Indicators of quantum chaos and the transition from few- to many-body systems

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Plan

Thermalization, many-body localization, scrambling of quantum information, many-body quantum scars, quantum-classical correspondence

- Indicators of Quantum Chaos:

STATIC
- Eigenvalues
- Eigenstates
- Observables

FROM FEW- TO MANY-BODY

From few- to many-body quantum systems
M. Schiulaz, M. Távora, LFS
Quantum Sci. Technol. **3**, 044006 (2018)

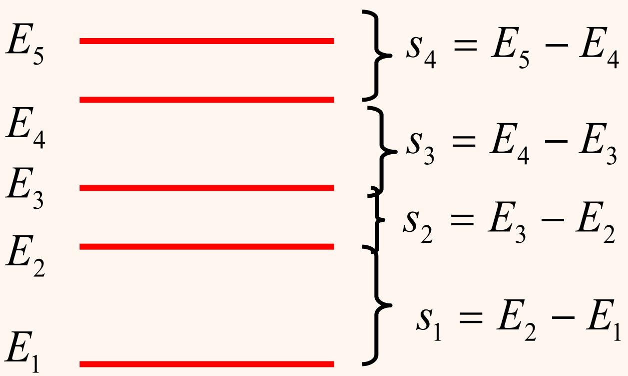
How many particles make up a chaotic many-body quantum system?
G. Zisling, LFS, Y. Bar Lev
arXiv:2012.14436

DYNAMICS
- OTOC
- Correlation hole

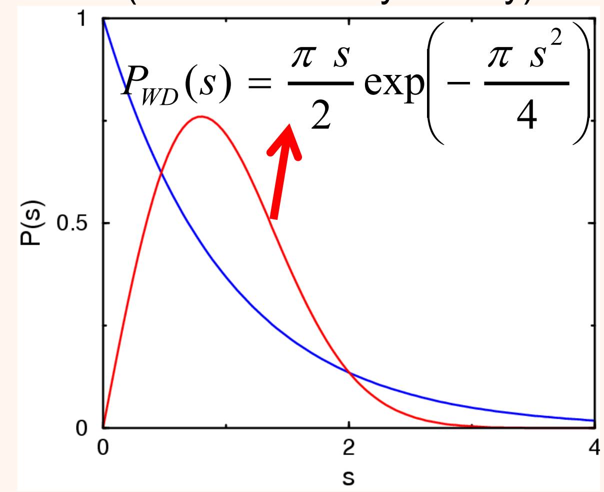
Level Statistics

Full random matrices from the GOE: real and symmetric
(Wigner in the 50's to describe statistically the spectra of heavy nuclei)

Level spacing distribution



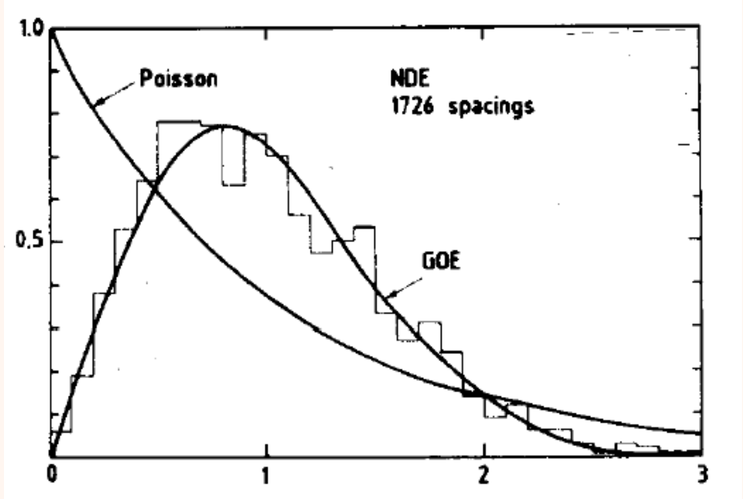
Wigner-Dyson distribution
(time reversal symmetry)



Eigenvalues are correlated
Level repulsion
Rigid spectrum

Level Spacing Distribution and Nuclei

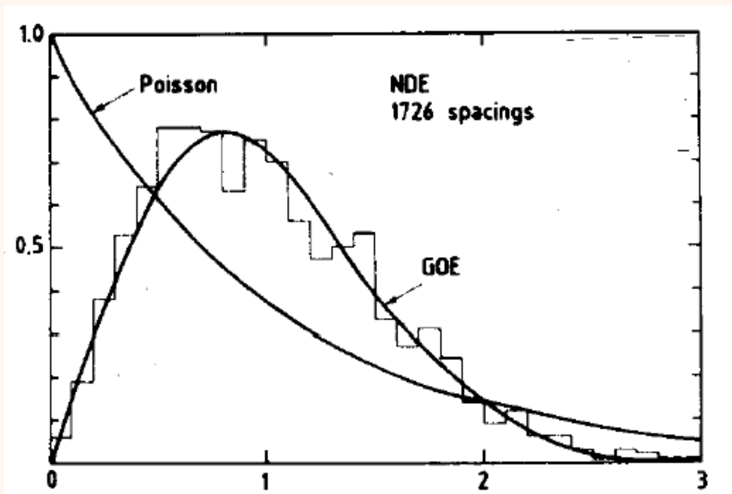
Nearest neighbor spacing distribution for the "**Nuclear Data Ensemble**" comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology

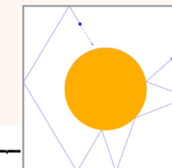
Level Spacing Distribution and Chaos

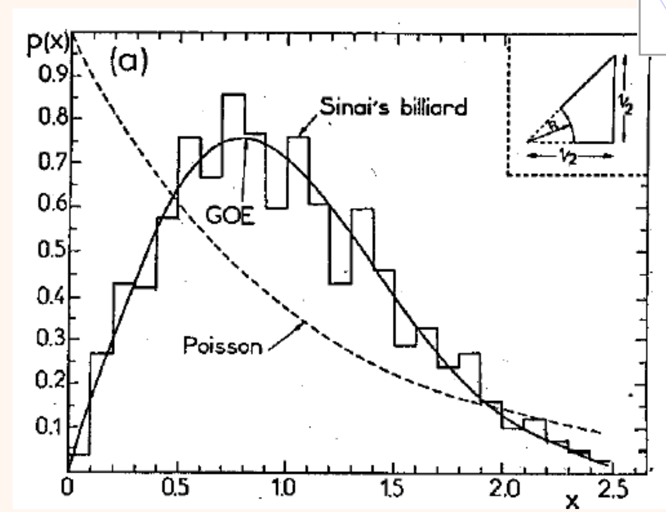
Nearest neighbor spacing distribution for the "**Nuclear Data Ensemble**" comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology

The nearest neighbor spacing distribution versus s for the ***quantum* Sinai billiard**. The histogram comprises about 1000 consecutive eigenvalues.





Classical chaos – Level Statistics
Quantum chaos = signatures of chaos

Correspondence established – few degrees of freedom

1D Spin-1/2 Systems

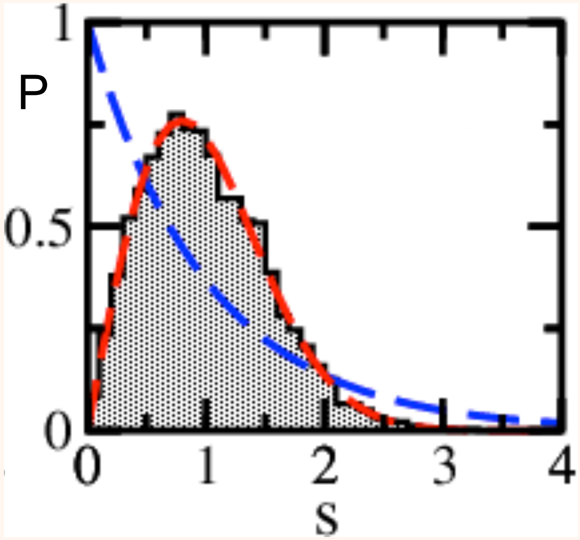
Integrable system:

XXZ model

$$H = \sum_{n=1}^{L-1} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right)$$

Interaction between next-nearest neighbors

$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) + \\ + \lambda \sum_{n=1}^{L-2} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y \right)$$



Many-body localization

$$H = \sum_{n=1}^{L} \frac{\varepsilon_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right)$$

Wigner-Dyson Distribution and Integrability

- We can construct integrable Hamiltonians with WD distribution

Relaño, Dukelsky, Gómez, Retamosa
PRE **70**, 026208 (2004)

- Finite-size effect: Localization length is larger than the system size

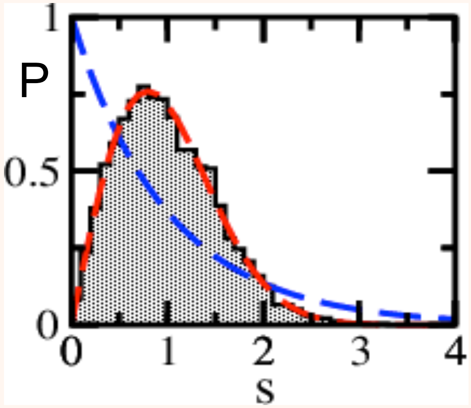
1D Anderson model

$$H = \sum_{n=1}^{L} \epsilon_n c_n^\dagger c_n - J \sum_{n=1}^{L-1} (c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n)$$

1D Aubry-André model

$$H = \sum_{j=1}^{L} h \cos[(\sqrt{5}-1)\pi j + \phi] c_j^\dagger c_j - J \sum_{j=1}^{L-1} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

Tight-binding models

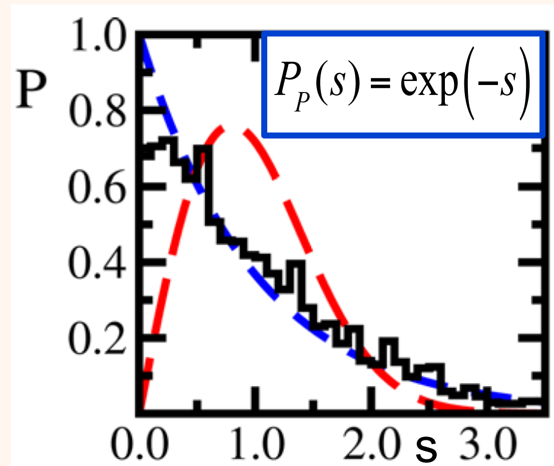


Torres, Bermudez, LFS
PRE**100**, 022142 (2019)

Poisson distribution

- **Chaotic** models but mixed symmetries:

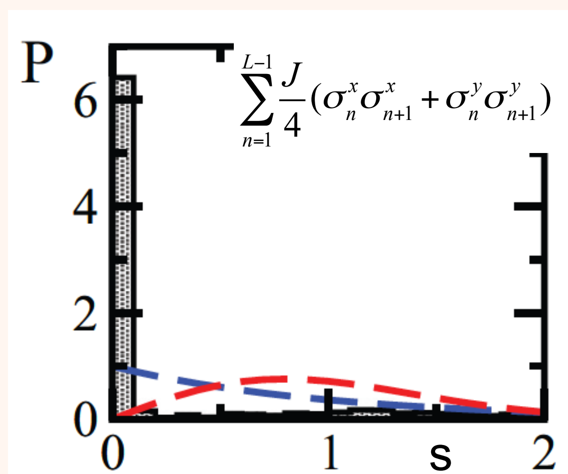
$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4}(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) +$$
$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4}(\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y)$$



AJP**80**, 246 (2012)

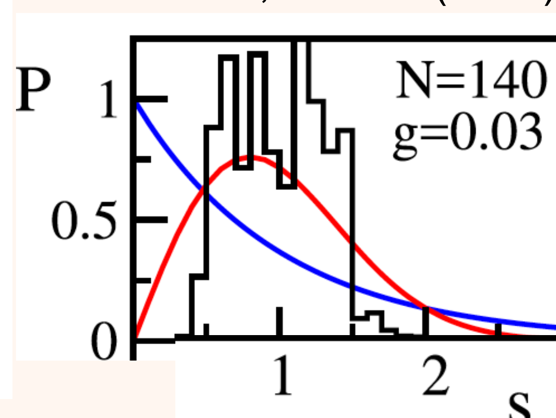
Integrable models:

- Degeneracies
- Picket-fence spectrum



PRE**88,** 032913 (2013)

NJP**20**, 113039 (2018)



1D Spin-1/2 Systems

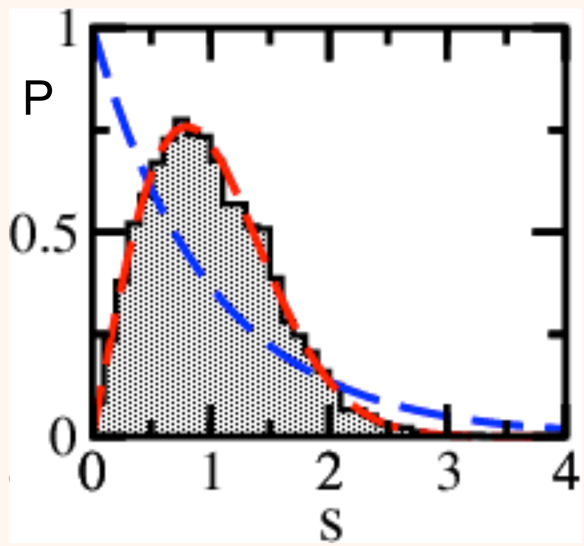
Integrable system:

