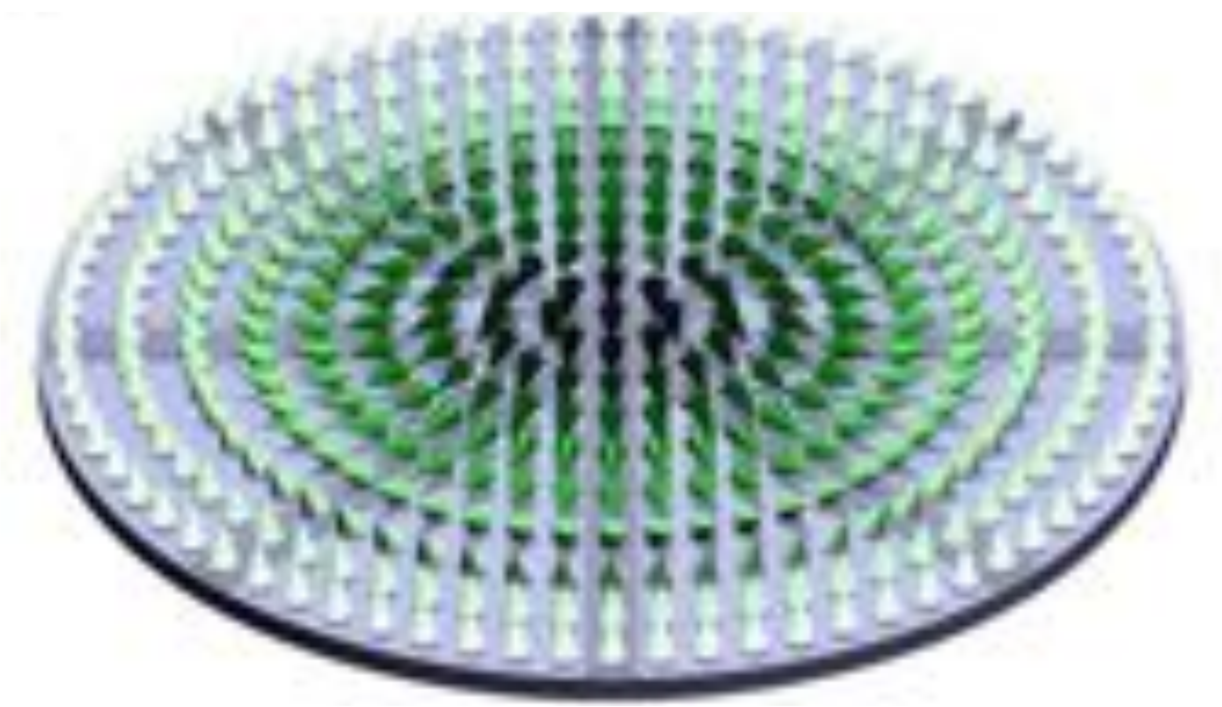


$$\hat{\mathbf{d}}(\mathbf{k}) = \frac{1}{\sqrt{\sum_i d_i^2}} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

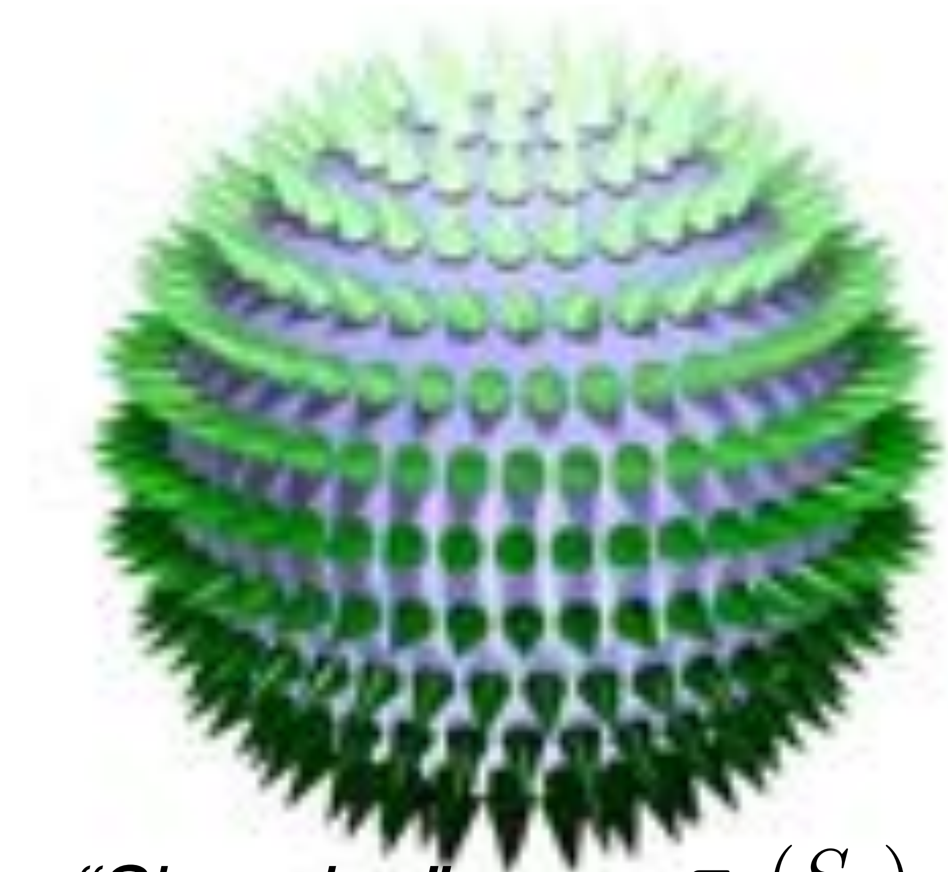
e.g. for more about this model, see for example "Topological Insulators and Superconductors" by Bernevig and Hughes

Skyrmions

Vector field over a 2D plane



Pseudo-spin space



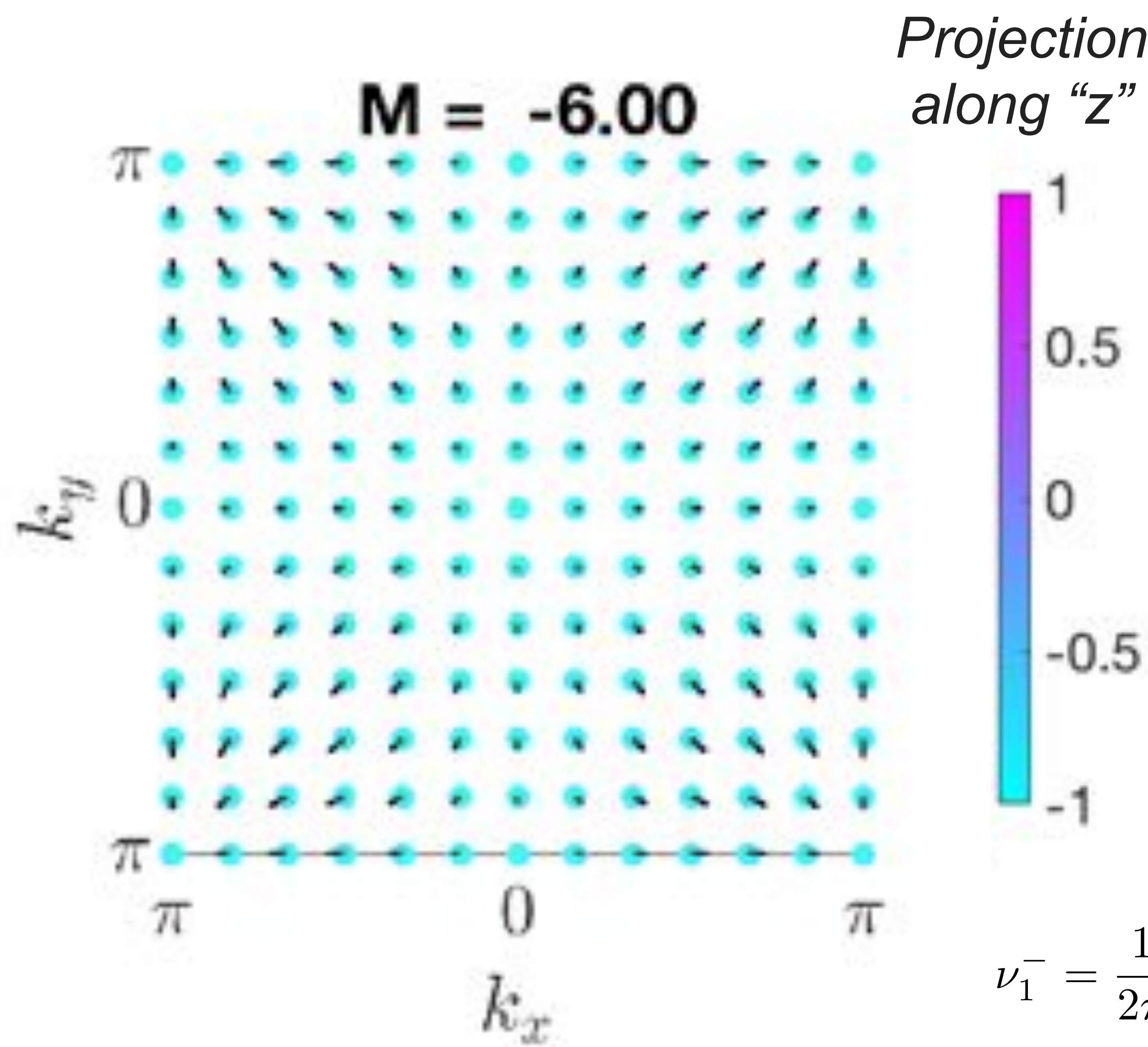
"Skyrmion" $\pi_2(S_2) = \mathbb{Z}$

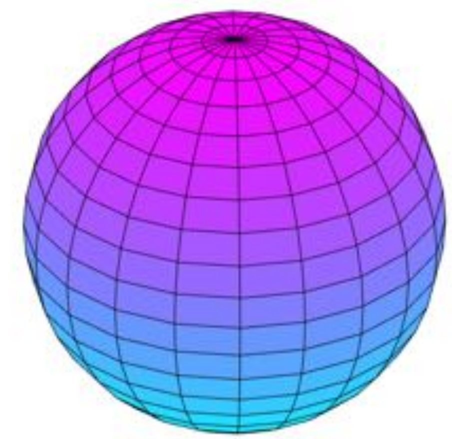
How many times does the vector field (associated with the Hamiltonian) wrap over the psuedo-spin sphere?

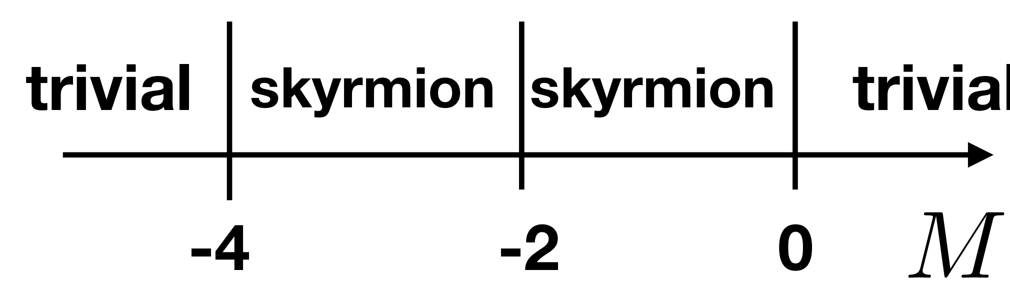
$$\nu_1^- = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathrm{d}^2\mathbf{k}\cdot\Omega_- = \frac{1}{4\pi}\int_{\mathrm{BZ}} \mathrm{d}^2\mathbf{k}\epsilon^{abc}\hat{d}_a\partial_{k_x}\hat{d}_b\partial_{k_y}\hat{d}_c$$

Example: 2-Band Lattice Chern Insulator Model

$$H = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + (2 + M - \cos(k_x) - \cos(k_y))\sigma_z$$







$$\nu_1^- = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathrm{d}^2\mathbf{k}\cdot\Omega_- = \frac{1}{4\pi}\int_{\mathrm{BZ}} \mathrm{d}^2\mathbf{k}\,\epsilon^{abc}\hat{d}_a\partial_{k_x}\hat{d}_b\partial_{k_y}\hat{d}_c$$

e.g. for more about this model, see for example "Topological Insulators and Superconductors" by Bernevig and Hughes

Summary: First Chern Number

- A 2D Topological Invariant (of a vector bundle)
- e.g. integral of Berry curvature over 2D BZ
- Counts Number of “Magnetic” Monopoles Enclosed
- For 2-band models, gives “skyrmion” (winding) number

Example Models

- 2-Band Lattice Chern Insulator

$$H = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + (2 + M - \cos(k_x) - \cos(k_y))\sigma_z$$

- Landau levels
- Harper-Hofstadter Model

$$\mathcal{H} = J \sum_{m,n} (\hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} + e^{i2\pi\Phi m}\hat{a}^\dagger_{m,n+1}\hat{a}_{m,n}) + \text{h.c.}$$

- Haldane model (tight-binding honeycomb lattice with *TRS-breaking*)…..

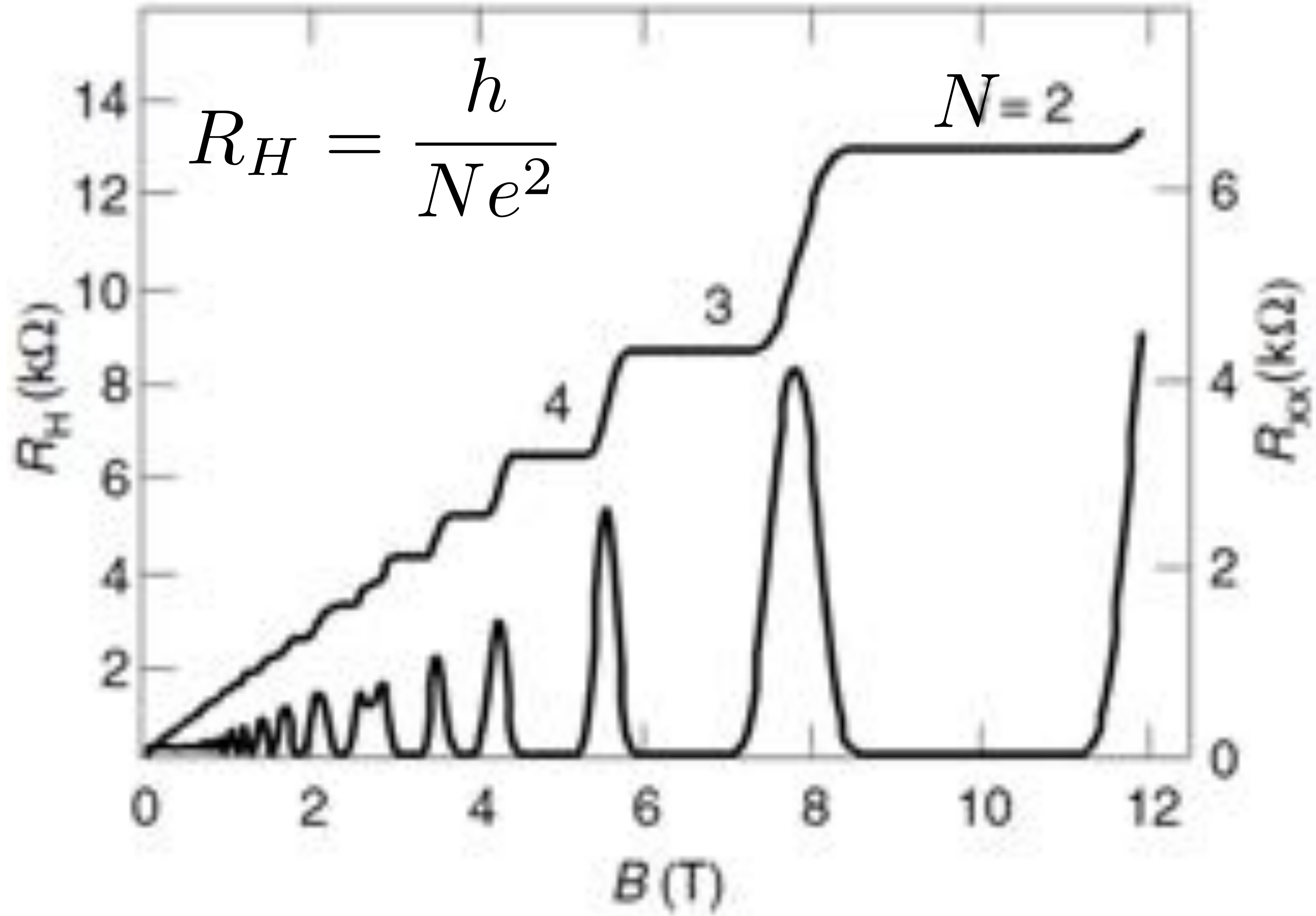
Klaus von Klitzing

Physical Consequences:

2D Quantum Hall Effect

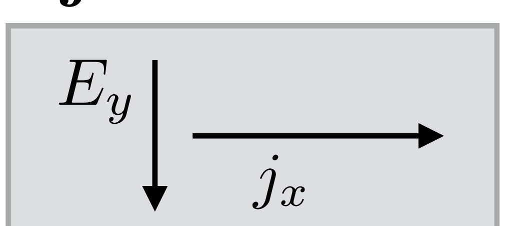
$$V_H = R_H I$$

2D electron gas in a perpendicular magnetic field



N.B. Alternatively: $\mathbf{j} = \sigma \mathbf{E}$

$$j_x = \frac{Ne^2}{h} E_y$$

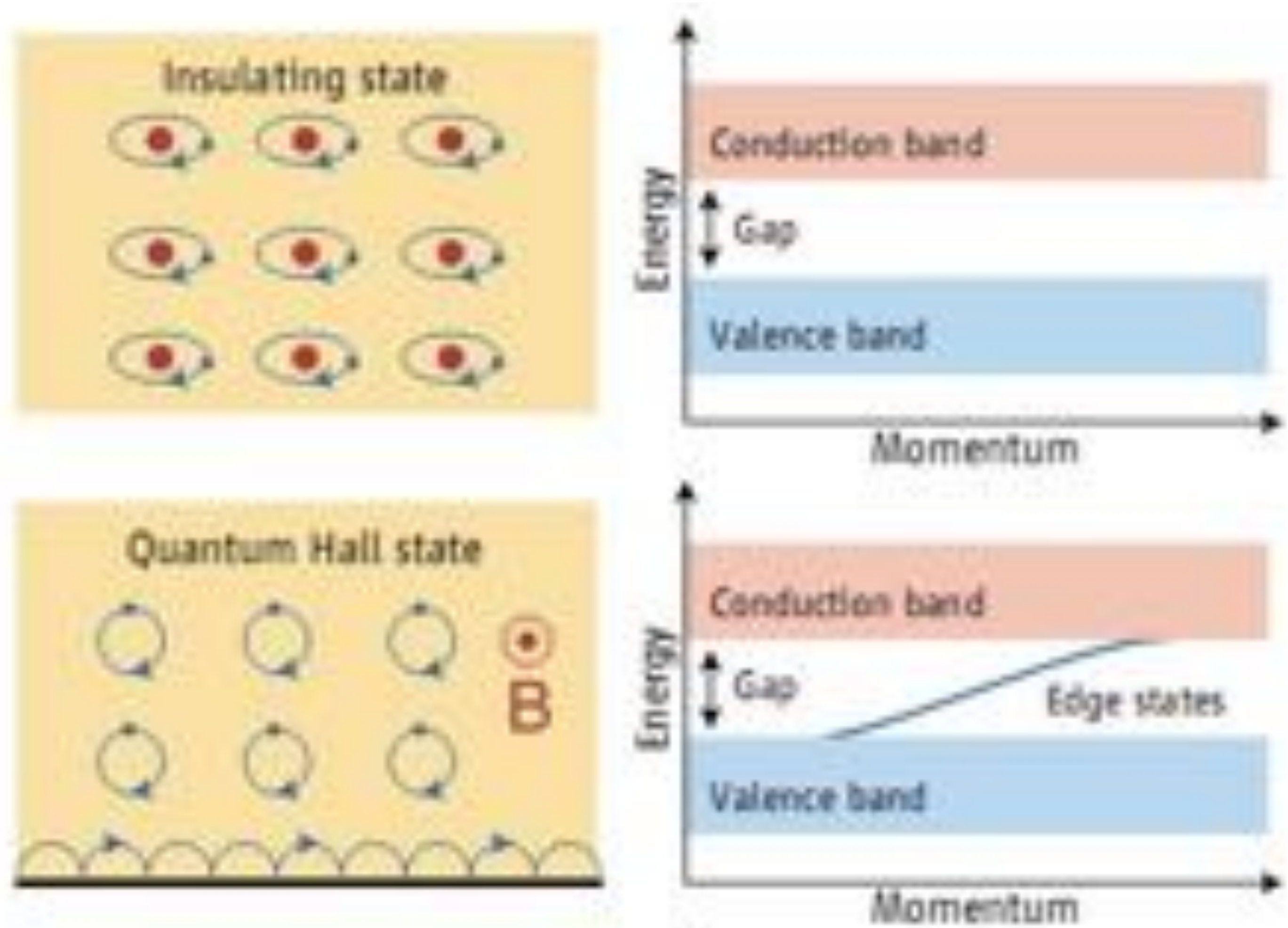


$$N = \sum_{n \in \text{occ.}} \nu_1^n$$ topological first Chern numbers

Physical Consequences:

One-Way Topological Edge States

Figure from
C. L. Kane & E. J. Mele, Science 314, 5806, 1692 (2006)

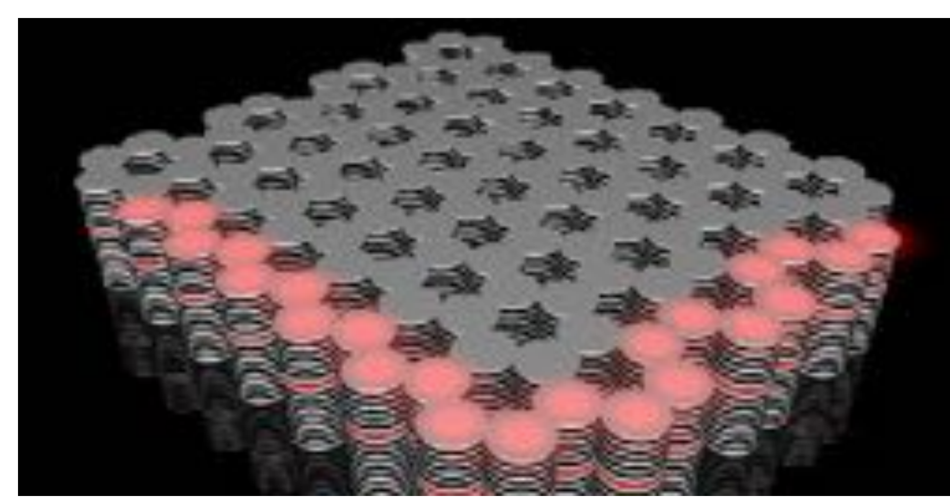


Bands are topologically -trivial

Bands have non-zero Chern numbers

Bulk-boundary correspondence

Polaritons: Klembt et al. Nature 562, 552(2018)



Engineering Chern bands in cold atoms/photonics:

- Cold atoms review: Cooper et al., Rev. Mod. Phys. 91, 015005 (2019)
- Photonics review: T. Ozawa, et al., Rev. Mod. Phys. 91, 015006 (2019)

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3. How can we explore 4D Quantum Hall with quantum simulation?
    - *(Topological Pumping)*
    - *Connectivity*
    - *Synthetic Dimensions*

Analogy with Magnetic Fields

Berry connection	Magnetic vector potential
$\mathcal{A}_n(\mathbf{k}) = i\langle u_{n,\mathbf{k}}\vert\frac{\partial}{\partial\mathbf{k}}\vert u_{n,\mathbf{k}}\rangle$	$\mathbf{A}(\mathbf{r})$
Berry curvature	**Magnetic field**
$\Omega_n(\mathbf{k}) = \nabla \times \mathcal{A}_n(\mathbf{k})$	$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$
Berry phase	**Magnetic Flux**
$\gamma_n = \oint_{\mathcal{C}} d\mathbf{k}\cdot\mathcal{A}_n(\mathbf{k}) = \int_{\mathcal{S}} d\mathbf{S}\cdot\Omega_n(\mathbf{k})$	$\Phi = \int_{\mathcal{S}} \mathrm{d}\mathbf{S}\cdot\mathbf{B}(\mathbf{r})$
Chern number	**No/ magnetic monopoles**
$\nu_1^n = \frac{1}{2\pi}\int_{BZ} \mathrm{d}^2\mathbf{k}\cdot\Omega_n(\mathbf{k})$	$N = \frac{1}{\Phi_0}\int_{\mathcal{S}_{\mathrm{tot}}} d\mathbf{S}\cdot\mathbf{B}(\mathbf{r})$