XXZ model

$$H=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

Interaction between next-nearest neighbors

$$H_{NN}+H_{NNN}=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)+$$
$$+\lambda\sum_{n=1}^{L-2}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+2}^z+\sigma_n^x\sigma_{n+2}^x+\sigma_n^y\sigma_{n+2}^y)$$

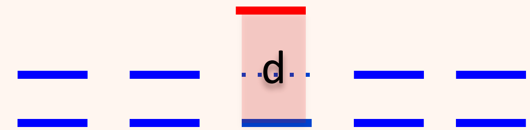


Many-body localization

$$H=\sum_{n=1}^{L}\frac{\varepsilon_n}{2}\sigma_n^z+\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

Speck of Chaos

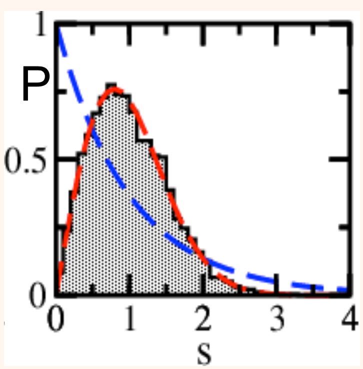
$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1})$$

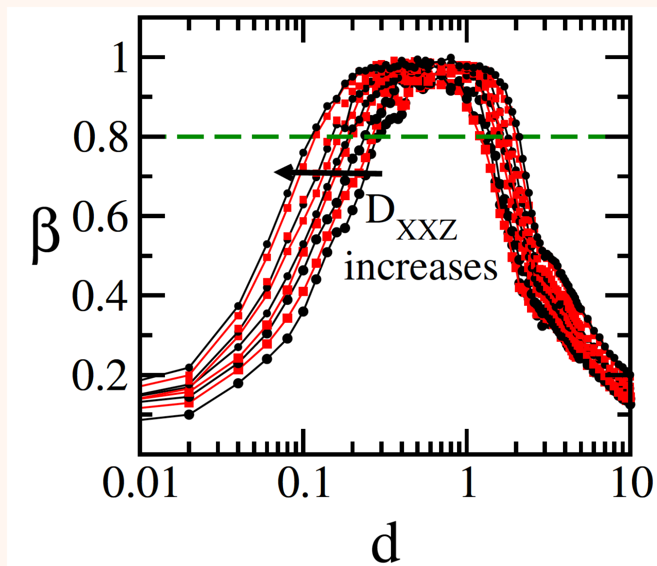


LFS,
JPA **37**, 4723 (2004)

$\beta \sim 1$ **chaos**

$$P_{WD}(s) = \frac{\pi\, s}{2}\exp\left(-\frac{\pi\, s^2}{4}\right)$$





(local perturbation)

Gubin & LFS
AJP **80**, 246 (2012)

$$P(s) = (\beta+1)bs^{\beta}\exp(-bs^{\beta+1})$$

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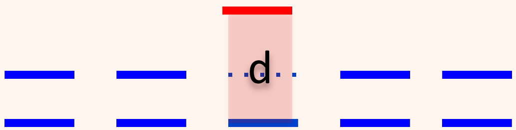
Speck of Chaos

$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1})$$



LFS,
JPA **37**, 4723 (2004)

Chaos: Level statistics, chaotic eigenstates,
diagonal and off-diagonal elements of O
Chaos is the mechanism for **thermalization**
Chaos is the condition for the validity of ETH

Torres & LFS
PRE **89**, 062110 (2014)

Ballistic quantum transport

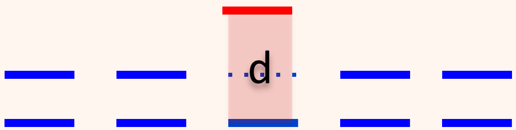
Brenes, Mascarenhas,
Rigol & Goold
PRB **98**, 235128 (2018)

M. Znidaric
PRL**125**, 180605 (2020)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

Speck of Chaos

$$H_{XXZ} = \boxed{d_{L/2} S^z_{L/2}} + J \sum_{n=1}^{L-1} (S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + \Delta S^z_n S^z_{n+1})$$



LFS,
JPA **37**, 4723 (2004)

$$H_{ZZ} = \boxed{d_{L/2} S^z_{L/2}} + J h_x \sum_{n=1}^{L-1} S^x_n - J \sum_{n=1}^{L-1} S^z_n S^z_{n+1}$$

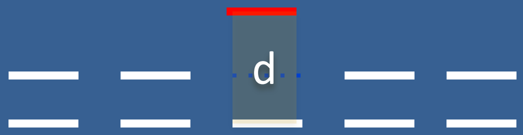
Ising model in a transverse field
Spin-1/2

$$H_{S1} = \boxed{d_{L/2} S^z_{L/2}} + J \sum_{n=1}^{L-1} (S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + S^z_n S^z_{n+1})$$
$$+ J \sum_{n=1}^{L-1} ((S^x_n S^x_{n+1})^2 + (S^y_n S^y_{n+1})^2 + (S^z_n S^z_{n+1})^2)$$

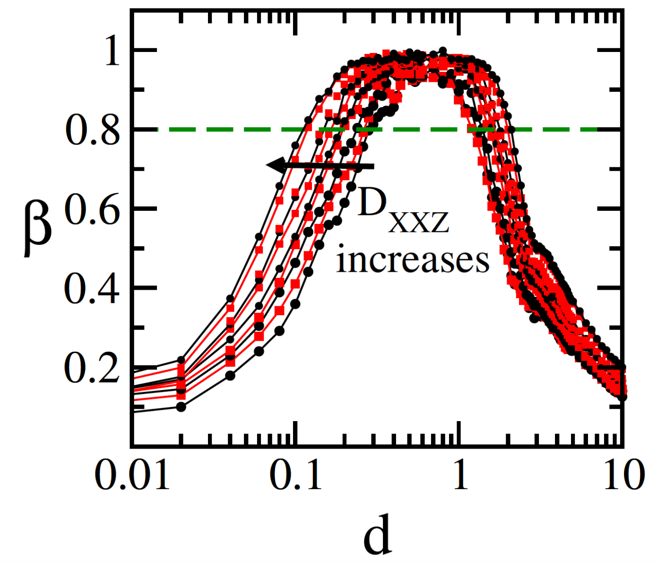
Lai-Sutherland model
Spin-1

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

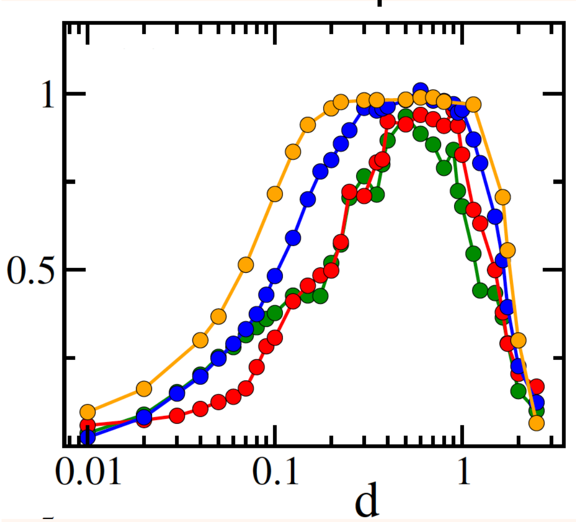
Speck of Chaos



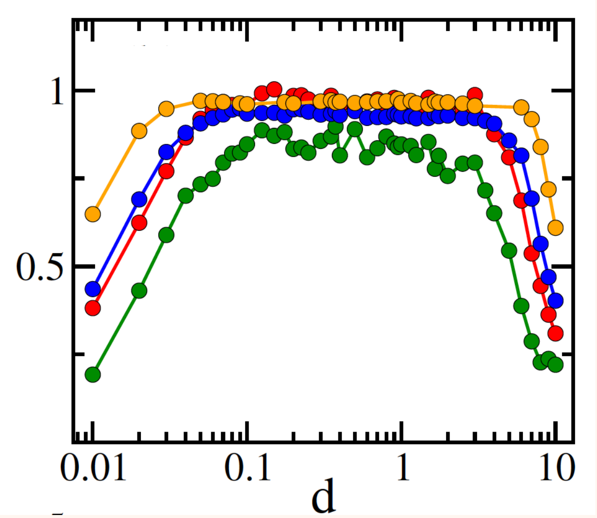
XXZ + defect
Spin-1/2

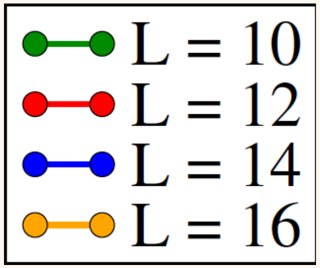


Ising + defect
in a transverse field
Spin-1/2



Lai-Sutherland + defect
Spin-1





$$P(s) = (\beta + 1) b s^{\beta} \exp(-b s^{\beta+1})$$

$\beta \sim 1$ **chaos**

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

Lea F. Santos, Yeshiva University

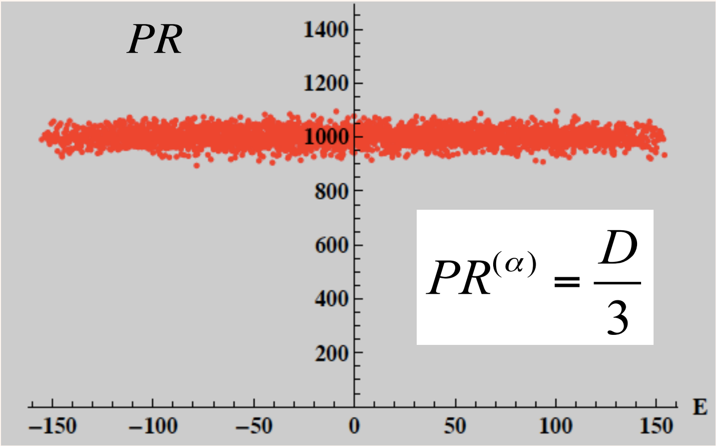
Hamiltonian matrix: Spin-1/2 model NN couplings

	110000	101000	100100	100010	100001	011000	010100	010010	010001	001100	001010	001001	000110	000101	000011
110000	H_{11}	J/2	0	0	0	0	0	0	0	0	0	0	0	0	0
101000	J/2	H_{22}	J/2	0	0	J/2	0	0	0	0	0	0	0	0	0
100100	0	J/2	H_{33}	J/2	0	0	J/2	0	0	0	0	0	0	0	0
100010	0	0	J/2	H_{44}	J/2	0	0	J/2	0	0	0	0	0	0	0
100001	0	0	0	J/2	H_{55}	0	0	0	J/2	0	0	0	0	0	0
011000	0	J/2	0	0	0	H_{66}	J/2	0	0	0	0	0	0	0	0
010100	0	0	J/2	0	0	J/2	H_{77}	J/2	0	J/2	0	0	0	0	0
010010	0	0	0	J/2	0	0	J/2	H_{88}	J/2	0	J/2	0	0	0	0
010001	0	0	0	0	J/2	0	0	J/2	H_{99}	0	0	J/2	0	0	0
001100	0	0	0	0	0	0	J/2	0	0	H_{1010}	J/2	0	0	0	0
001010	0	0	0	0	0	0	0	J/2	0	J/2	H_{1111}	J/2	J/2	0	0
001001	0	0	0	0	0	0	0	0	J/2	0	J/2	H_{1212}	0	J/2	0
000110	0	0	0	0	0	0	0	0	0	0	J/2	0	H_{1313}	J/2	0
000101	0	0	0	0	0	0	0	0	0	0	0	J/2	J/2	H_{1414}	J/2
000011	0	0	0	0	0	0	0	0	0	0	0	0	0	J/2	H_{1515}

Eigenstates

Eigenstates of full random matrices are random vectors

$$|\alpha\rangle = \sum_{n=1}^{D} C_n^{\alpha} |n\rangle \qquad PR^{(\alpha)} \equiv \frac{1}{\sum_{n=1}^{D} |C_n^{(\alpha)}|^4}$$



$$|\alpha\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \ldots \end{pmatrix}$$

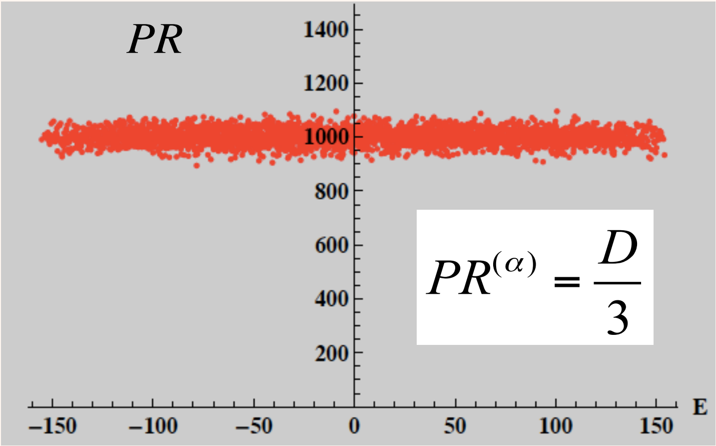
Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

Eigenstates

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)

Eigenstates of full random matrices are random vectors

$$|\alpha\rangle = \sum_{n=1}^{D} C_n^{\alpha} |n\rangle \qquad PR^{(\alpha)} \equiv \frac{1}{\sum_{n=1}^{D} |C_n^{(\alpha)}|^4}$$

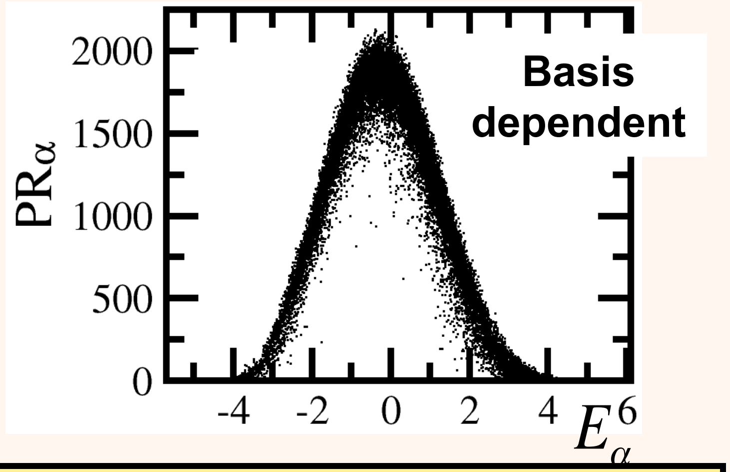


(mean-field basis)

$$H_{NN} + H_{NNN} =$$

$$\sum_{n=1}^{L-1} \frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) +$$

$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4}(\Delta\sigma_n^z\sigma_{n+2}^z + \sigma_n^x\sigma_{n+2}^x + \sigma_n^y\sigma_{n+2}^y)$$



Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

Structure of Initial State

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)
Torres, Vyas, LFS
NJP**16**, 063010 (2014)

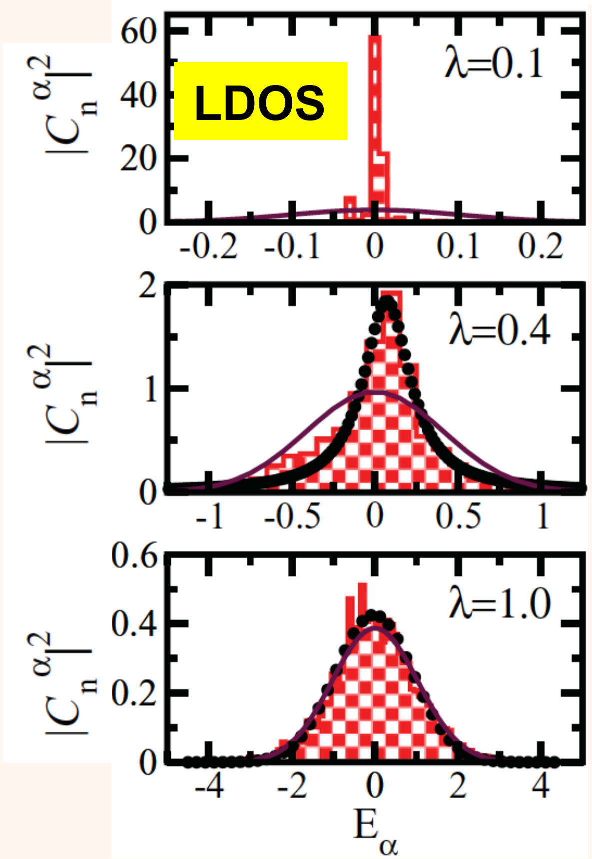
Perturbed Hamiltonian | Unperturbed Hamiltonian | Perturbation

$$\boxed{H = H_0 + \lambda V}$$

Energy Eigenbasis $|\alpha\rangle$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$\rho_{ini}(E) = \sum_\alpha \left|C_\alpha^{ini}\right|^2 \delta(E - E_\alpha)$$



Chaotic Hamiltonian/
Chaotic Eigenstates

Strong Perturbation

Middle of the Spectrum

$$E_{ini} = \langle \Psi(0) | H | \Psi(0) \rangle \sim 0$$

$$|\Psi(0)\rangle = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \ldots \end{pmatrix}$$

Uncorrelated components

Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

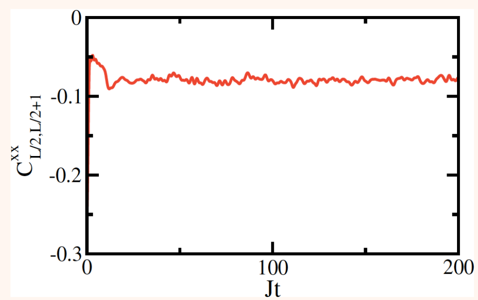
From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics,
L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol,
Adv. Phys. **65**, 239 (2016)

Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle \qquad O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

Equilibration:



Components are small and **uncorrelated**
Lack of degeneracies: eigenvalues are **correlated**
Off-diagonal elements of local observables are small

Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \boxed{\sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}}$$

Infinite time average Thermodynamic average

$$\boxed{\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \xleftrightarrow{=?} O_{micro} \equiv \frac{1}{\mathcal{N}_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}}$$

$$\langle \alpha | O | \alpha \rangle$$

depends on the initial conditions depends only on the energy

ETH: the expectation values $O_{\alpha\alpha}$ of few-body observables do not fluctuate for eigenstates close in energy

Chaos guarantees thermalization, ETH

Chaotic states

Peres Lattice

PRL **53**, 1711 (1984)

$$PR^{(\alpha)} = 1/\sum_{i=1}^{D} |c_i^{(\alpha)}|^4$$

$$\boxed{O_{\alpha\alpha} = \langle \alpha | O | \alpha \rangle}$$

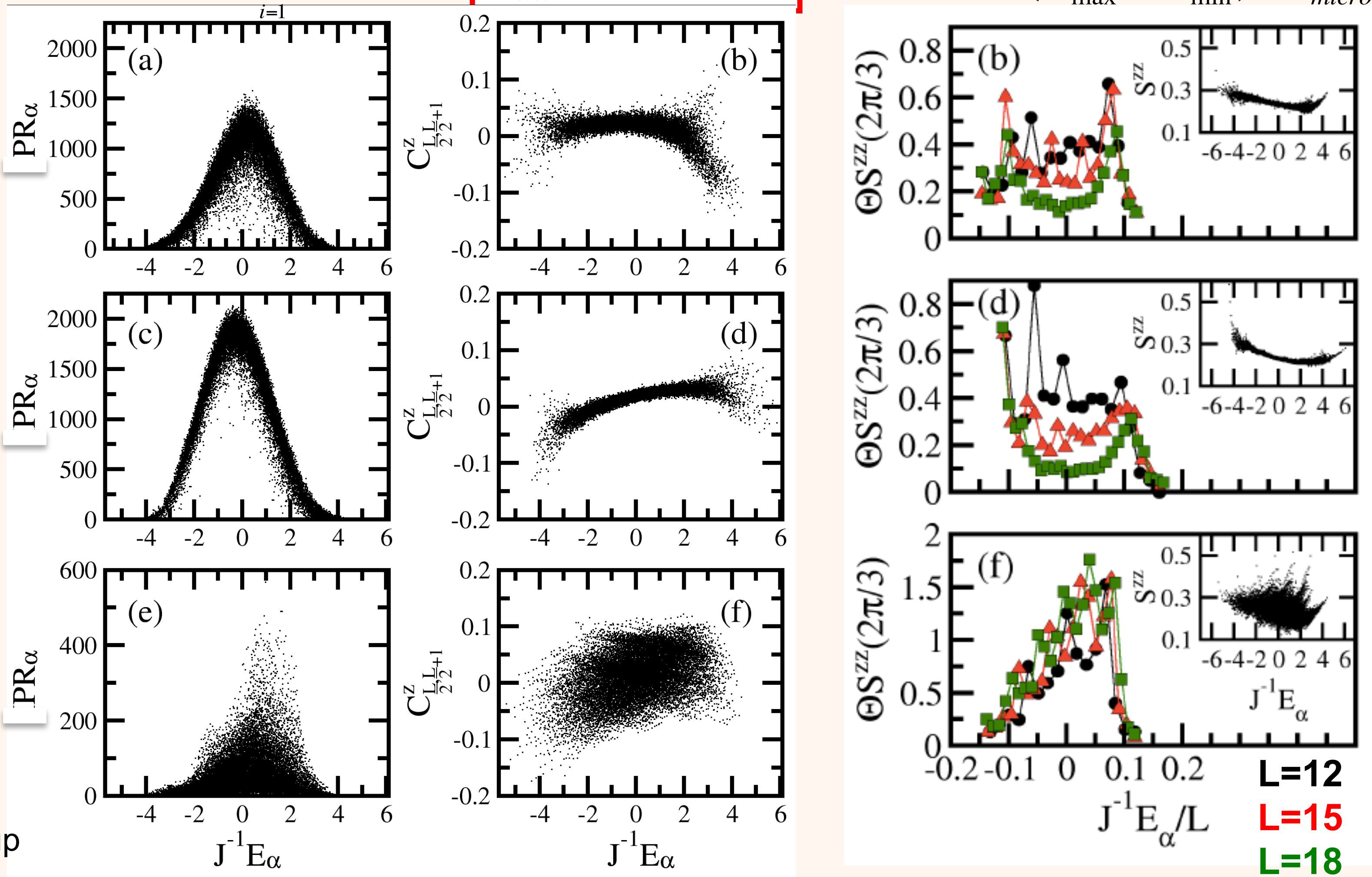
$$\Theta O = (O_{\max} - O_{\min})/O_{micro}$$

Chaotic Single-Defect Model

Chaotic NNN Model

Integrable XXZ Model

L=18, 1/3 up



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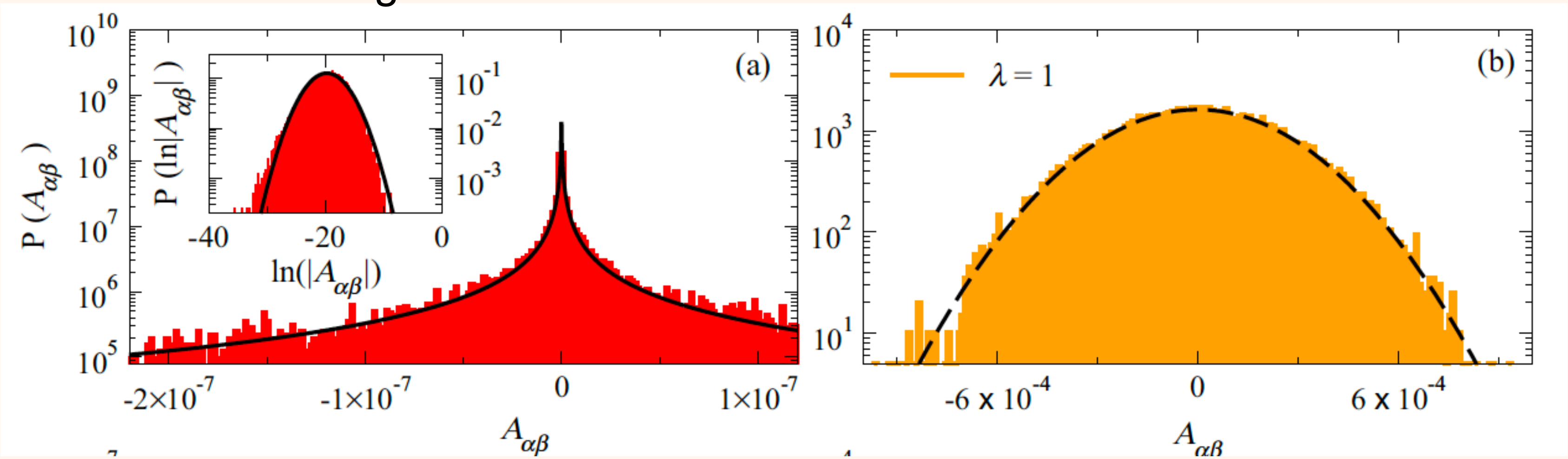
PRE **89**, 062110 (2014)

Off-diagonal elements

Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)
(Gaussian distribution)

$$\boxed{O_{\beta\alpha} = \langle \beta | O | \alpha \rangle}$$

Integrable model   Chaotic model



LeBlond, Mallayya, Vidmar, Rigol
PRE **100**, 062134 (2019)