Classical Particle in a magnetic field

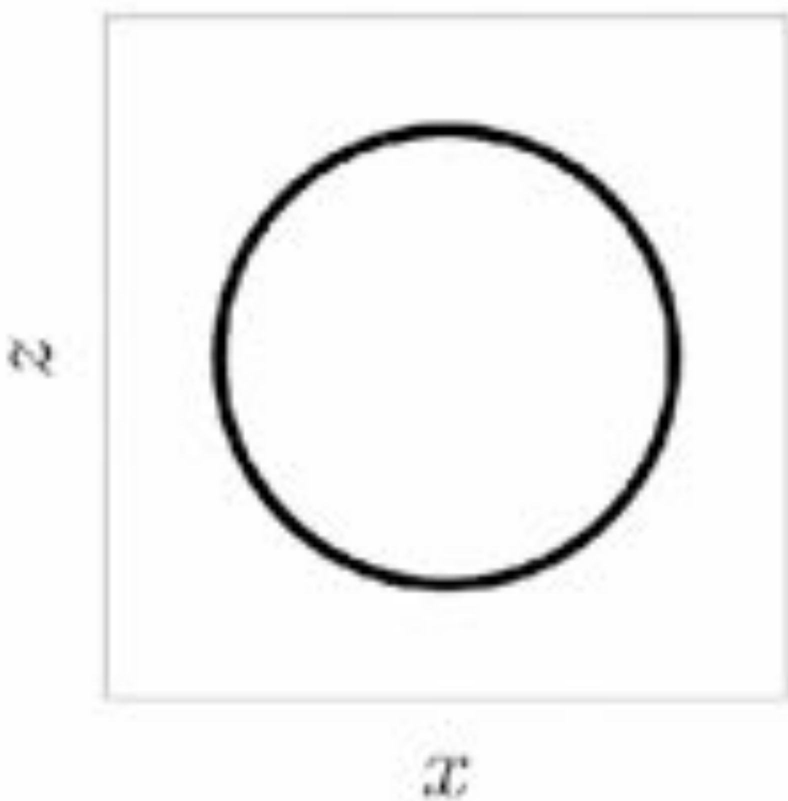
$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

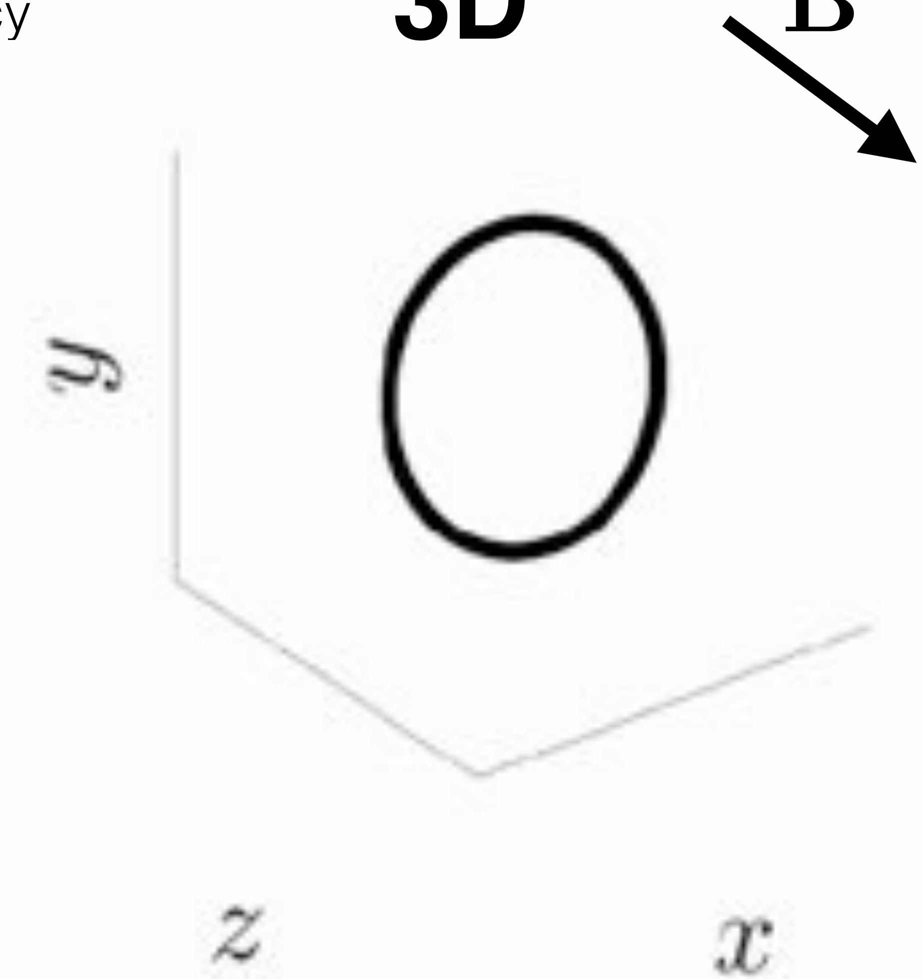
2D ⊗ $\mathbf{B}$

Cyclotron frequency

$$\omega = \frac{q|B|}{m}$$

3D $\mathbf{B}$





$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$B_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

$$F_\mu = q v_\nu B_{\mu\nu}$$

$$B_{xz}$$

$$x = \cos(\omega t), z = \sin(\omega t)$$

$$B_{xy}, B_{xz}, B_{yz} \rightarrow B_{x'z'}$$

$$x' = \cos(\omega t), z' = \sin(\omega t)$$

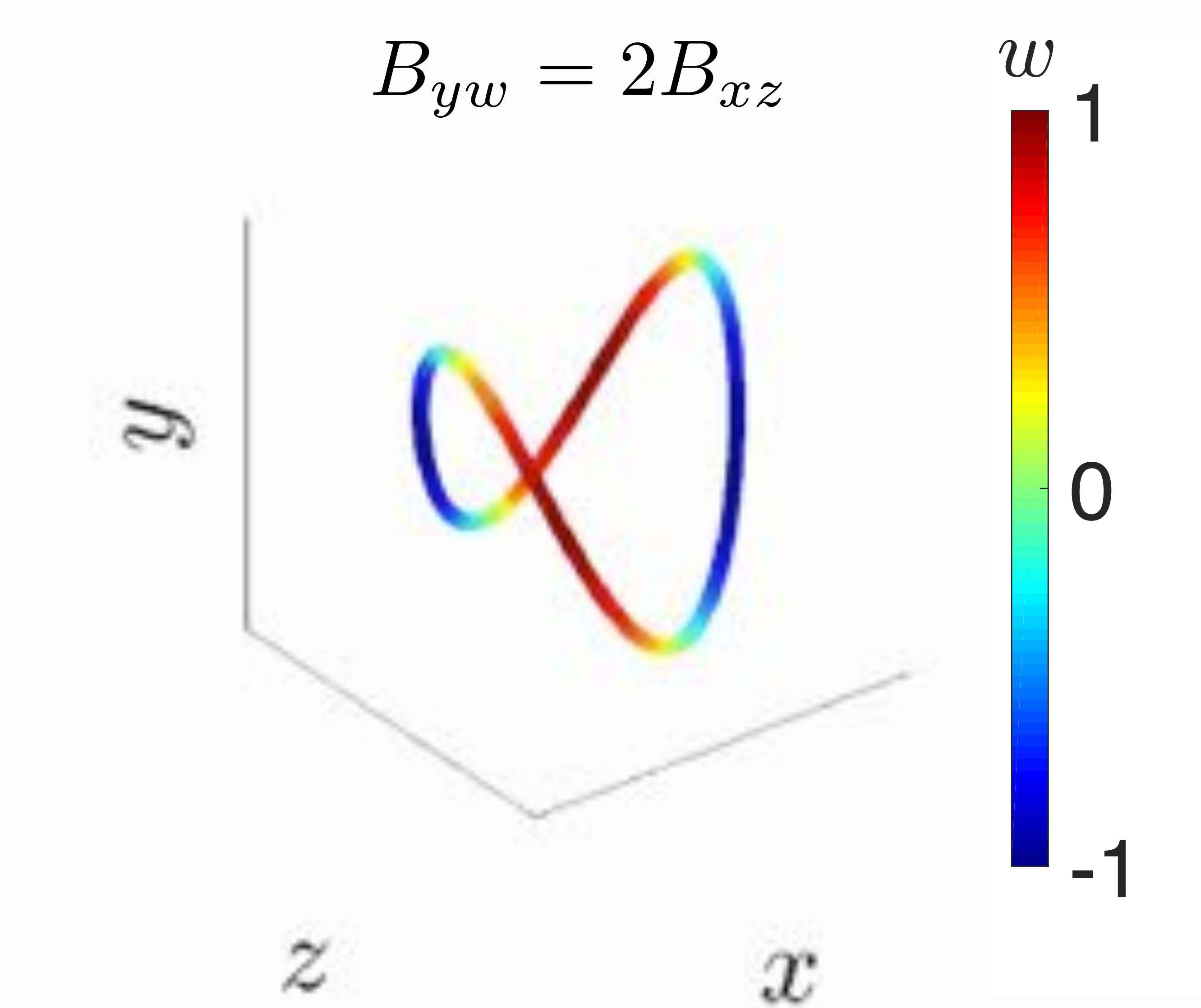
Classical Particle in a magnetic field

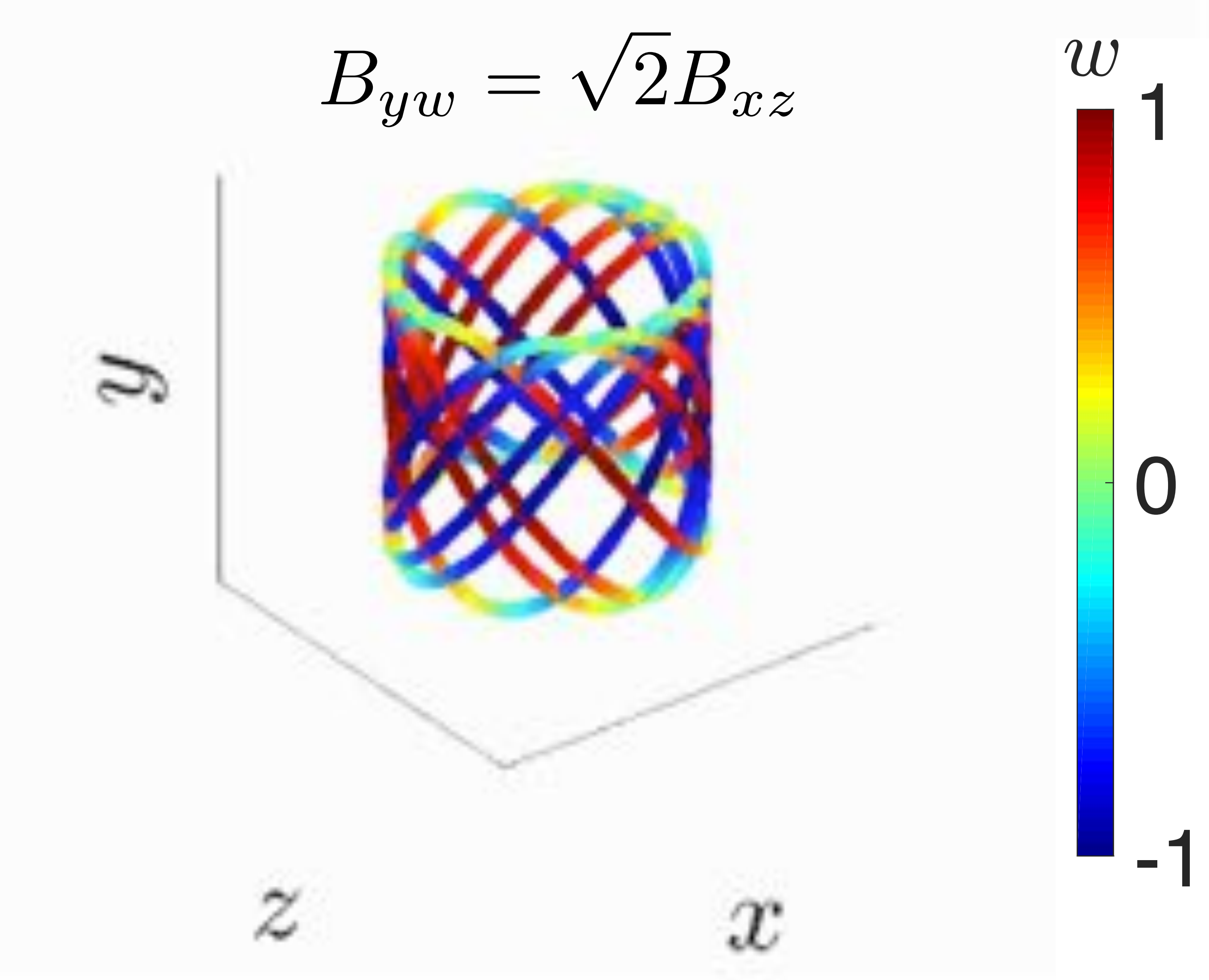
4D $B_{xy}, B_{xz}, B_{xw}, B_{yz}, B_{yw}, B_{zw}$