Defect models
Off-diagonal elements

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

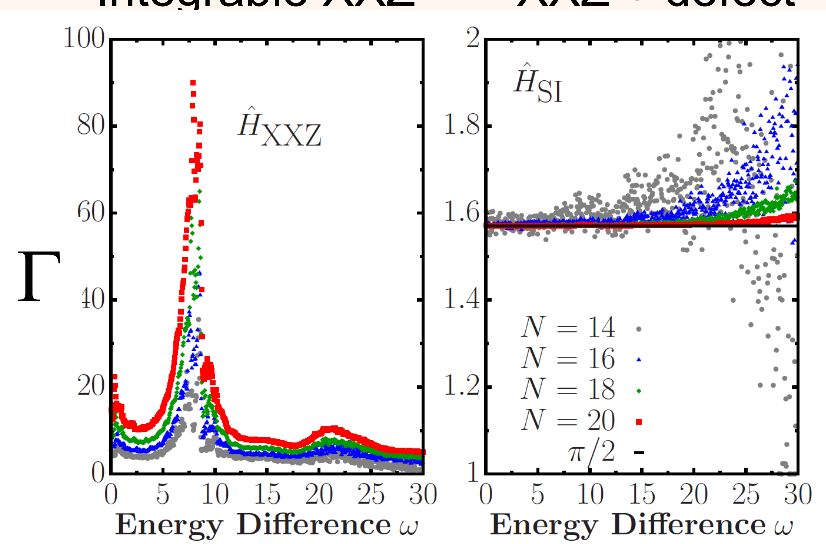
$$\Gamma(\omega) = \frac{\overline{|\langle \alpha | O | \beta \rangle|^2}}{\overline{|\langle \alpha | O | \beta \rangle|}^2}$$

$\Gamma(\omega)=\pi/2$

means Gaussian distribution

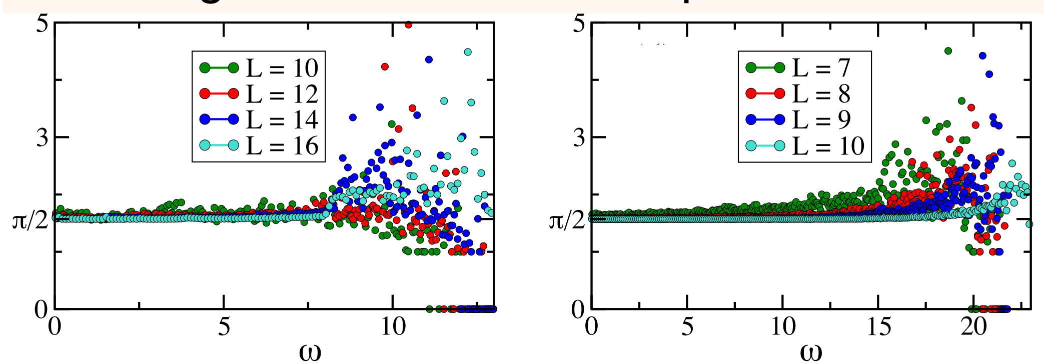
Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)
(Gaussian distribution)

Integrable XXZ   XXZ + defect



Brenen, Goold, Rigol
PRB **102**, 075127 (2020)

Ising + defect   Spin 1 + defect



Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

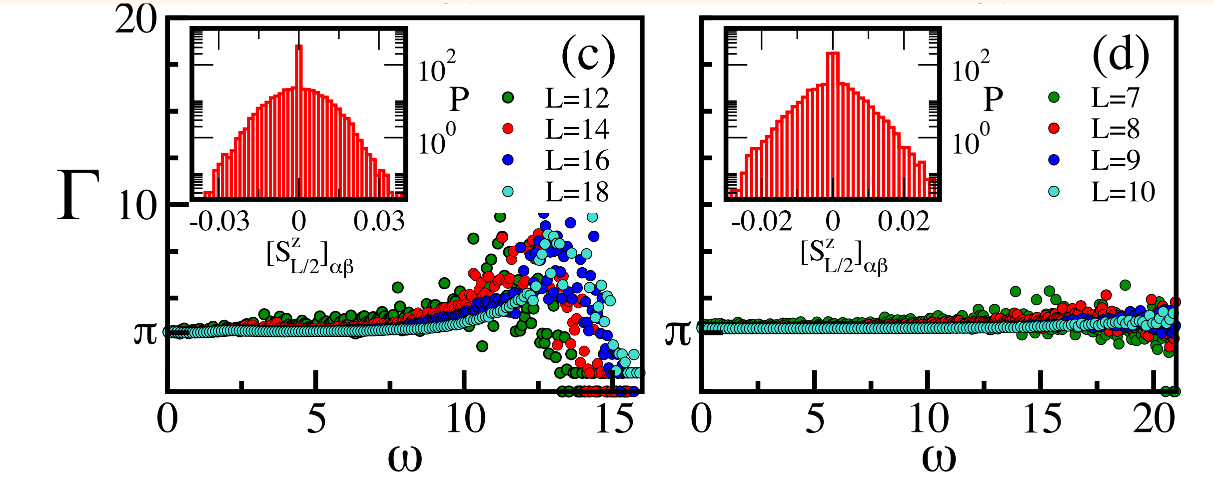
Off-diagonal elements and symmetries

$$\Gamma(\omega) = \overline{\left|\langle \alpha | S^z_{L/2} | \beta \rangle\right|^2} \Big/ \overline{\left|\langle \alpha | S^z_{L/2} | \beta \rangle\right|}^2$$

No need for unfolding
Detect chaos despite symmetries

NN+NNN

Spin 1 + border



Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

Lea F. Santos, Yeshiva University

From Few- to Many-Body Quantum Systems From a Chaos Perspective

From few- to many-body quantum systems
M. Schiulaz, M. Távora, LFS
Quantum Sci. Technol. **3**, 044006 (2018)

How many particles make up a chaotic many-body quantum system?
G. Zisling, LFS, Y. Bar Lev
arXiv:2012.14436

What is special about 4?

From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz, G. Zürn, S. Murmann, I. Brouzos, T. Lompe, S. Jochim

Science **342**, 457 (2013)

Knowing when a physical system has reached sufficient size for its macroscopic properties to be well described by many-body theory is difficult. We investigated the crossover from few- to many-body physics by studying quasi–one-dimensional systems of ultracold atoms consisting of a single impurity interacting with an increasing number of identical fermions. We measured the interaction energy of such a system as a function of the number of majority atoms for different strengths of the interparticle interaction. As we increased the number of majority atoms one by one, we observed fast convergence of the normalized interaction energy toward a many-body limit calculated for a single impurity immersed in a Fermi sea of majority particles.

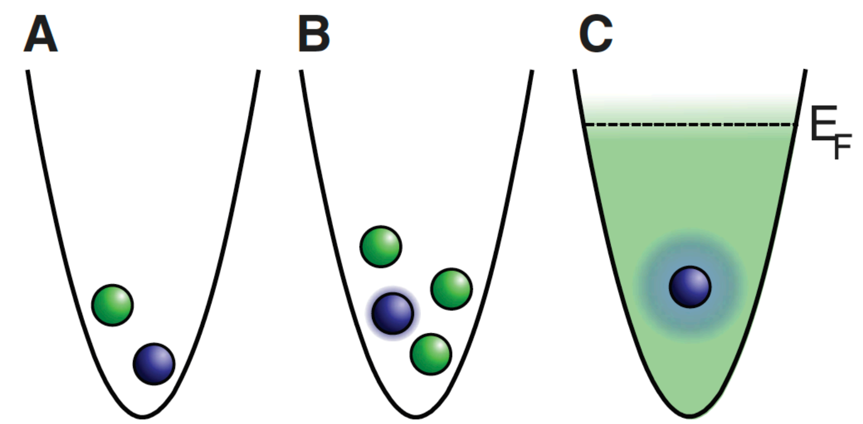


Fig. 1. From few to many. A single impurity (blue) interacting with one, few, and many fermions (green)

What is special about 4?

Statistical properties of fermionic molecular dynamics
J. Schnack, H. Feldmeier
Nuclear Physics A **601,** 181 (1996)

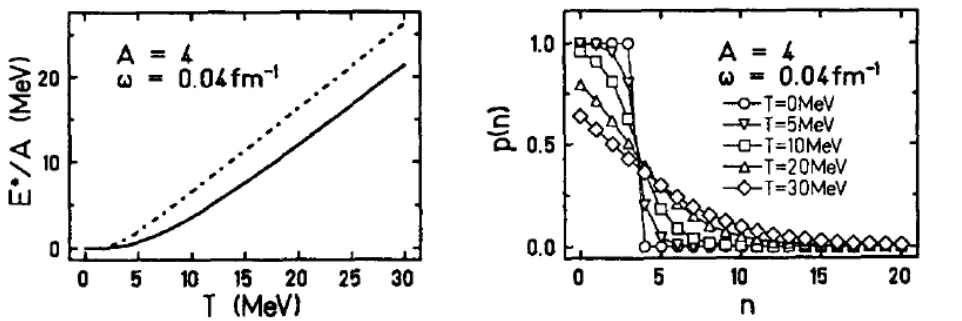


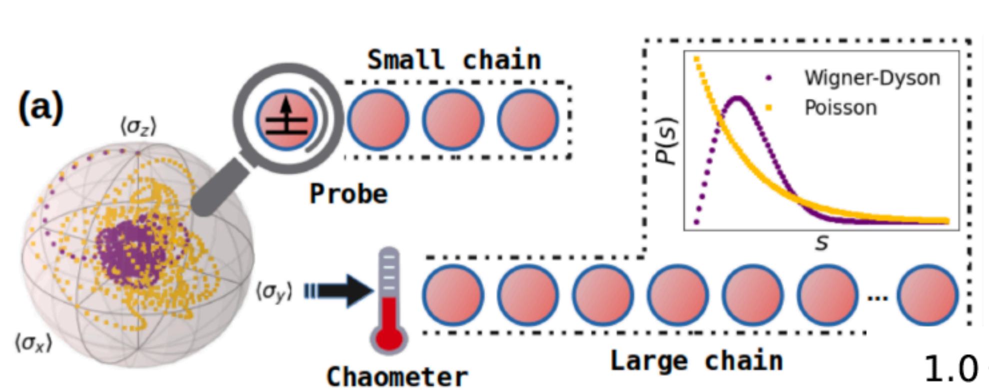
Fig. 1. A system of four fermions in a common oscillator described by the canonical ensemble. L.h.s.: Excitation energy as a function of temperature (solid line). The dashed-dotted line shows the result for a product state (Boltzmann statistics). R.h.s.: Occupation numbers $p(n)$ of the oscillator eigenstates for five temperatures (Eq. (16)). The lines are drawn as a guide for the eye.

Towards a statistical theory of finite Fermi systems and compound states:
Random two-body interaction approach
V. V. Flambaum, F. M. Izrailev, and G. Casati
Physical Review E. **54**, 2136 (1996)

Structure of compound states in the chaotic spectrum of the Ce atom: Localization properties, matrix elements, and enhancement of weak perturbation [*cerium atom, 4 valence electrons*]
Flambaum, Gribakina, Gribakin, Kozlov
PRA **50**, 267 (1994)

What is special about 3-4?

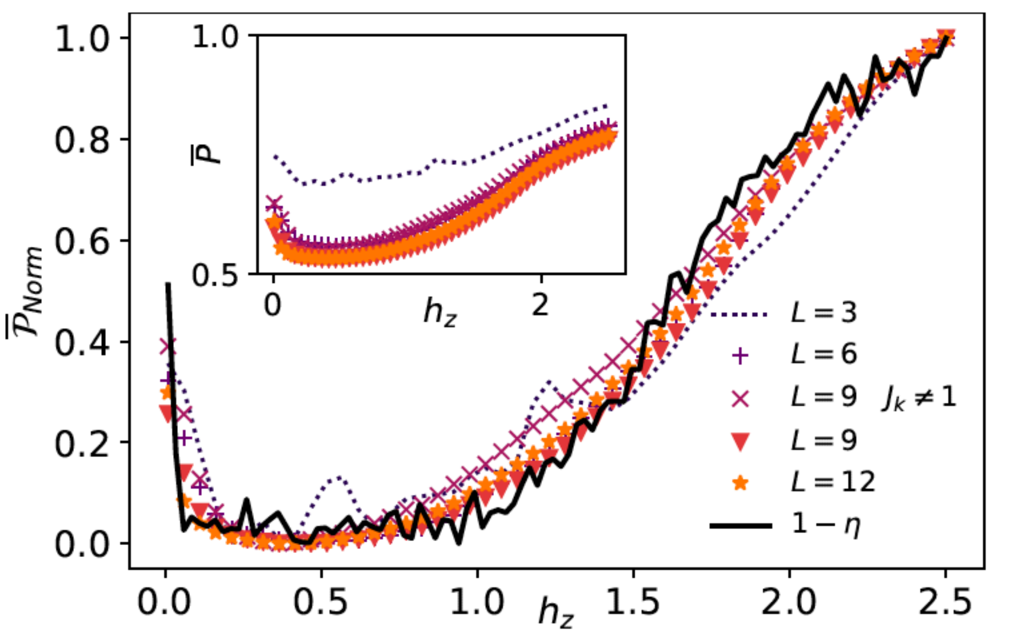
Quantum chaos, equilibration and control in extremely short spin chains
Nicolás Mirkin and Diego Wisniacki
arXiv:2006.14468



$$H = \sum_{k=1}^{L} \left(h_x \hat{\sigma}_k^x + h_z \hat{\sigma}_k^z\right) - \sum_{k=1}^{L-1} J_k \hat{\sigma}_k^z \hat{\sigma}_{k+1}^z$$

$$\overline{\mathcal{P}} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{T} \int_0^T \mathrm{Tr}\left[\tilde{\rho}_i^2(t)\right] dt \right)$$

$$\overline{\mathcal{P}}_{Norm} = \frac{\overline{\mathcal{P}} - \min\left(\overline{\mathcal{P}}\right)}{\max\left(\overline{\mathcal{P}}\right) - \min\left(\overline{\mathcal{P}}\right)}$$



Quantum Chaos
From few- to many-body quantum systems

Spin-1/2 model:

$$H=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

excitations

$$|\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

Map into hardcore bosons:

$$H=\sum_{n=1}^{L}\left[V\left(b_n^+b_n-\frac{1}{2}\right)\left(b_{n+1}^+b_{n+1}-\frac{1}{2}\right)-t\left(b_n^+b_{n+1}+h.c.\right)\right]$$

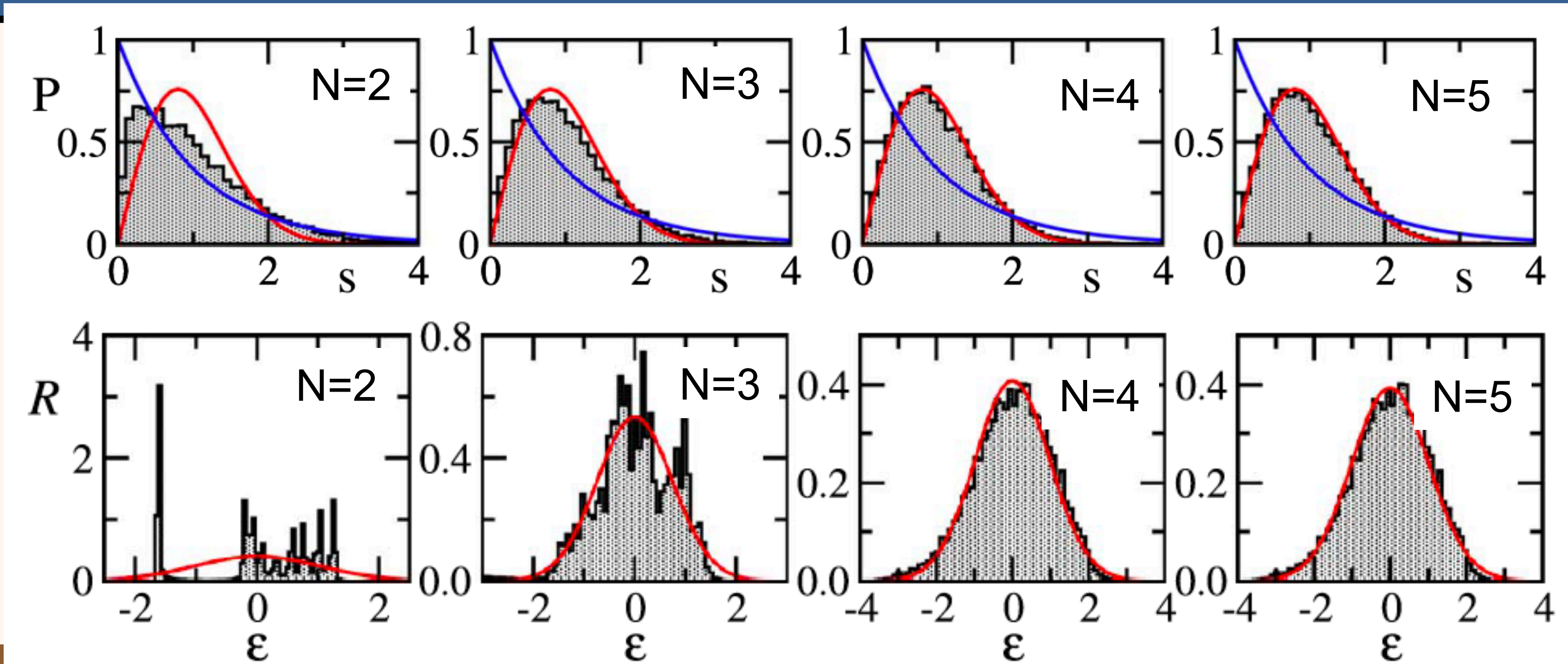
particles

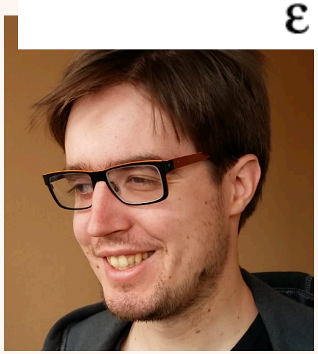
$$|1110000000\rangle$$

From few- to many-body quantum systems

M. Schiulaz, M. Távora, LFS

Quantum Sci. Technol. **3**, 044006 (2018)





$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4}\left(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)$$

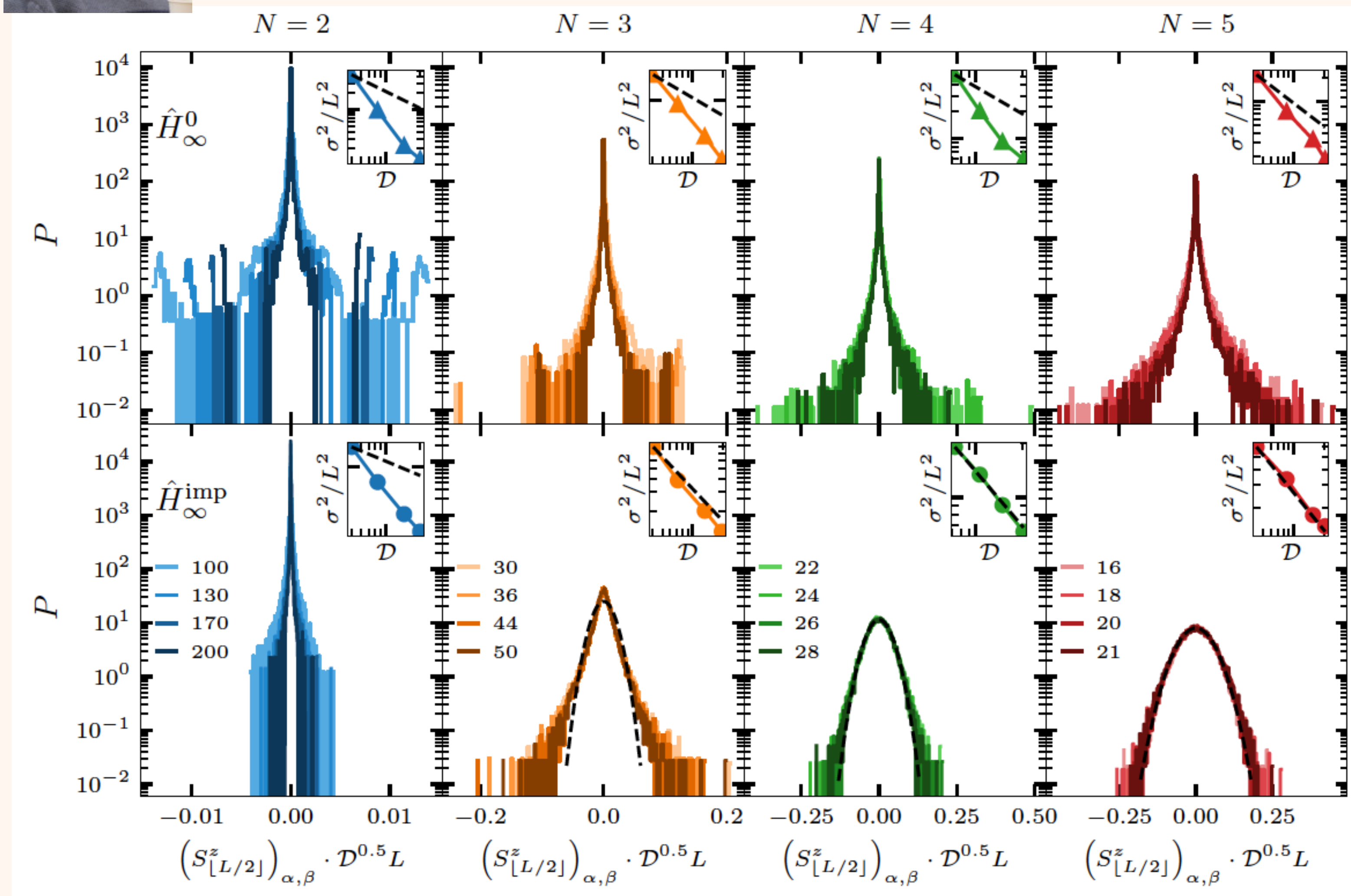
$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4}\left(\Delta\sigma_n^z\sigma_{n+2}^z + \sigma_n^x\sigma_{n+2}^x + \sigma_n^y\sigma_{n+2}^y\right)$$

Lea F. Santos, Yeshiva University

How many particles make up a chaotic many-body quantum system?

G. Zisling, LFS, Y. Bar Lev

arXiv:2012.14436



integrable
1D XXZ

chaotic
single-defect

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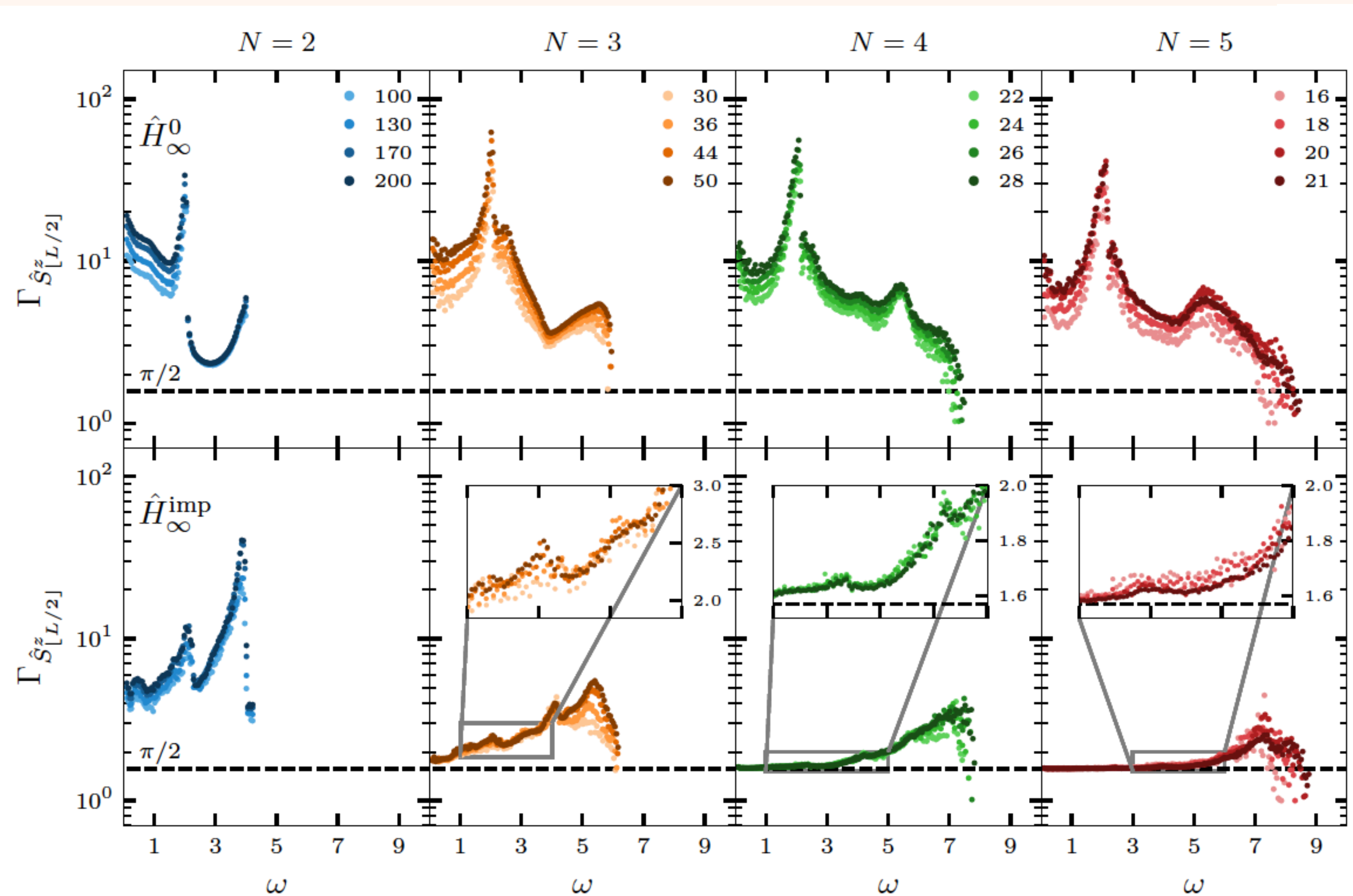