Not always possible to rotate axes so that only one component is non-zero

e.g. $B_{xz}, B_{yw} \neq 0$ $\quad \omega = \dfrac{qB_{xz}}{m}, \quad \omega' = \dfrac{qB_{yw}}{m}$

$x = \cos(\omega t), z = \sin(\omega t),$
$y = \cos(\omega' t), w = \sin(\omega' t)$





Quantum Hall Effects

2D

2D system in a perpendicular magnetic field

$$B_{xz} \neq 0$$

→

Topological first Chern number

$$\nu_1^{zx}$$

3D

3D system with

$$B_{xy}, B_{xz}, B_{yz} \to B_{x'z'}$$

→

Triad of 3D first Chern numbers

$$\nu_1^{xy}, \nu_1^{zx}, \nu_1^{yz} \to \nu_1^{x'z'}$$

4D

Minimal 4D system with

$$B_{xz}, B_{yw} \neq 0$$

(more generally, up to 6 planes)

→

$$\nu_1^{zx}, \nu_1^{yw}$$

(Simple example of) topological *second* Chern number

$$\nu_2 = \nu_1^{zx}\nu_1^{yw}$$

Second Chern Number

$$\nu_2 = \frac{1}{8\pi^2}\int_{4\mathrm{DBZ}} \Omega \wedge \Omega \in \mathbb{Z}$$

$$= \frac{1}{4\pi^2}\int_{4\mathrm{DBZ}} \left[\Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{zy} + \Omega^{zx}\Omega^{yw}\right] \mathrm{d}^4 k$$

c.f. $\nu_1 = \frac{1}{2\pi}\int_{\mathbb{T}^2} \Omega$

Zhang et al, Science 294, 823 (2001),
Qi et al, Phys. Rev. B 78, 195424 (2008)…..
Sugawa et al, Science, 360, 1429 (2018)

Generalize to degenerate bands by tracing over

And then the third Chern number in 6D…

for 6DQH see Petrides, HMP, Zilberberg Phys. Rev. B 98, 125431 (2018) and references there-in

Topological Nonlinear Quantum Hall Response

$$j_\mu = \frac{q^3}{2h^2}\varepsilon^{\mu\gamma\delta\nu} E_\nu B_{\gamma\delta}\nu_2$$

c.f.

$$j_x \propto E_y \nu_1$$

Zhang et al, Science 294, 823 (2001)….
HMP, Zilberberg, Ozawa, Carusotto & Goldman, PRL 115, 195303 (2015)
HMP, Zilberberg, Ozawa, Carusotto & Goldman, PRB 93, 245113 (2016)

Connection to Homotopy

Minimal <u>four-band</u> model:

Qi et al, Phys. Rev. B 78, 195424 (2008)…..

$$H(\mathbf{k}) = \varepsilon(\mathbf{k})\Gamma_0 + \mathbf{d}(\mathbf{k}) \cdot \mathbf{\Gamma}$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \Gamma_4 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \Gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

Normalized 5D “pseudo-spin” vector

$$\hat{\mathbf{d}}(\mathbf{k}) = \frac{1}{\sqrt{\sum_i d_i^2}} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix}$$

$$\begin{aligned} \nu_2^- &= \frac{1}{8\pi^2} \int_{\mathrm{BZ}} \mathrm{tr}(\Omega_- \wedge \Omega_-), \\ &= \frac{3}{8\pi^2} \int_{\mathrm{BZ}} d^4\mathbf{k} \epsilon^{abcde} \hat{d}_a \partial_{k_x} \hat{d}_b \partial_{k_y} \hat{d}_c \partial_{k_z} \hat{d}_d \partial_{k_w} \hat{d}_e \end{aligned}$$

How many times do we wrap over the 4-sphere in the 4D BZ?

Summary: Second Chern Number

- A 4D Topological Invariant
- e.g. integral of trace of wedge product of Berry curvature over 4D BZ
- Can Count Number of “Yang” Monopoles Enclosed

c.f. Sugawa et al, Second Chern number of a quantum-simulated non-Abelian Yang monopole, Science, 360, 1429, (2018)

- For 4-band models, gives 4D “skyrmion” (winding) number

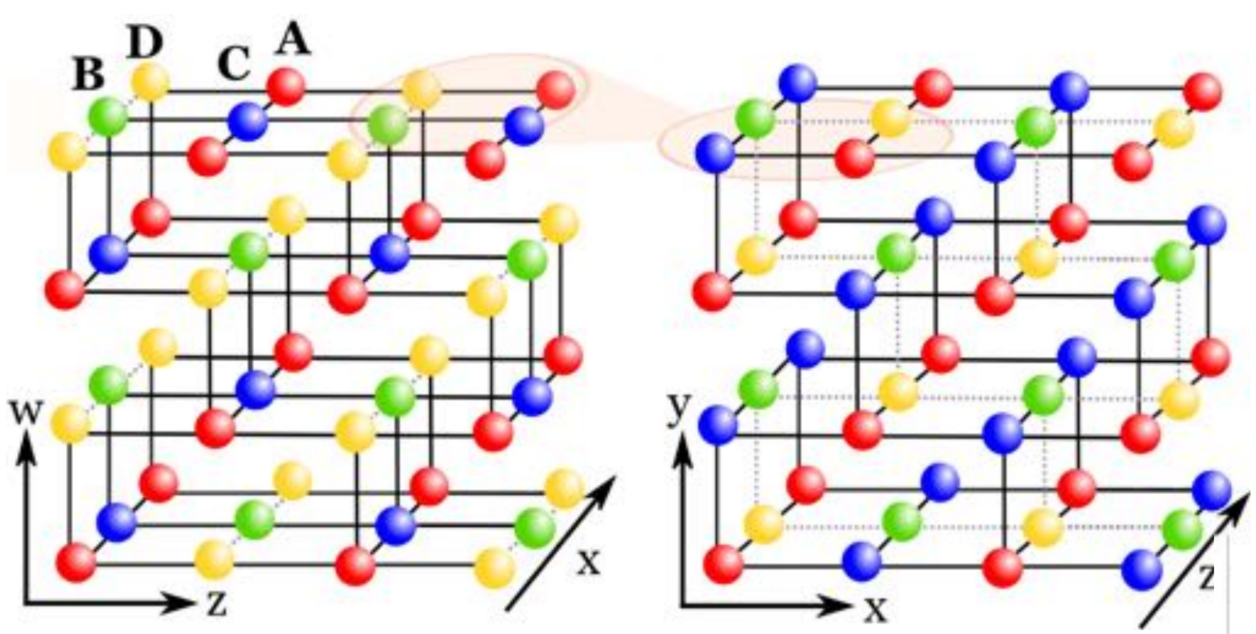
Example Models

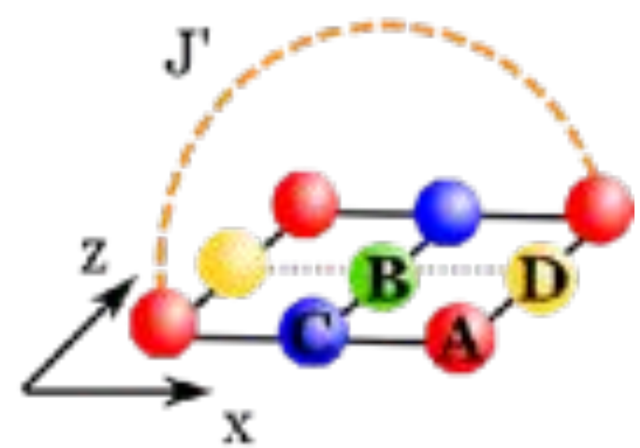
- 4D Landau levels
- 4D Harper-Hofstadter Model
- Qi/ Zhang/ Hughes Model

Qi et al, Phys. Rev. B 78, 195424 (2008).....

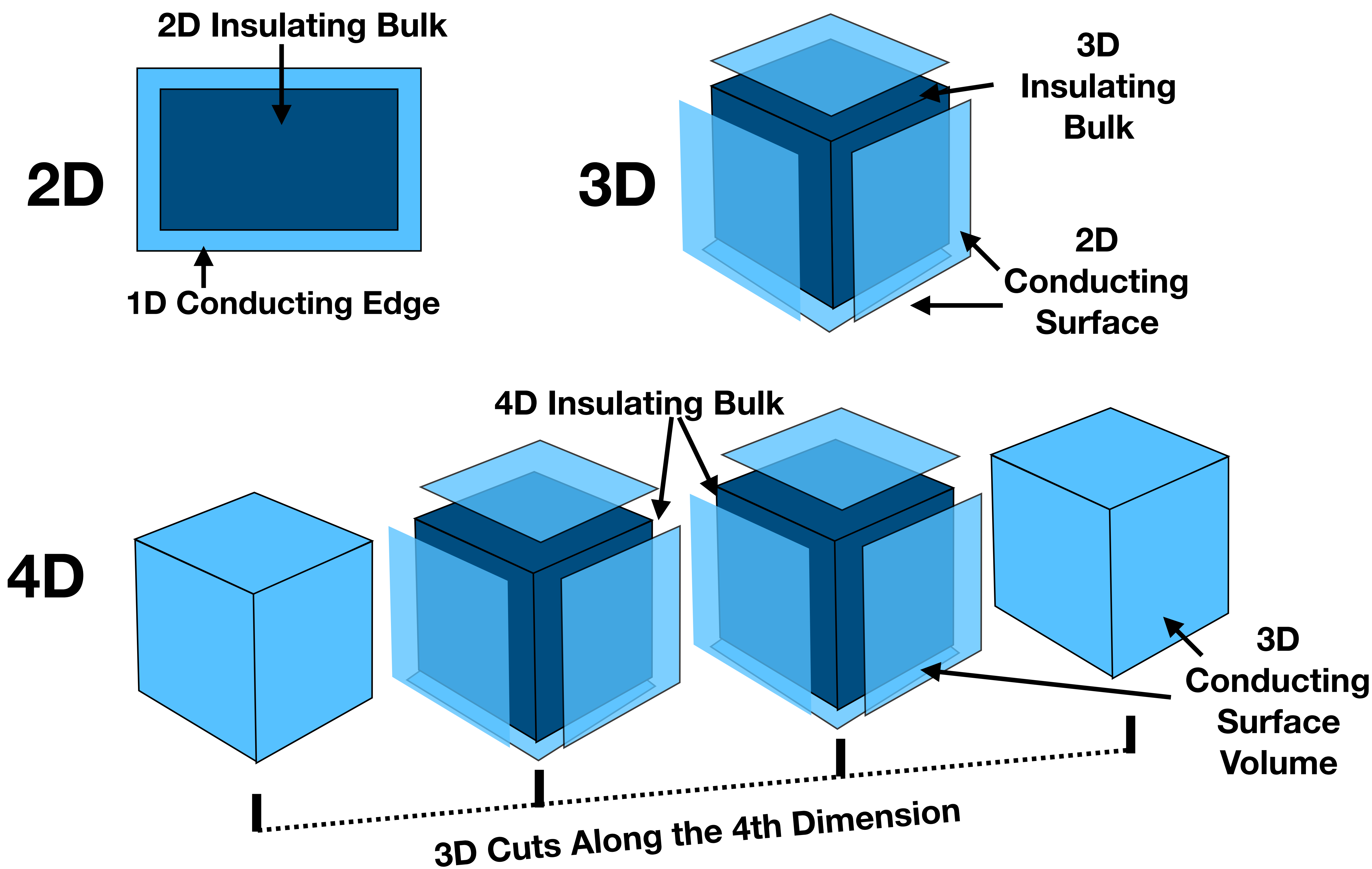
- 4D Modified Brickwall Model

HMP Phys. Rev. B 101, 205141 (2020)





Bulk-Boundary Correspondence



Aside: Symmetries…

"Periodic table" of gapped phases of quadratic fermionic Hamiltonians without extra symmetries

Class	Symmetry: Time-reversal	Particle-hole	Chiral	Dimensionality d: 1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

← **Quantum Hall**

← **SSH Model**

← Topological Superconductors

← Topological Insulators/ quantum spin Hall

Result of squaring the symmetry operator (0=symmetry is broken)

Possible values of topological invariant:
0 : always trivial
$\mathbb{Z}$: an integer
$\mathbb{Z}_2$: 0,1

Kitaev, arXiv:0901.2686
Ryu et al., New J. Phys. 12, 065010 (2010)
Chiu, et al., RMP 88, 035005, (2016)

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    - *(Topological Pumping)*
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Approach 1: 2D Topological Pumping

4D Quantum Hall Model

$$\hat{H}(x, y, z, w)$$

Fourier Transform wrt 2 coordinates

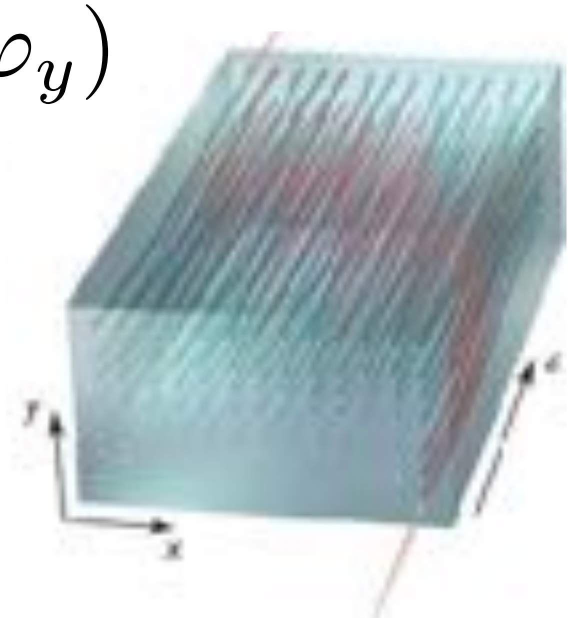
Mathematical Mapping

$$\sum_{k_z, k_w} \hat{H}(x, y, k_z, k_w)$$

Replace with periodic parameters

2D Topological Pump

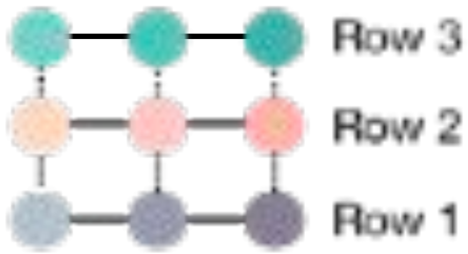
$$\hat{H}_{2D}(x, y, \varphi_x, \varphi_y)$$



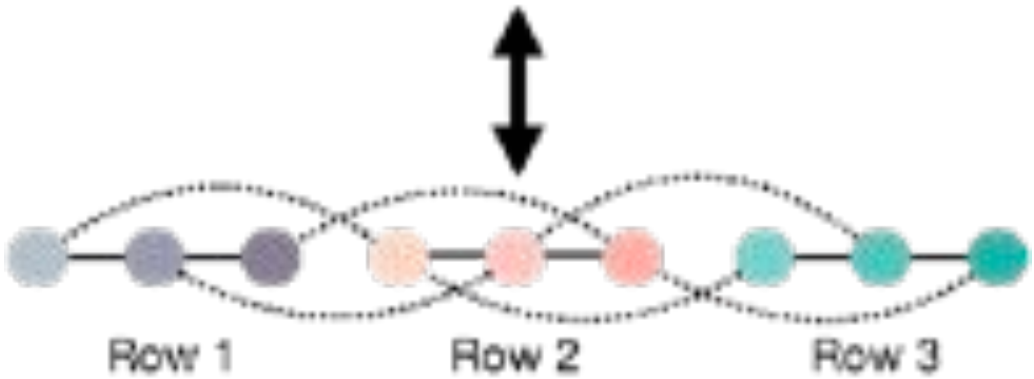
Extension of Thouless pumping (D. J. Thouless, Phys. Rev. B 27, 6083 (1983))
Proposal: Y. E. Kraus et al., Phys. Rev. Lett. 111, 226401 (2013)

Expt with cold atoms: Lohse, Schweizer, **HMP**, Zilberberg, Bloch, Nature 553, 55–58 (2018)
Expt with photons: O. Zilberberg et al., Nature 553, 59 (2018)

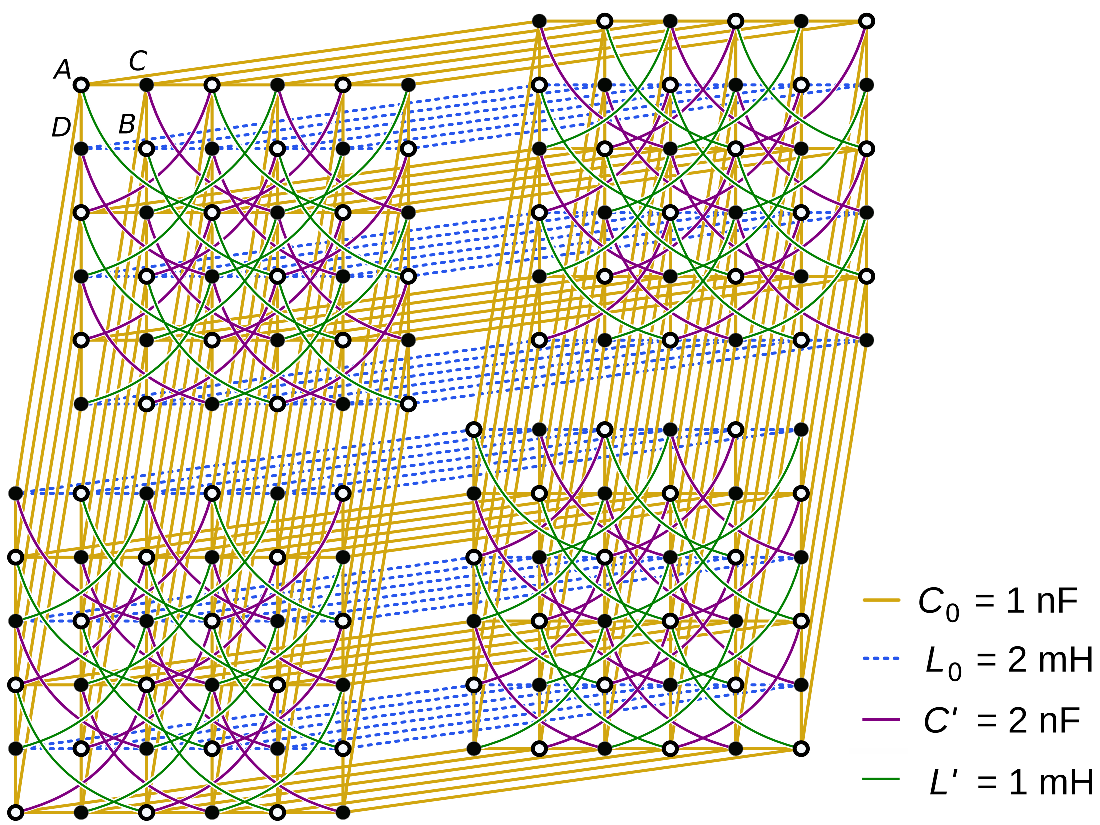
Approach 2: Circuit Connectivity

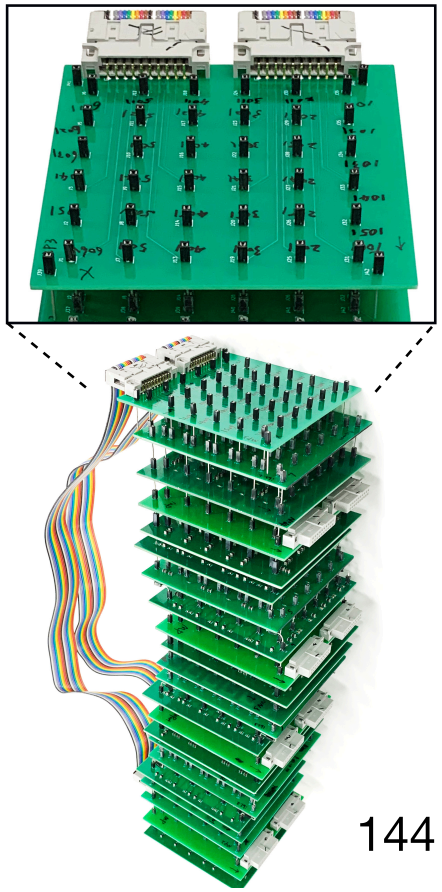


2D Lattice



2D Lattice Embedded into 1D Chain



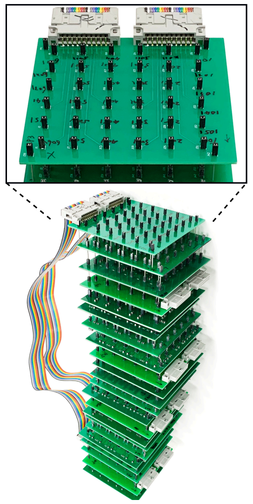


4D Lattice Embedded into 3D Stack of Circuit Boards

144 sites (6x2x6x2 with some PBCs)

Y. Wang, **HMP**, B. Zhang and Y. Chong, Nature Communications 11, 2356 (2020)
Yu et al. National Science Review, 7(8),1288-1295 (2020)…

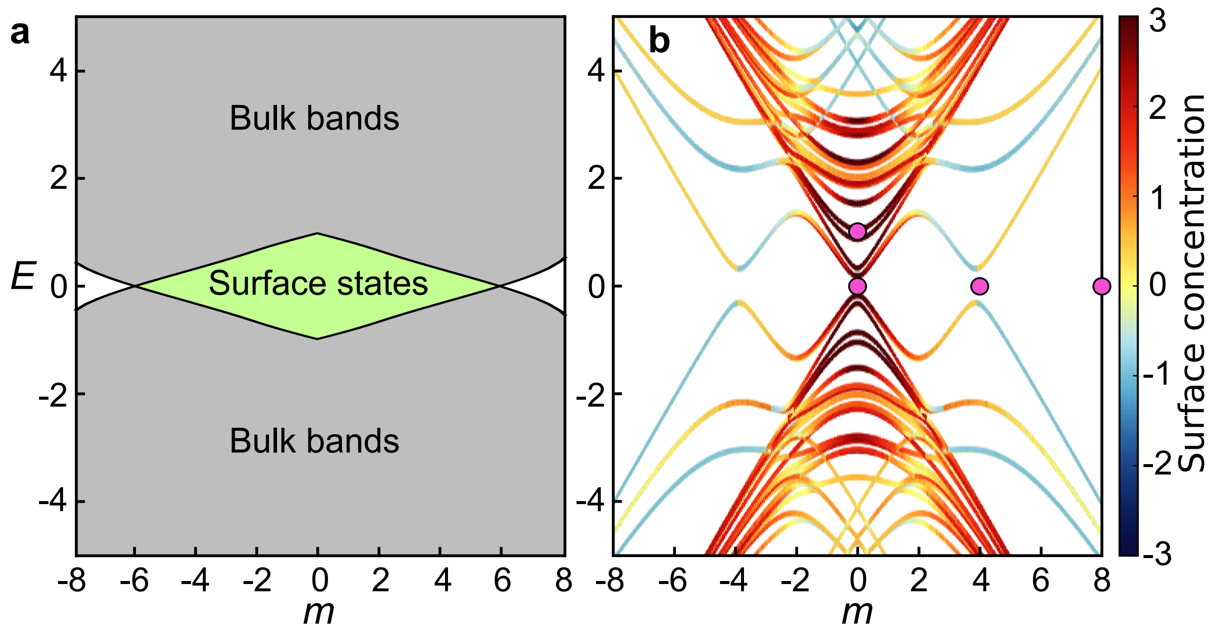
Approach 2: Circuit Connectivity



4D Topological Circuit

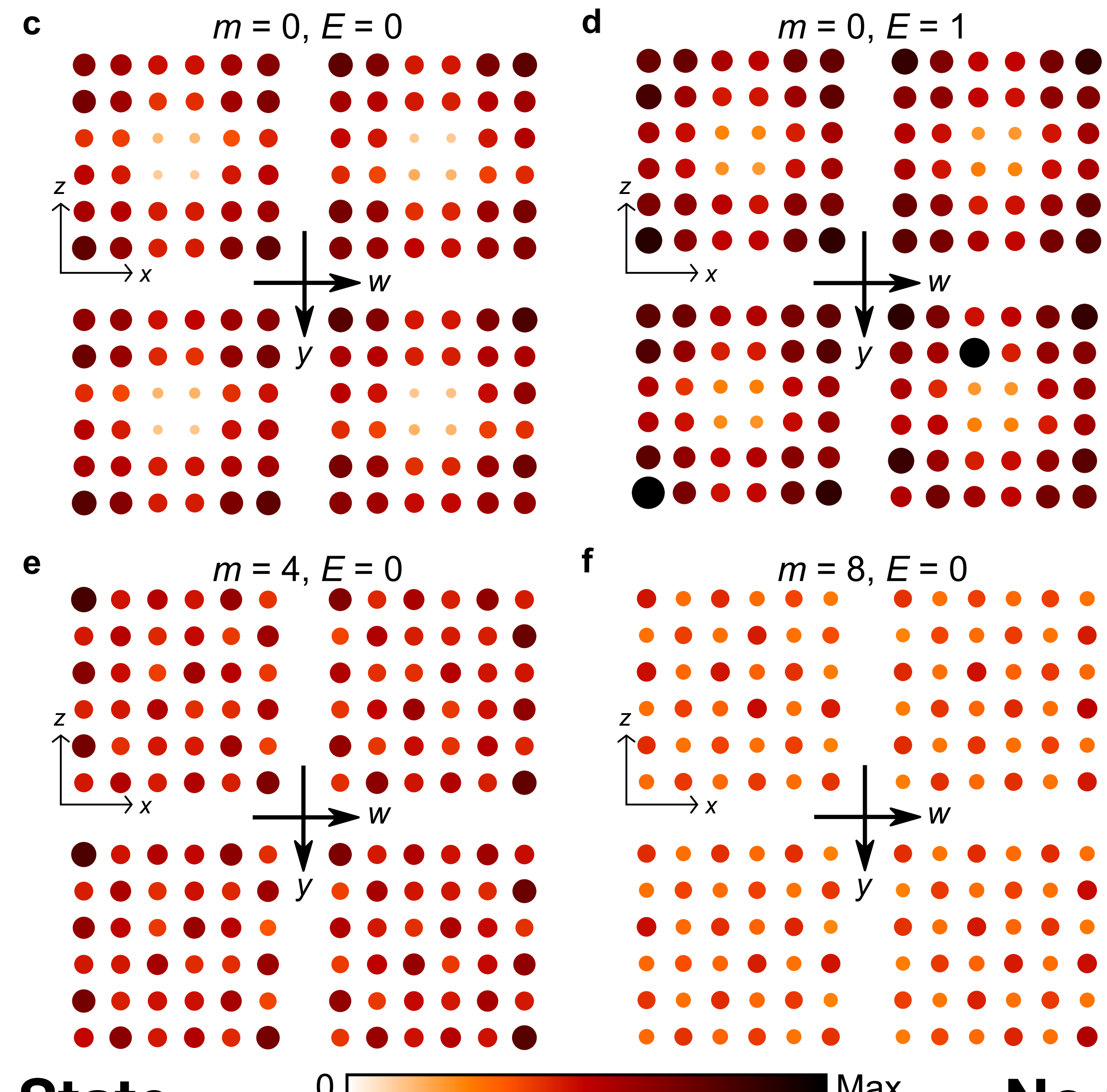
Grounding impedance is related to LDOS, so can probe properties of states

C. H. Lee et al, Comm. Phys. 1 (2018).



Tuning parameter

3D Surface states



Bulk State

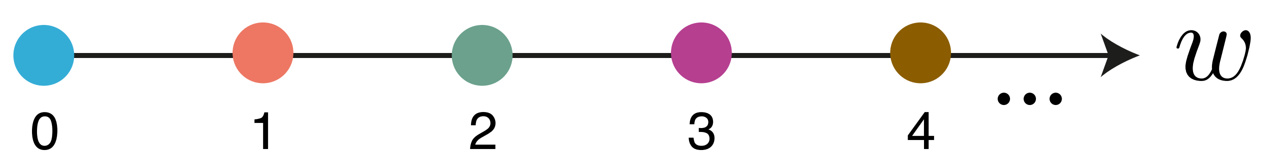
No State

Also observed robustness and emergence of surface states

Y. Wang, **HMP**, B. Zhang and Y. Chong, Nature Communications 11, 2356 (2020)

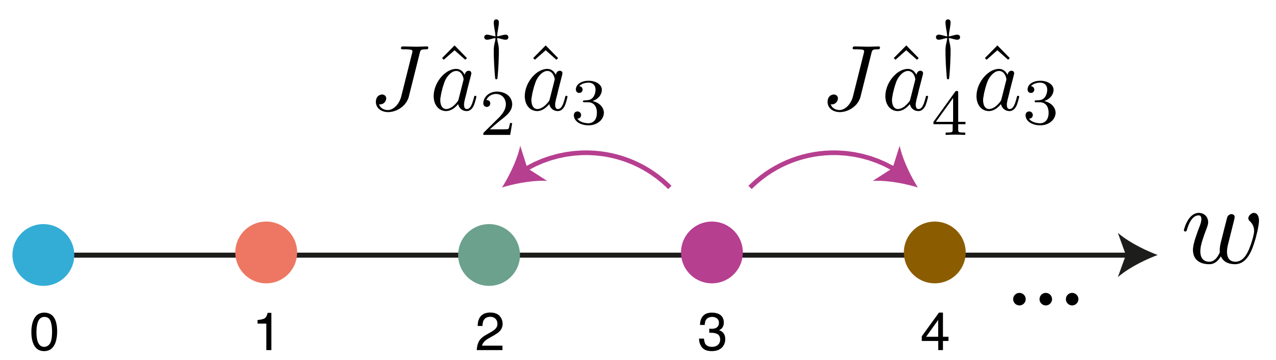
Approach 3: Synthetic dimensions

1. Identify a set of states and reinterpret as sites in a synthetic dimension



Boada et al., PRL, 108, 133001 (2012),
Celi et al., PRL, 112, 043001 (2014)

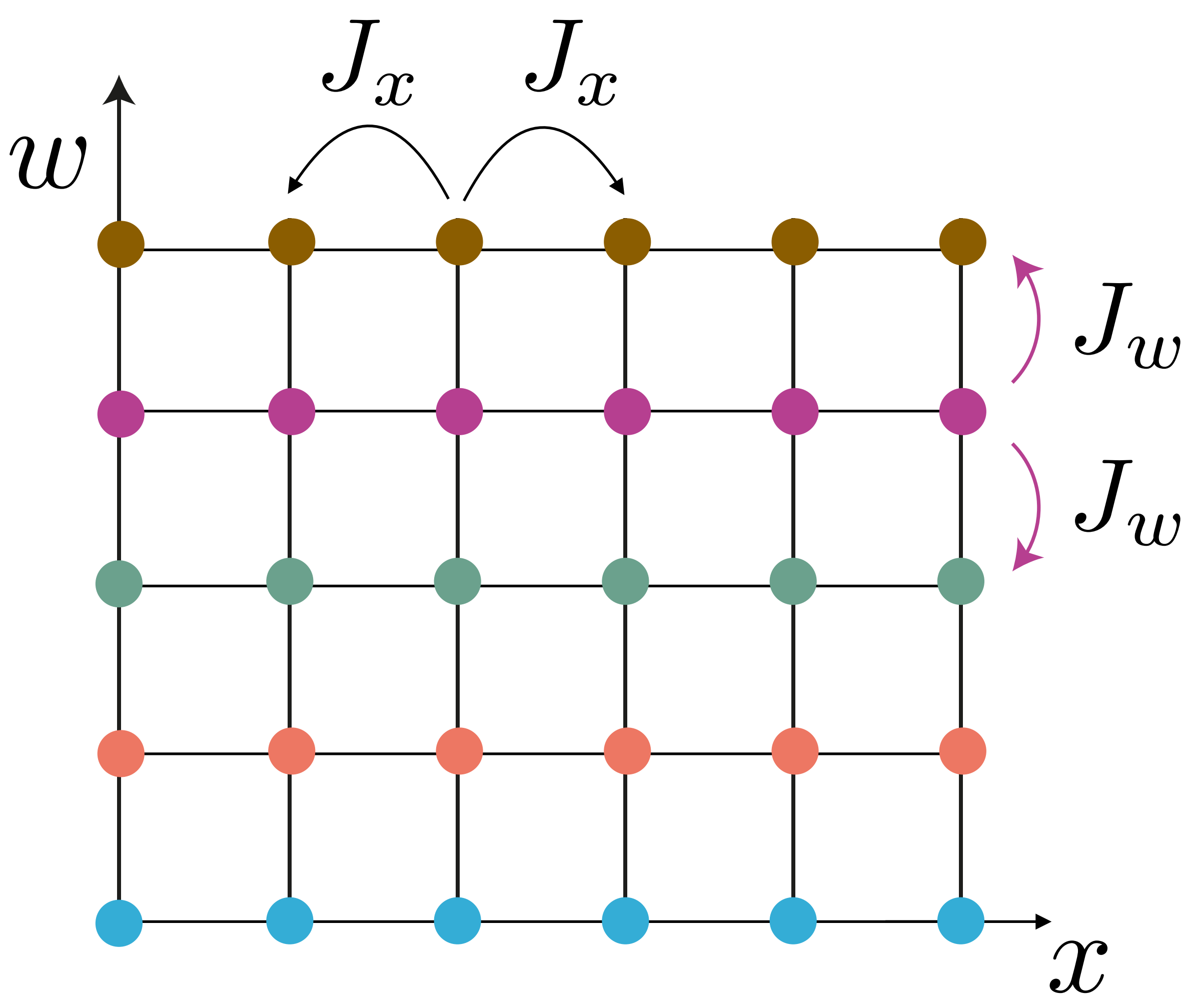
2. Couple these modes to simulate a tight-binding "hopping"



Simulates a particle on a 1D lattice

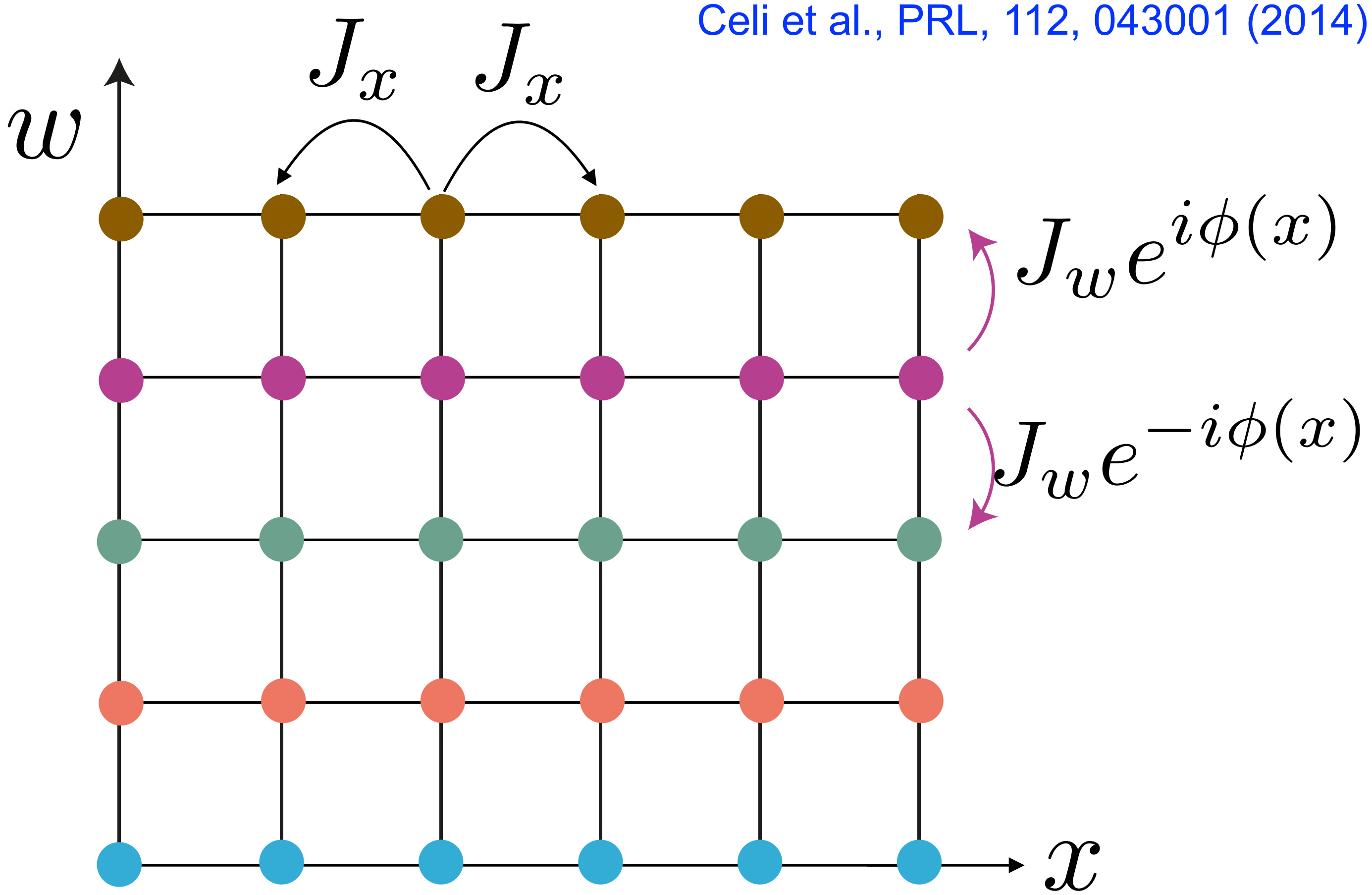
Approach 3: Synthetic dimensions

3. Add a second (real or synthetic) spatial dimension



For example: give a phase to the synthetic “hopping” that depends on the other co-ordinate

Boada et al., PRL, 108, 133001 (2012),
Celi et al., PRL, 112, 043001 (2014)



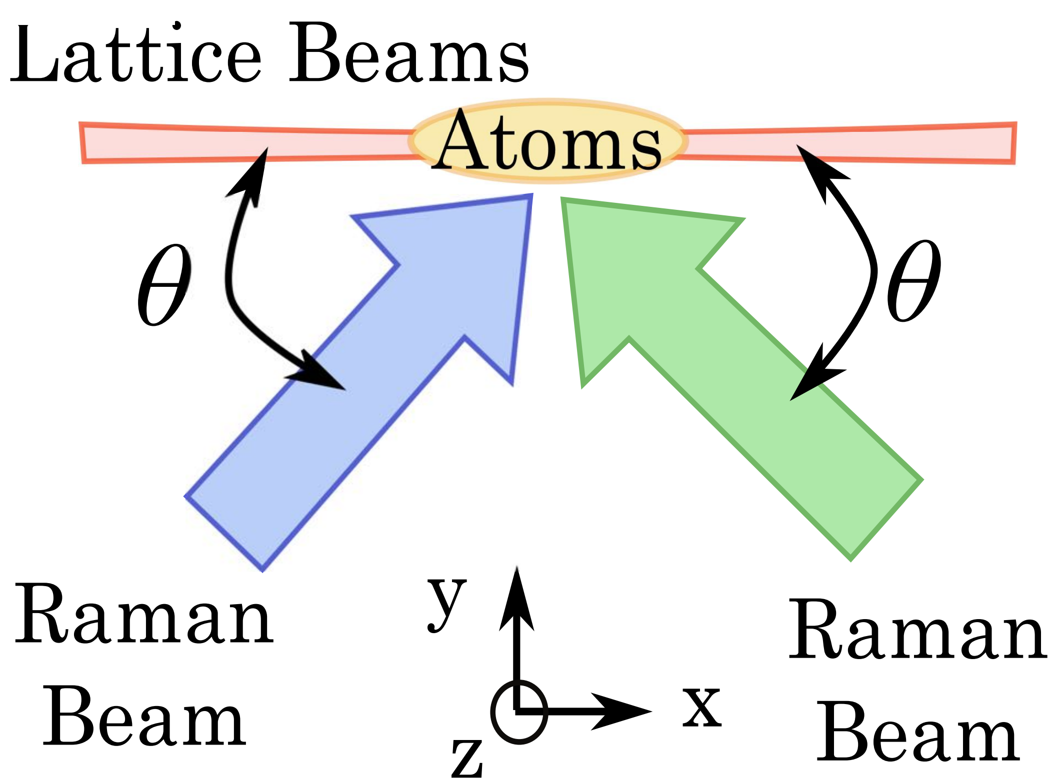
Simulates a magnetic field:
Harper-Hofstadter model

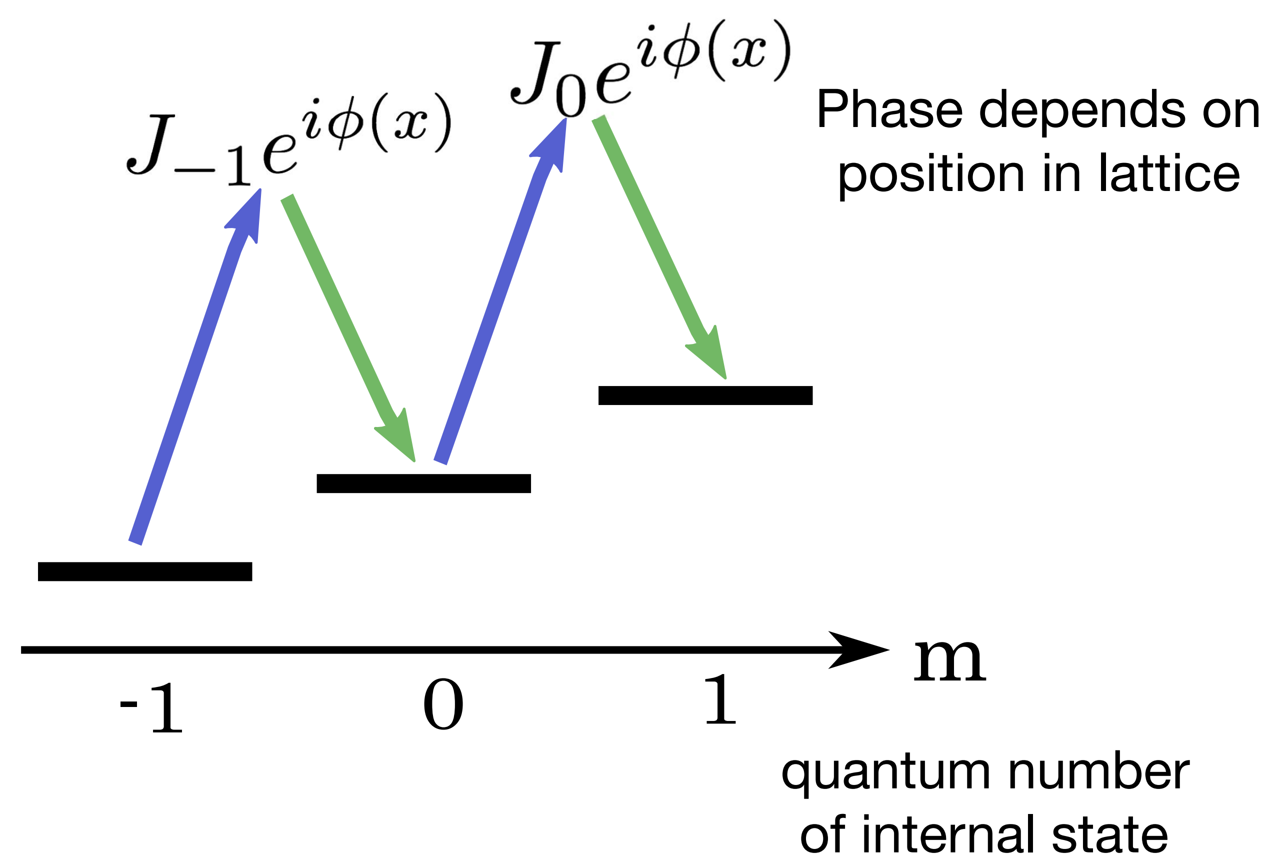
$$\mathcal{H} = J \sum_{m,n} (\hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} + e^{i2\pi\Phi m}\hat{a}^\dagger_{m,n+1}\hat{a}_{m,n}) + \text{h.c.}$$

Synthetic dimension with internal atomic states

Ingredients:

1. Reinterpret states as sites in synthetic dimension -> **Internal atomic states**
2. Couple states to simulate a “hopping” term -> **Raman beams**





Experiments

Florence: Mancini et al, Science, 349, 1510 (2015), Livi et al, Phys. Rev. Lett. 117, 220401 (2016)
Maryland: Stuhl et al. Science, 349, 1514 (2015)
Boulder: Kolkowitz et al, Nature, 542, 66 (2017)
Paris: Chalopin et al, Nature Phys, 16, 1017 (2020)

They observed skipping orbits in (1 real + 1 synthetic)-D…

A lot of recent progress

Review: T. Ozawa & HMP, Nature Reviews Physics 1, 349 (2019)

ATOMS & MOLECULES

Atomic states: Celi et al., PRL, 112, 043001 (2014), Mancini et al, Science, 349, 1510 (2015), Stuhl et al. Science, 349, 1514 (2015)…

Momentum states of atoms: An, Meier, Galway, Sci. Adv. e1602685 (2017), Viebahn et al, PRL 122 (11), 110404 (2019)….

Harmonic trap states of atoms: HMP et al., PRA 95, 023607 (2017), Salerno, HMP et al, Phys. Rev. X 9, 041001 (2019)

Rotational States of ultracold molecules: Sundar, Gadway & Hazzard, Sci. Rep. 8, 3422 (2018) Sundar et al, PRA, 99, 013624 (2019)

PHOTONS

Optomechanics: Schmidt et al, Optica 2, 7, 635 (2015)

Photons in Optical cavities: Luo et al, Nature Comm. 6, 7704, (2015)

Frequency modes: Ozawa, HMP, Goldman, Zilberberg, & Carusotto, PRA 93, 043827 (2016), Yuan, et al, Optics Letters 41, 4, 741 (2016)…..
… Yuan et al, Photon. Res. 8(9), B8-B14 (2020), Tusnin et al, PRA, 102, 023518 (2020)
Dutt et al. Nature Communications 10, 3122 (2019), Dutt et al Science 367, 59 (2020)

Angular co-ordinate of ring resonator: Ozawa & Carusotto, PRL, 118, 013601 (2017)

Arrival time of pulses Schreiber, A. et al. Phys. Rev. Lett. 104, 050502 (2010).
Wimmer, HMP, Carusotto & Peschel, Nat. Phys. 13, 545 (2017),
Chen, C. et al. Phys. Rev. Lett. 121, 100502 (2018)….

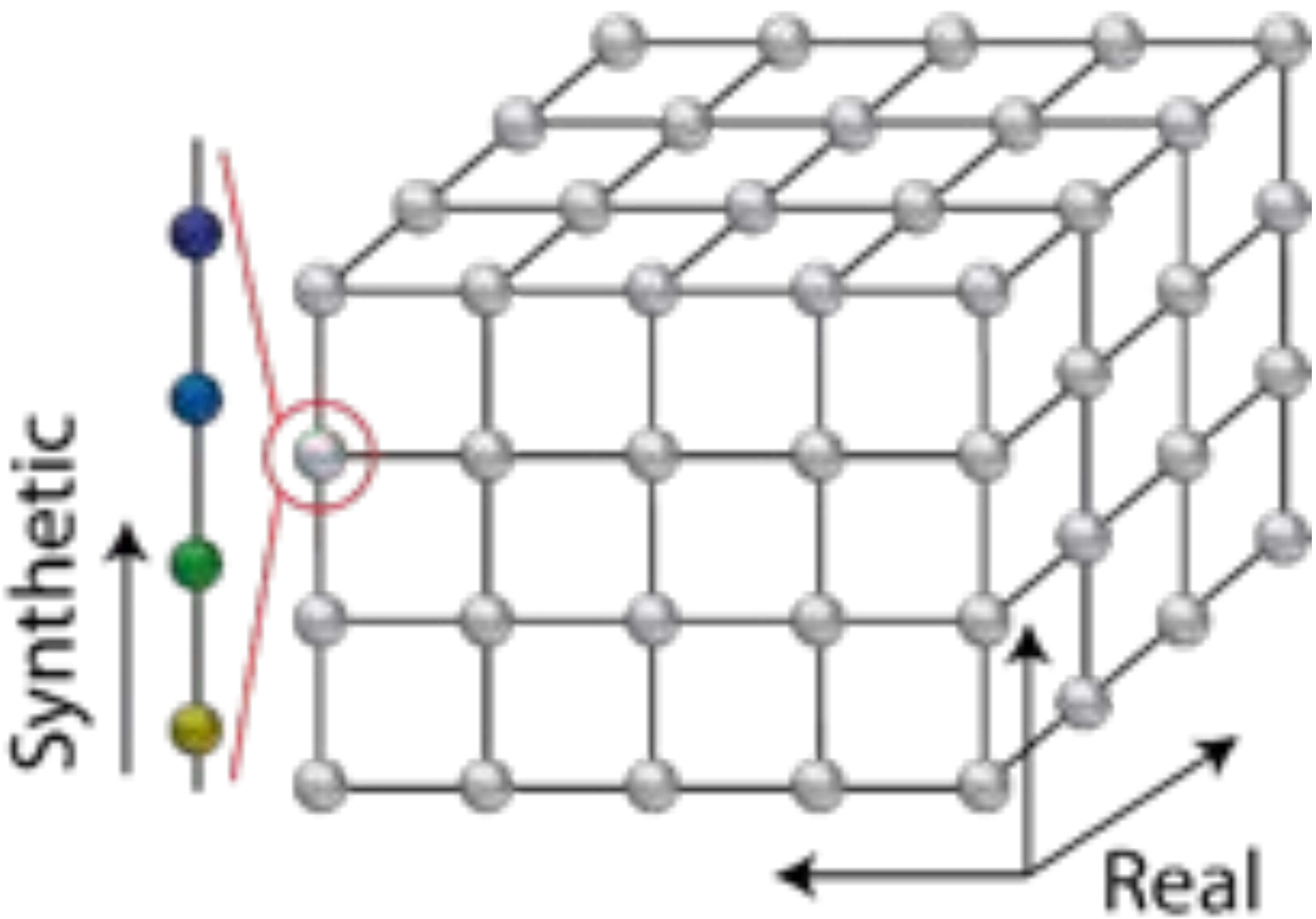
Spatial modes of waveguide array: Lustig et al., Nature, 567, 356 (2019)

OTHER

Mesoscopic Nanomagnet-Ring system: HMP, Ozawa & Schomerus, PRR, 2, 032017(R) (2020)

Floquet states: Martin, Refael, & Halperin, PRX 7, 041008 (2017)…

Future Experiments? Synthetic Dimensions?



4D Quantum Hall effect with synthetic dimensions:
HMP, Zilberberg, Ozawa, Carusotto & Goldman, Phys. Rev. Lett. 115, 195303 (2015)
T. Ozawa, **HMP**, N. Goldman, O. Zilberberg, and I. Carusotto, Phys. Rev. A 93, 043827 (2016)

Topological Pumping

- Experiments in 1D (mapped from 2D) and 2D (mapped from 4D)
- External parameters
- Topology after a pump cycle $x(T) \propto \nu_1$
- Limited dynamics

Circuit connectivity

- Experiment in "4D"!
- Easy to scale, and very accessible
- Probing of surface states
- Classical circuits

Synthetic Dimensions

- Experiments not yet up to 4D
- Each implementation quite different
- Topology in current response $j_x \propto E_y \nu_1$
- Can be truly quantum
- Interactions!?

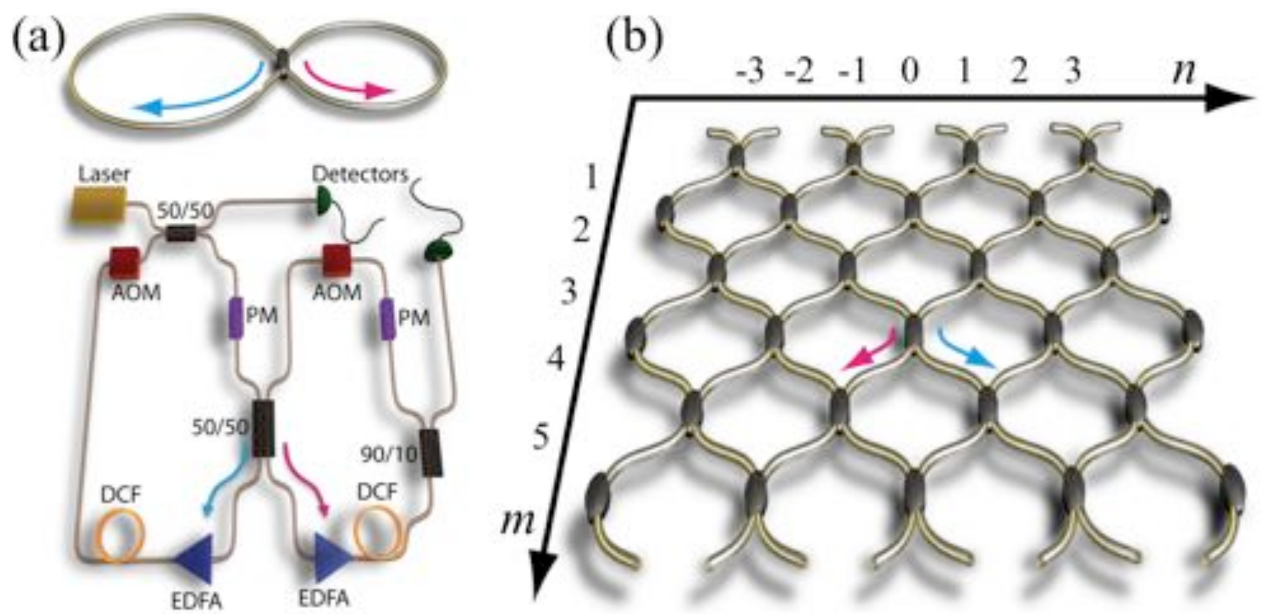
Aside: a few other recent projects…

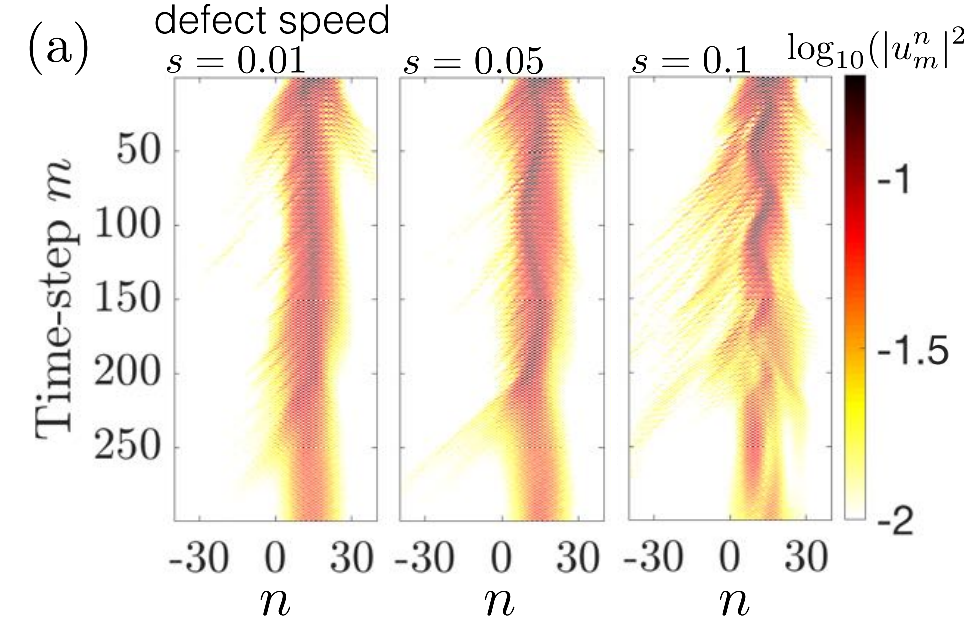
Analogue Superfluidity in an Optical Mesh Lattice

Expt + theory:
Wimmer, Monika, Carusotto, Peschel, **HMP**, arXiv:2008.04663

time-multiplexing: a 1D synthetic dimension

Schreiber, A. et al. Phys. Rev. Lett. 104, 050502 (2010).

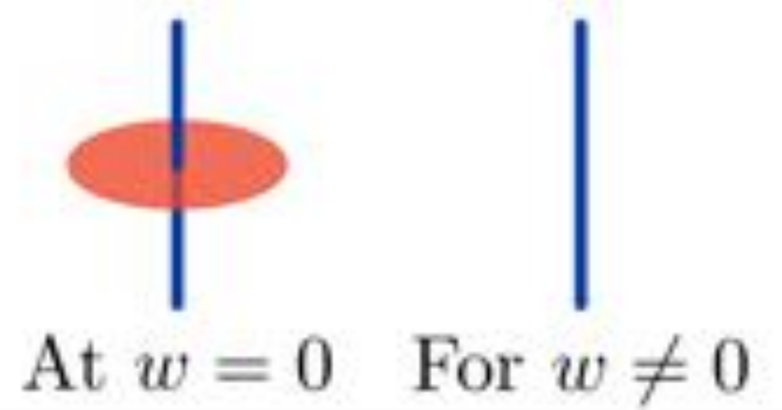




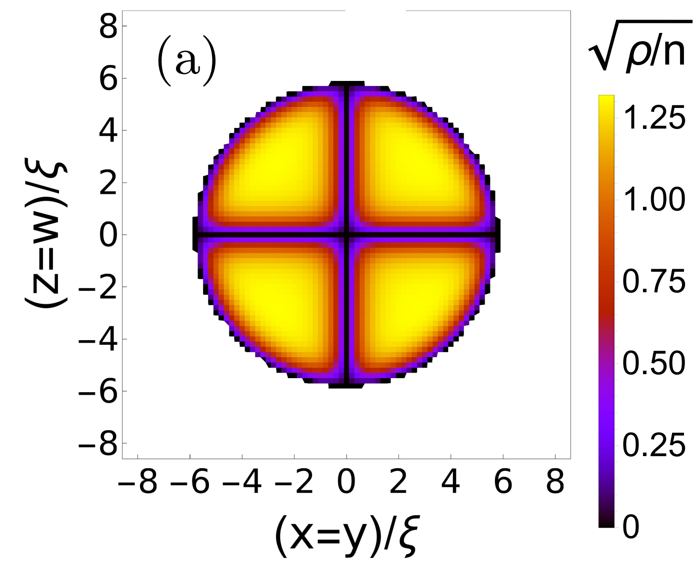
What happens to superfluid vortices in 4D?

Theory:
McCanna, **HMP**, arXiv:2005.07485

Ben McCanna

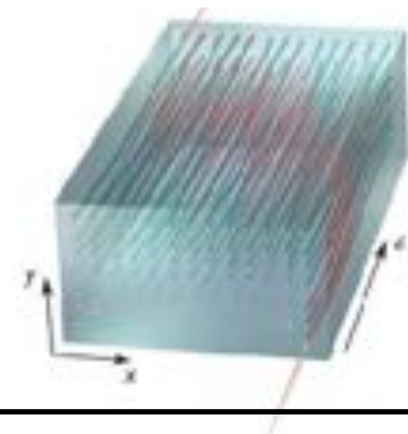


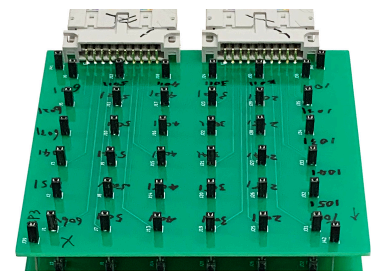
Vortex cores: Two intersecting 2D planes



Summary

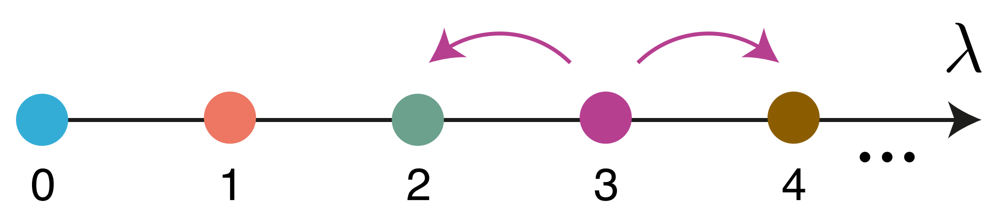
$$j_\mu = \frac{q^3}{2h^2}\varepsilon^{\mu\gamma\delta\nu}E_\nu B_{\gamma\delta}\nu_2$$





4D Topological Physics

Topological pumping, connectivity and synthetic dimensions for cold atoms and photonics



Future Prospects:

Quantum Simulation of 4D Lattices? Interactions?