$$= \frac{\overline{\left|\langle\alpha|O|\beta\rangle\right|^2}}{\overline{\left|\langle\alpha|O|\beta\rangle\right|}^2}$$

integrable 1D XXZ

chaotic single-defect

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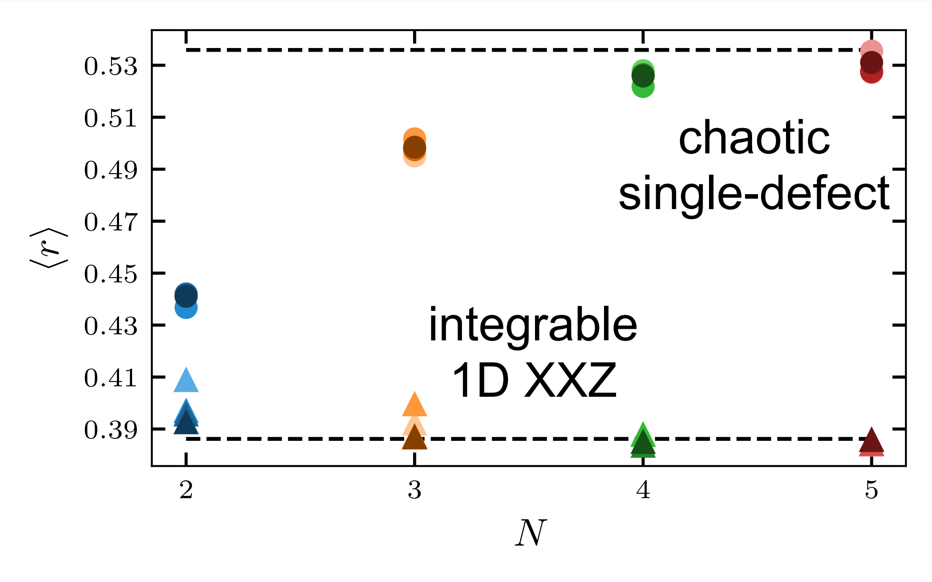


Ratio of consecutive level spacings

$$r_\alpha = \min\left(\frac{s_\alpha}{s_{\alpha-1}}, \frac{s_{\alpha-1}}{s_\alpha}\right)$$

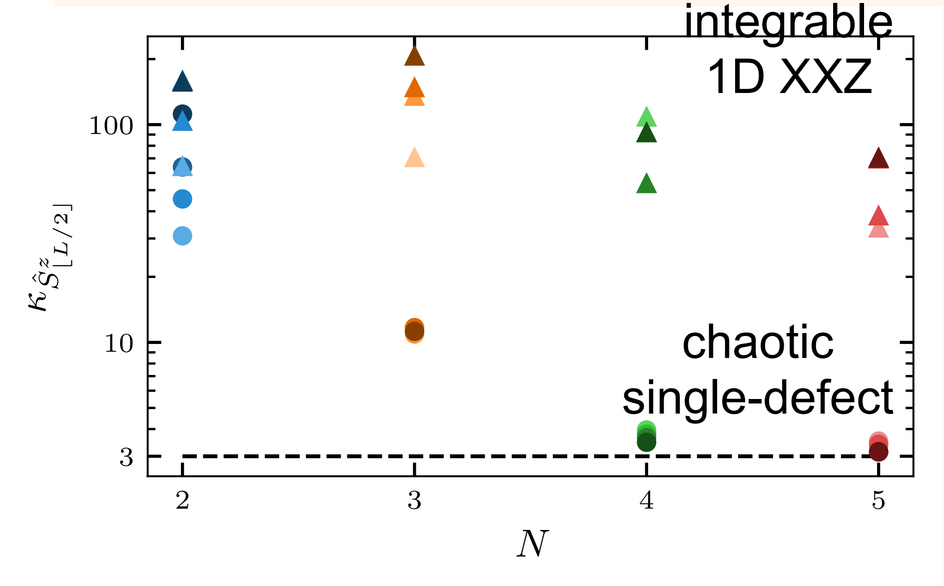
Chaos: <r> = 0.536

Poisson: <r> = 0.39

Kurtosis

$$\kappa_{\hat{O}} = \frac{1}{\sigma^4}\left\langle \left(O_{\alpha\beta} - \langle O_{\alpha\beta}\rangle\right)^4 \right\rangle$$

Chaos: κ=3



How many particles make up a chaotic many-body quantum system?

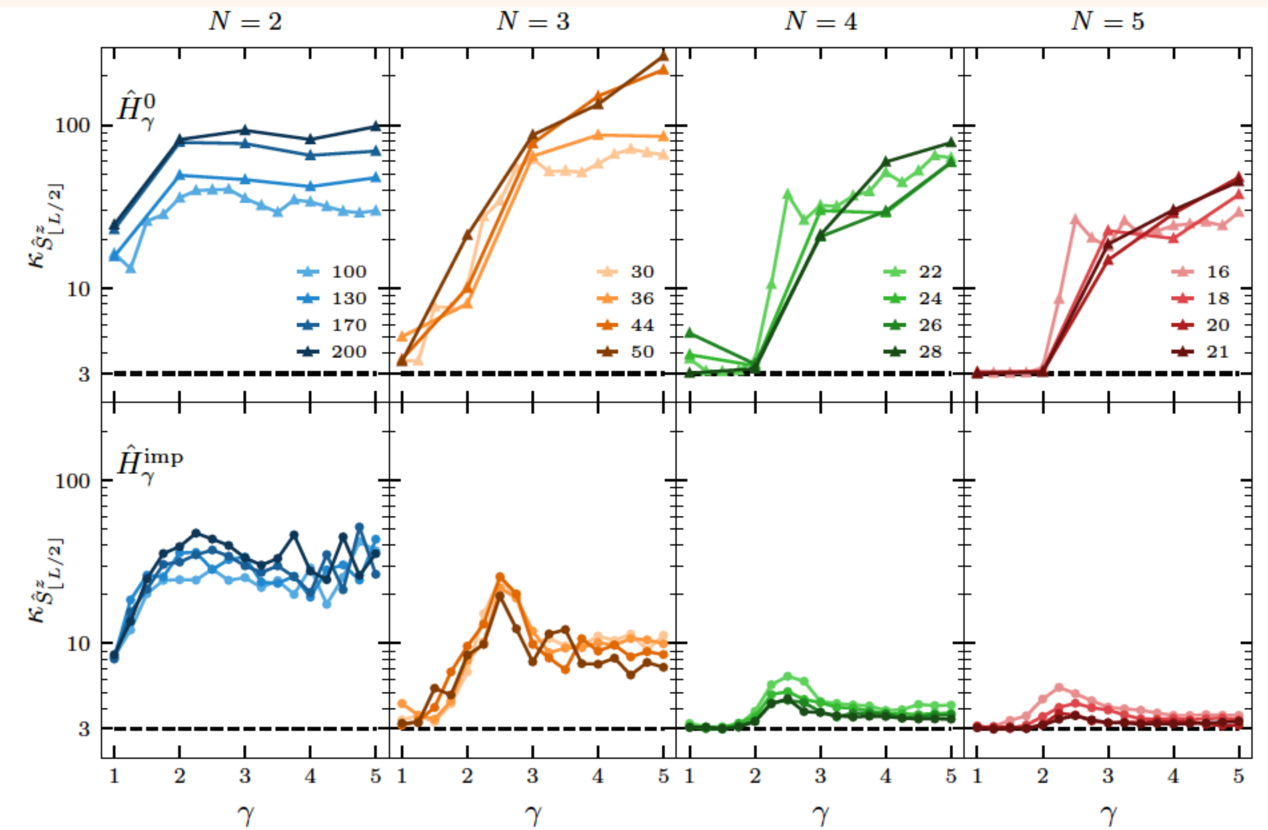
G. Zisling, LFS, Y. Bar Lev

arXiv:2012.14436

$$H_\gamma = \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \frac{J\left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z\right)}{(j-i)^\gamma} + h_{L/2} S_{L/2}^z$$

KURTOSIS = 3
means Gaussian

$$\kappa_{\hat{O}} = \frac{1}{\sigma^4}\left\langle \left(O_{\alpha\beta} - \langle O_{\alpha\beta}\rangle\right)^4 \right\rangle$$



No defect

With single defect

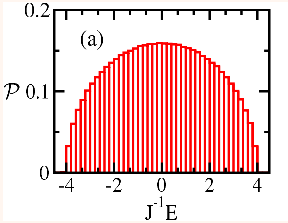
How many particles make up a chaotic many-body quantum system?

- Long range of interactions
- Larger spins
- Larger dimensions

-- What is special about N=4?

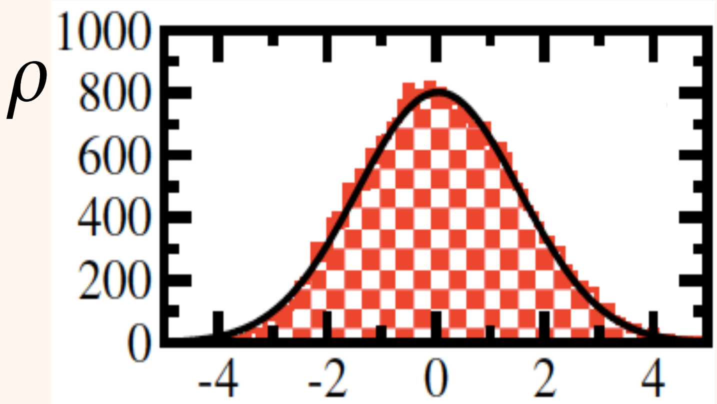
-- Is there a threshold or is it just a crossover?

Gaussian DOS



Wigner (1957)

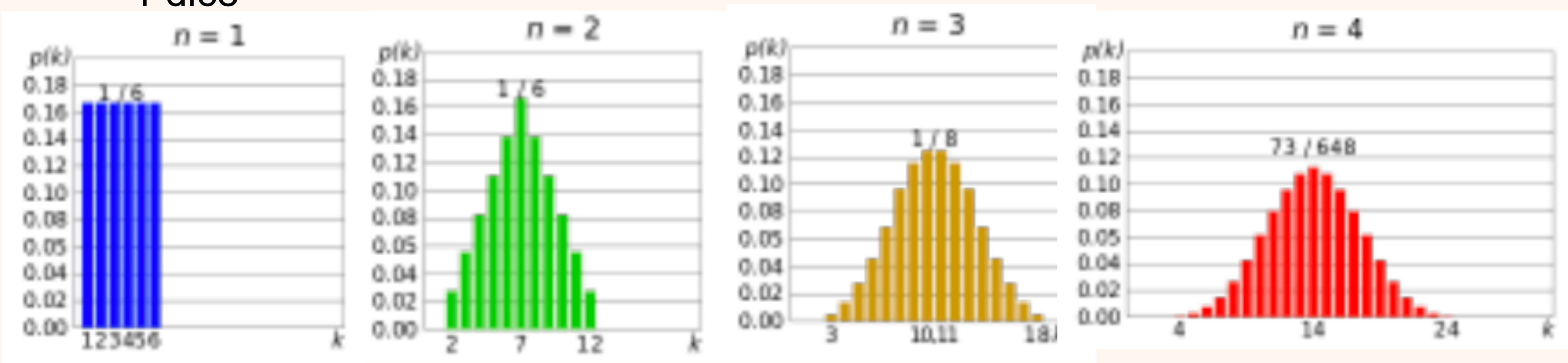
Many-body quantum systems with two-body interactions: Gaussian



French & Wong, PLB (1970)

1 dice

4 dice



From few- to many-body quantum systems

M. Schiulaz, M. Távora, LFS

Quantum Sci. Technol. **3**, 044006 (2018)

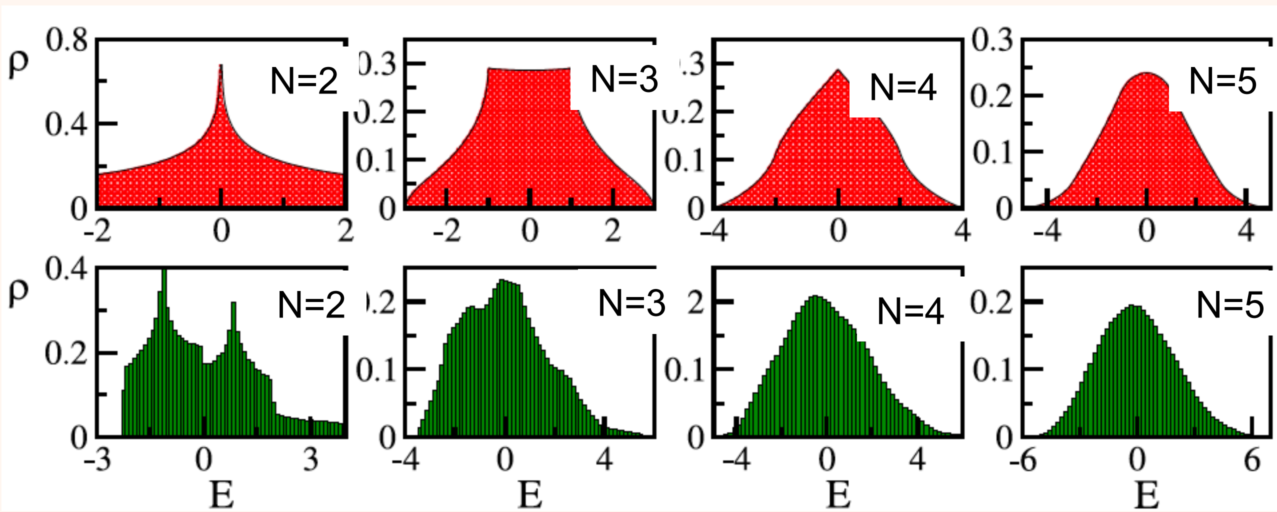
XX model

$$H_{XX} = \sum_{n=1}^{L-1} \frac{J}{4}(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

$$E_\alpha = J \sum_{i=1}^{N} \cos\left(\frac{2\pi k_i}{L}\right)$$

$$k_1 < k_2 < \cdots < k_N$$

$$k_i \in \{0, \pm 1, \pm 2, \cdots, \pm(L/2-1), L/2\}$$



NN+NNN model

$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) +$$

$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4}(\Delta\sigma_n^z\sigma_{n+2}^z + \sigma_n^x\sigma_{n+2}^x + \sigma_n^y\sigma_{n+2}^y)$$

DYNAMICS

From few- to many-body quantum systems

M. Schiulaz, M. Távora, LFS

Quantum Sci. Technol. **3**, 044006 (2018)

$$\left|\Psi(0)\right\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$W_j(t) = |\langle\phi_j|e^{-iHt}|\Psi(0)\rangle|^2$$

$$S_h(t) \equiv -\sum_j W_j(t)\ln W_j(t)$$

Participation 1st-order Rényi entropy
Participation Shannon entropy

measures the spread of the initial many-body state in the exponentially large many-body Hilbert space

Lea F. Santos, Yeshiva University

Instituto Técnico Superior, 2021

Spread in the many-body Hilbert space

$$|\Psi(0)\rangle = |\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle \qquad |\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$$

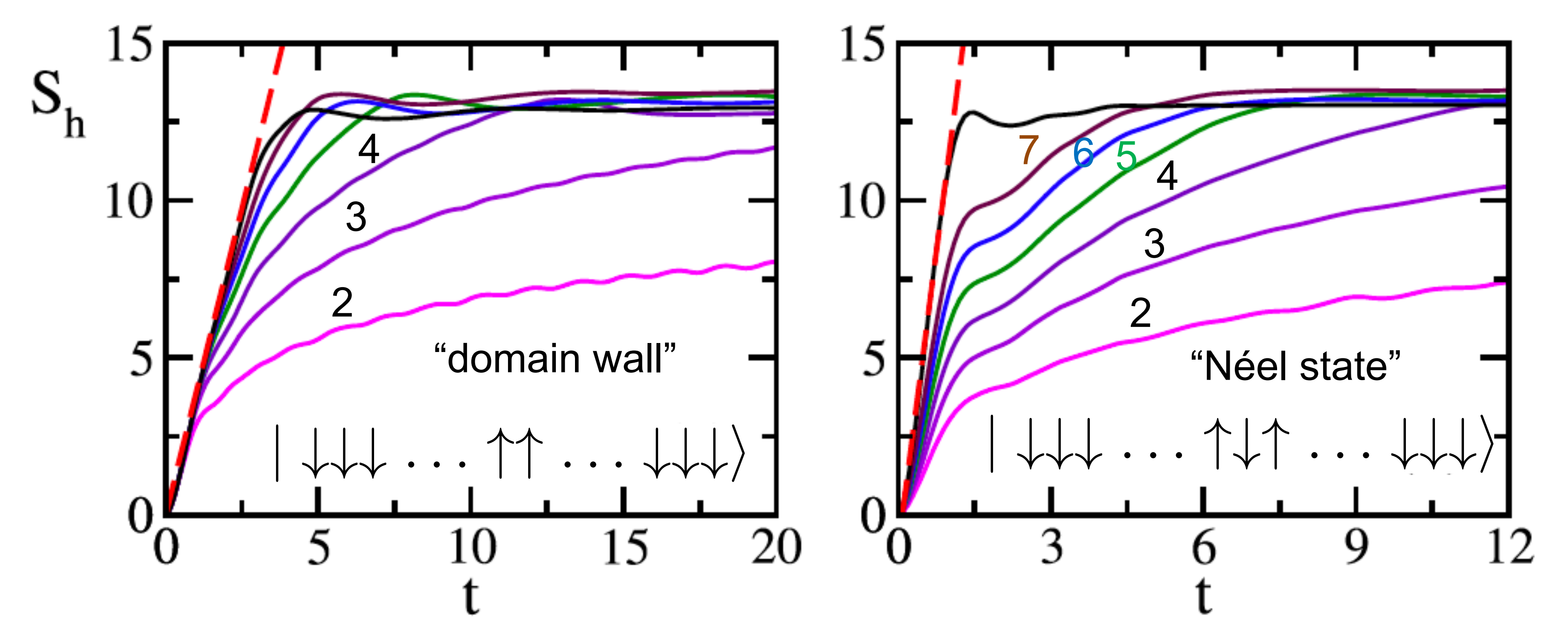
$$|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle \qquad \ldots \qquad \ldots$$

$$W_j(t) = |\langle\phi_j|e^{-iHt}|\Psi(0)\rangle|^2$$

$$S_h(t) \equiv -\sum_j W_j(t) \ln W_j(t)$$

From few- to many-body quantum systems

M. Schiulaz, M. Távora, LFS
Quantum Sci. Technol. **3**, 044006 (2018)



(L,N) = $(1400, 2), (184, 3), (72, 4), (44, 5), (32, 6), (28, 7), (22, 11)$

For N <5:

$$S_h \sim N \ln(t)$$

Thermalization time

Conclusions

- How many interacting particles make up a chaotic many-body quantum system?

- What is special about 4?
- Dependence on range of interactions, size of spins, dimension.
- Is there a threshold? Is it a transition or just a crossover?

- DYNAMICS & TRANSPORT
  Dynamics slows down once the particles get very far from each other.

From few- to many-body quantum systems
M. Schiulaz, M. Távora, LFS
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How many particles make up a chaotic many-body quantum system?
G. Zisling, LFS, Y. Bar Lev
arXiv:2012.14436

Manifestations of Quantum Chaos in the Dynamics

Level statistics is a good approach when we have access to the spectrum: Nuclear Physics

How about experiments with cold atoms and ion traps?

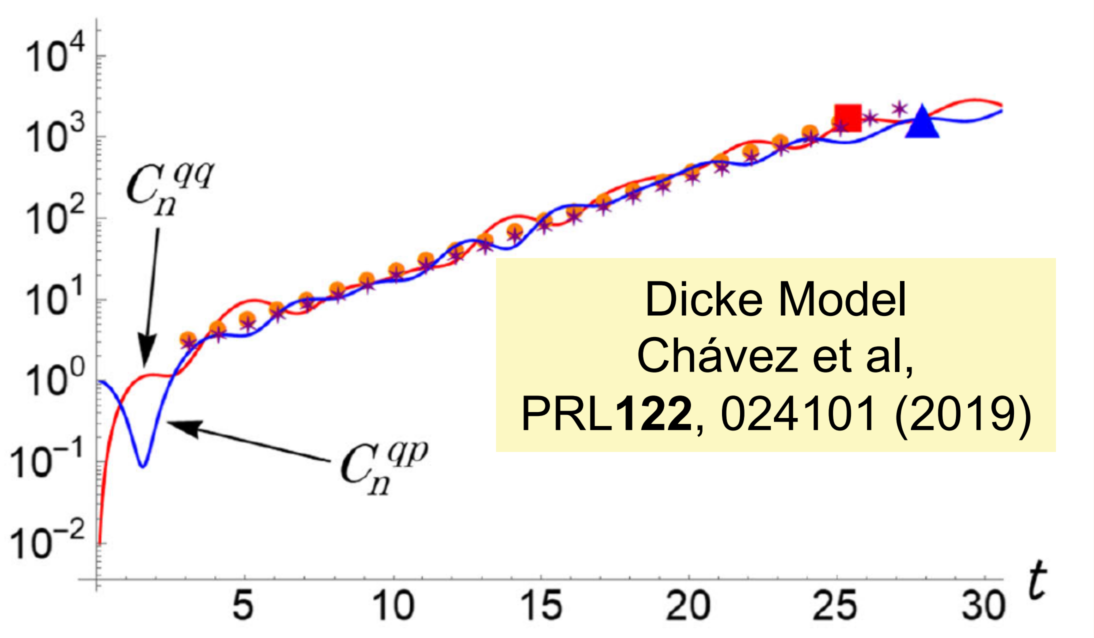
Dynamics: OTOC

Loschmidt Echo $$L(t) = \left| \left\langle \Psi(0) \,|\, e^{iH_2 t} e^{-iH_1 t} \,|\, \Psi(0) \right\rangle \right|^2$$

Out-of-time-ordered four-point correlator $$C(t) = -\left\langle [W(t), V(0)]^2 \right\rangle$$

$$C(t) = -\left\langle W^+(t) V^+(0) W(t) V(0) \right\rangle$$

Kicked Rotor
Rozenbaum, Ganeshan, Galitski,
PRL **118**, 086801(2017)



OTOC detects instability

Is the OTOC a detector of chaos?

(is the exponential growth of the OTOC at short times an indicator of chaos?)

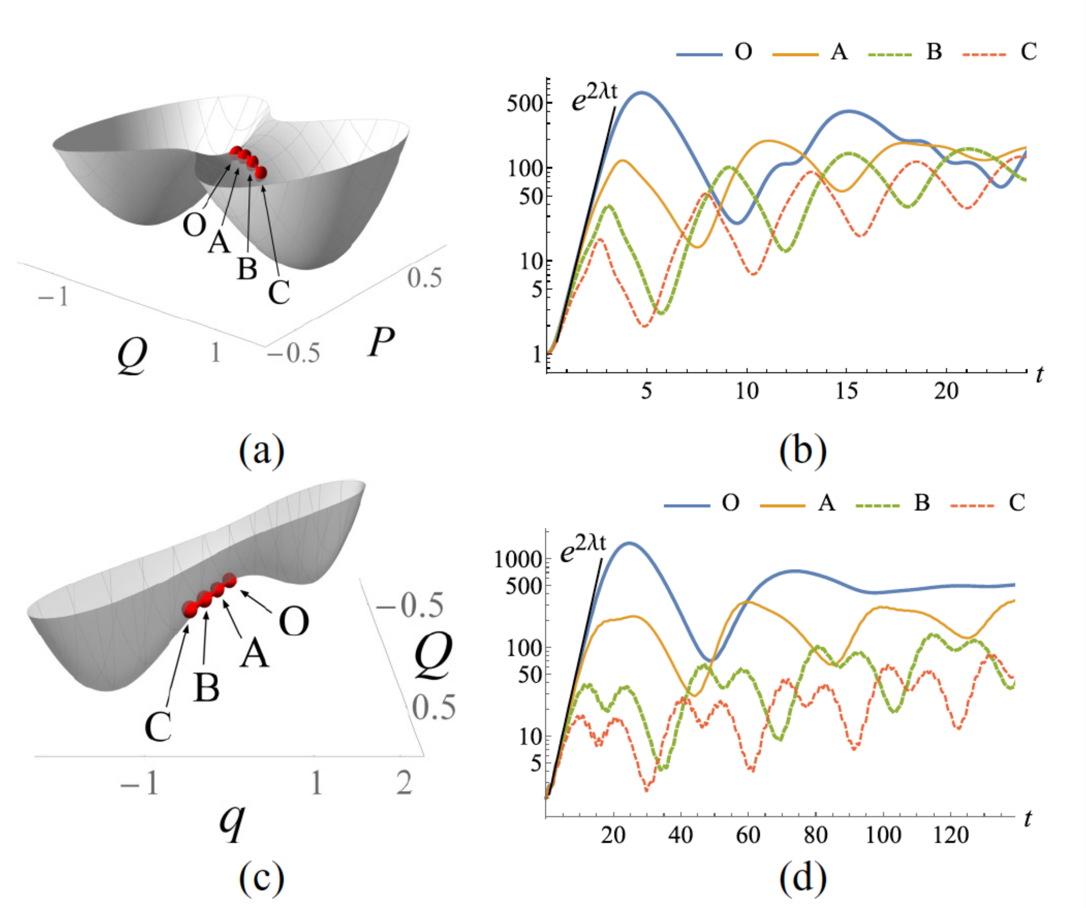
In classical systems:
Positive Lyapunov exponent does not necessarily imply chaos.

Example: **inverted simple pendulum**.
Its upright position corresponds to a stationary point that is unstable.
It has a positive LE, as any genuine chaotic system, but it is completely integrable.
The pendulum does not exhibit chaotic behaviors, such as nonperiodicity and mixing.

OTOC detects instability

Lipkin-Meshkov-Glick model

Dicke model in the regular regime



PRB 98, 134303 (2018)
PRL123,160401 (2019)
PRL124, 140602 (2020)
ArXiv:2007.04746

Pilatowsky et al
PRE 101, 010202(R) (2020)

Long-Time Dynamics: Correlation Hole

Manifestations of spectral correlations in the dynamics?

At long times, when the dynamics resolve the discreteness of the spectrum:
correlation hole

Survival Probability

$$SP(t) = \left| \langle \Psi(0) | \Psi(t) \rangle \right|^2 = \left| \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 e^{-iE_{\alpha}t} \right|^2 \cong \left| \int \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

$$C_{\alpha}^{ini} = \langle \alpha | \Psi(0) \rangle$$

$$\boldsymbol{\rho}_{ini}(E) = \sum_{\boldsymbol{\alpha}} \left| C_{\boldsymbol{\alpha}}^{ini} \right|^2 \boldsymbol{\delta}(E - E_{\boldsymbol{\alpha}})$$

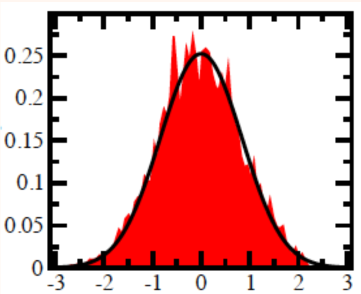
Energy distribution of the initial state

Initial States

$$|\Psi(0)\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

LDOS

ρ



E

Survival Probability

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

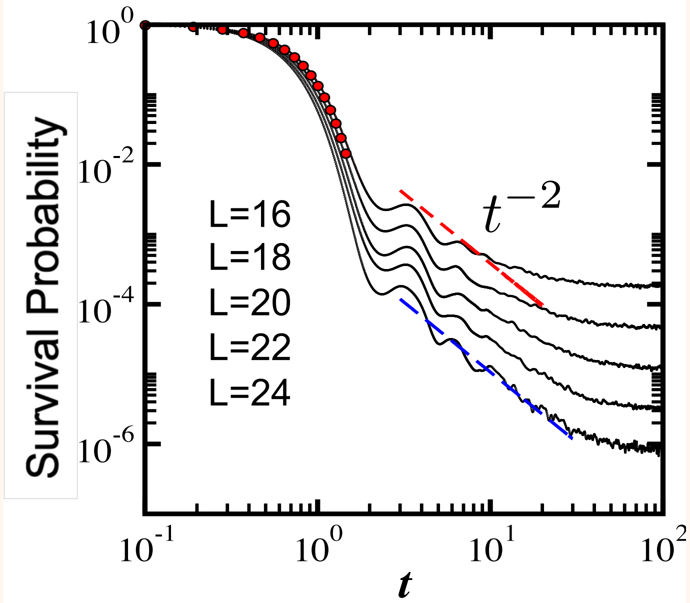
$$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$$

Initial States

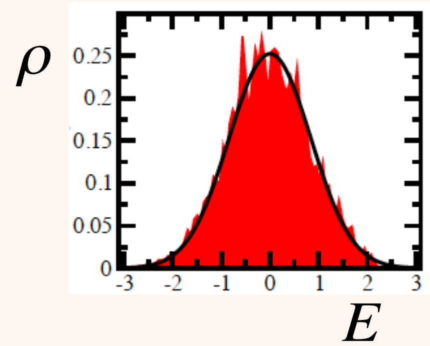
$$|\Psi(0)\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$\boldsymbol{\rho}_{ini}(E) = \sum_{\boldsymbol{\alpha}} \left|C_{\boldsymbol{\alpha}}^{ini}\right|^2 \boldsymbol{\delta}(E - E_{\boldsymbol{\alpha}})$$

Energy distribution of the initial state



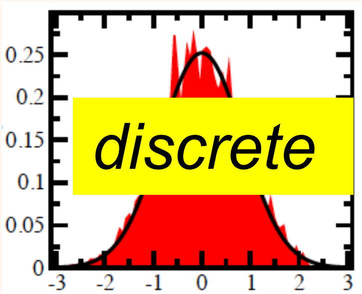
LDOS



Discrete Spectrum

$$SP(t) = \left|\langle \Psi(0) \,|\, \Psi(t) \rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

ρ



E

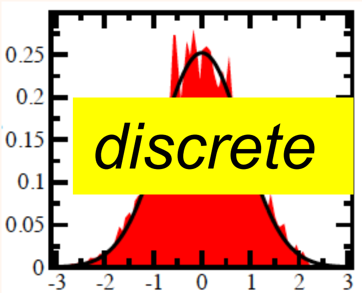
$$= \sum_{\alpha \neq \beta} \left|C_{\alpha}^{ini}\right|^2 \left|C_{\beta}^{ini}\right|^2 e^{-i(E_{\alpha}-E_{\beta})t} + \sum_{\alpha,\beta} \left|C_{\alpha}^{ini}\right|^4$$

$$C_{\alpha}^{ini} = \langle \alpha | \Psi(0) \rangle$$

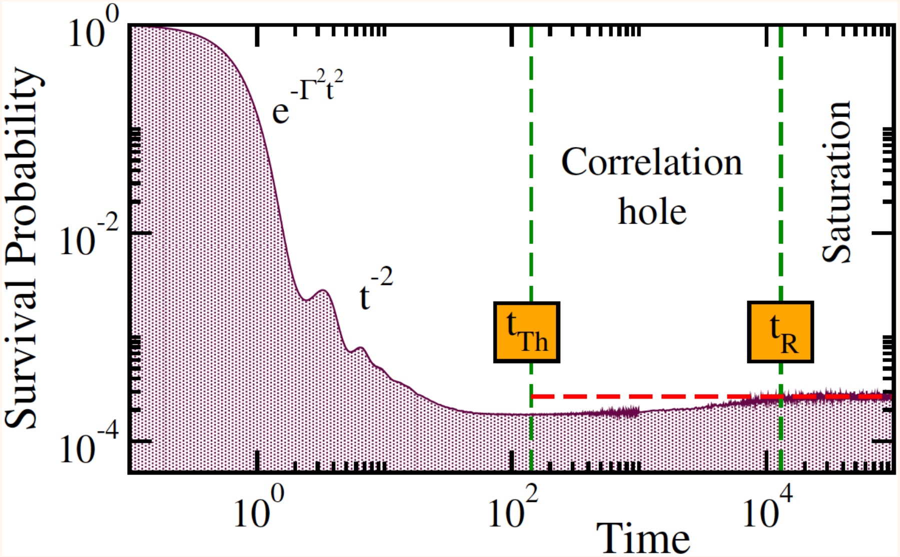
Correlation Hole

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left| \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t} \right|^2$$

ρ



E



$$\sum_{\alpha,\beta} \left|C_{\alpha}^{ini}\right|^4$$

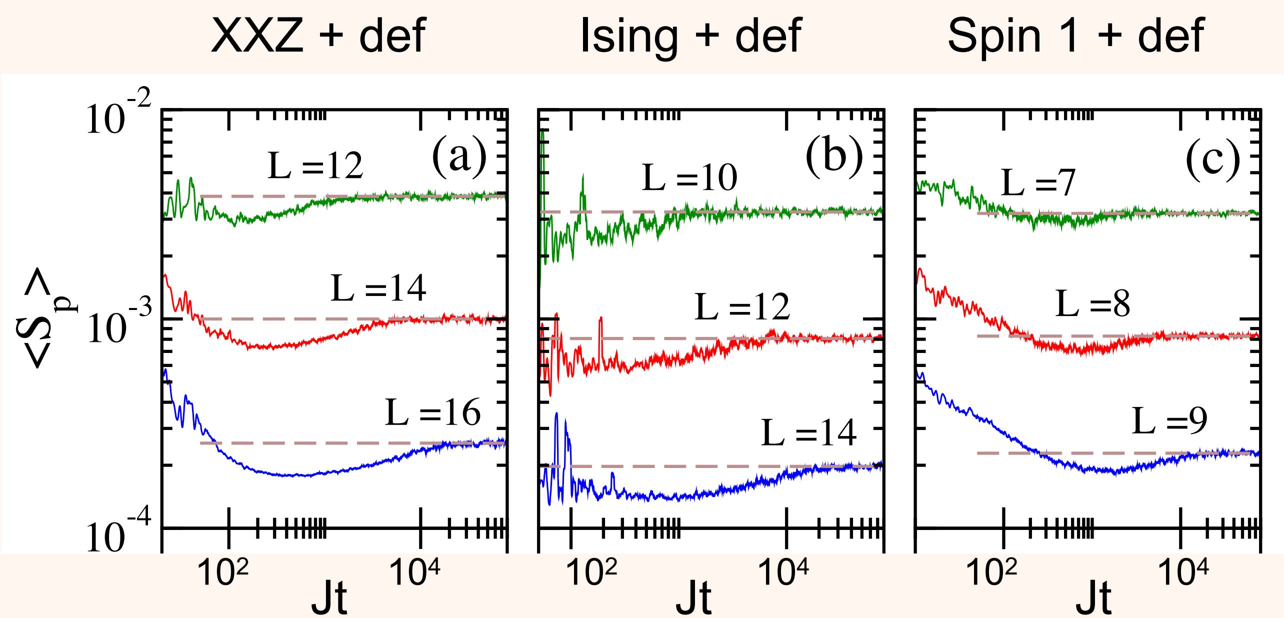
$$C_{\alpha}^{ini} = \langle \alpha | \Psi(0) \rangle$$

Schiulaz, Torres & LFS, PRB **99**, 174313 (2019)

Correlation hole = ramp

Correlation Hole: Single-Defect Models

No need for unfolding
Detect chaos despite symmetries



Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

$$|\Psi(0)\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

Correlation Hole and Experiments

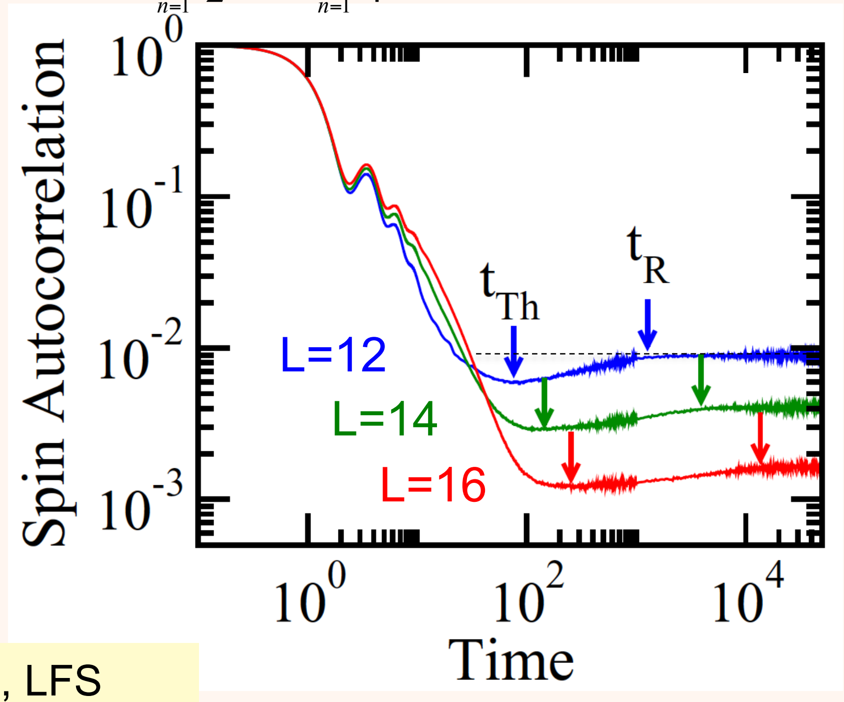
$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

- Small values: Precision

- Local observables

spin autocorrelation function

$$H = \sum_{n=1}^{L}\frac{\varepsilon_n}{2}\sigma_n^z + \sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)$$



Torres, Garcia, LFS
PRB **97**, 060303 (R) (2018)

Conclusions

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- DYNAMICS & TRANSPORT
  Dynamics slows down once the particles get very far from each other.

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M. Schiulaz, M. Távora, LFS
Quantum Sci. Technol. **3**, 044006 (2018)

How many particles make up a chaotic many-body quantum system?
G. Zisling, LFS, Y. Bar Lev
arXiv:2012.14436

Conclusions

Single-defect model

- ➢ Local perturbation in quantum many-body systems can lead to chaos.
- • XXZ + defect (Ballistic vs Diffusion)
- • Ising + transverse field + defect
- • Spin 1 + defect

- ➢ **Correlation hole + off-diagonal** elements detect chaos despite symmetries
  no unfolding, **no separation by symmetries**

- ➢ **Correlation hole**: a **dynamical** indicator of chaos (experiments – dynamics)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres