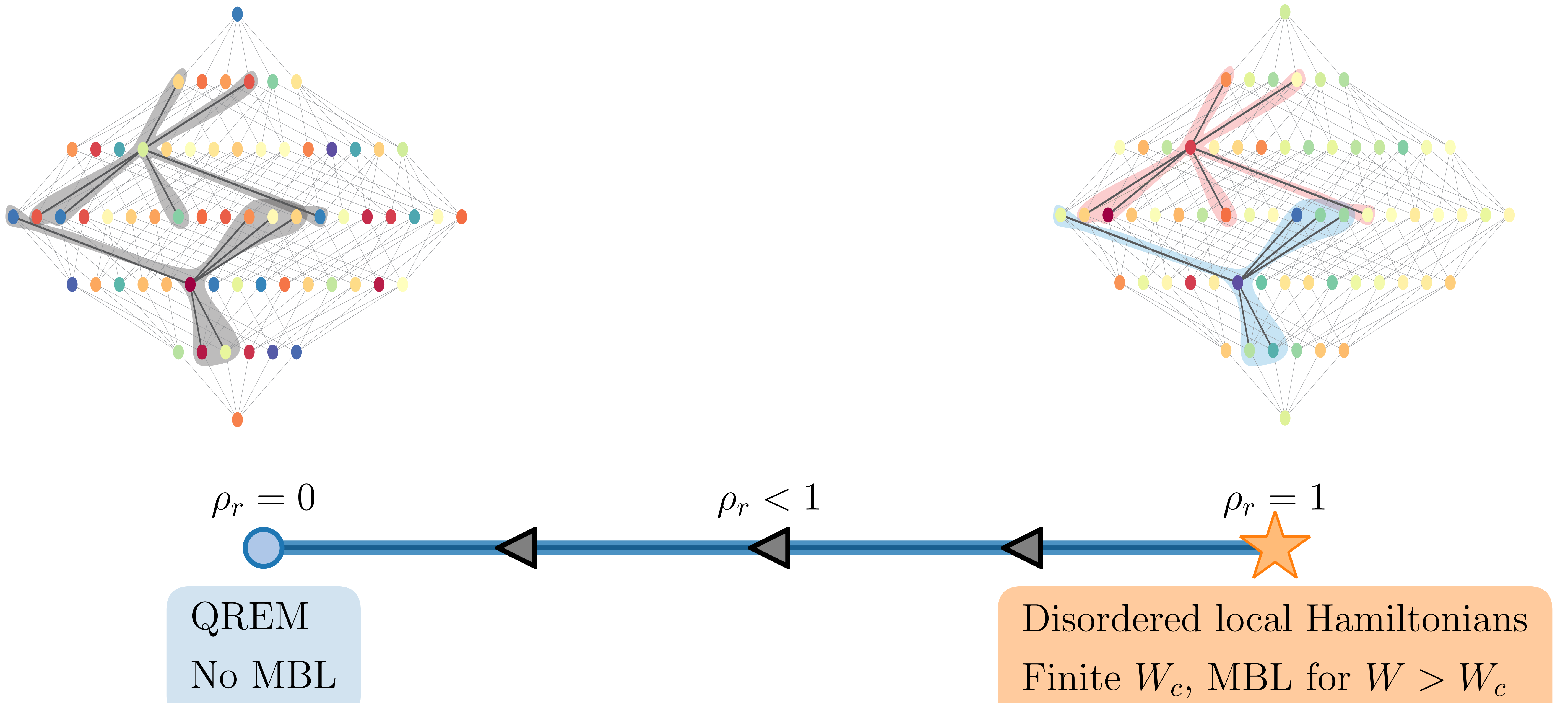
Many-body localisation: a tale of correlations and classical percolation on Fock space



$\rho_r = 0$

$\rho_r < 1$

$\rho_r = 1$

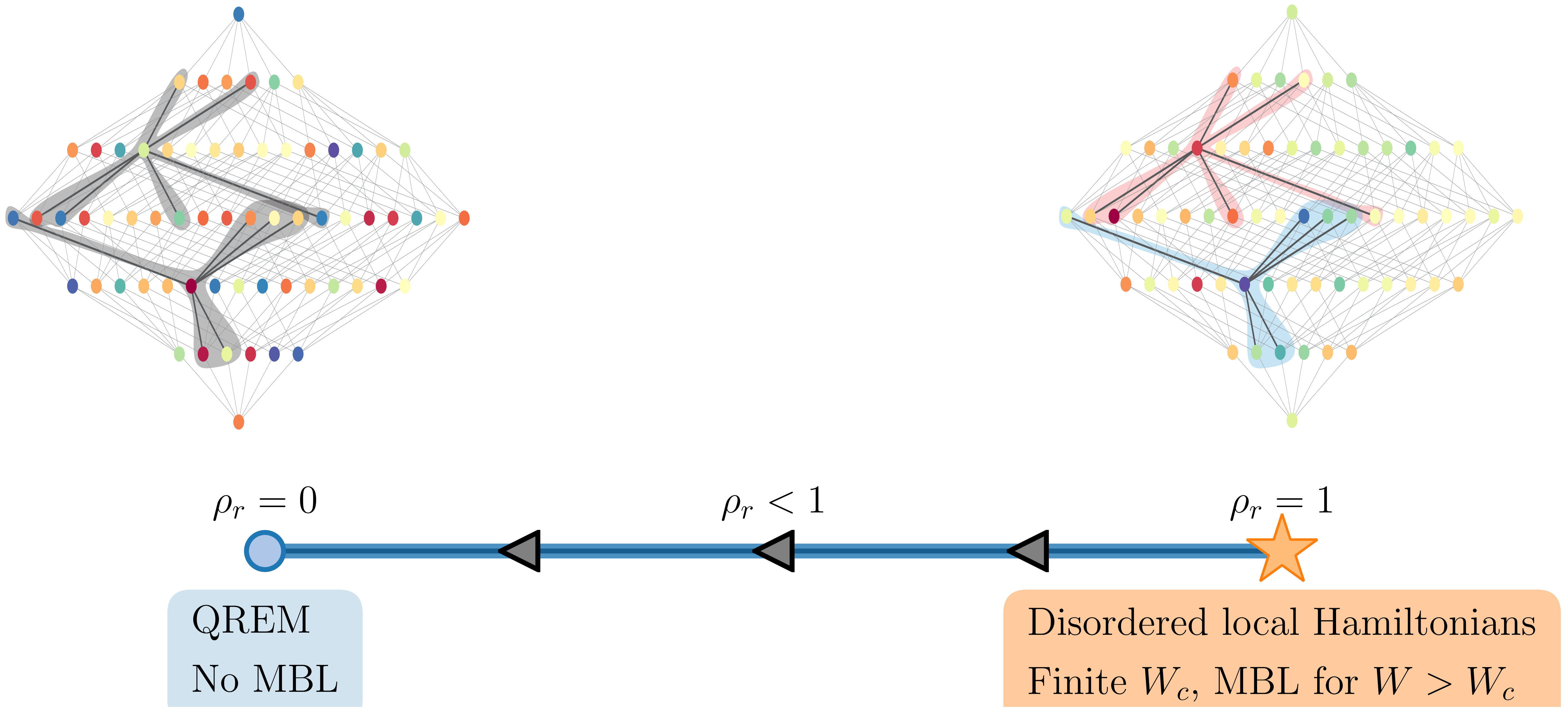
QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

Sthitadhi Roy
University of Oxford

QM^3 Seminar, January 18, 2021

Many-body localisation: a tale of correlations and classical percolation on Fock space



$\rho_r = 0$

$\rho_r < 1$

$\rho_r = 1$

QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

Sthitadhi Roy
University of Oxford

J. T. Chalker

D. E. Logan

Outline of the talk

- Lightning review of many-body localisation

Outline of the talk

- Lightning review of many-body localisation

- Fock-space correlations and origins of MBL
  - MBL on Fock-space — how and why ?
  - Why not standard Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

Outline of the talk

- Lightning review of many-body localisation

- **Fock-space correlations and origins of MBL**
  - MBL on Fock-space — how and why ?
  - Why not standard Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

- **Classical percolation in Fock space as a proxy for MBL**
  - Fock-space fragmentation due to local frozen degrees of freedom
  - Heuristic picture for the effect of correlations

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

Outline of the talk

- Lightning review of many-body localisation

- **Fock-space correlations and origins of MBL**
  - MBL on Fock-space — how and why ?
  - Why not standard Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

- **Classical percolation in Fock space as a proxy for MBL**
  - Fock-space fragmentation due to local frozen degrees of freedom
  - Heuristic picture for the effect of correlations

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

- **Anderson localisation on graphs with strongly correlated disorder**
  - Disorder correlations analogous to Fock-space correlations
  - Arguably a more controlled setting

Phys. Rev. Lett. 125, 250402 (2020)

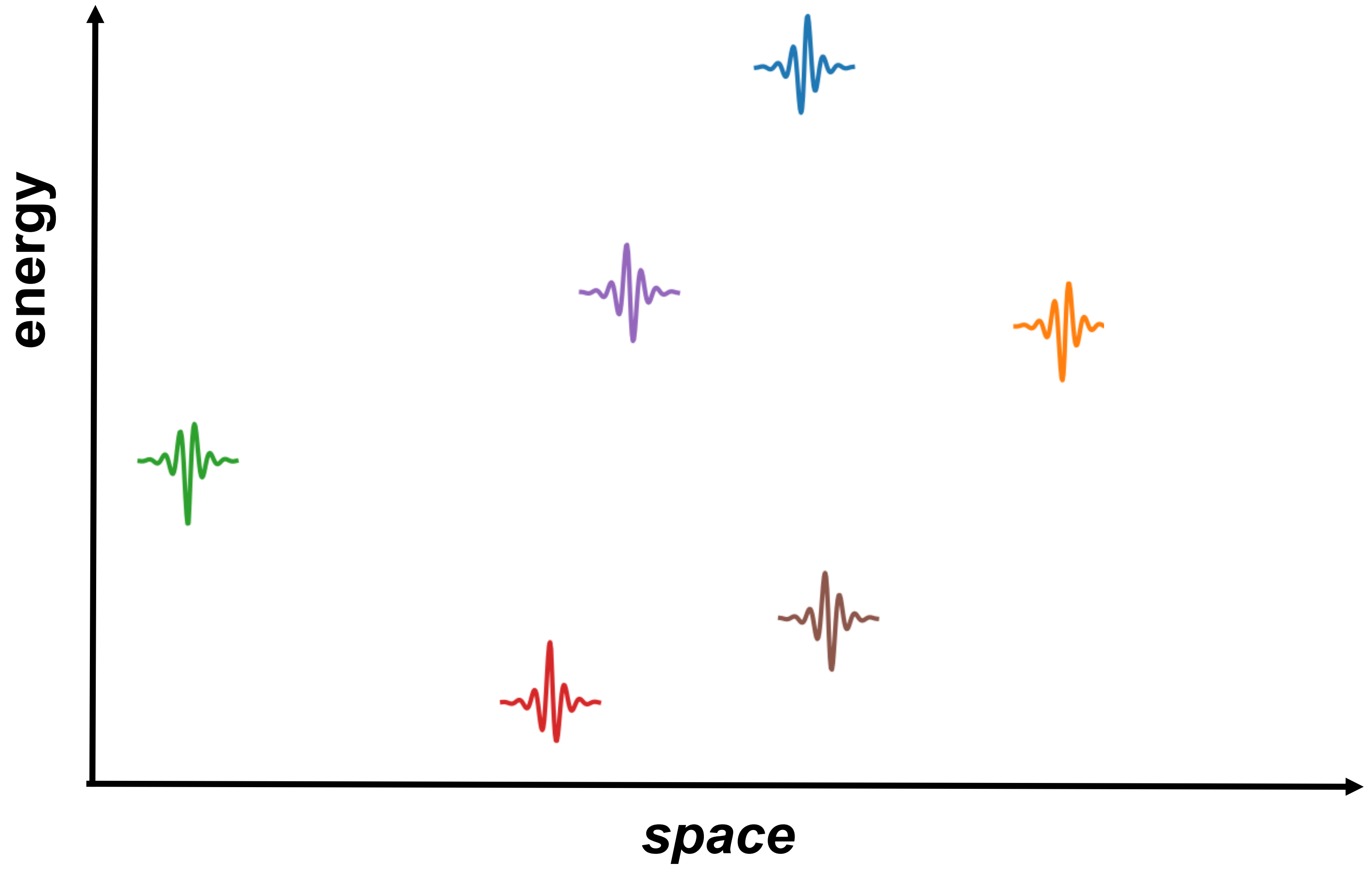
Many-body localisation

Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

Many-body localisation

Microscopics ↔ Statistical Mechanics ↔ Thermodynamics

Can quantum systems fail to thermalise ?



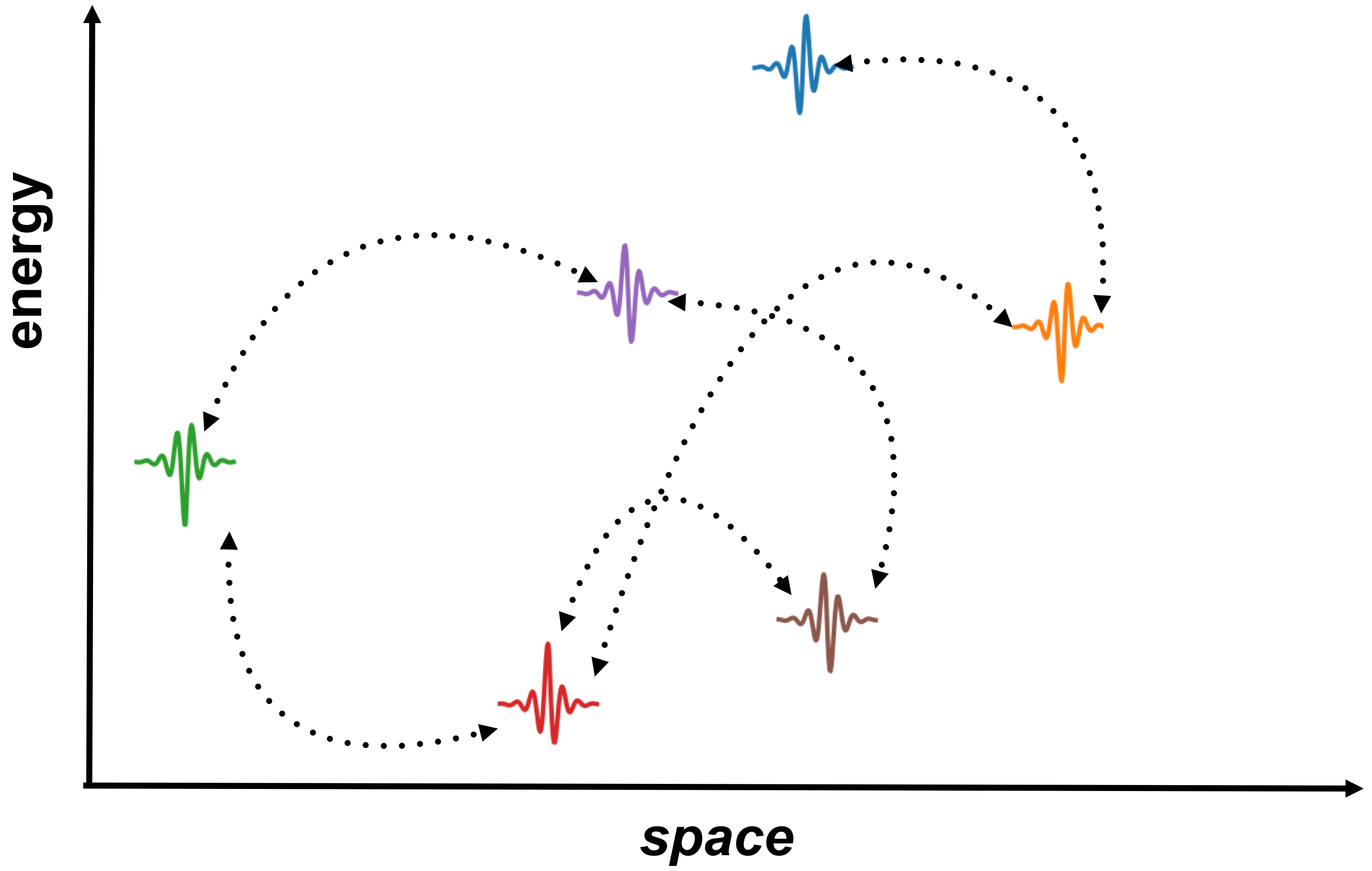
Anderson localisation (1958)

Exponential localisation of non-interacting quantum particles on a disordered lattice

Many-body localisation

Microscopics ↔ Statistical Mechanics ↔ Thermodynamics

Can quantum systems fail to thermalise ?



Many-body localisation: Fate of Anderson localisation upon adding interactions between quantum particles

Anderson localisation (1958)

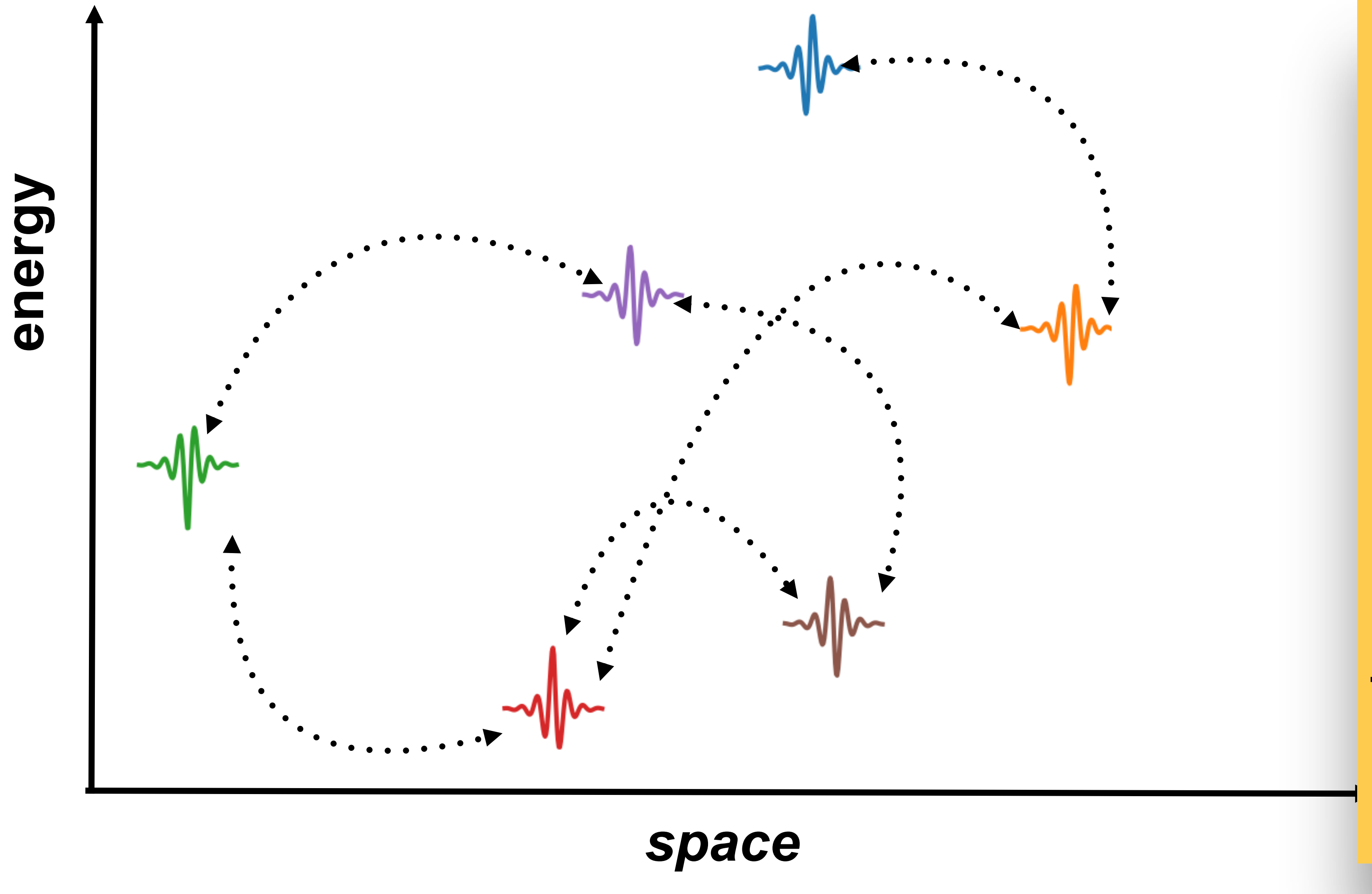
Exponential localisation of non-interacting quantum particles on a disordered lattice

[Gornyi et al., Basko et al., Oganesyan+Huse, Znidaric et al., Pal+Huse, Kjäll et al., Luitz et al., Nandkishore+Huse, Abanin+Papic, Vasseur+Potter+Parameswaran, Vosk+Huse+Altman, SR+Logan+Chalker, ...]

Many-body localisation

Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

Can quantum systems fail to thermalise ?



- — How do they admit a statistical mechanics description?
- — Non-thermal ensembles that govern stationary states after quenches
- — Temporal approach to such ensembles
- — Suppressed transport and propagation of quantum information
- — Novel phases of matter protected by localisation, infinite-temperature glasses, time-crystals…

Anderson localisation (1958)

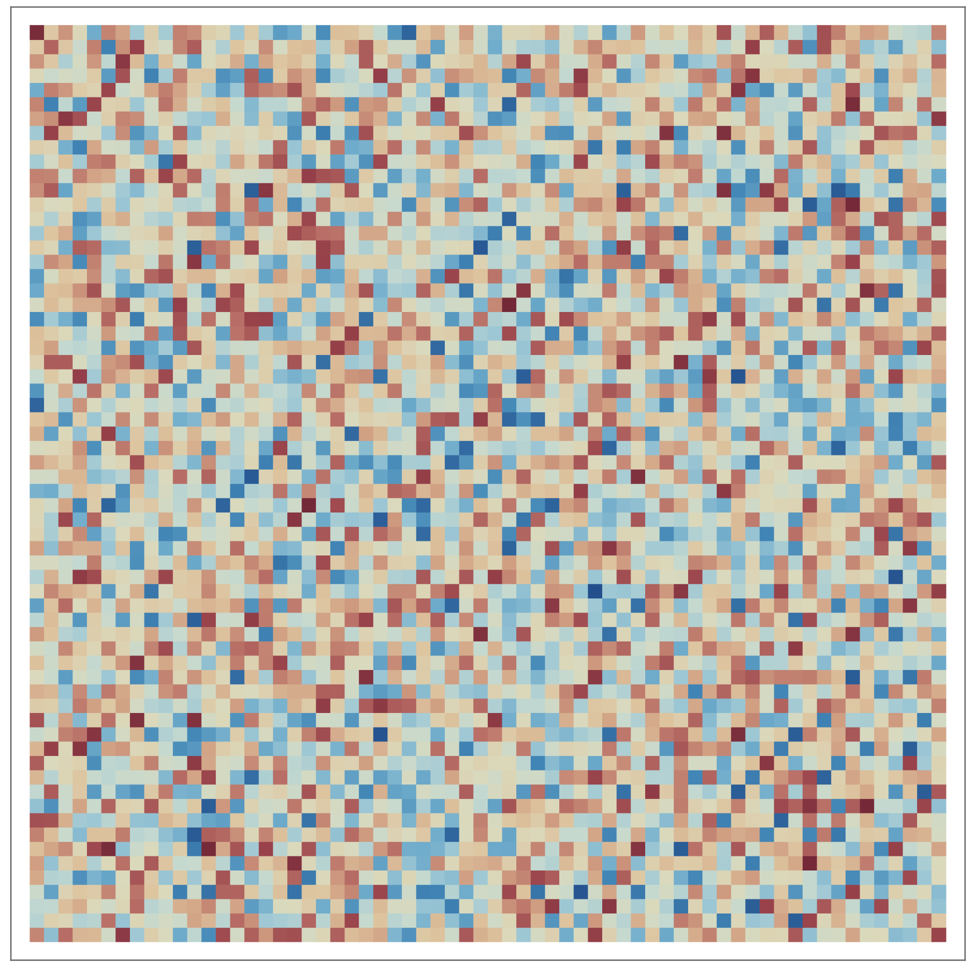
Exponential localisation of non-interacting quantum particles on a disordered lattice

[Gornyi et al., Basko et al., Oganesyan+Huse, Znidaric et al., Pal+Huse, Kjäll et al., Luitz et al., Nandkishore+Huse, Abanin+Papic, Vasseur+Potter+Parameswaran, Vosk+Huse+Altman, SR+Logan+Chalker, ….]

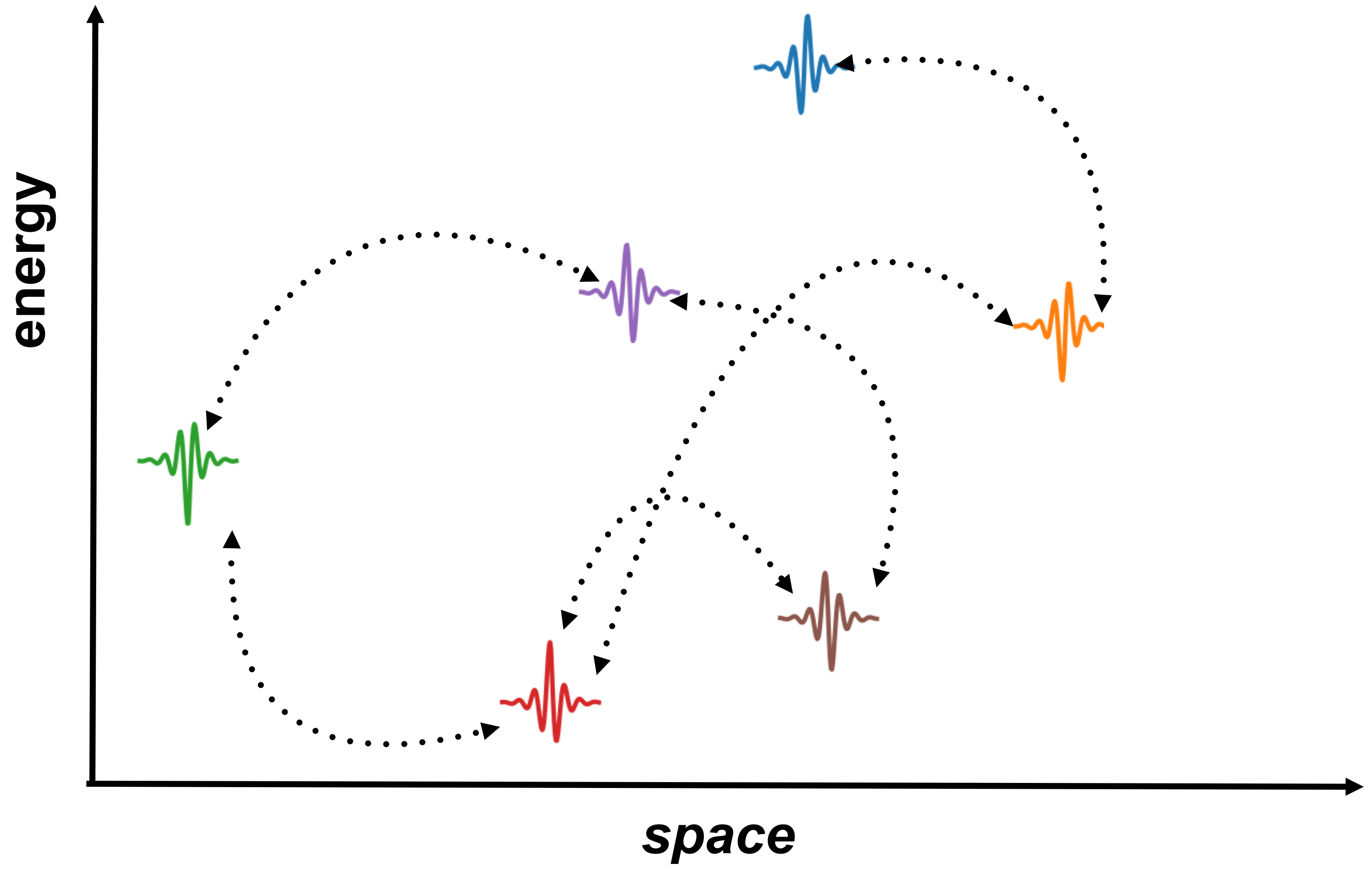
Many-body localisation

Microscopics ↔ Statistical Mechanics ↔ Thermodynamics

Minimal model — random matrix Hamiltonian: $H_{\mathrm{RMT}} =$



Can quantum systems fail to thermalise ?



Many-body localisation: Fate of Anderson localisation upon adding interactions between quantum particles

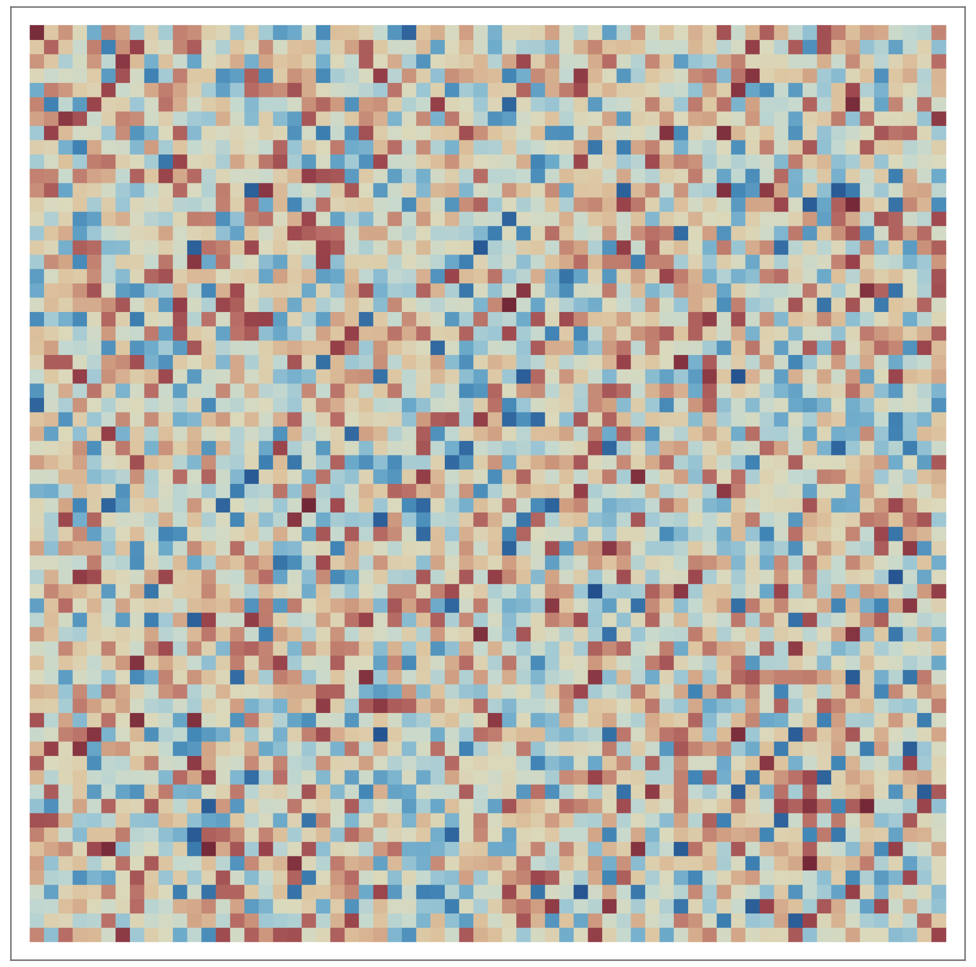
Anderson localisation (1958)

Exponential localisation of non-interacting quantum particles on a disordered lattice

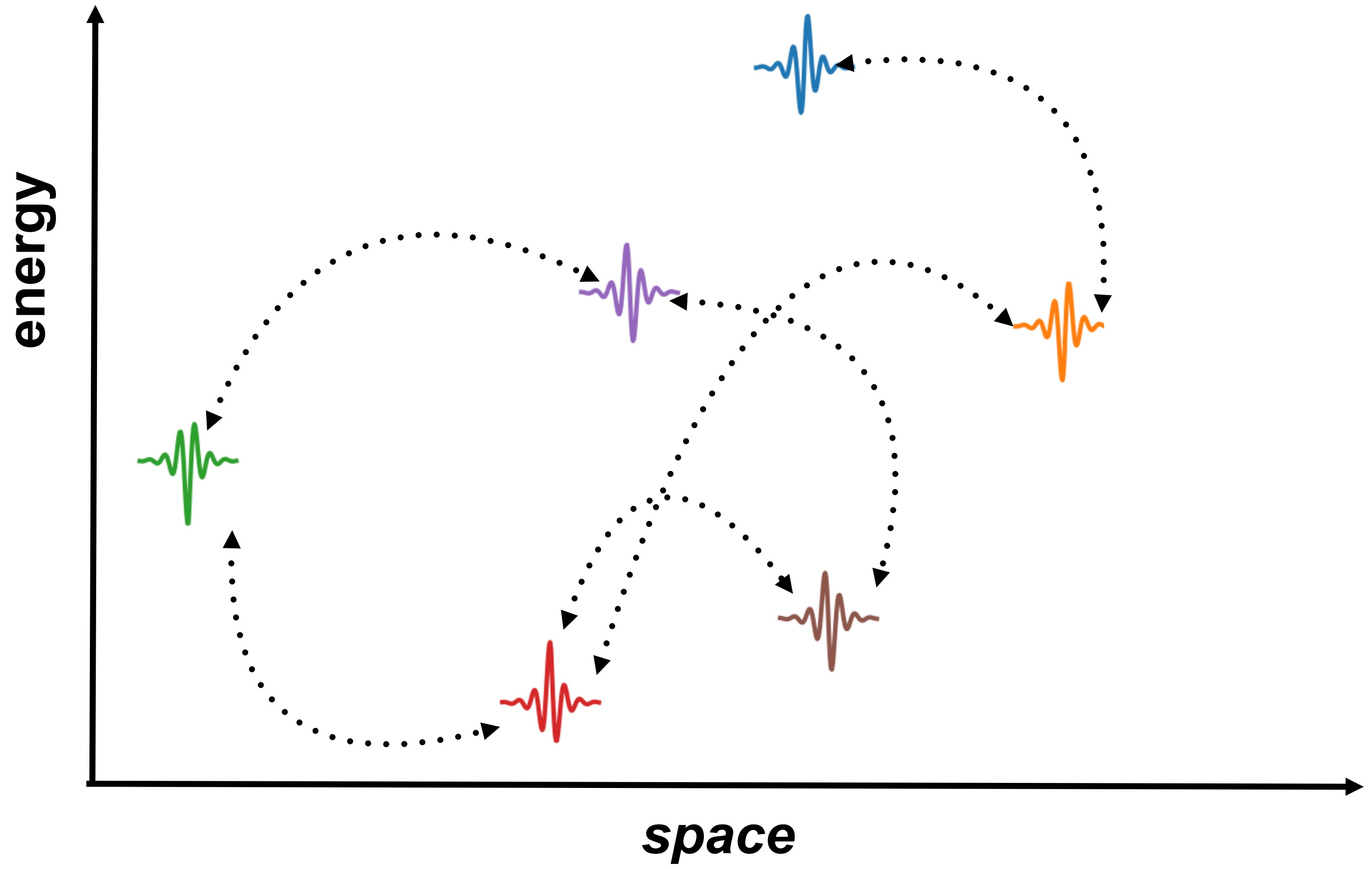
Many-body localisation

Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

Minimal model — random matrix Hamiltonian: $H_{\mathrm{RMT}} =$



Can quantum systems fail to thermalise ?



Anderson localisation (1958)

Exponential localisation of non-interacting quantum particles on a disordered lattice

Many-body localisation: Fate of Anderson localisation upon adding interactions between quantum particles

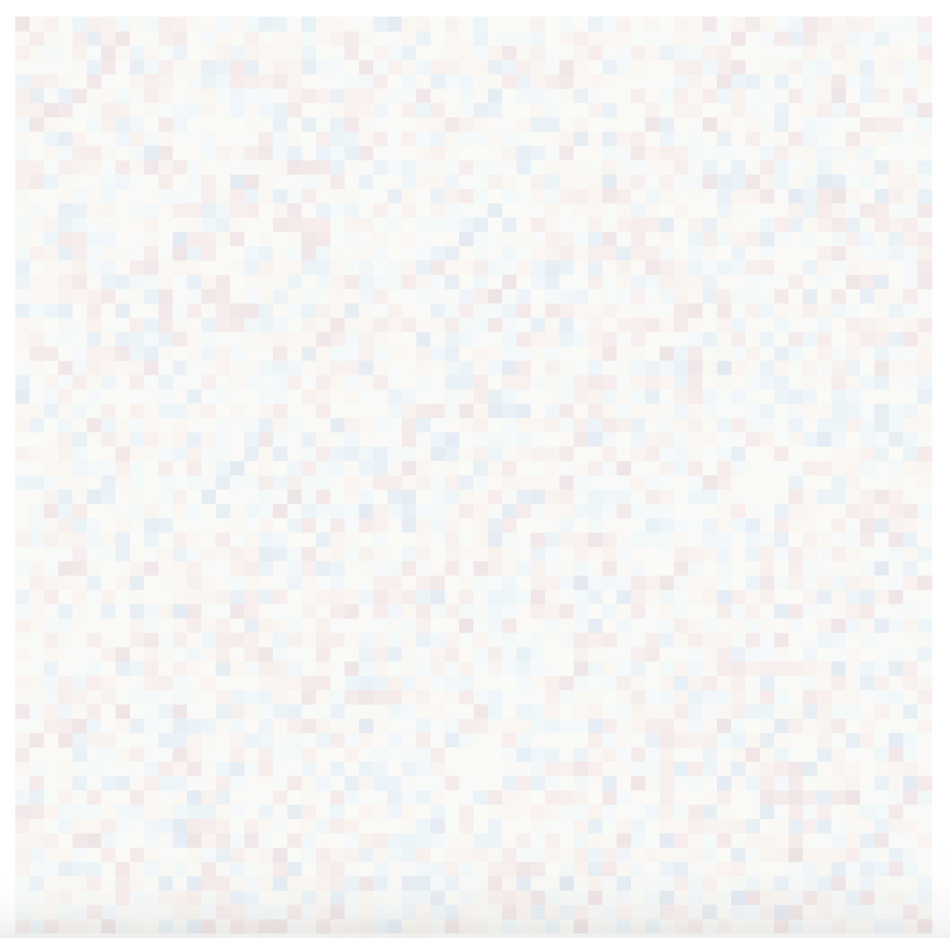
Minimal many-body Hamiltonian with a stable MBL phase?

$H_{\mathrm{MBL}} =$?

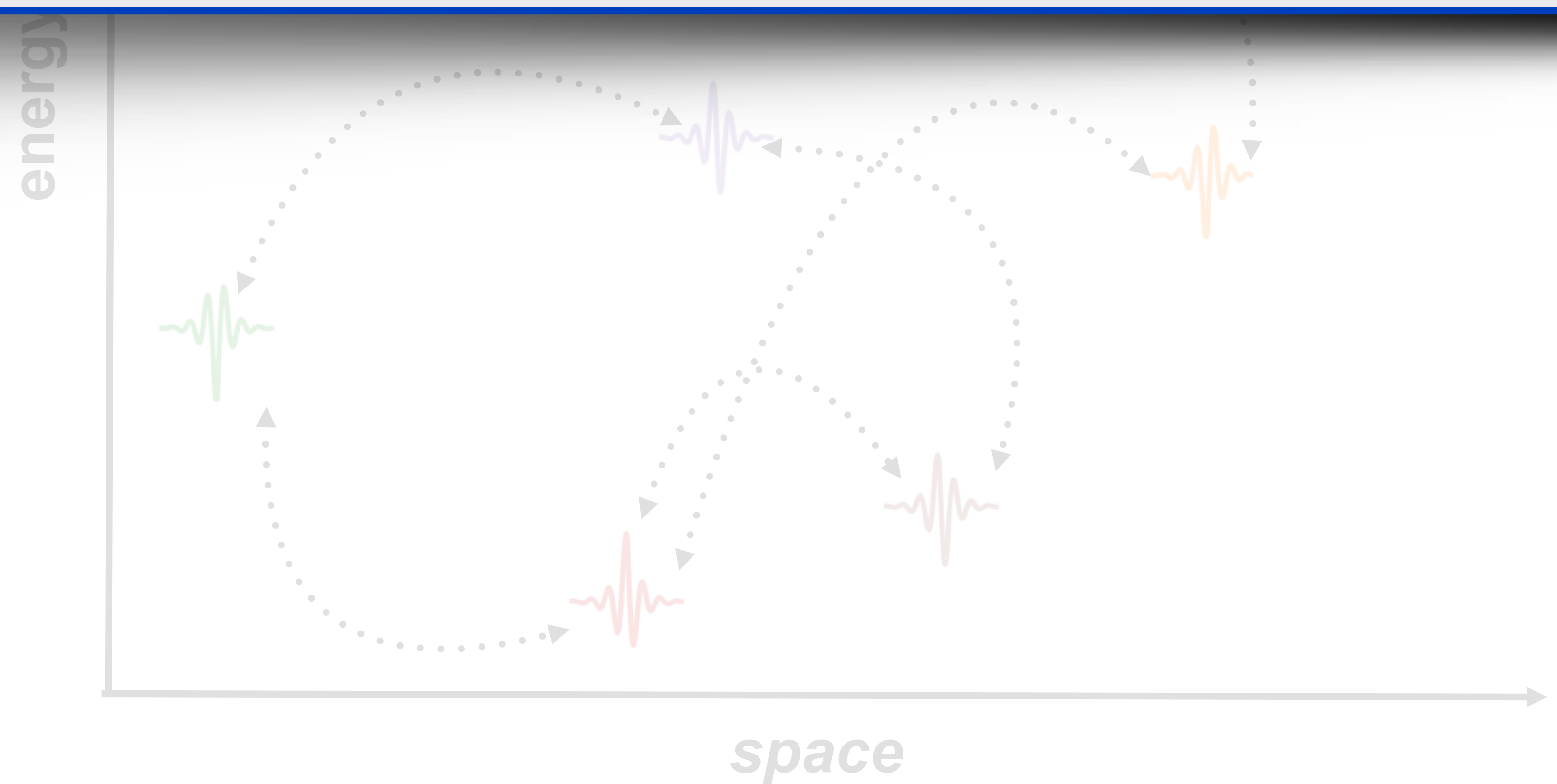
Many-body localisation

Microscopics ⟷ Statistical Mechanics ⟷ Thermodynamics

Minimal model — random matrix Hamiltonian: $H_{\mathrm{RMT}} =$



Minimal ingredients in a many-body Hamiltonian for a stable and robust MBL phase ?



Anderson localisation (1958)

Exponential localisation of non-interacting quantum particles on a disordered lattice

adding interactions between quantum particles

Minimal many-body Hamiltonian
with a stable MBL phase?

$H_{\mathrm{MBL}} =$?

MBL on Fock space

- Lightning review of many-body localisation

- Fock-space correlations and origins of MBL
  - MBL on Fock-space — how and why ?
  - Why not standard Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

- Classical percolation in Fock space as a proxy for MBL
  - Fock-space fragmentation due to local frozen degrees of freedom
  - Heuristic picture for the effect of correlations

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

- Anderson localisation on graphs with strongly correlated disorder
  - Disorder correlations analogous to Fock-space correlations
  - Arguably a more controlled setting

arxiv:2007.10357, Phys. Rev. Lett. (In press)

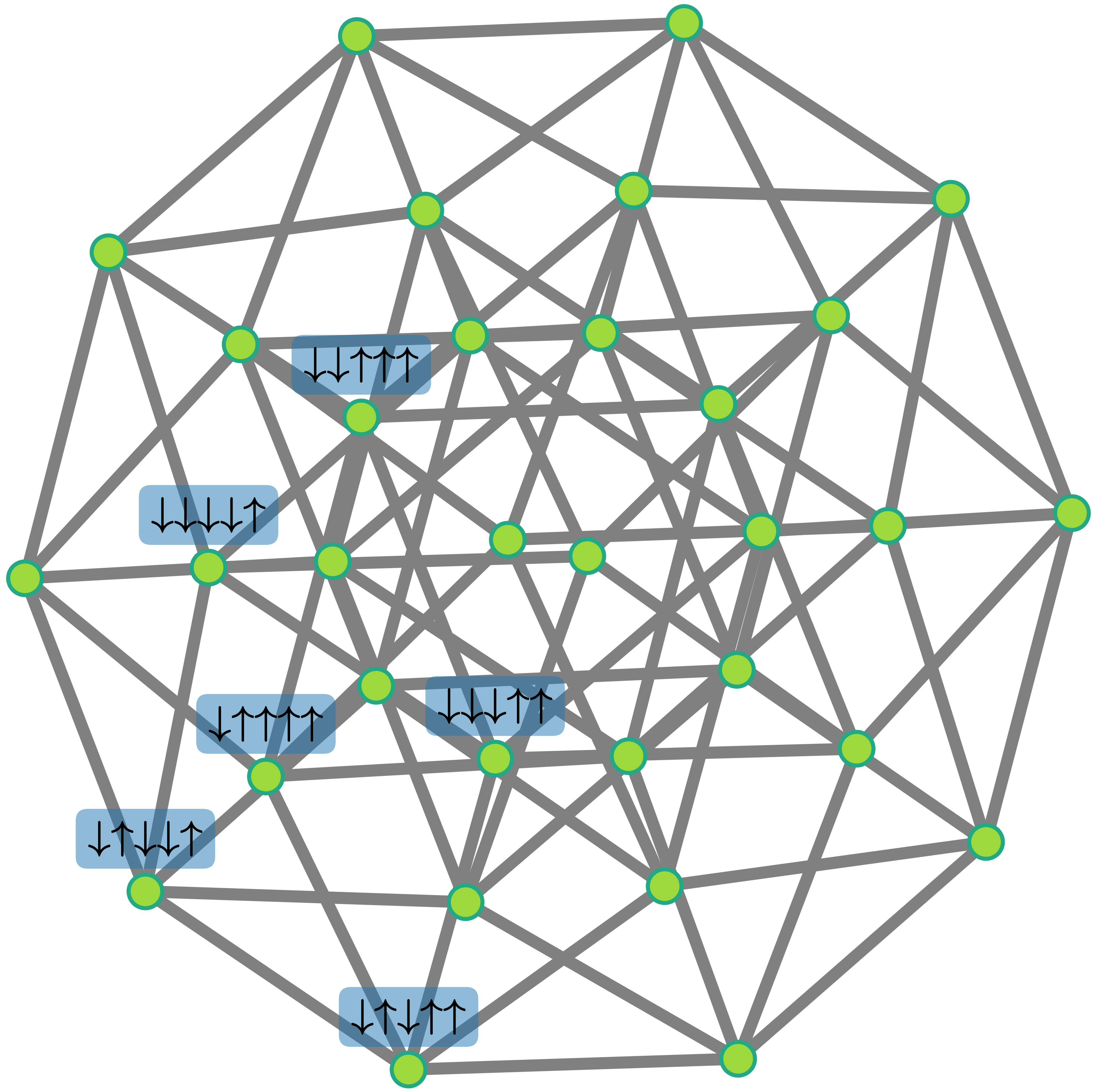
MBL on Fock space

Any many-body Hamiltonian = tight-binding Hamiltonian on the Fock-space graph

$$H = H_{\mathrm{diag}}[\{\sigma_i^z\}] + \Gamma \sum_{i=1}^{N} \sigma_i^x$$

With $|I\rangle \equiv$ Fock-basis state $\equiv \sigma^z$- product state

$$H = \sum_I \mathscr{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathscr{T}_{IK} |I\rangle\langle K|$$



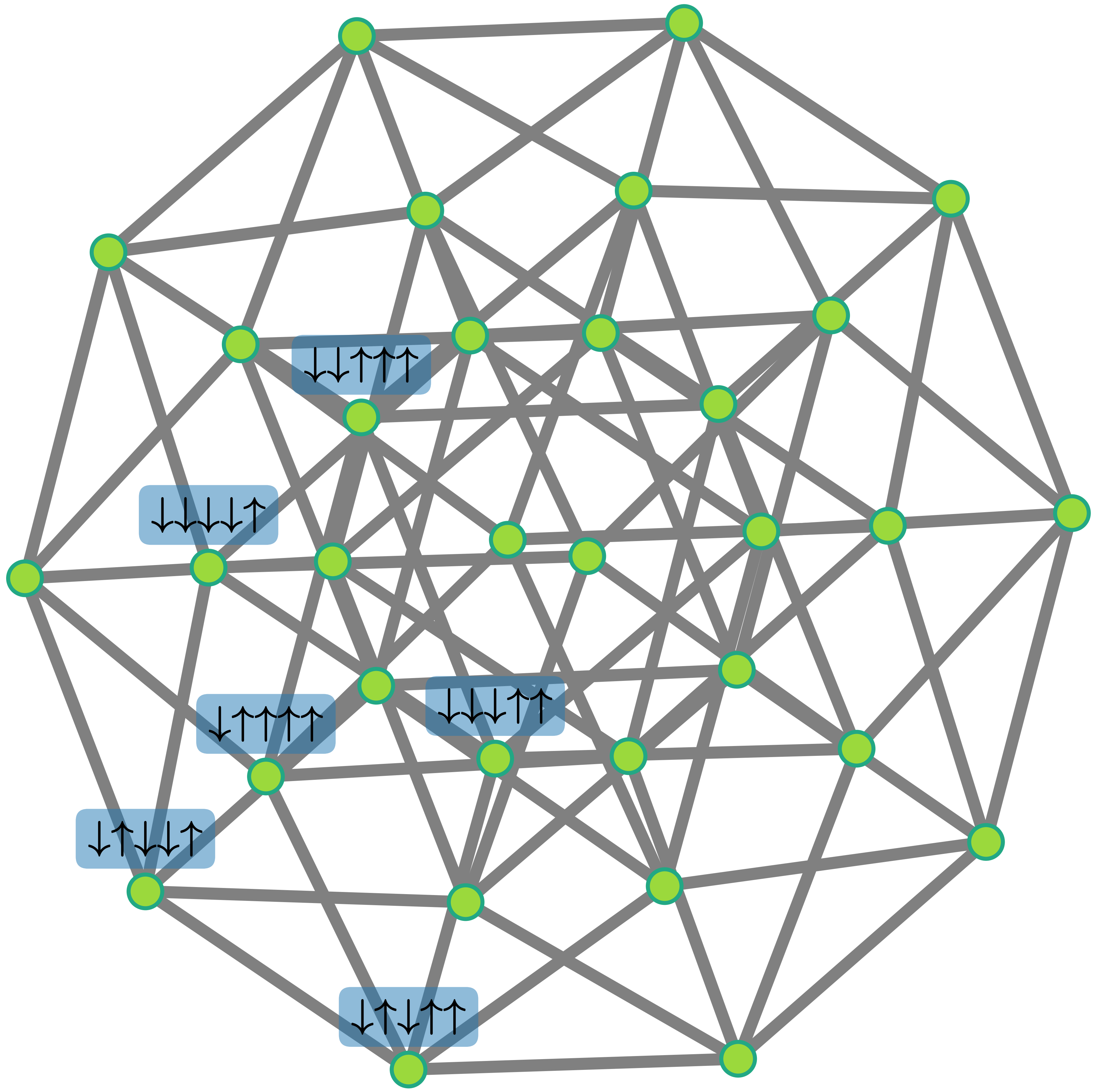
MBL on Fock space

Any many-body Hamiltonian = tight-binding Hamiltonian on the Fock-space graph

$$H = H_{\text{diag}}[\{\sigma_i^z\}] + \Gamma \sum_{i=1}^{N} \sigma_i^x$$

With $|I\rangle \equiv$ Fock-basis state $\equiv \sigma^z$- product state

$$H = \sum_{I} \mathscr{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathscr{T}_{IK} |I\rangle\langle K|$$



- Dimension of the graph exponentially large in system size: $N_{\mathscr{H}} \sim \exp(N)$
- Connectivities on the graph typically extensive: $\sum_K \mathscr{T}_{IK}^2 \sim N$
- Effective variance of the Fock-space site energies also extensive:

$$\langle \mathscr{E}_I^2 \rangle - \langle \mathscr{E}_I \rangle^2 \sim N$$

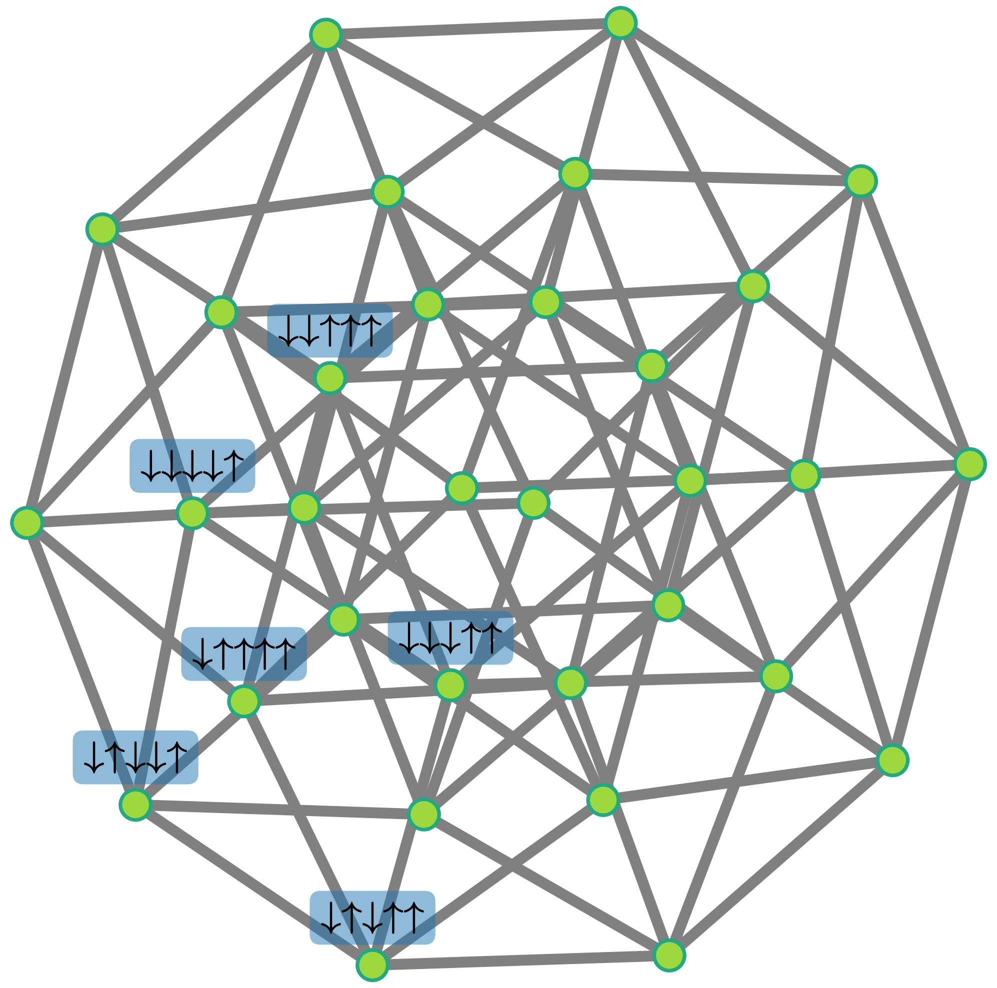
MBL on Fock space

Quantum Random Energy Model: uncorrelated random energies on every spin-configuration

$$H_{\mathrm{QREM}} = \sum_{I} \mathcal{E}_I |I\rangle\langle I| + \Gamma \sum_{i} \sigma_i^x$$

Independent Gaussian random numbers:

$$\langle \mathcal{E}_I \mathcal{E}_K \rangle = \delta_{IK} N W$$



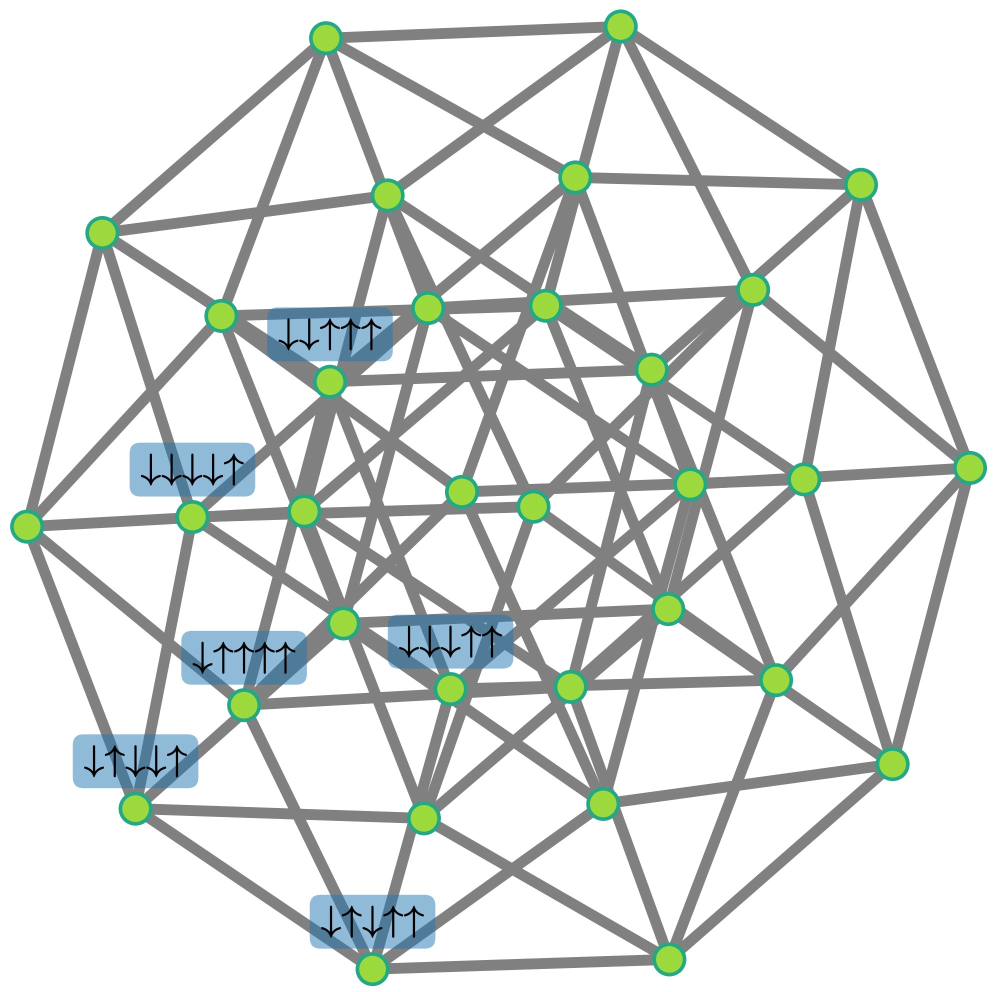
MBL on Fock space

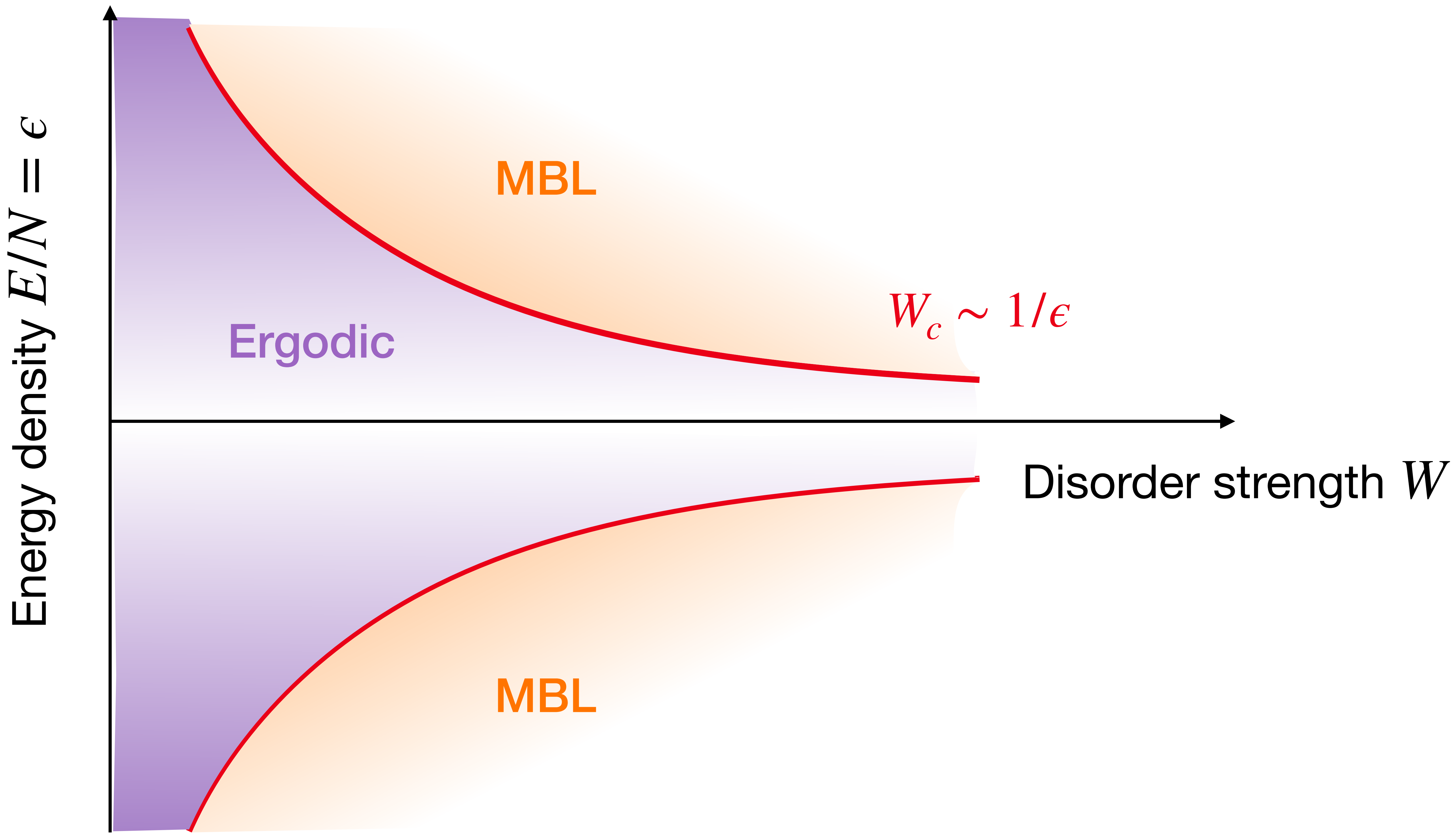
Quantum Random Energy Model: uncorrelated random energies on every spin-configuration

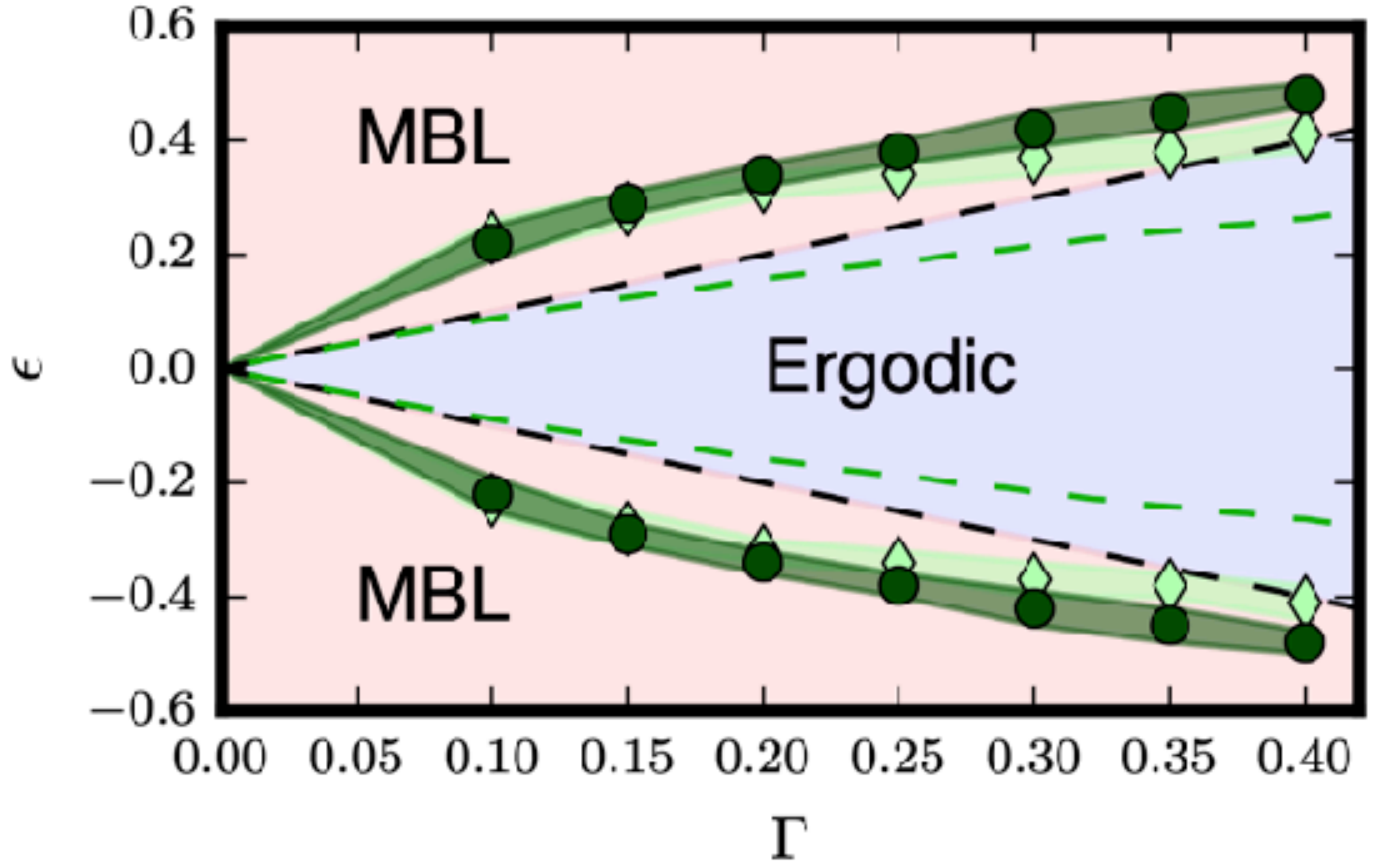
$$H_{\mathrm{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Independent Gaussian random numbers:

$$\langle \mathscr{E}_I \mathscr{E}_K \rangle = \delta_{IK} N W$$



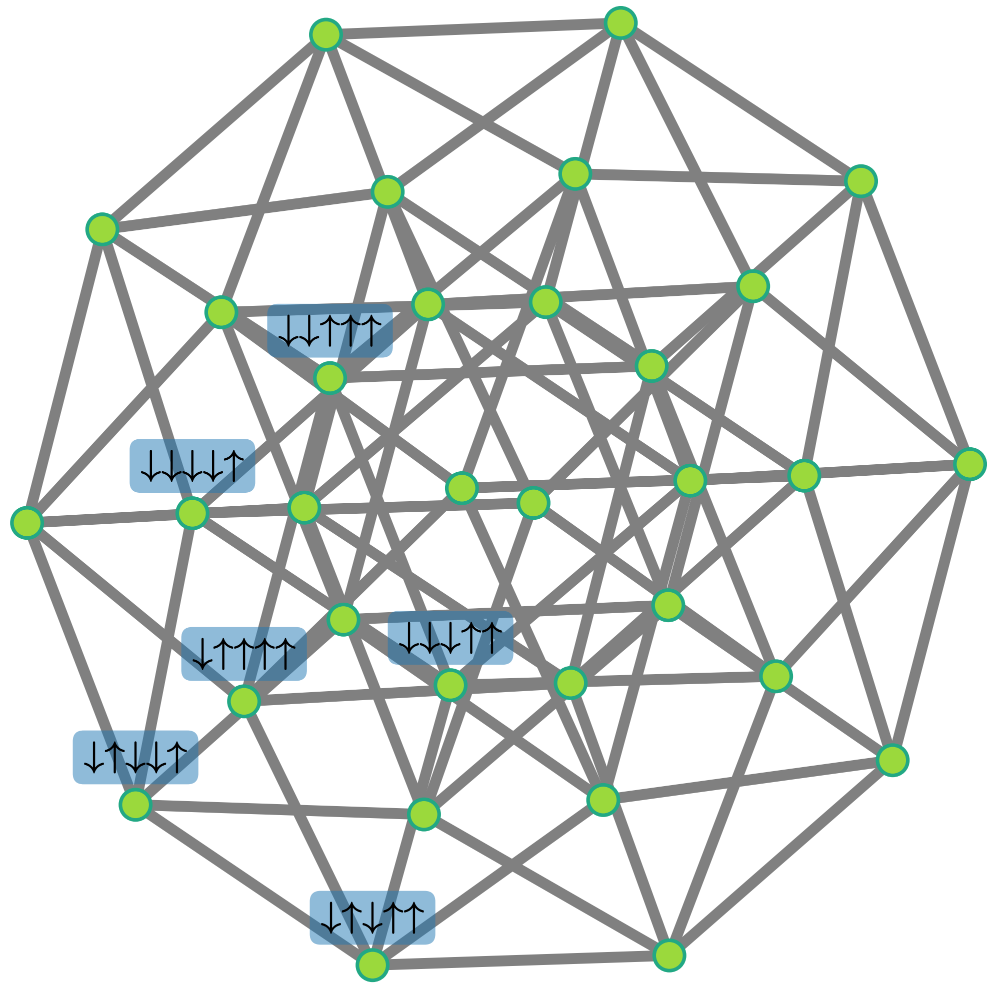




Badlwin, Laumann, Pal, Scardicchio, PRB (2016)
Biroli, Facoetti, Schiró,Tarzia, Vivo, arxiv:2009.09817

MBL on Fock space

Quantum Random Energy Model: uncorrelated random energies on every spin-configuration



$$H_{\text{QREM}} = \sum_I \mathcal{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Independent Gaussian random numbers:

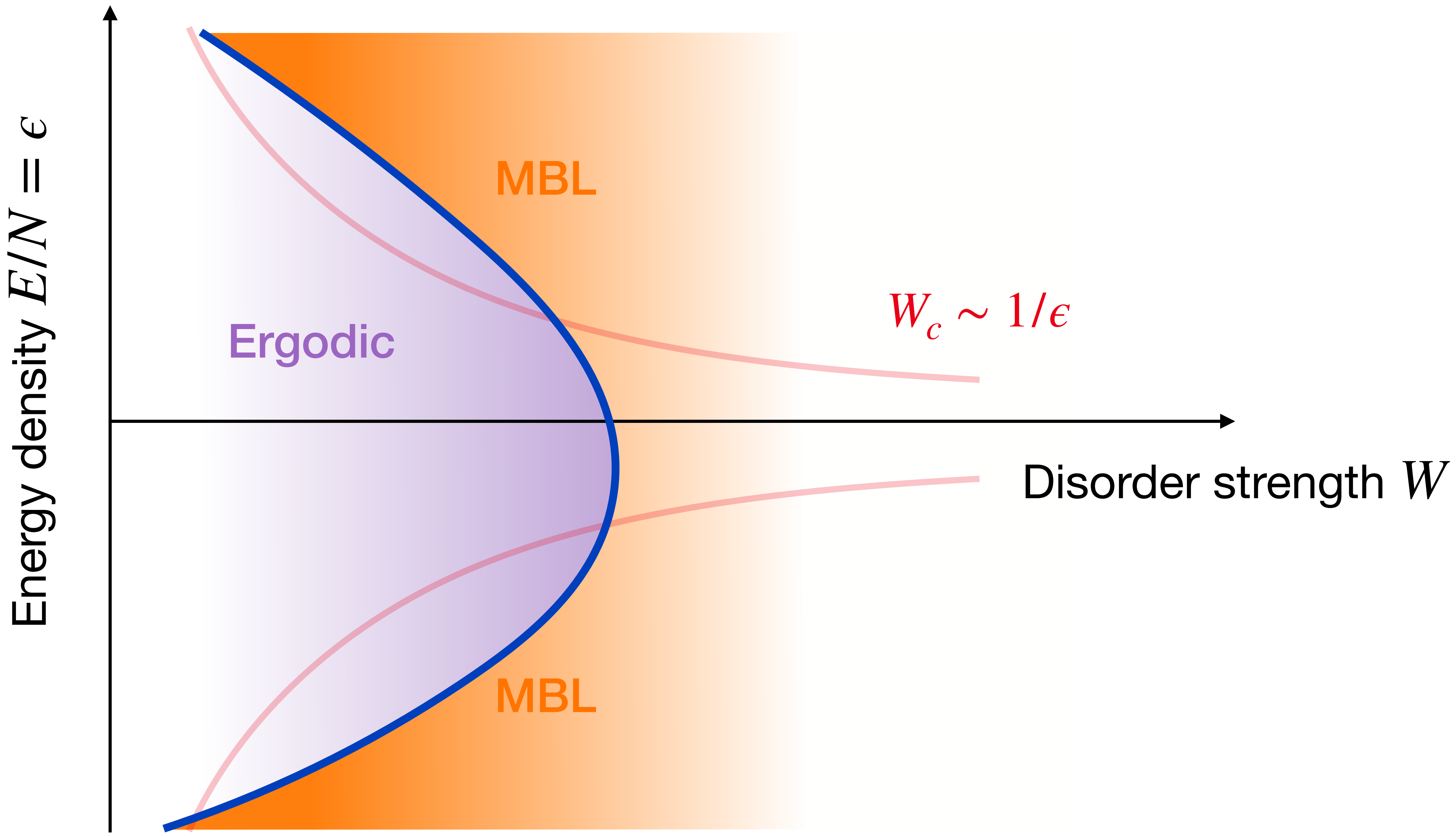
$$\langle \mathcal{E}_I \mathcal{E}_K \rangle = \delta_{IK} N W$$

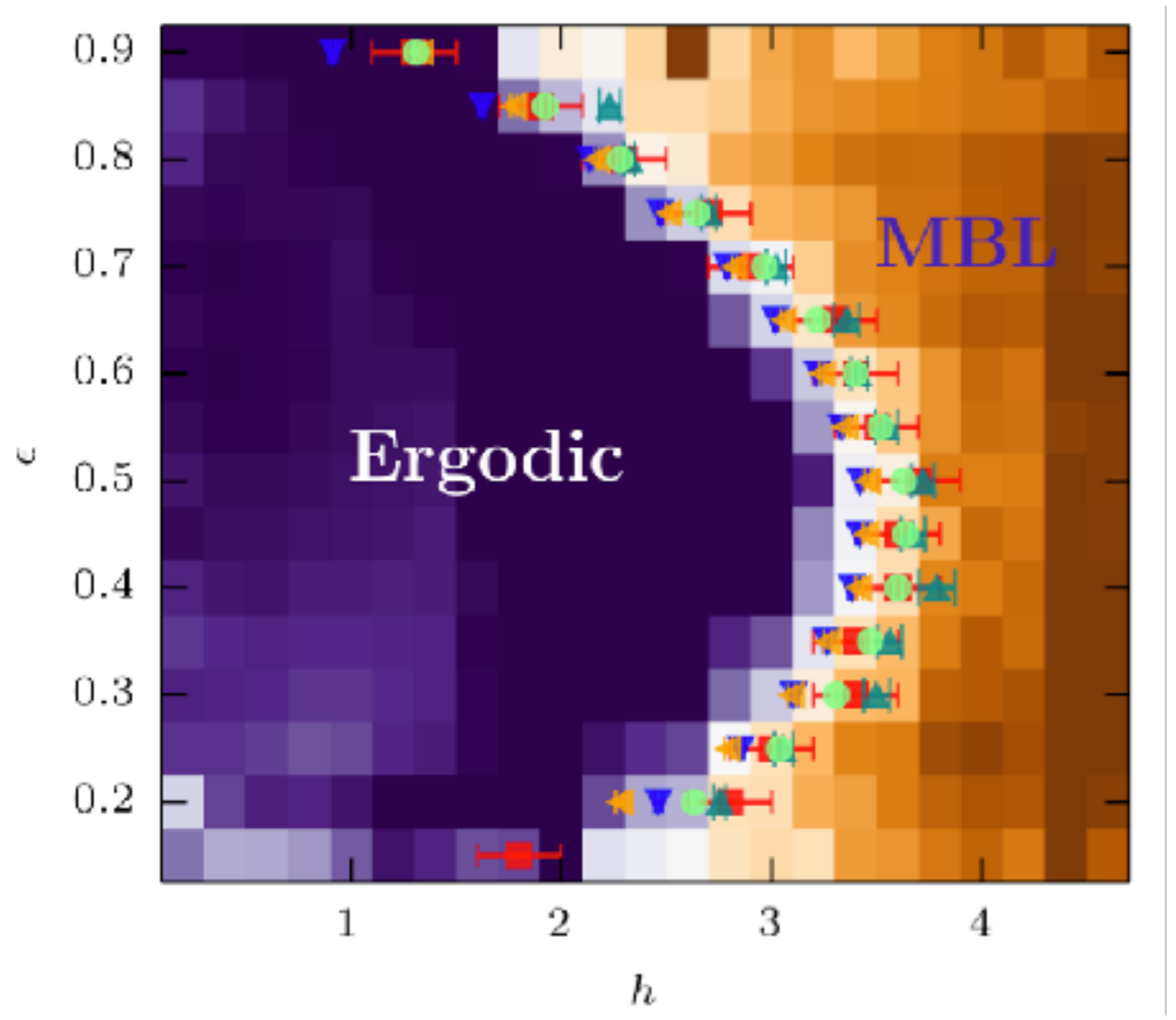
Disordered quantum spin chain

$$H_{\text{TFI}} = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z] + \Gamma \sum_i \sigma_i^x$$

Effective disorder on the Fock space is correlated:

$$\langle \mathcal{E}_I \mathcal{E}_K \rangle \neq 0$$





Luitz, Laflorencie, Alet, PRB (2015)

MBL on Fock space

Quantum Random Energy Model: uncorrelated random energies on every spin-configuration

$$H_{\mathrm{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Independent Gaussian random numbers:

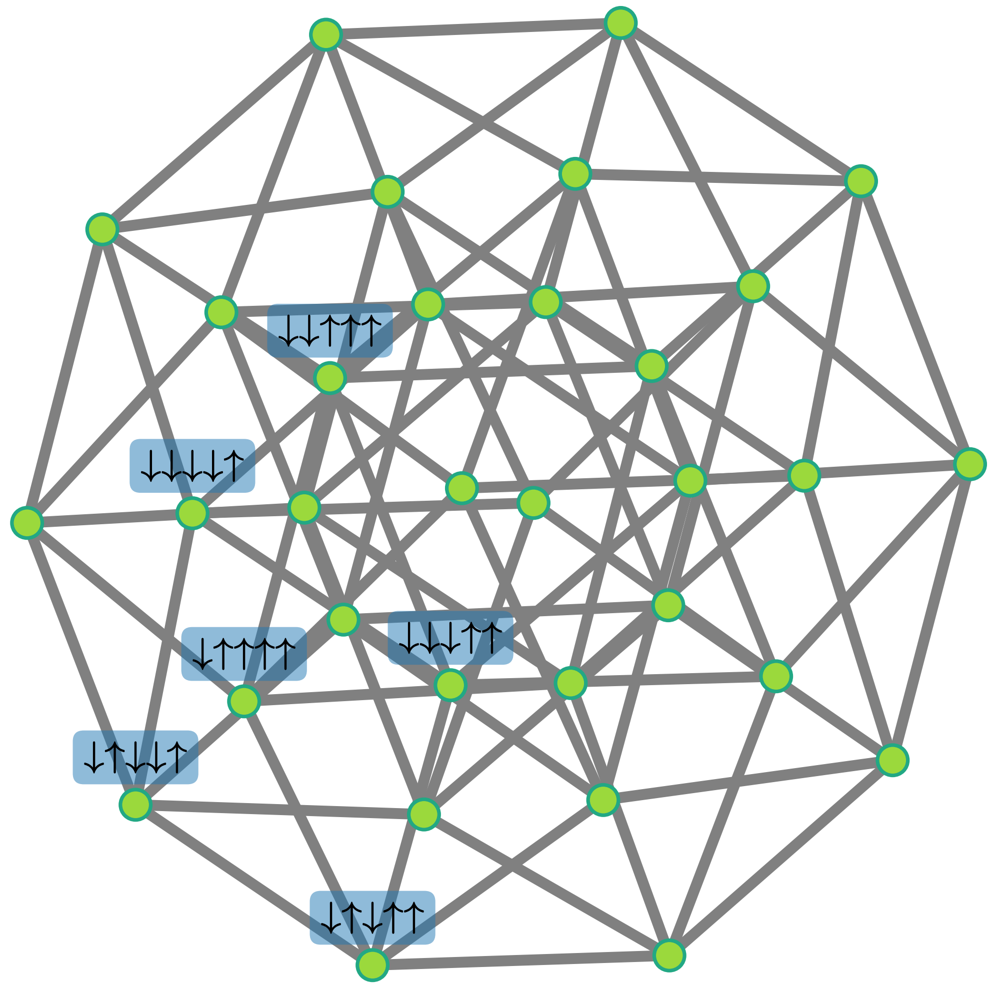
$$\langle \mathscr{E}_I \mathscr{E}_K \rangle = \delta_{IK} N W$$

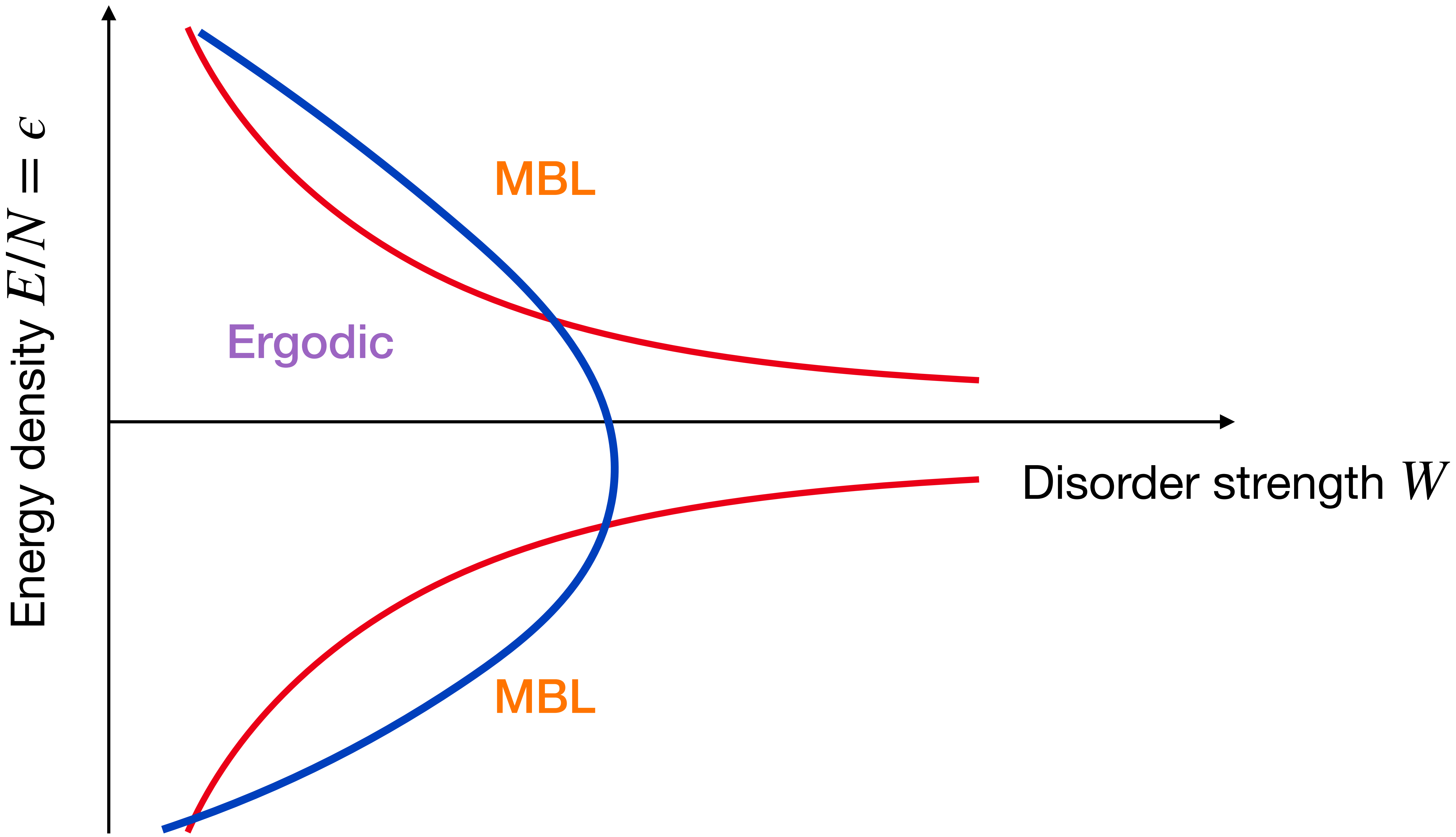
Disordered quantum spin chain

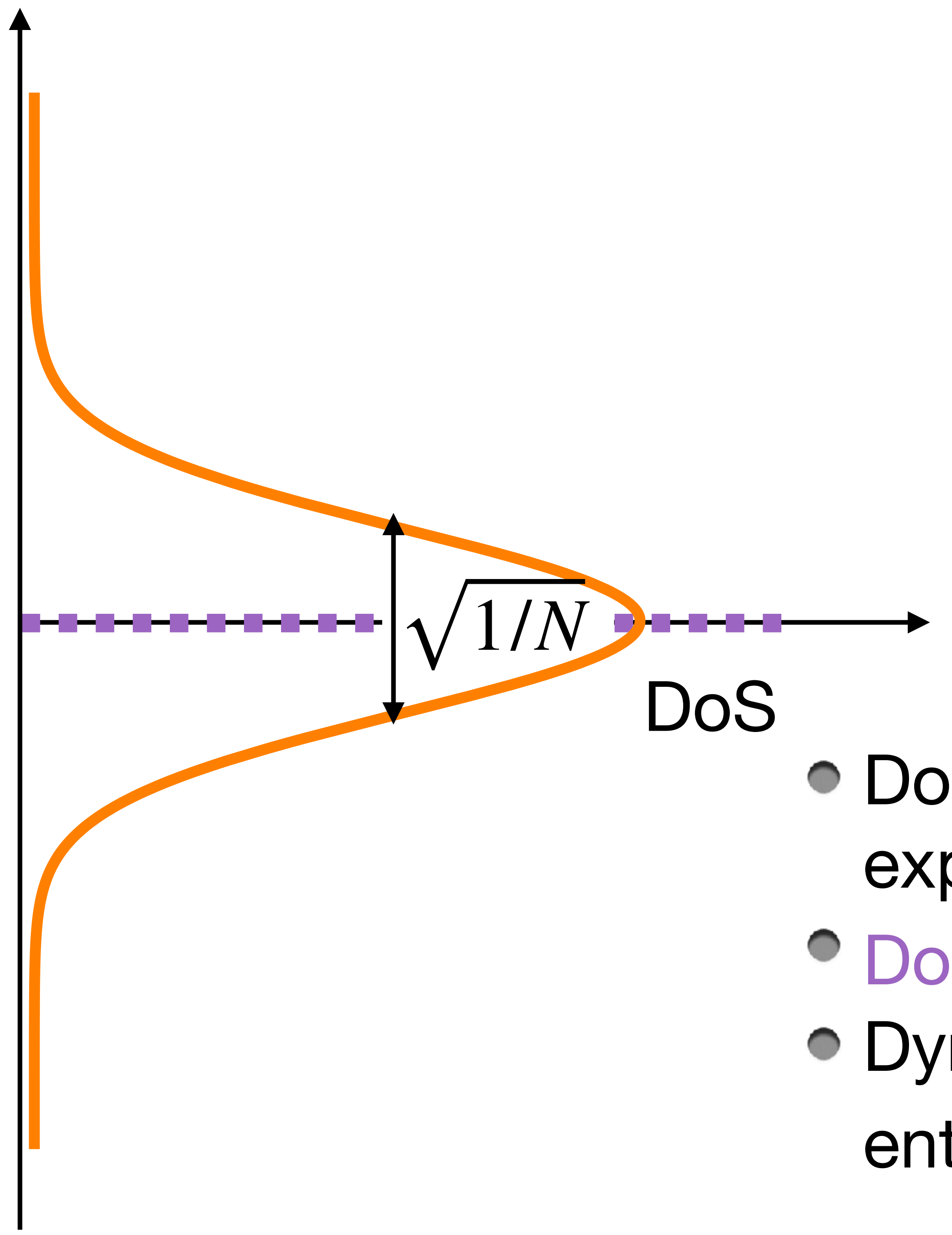
$$H_{\mathrm{TFI}} = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z] + \Gamma \sum_i \sigma_i^x$$

Effective disorder on the Fock space is correlated:

$$\langle \mathscr{E}_I \mathscr{E}_K \rangle \neq 0$$







- DoS in at any finite energy density is exponentially small
- DoS $\equiv \delta(\epsilon)$ in the $N \to \infty$ limit
- Dynamics in general controlled entirely by nature of states at $\epsilon = 0$

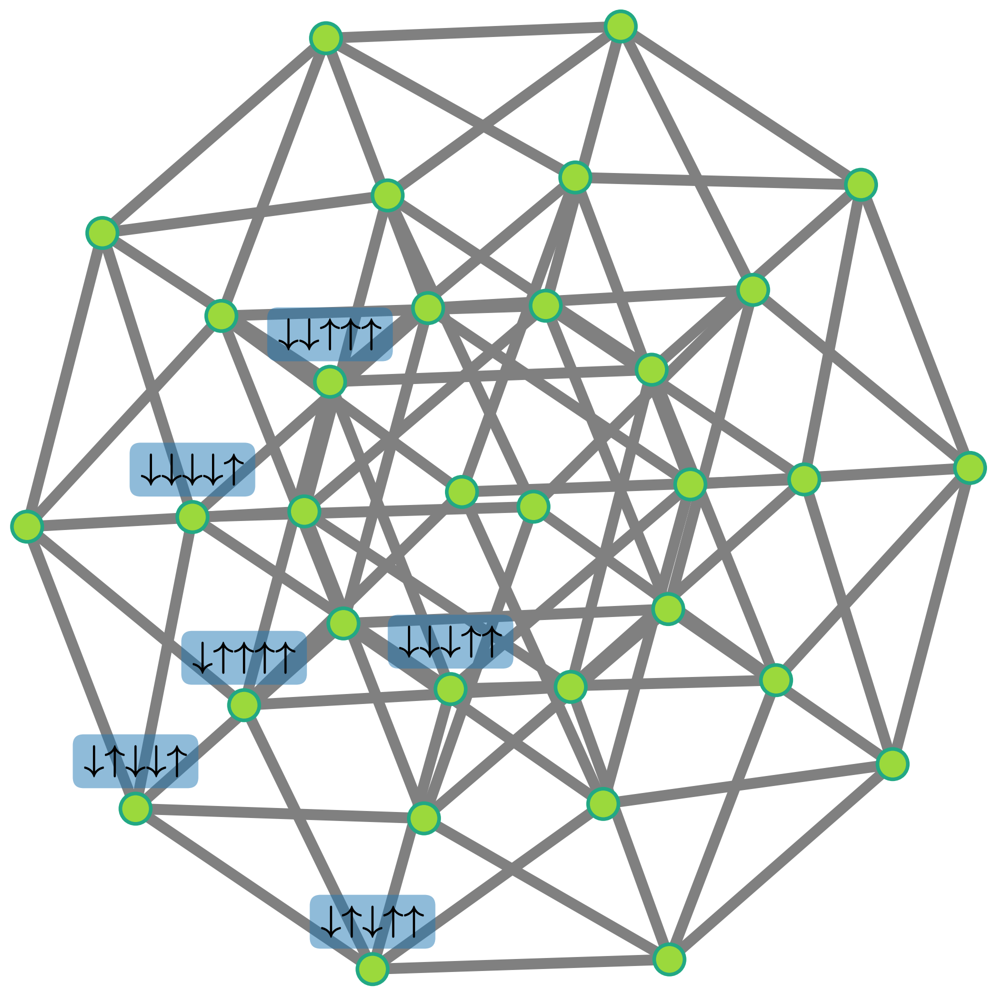
MBL on Fock space

Quantum Random Energy Model: uncorrelated random energies on every spin-configuration

$$H_{\mathrm{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Independent Gaussian random numbers:

$$\langle \mathscr{E}_I \mathscr{E}_K \rangle = \delta_{IK} N W$$



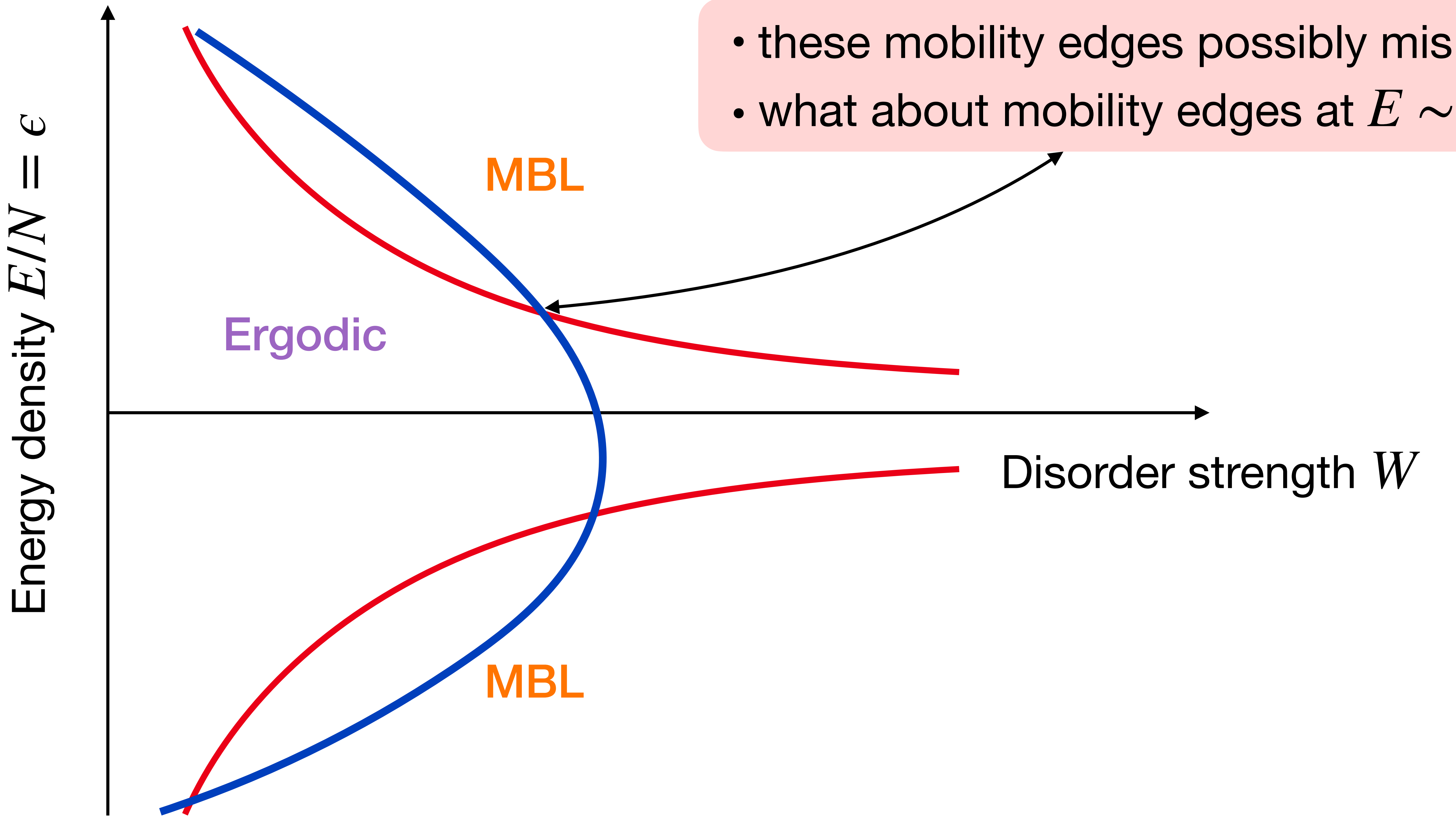
Disordered quantum spin chain

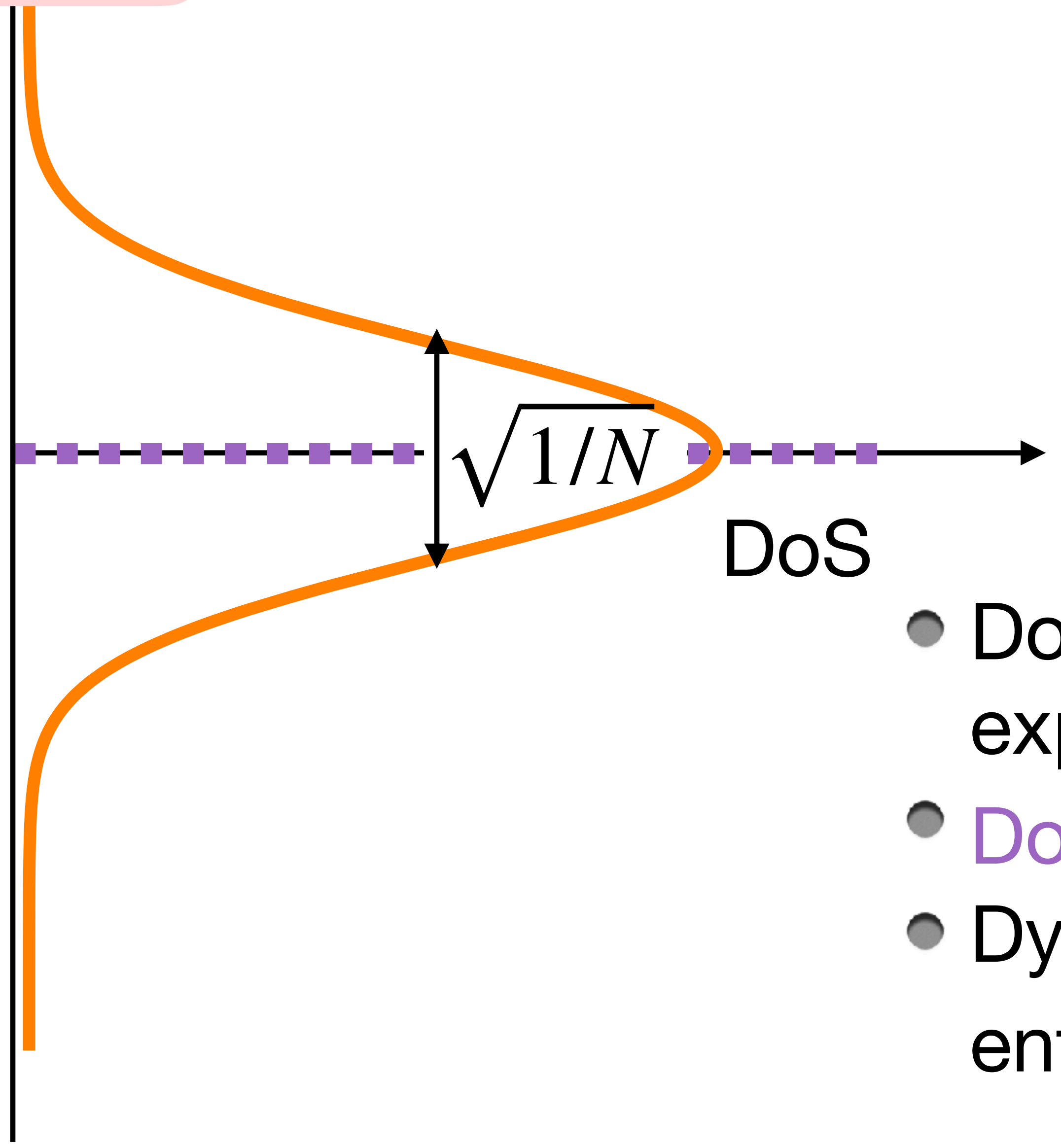
$$H_{\mathrm{TFI}} = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z] + \Gamma \sum_i \sigma_i^x$$

Effective disorder on the Fock space is correlated:

$$\langle \mathscr{E}_I \mathscr{E}_K \rangle \neq 0$$

- these mobility edges possibly misleading
- what about mobility edges at $E \sim \sqrt{N}$?





- DoS in at any finite energy density is exponentially small
- DoS $\equiv \delta(\epsilon)$ in the $N \to \infty$ limit
- Dynamics in general controlled entirely by nature of states at $\epsilon = 0$

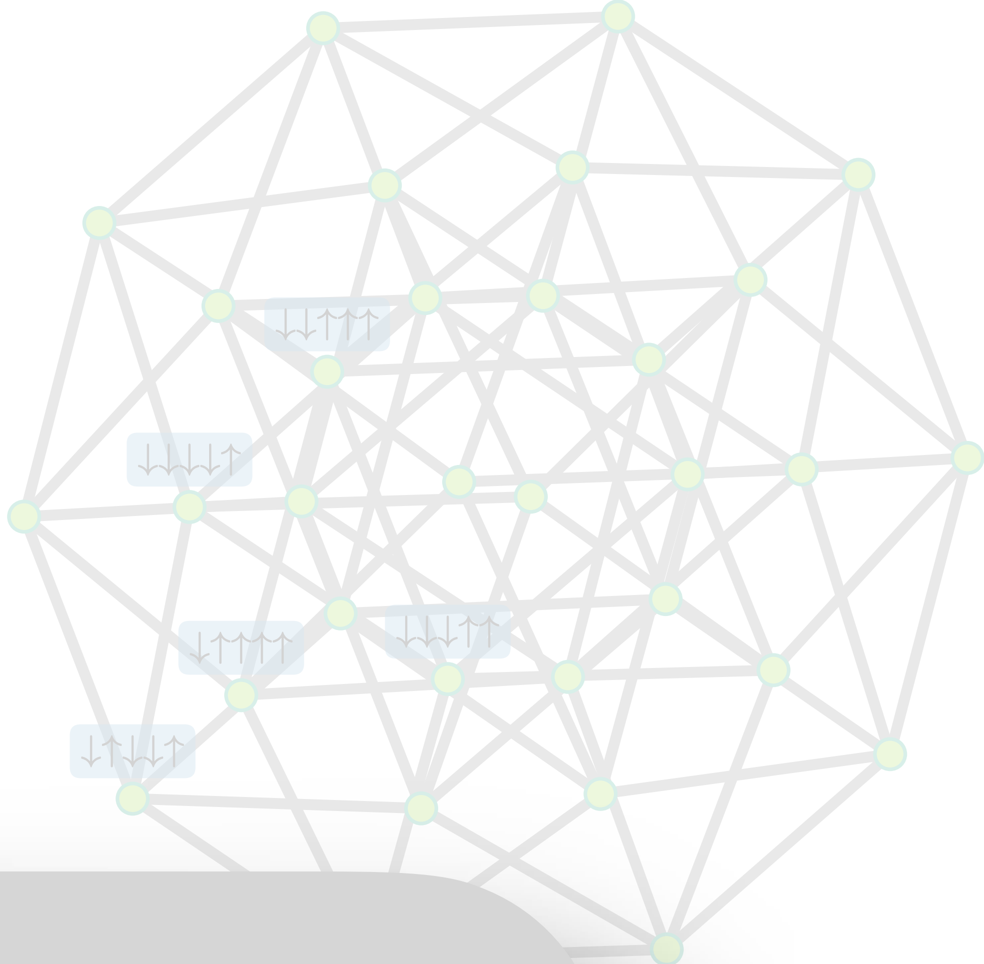
MBL on Fock space

Quantum Random Energy Model: uncorrelated random energies on every spin-configuration

$$H_{\text{QREM}} = \sum_I \mathcal{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

Independent Gaussian random numbers:

$$\langle \mathcal{E}_I \mathcal{E}_K \rangle = \delta_{IK} N W$$



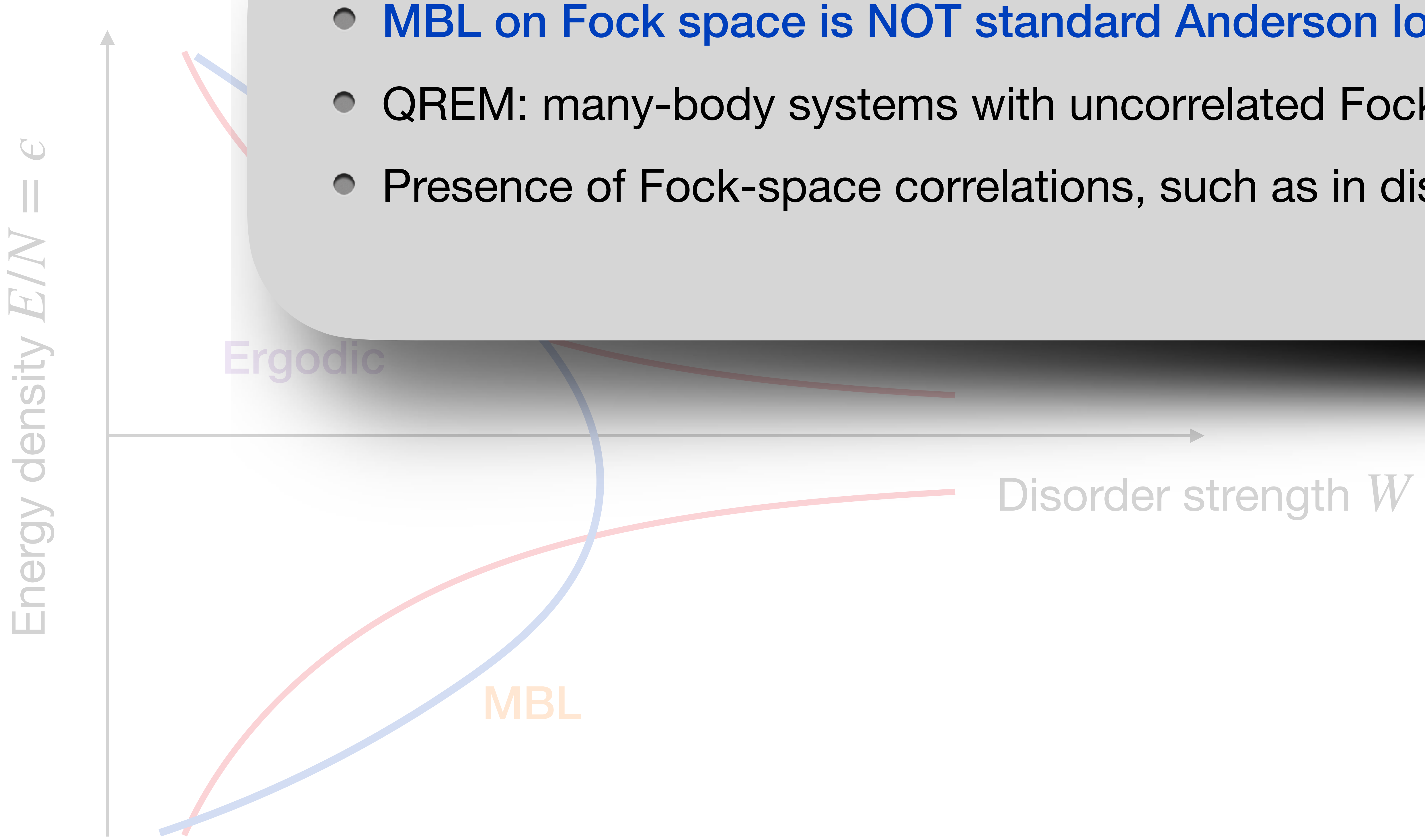
Disordered quantum spin chain

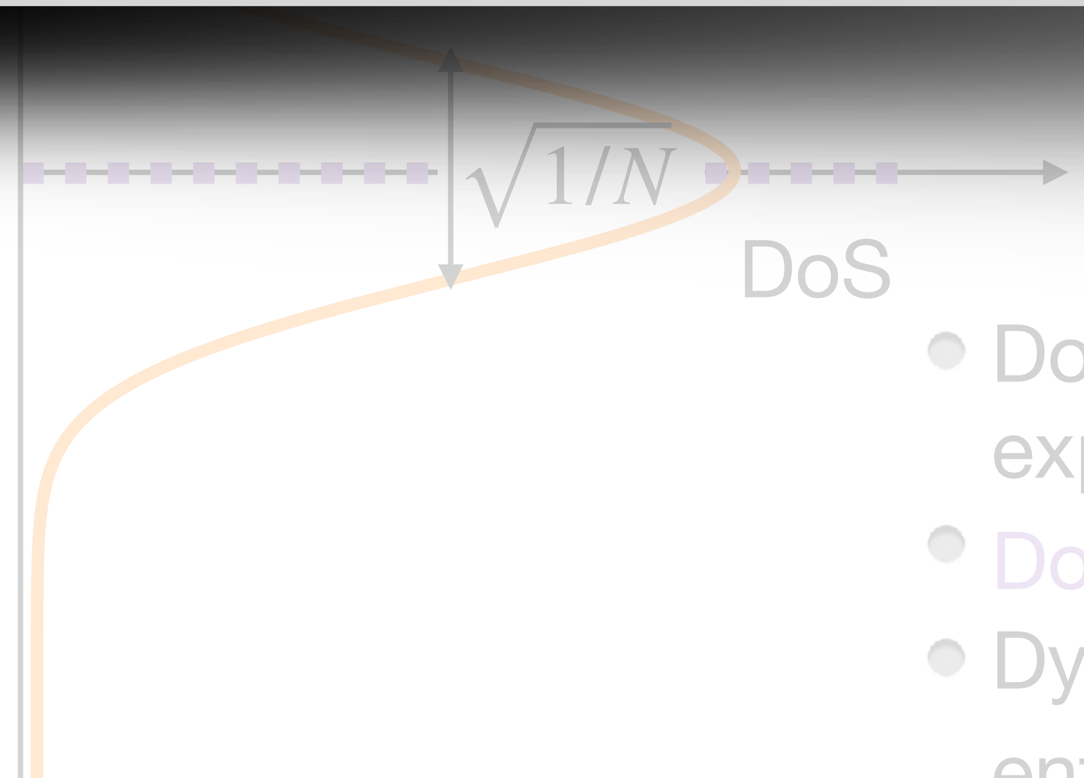
$$H_{\text{TFI}} = \sum [J_i\sigma_i^z\sigma_{i+1}^z + h_i\sigma_i^z] + \Gamma \sum \sigma_i^x$$

Effective disorder on the Fock space is correlated:

$$\langle \mathcal{E}_I \mathcal{E}_K \rangle \neq 0$$

- **MBL on Fock space is NOT standard Anderson localisation problem on a high-dimensional graph**
- QREM: many-body systems with uncorrelated Fock-space disorder doesn't host a MBL phase
- Presence of Fock-space correlations, such as in disordered local Hamiltonians crucial





- DoS in at any finite energy density is exponentially small
- DoS $\equiv \delta(\epsilon)$ in the $N \to \infty$ limit
- Dynamics in general controlled entirely by nature of states at $\epsilon = 0$

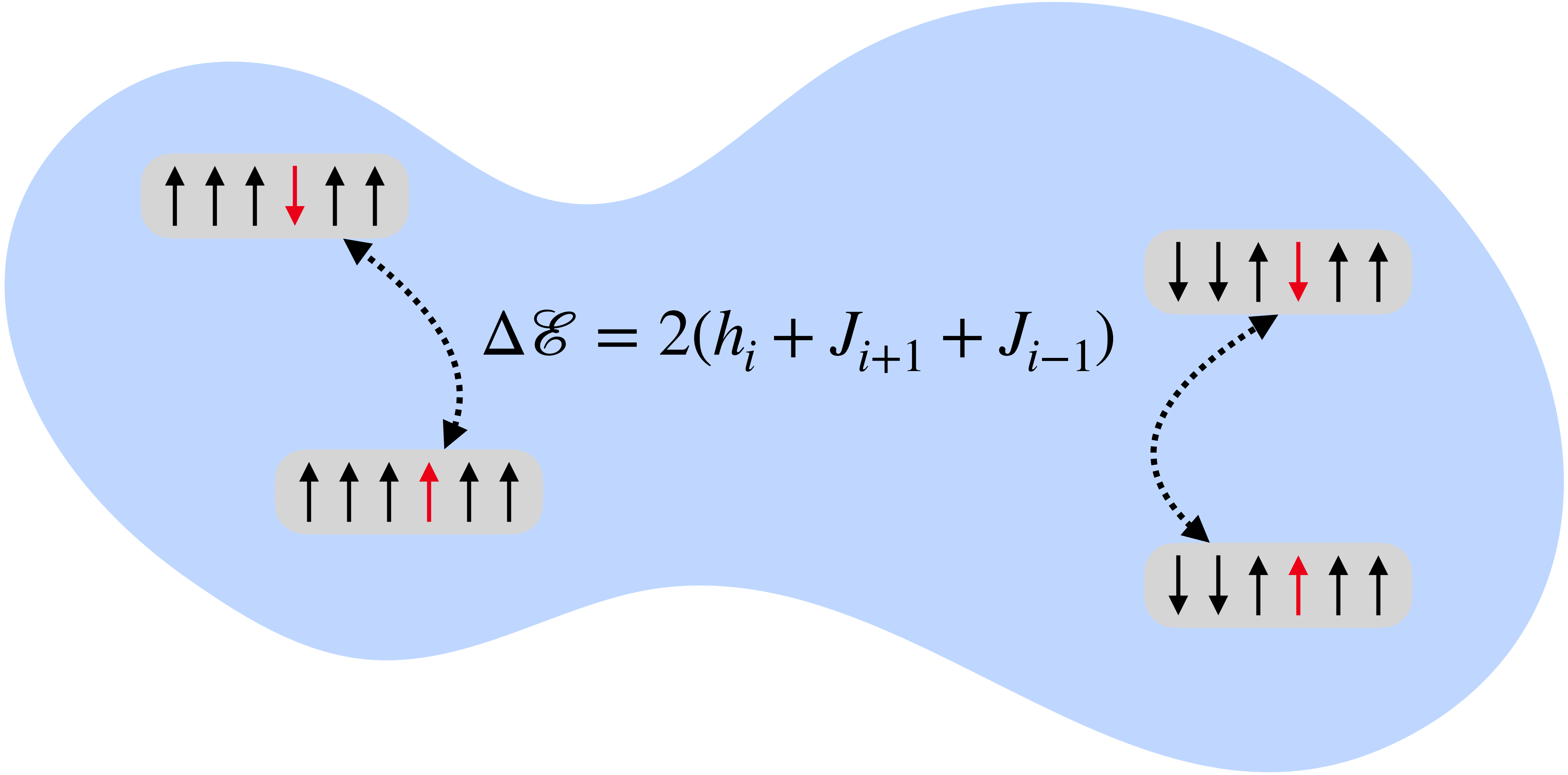
Correlations in Fock-space site energies

Diagonal part of the Hamiltonian for a system with local interactions

$$H_{\text{diag}} = \sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i=1} J_i \sigma_i^z \sigma_{i+1}^z$$

— number of random numbers required is polynomially large in N

— they constitute the $\sim \exp(N)$ Fock-space site energies $\{\mathscr{E}_I\}$



$\sim \exp(N)$ such linked pairs in Fock space

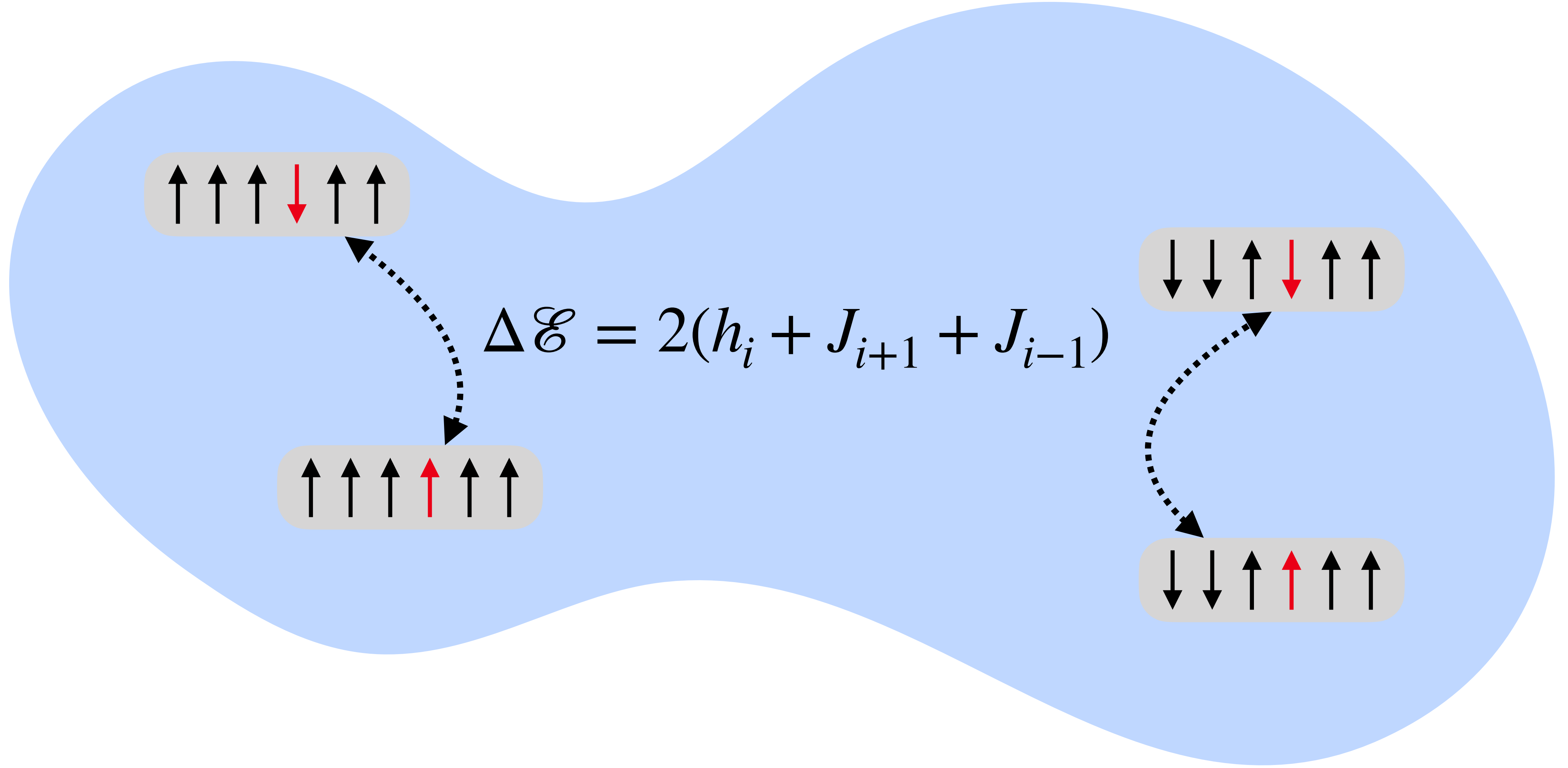
Correlations in Fock-space site energies

Diagonal part of the Hamiltonian for a system with local interactions

$$H_{\text{diag}} = \sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i=1} J_i \sigma_i^z \sigma_{i+1}^z$$

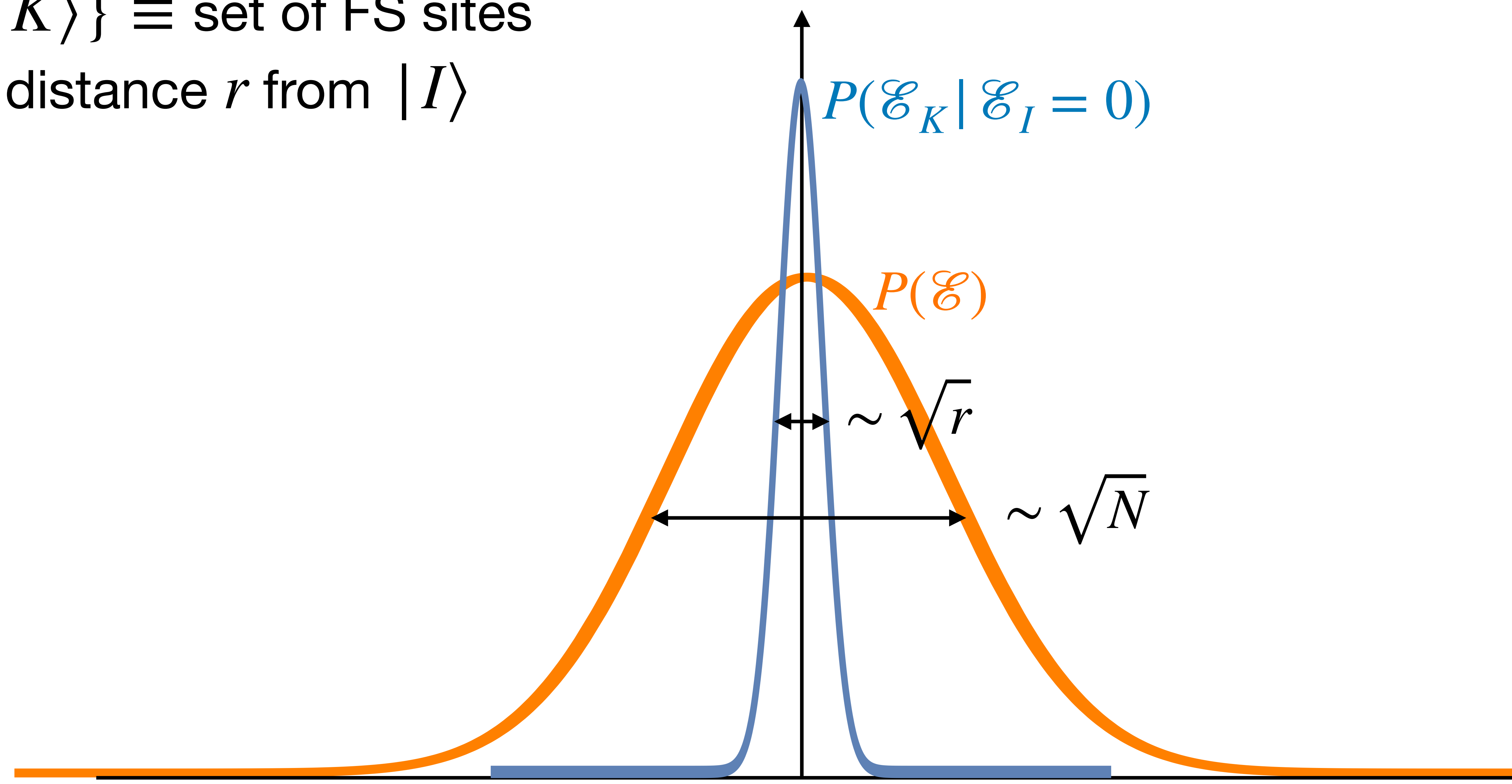
— number of random numbers required is polynomially large in N

— they constitute the $\sim \exp(N)$ Fock-space site energies $\{\mathscr{E}_I\}$



$\sim \exp(N)$ such linked pairs in Fock space

$\{|K\rangle\} \equiv$ set of FS sites
at distance r from $|I\rangle$



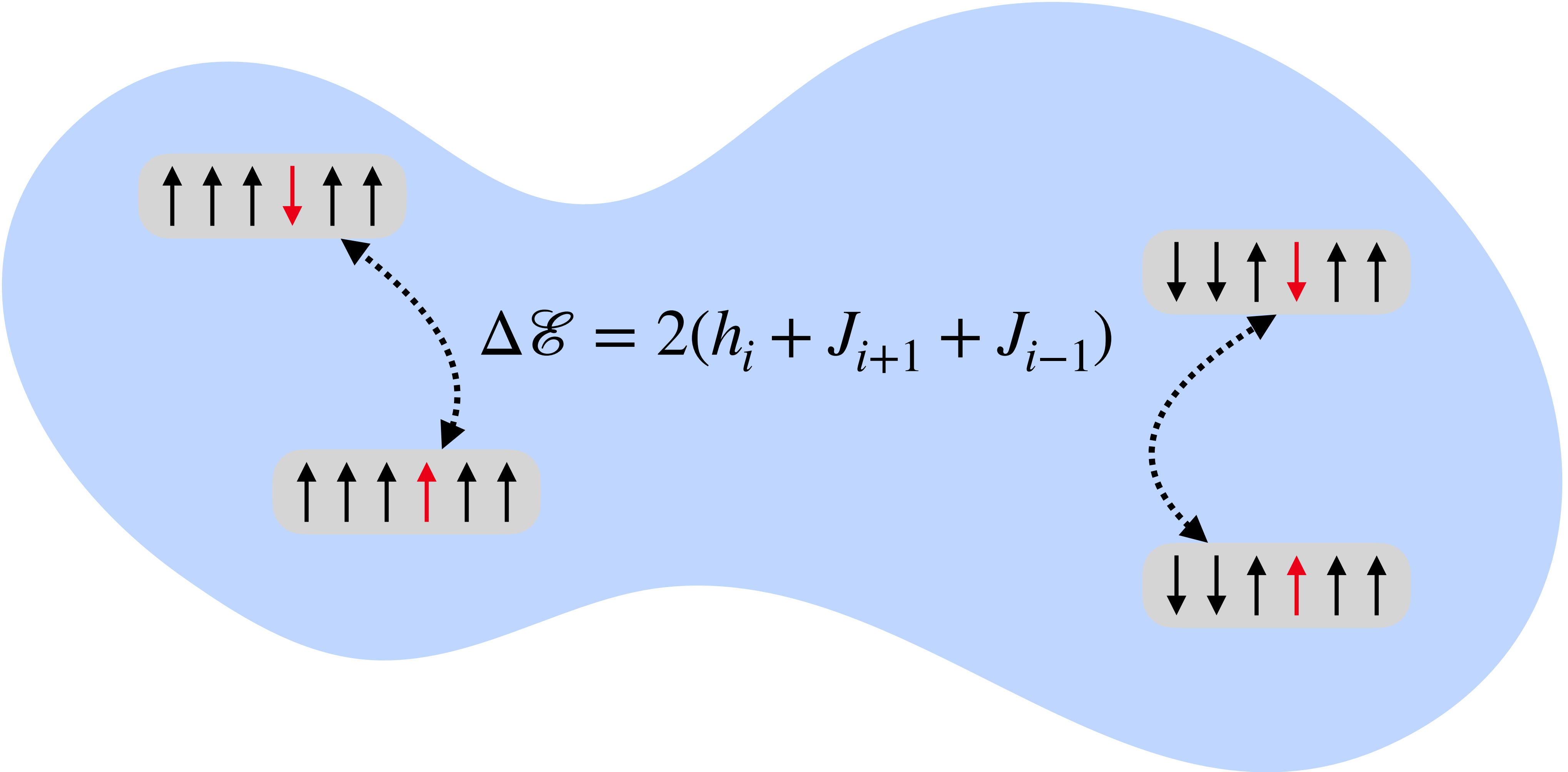
★ Energies of Fock-space sites at finite distance from each other the Fock-space graph are completely slaved to each other

Correlations in Fock-space site energies

Problem fully specified by the **joint distribution** of the site energies

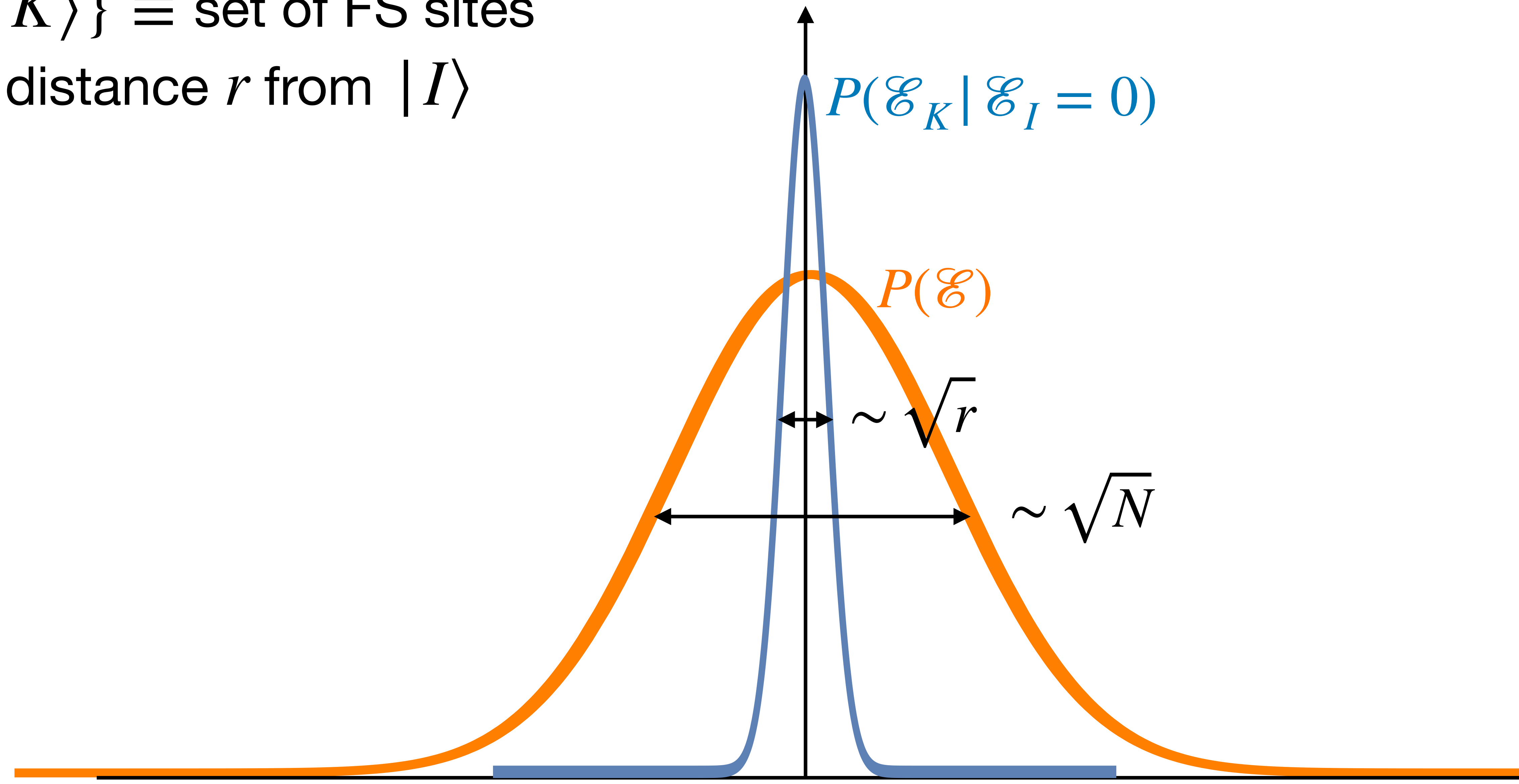
$$P_{N_{\mathscr{H}}}(\{\mathscr{E}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|\mathbf{C}|}} \exp\left[-\frac{1}{2}\vec{\mathscr{E}}^T\cdot\mathbf{C}^{-1}\cdot\vec{\mathscr{E}}\right]$$

- Covariance matrix $\mathbf{C}$ completely specifies the distribution
- matrix elements of $\mathbf{C}$ depend on the distance between the sites



$\sim \exp(N)$ such linked pairs in Fock space

$\{|K\rangle\} \equiv$ set of FS sites
at distance r from $|I\rangle$



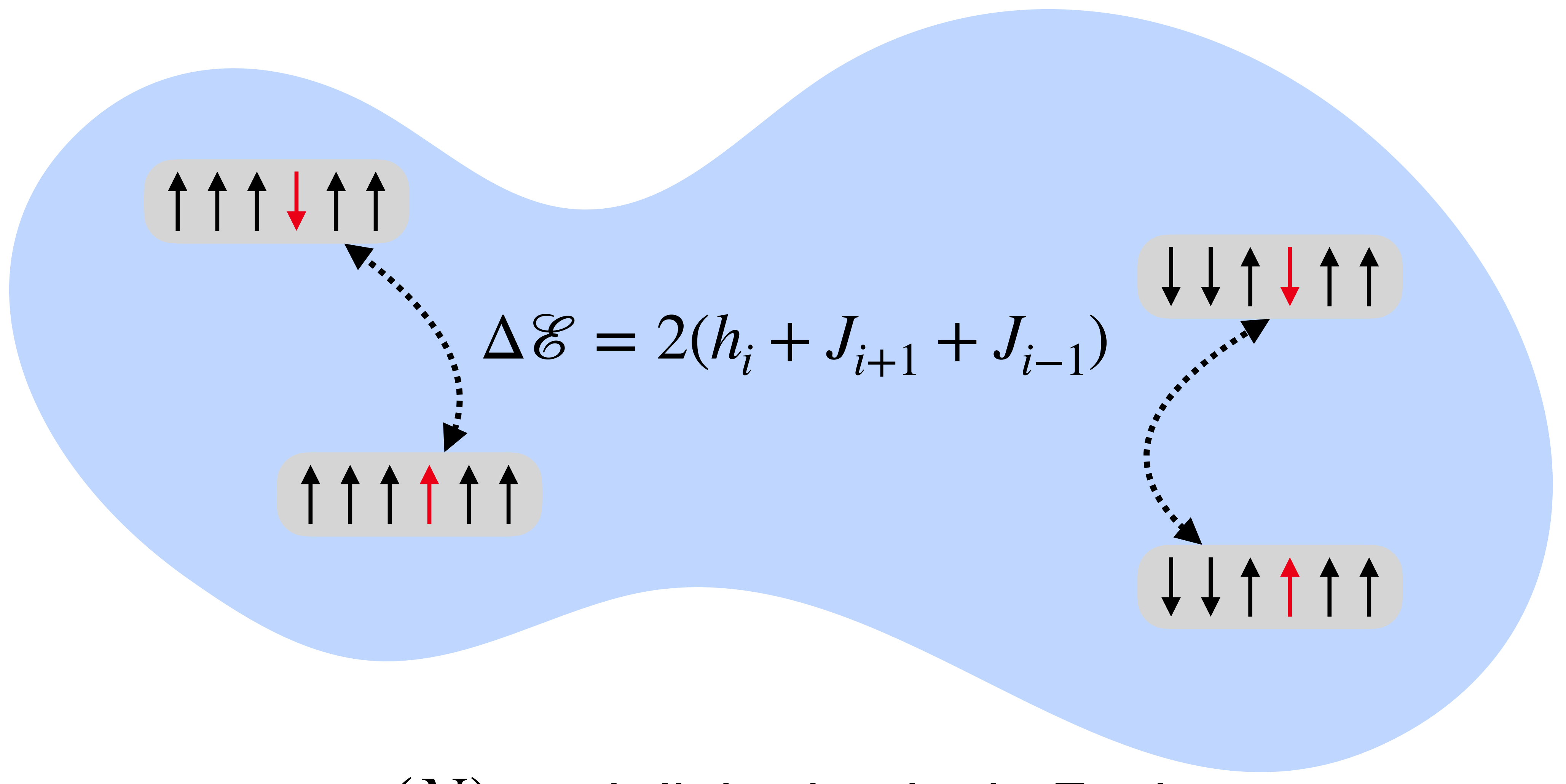
★ Energies of Fock-space sites at finite distance from each other the Fock-space graph are completely slaved to each other

Correlations in Fock-space site energies

Problem fully specified by the **joint distribution** of the site energies

$$P_{N_{\mathscr{H}}}(\{\mathscr{E}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|\mathbf{C}|}} \exp\left[-\frac{1}{2}\vec{\mathscr{E}}^T\cdot\mathbf{C}^{-1}\cdot\vec{\mathscr{E}}\right]$$

- Covariance matrix $\mathbf{C}$ completely specifies the distribution
- matrix elements of $\mathbf{C}$ depend on the distance between the sites



$\sim \exp(N)$ such linked pairs in Fock space

General form of the covariance

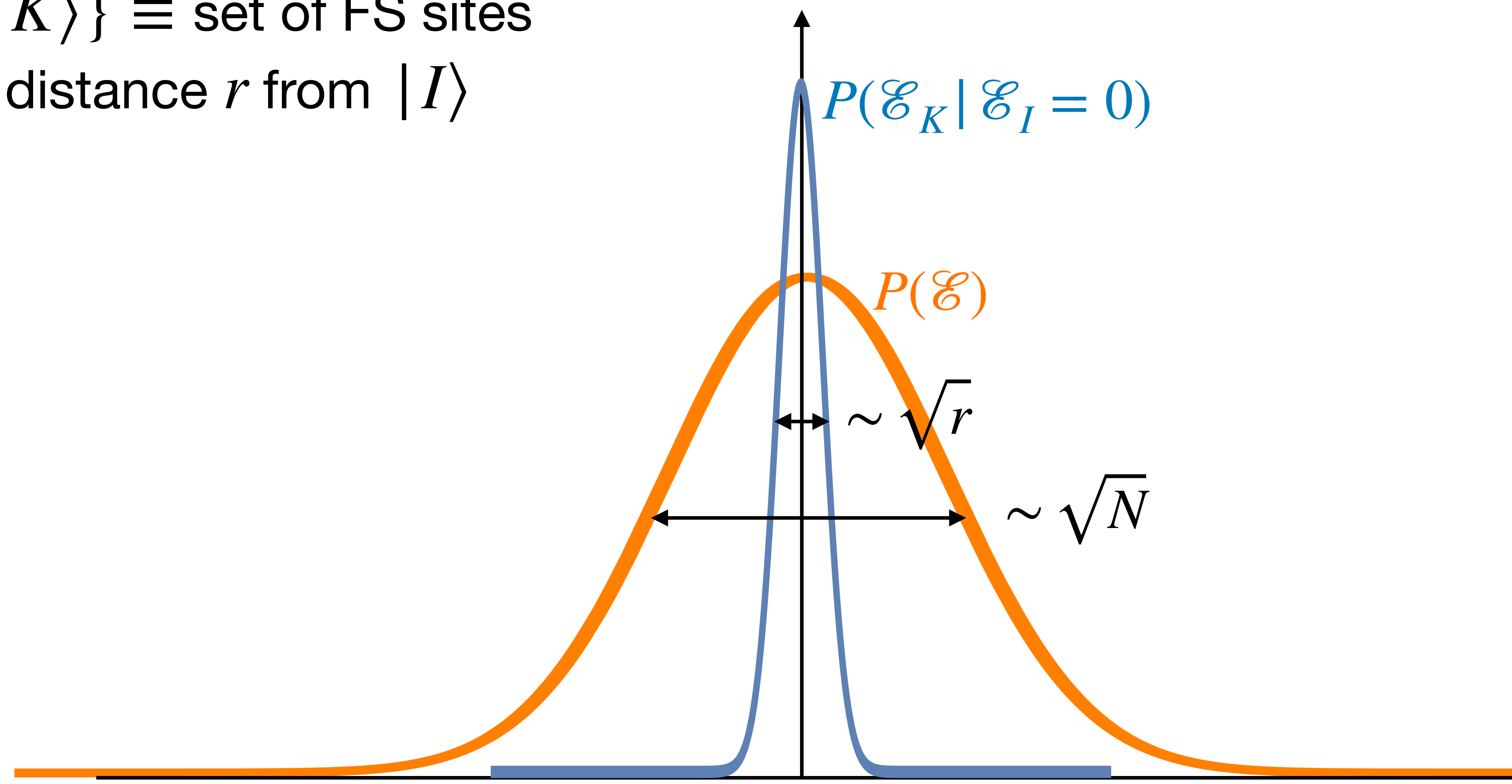
$$C(r) = W^2 N\rho(r,N); \quad \rho(r=0,\,N)=1$$

Effective disorder

Scaling with system size

All dependence on Hamming distance

$\{|K\rangle\} \equiv$ set of FS sites at distance r from $|I\rangle$



★ $\rho(r,N)$ generally a p-order polynomial of r/N for a p-spin system

$\Rightarrow \rho(r,N)\to 1$ for $r/N\to 0$

★ Energies of Fock-space sites at finite distance from each other the Fock-space graph are completely slaved to each other

Correlations in Fock-space site energies

Problem fully specified by the **joint distribution** of the site energies

$$P_{N_{\mathscr{H}}}(\{\mathscr{E}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|\mathbf{C}|}} \exp\left[-\frac{1}{2}\vec{\mathscr{E}}^T \cdot \mathbf{C}^{-1} \cdot \vec{\mathscr{E}}\right]$$

- Covariance matrix $\mathbf{C}$ completely specifies the distribution
- matrix elements of $\mathbf{C}$ depend on the distance between the sites

General form of the covariance

$$C(r) = W^2 N \rho(r, N); \quad \rho(r=0, N) = 1$$

Effective disorder

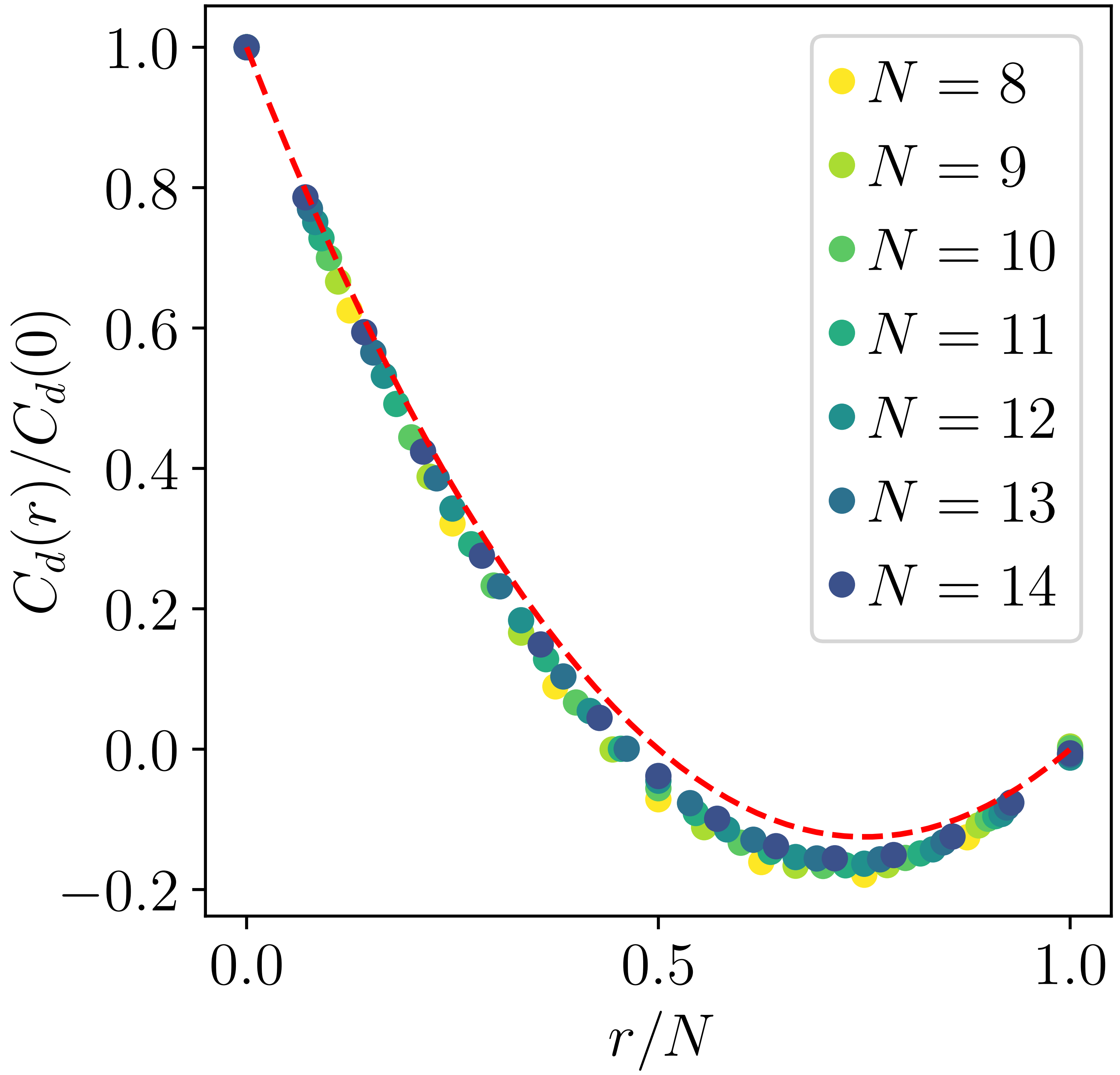
Scaling with system size

All dependence on Hamming distance

★ $\rho(r, N)$ generally a p-order polynomial of r/N for a p-spin system
$\Rightarrow \rho(r, N) \to 1$ for $r/N \to 0$

★ Energies of Fock-space sites at finite distance from each other the Fock-space graph are completely slaved to each other

$$H_{\text{diag}} = \sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i=1} J_i \sigma_i^z \sigma_{i+1}^z$$



$$\rho(r, N) = a\left(1 - \frac{2r}{N}\right)^2 + (1-a)\left(1 - \frac{2r}{N}\right)$$

Correlations in Fock-space site energies

Problem fully specified by the **joint distribution** of the site energies

$$P_{N_{\mathscr{H}}}(\{\mathscr{E}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|\mathbf{C}|}} \exp\left[-\frac{1}{2}\vec{\mathscr{E}}^T \cdot \mathbf{C}^{-1} \cdot \vec{\mathscr{E}}\right]$$

— Covariance matrix $\mathbf{C}$ completely specifies the distribution
— matrix elements of $\mathbf{C}$ depend on the distance between the sites

General form of the covariance

$$C(r) = W^2 N \rho(r,N); \quad \rho(r=0,\, N) = 1$$

Effective disorder

Scaling with system size

All dependence on Hamming distance

★ $\rho(r,N)$ generally a p-order polynomial of r/N for a p-spin system
$\Rightarrow \rho(r,N) \to 1$ for $r/N \to 0$

★ Energies of Fock-space sites at finite distance from each other the Fock-space graph are completely slaved to each other

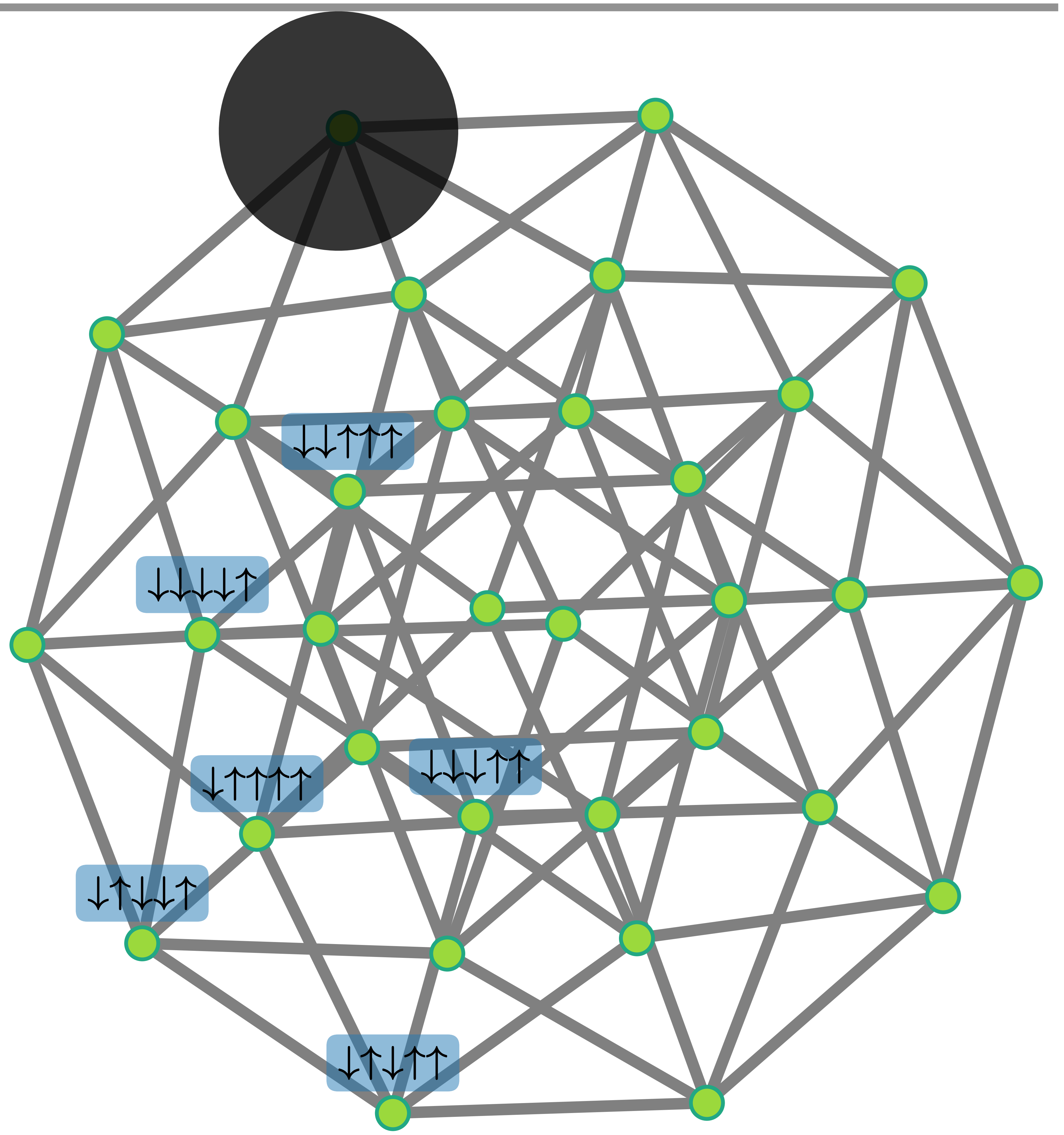
For the distribution to be stable in the $N \to \infty$ limit, one needs to rescale

$$\tilde{\mathscr{E}} = \mathscr{E}/\sqrt{N}$$

$$P_{N_{\mathscr{H}}}(\{\tilde{\mathscr{E}}_I\}) = \frac{1}{\sqrt{(2\pi)^{N_{\mathscr{H}}}|W^2\rho|}} \exp\left[-\frac{1}{2}\vec{\tilde{\mathscr{E}}}^T \cdot (W^2\rho)^{-1} \cdot \vec{\tilde{\mathscr{E}}}\right]$$

Self-consistent theory of MBL

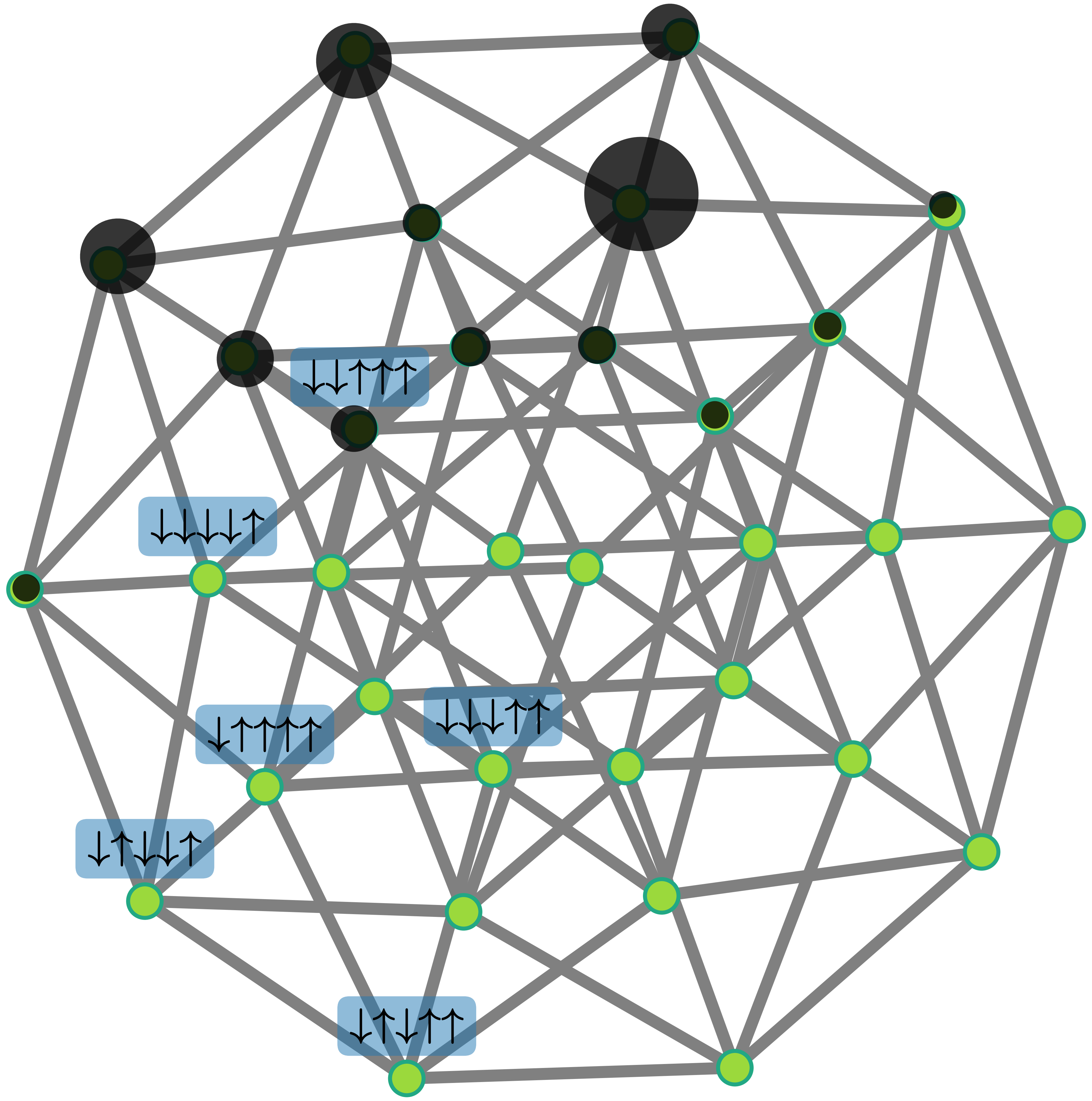
How does a state initially localised on a Fock-space site spread out ?



Self-consistent theory of MBL

How does a state initially localised on a Fock-space site spread out ?

Encoded in the local Fock-space propagator: $G_I(t) = -i\Theta(t)\langle I|e^{-iHt}|I\rangle$



Self-consistent theory of MBL

How does a state initially localised on a Fock-space site spread out ?

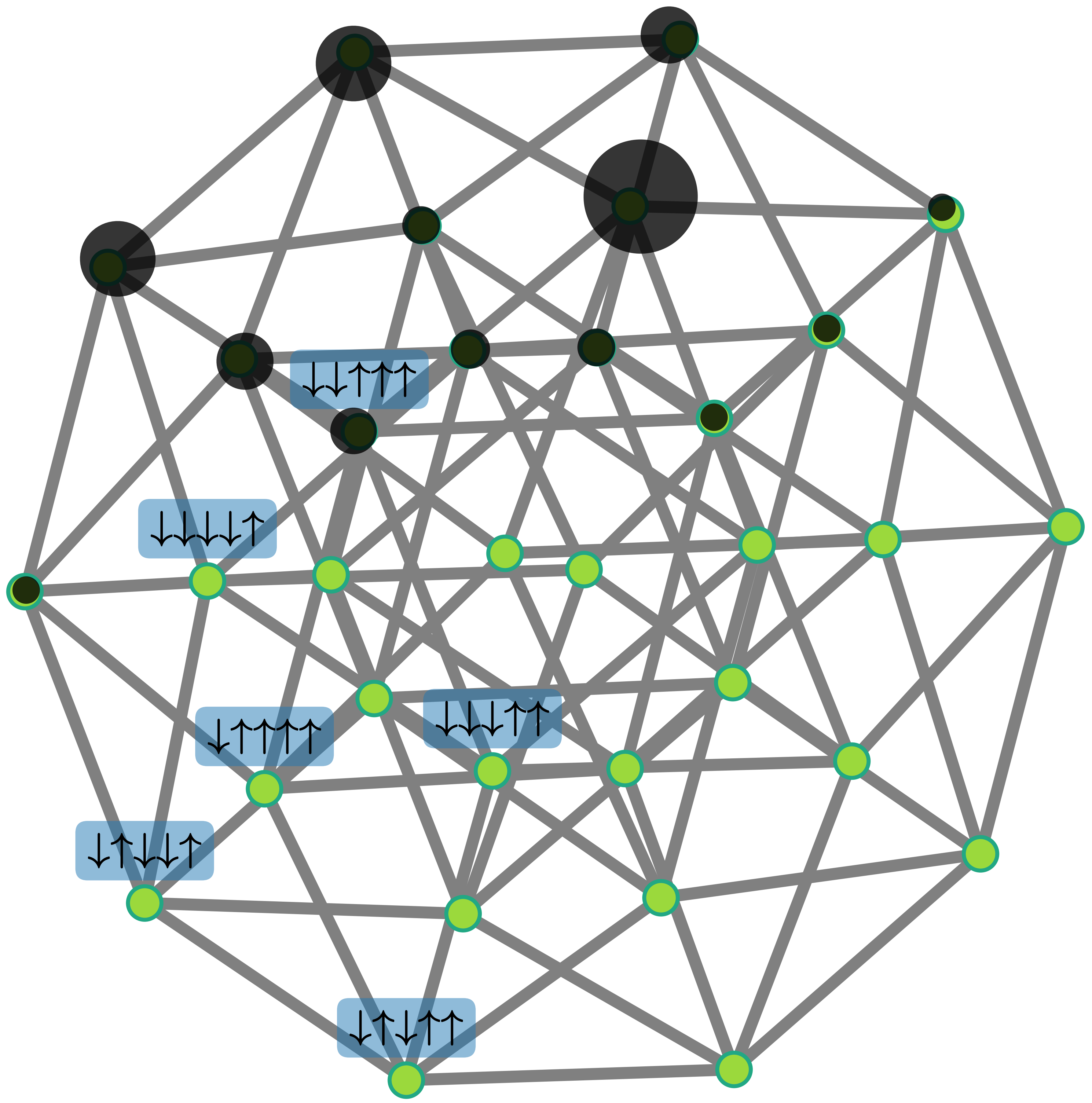
Encoded in the local Fock-space propagator: $G_I(t) = -i\Theta(t)\langle I|e^{-iHt}|I\rangle$

In the frequency domain: $G_I(\omega) = \langle I|(\omega + i\eta - H)^{-1}|I\rangle$

$$= [\omega^+ - \mathscr{E}_I - S_I(\omega)]^{-1}$$

Self-energy: $S_I(\omega) = X_I(\omega) - i\Delta_I(\omega)$

Rate of loss of probability from site I into states at energy ω



Self-consistent theory of MBL

How does a state initially localised on a Fock-space site spread out ?

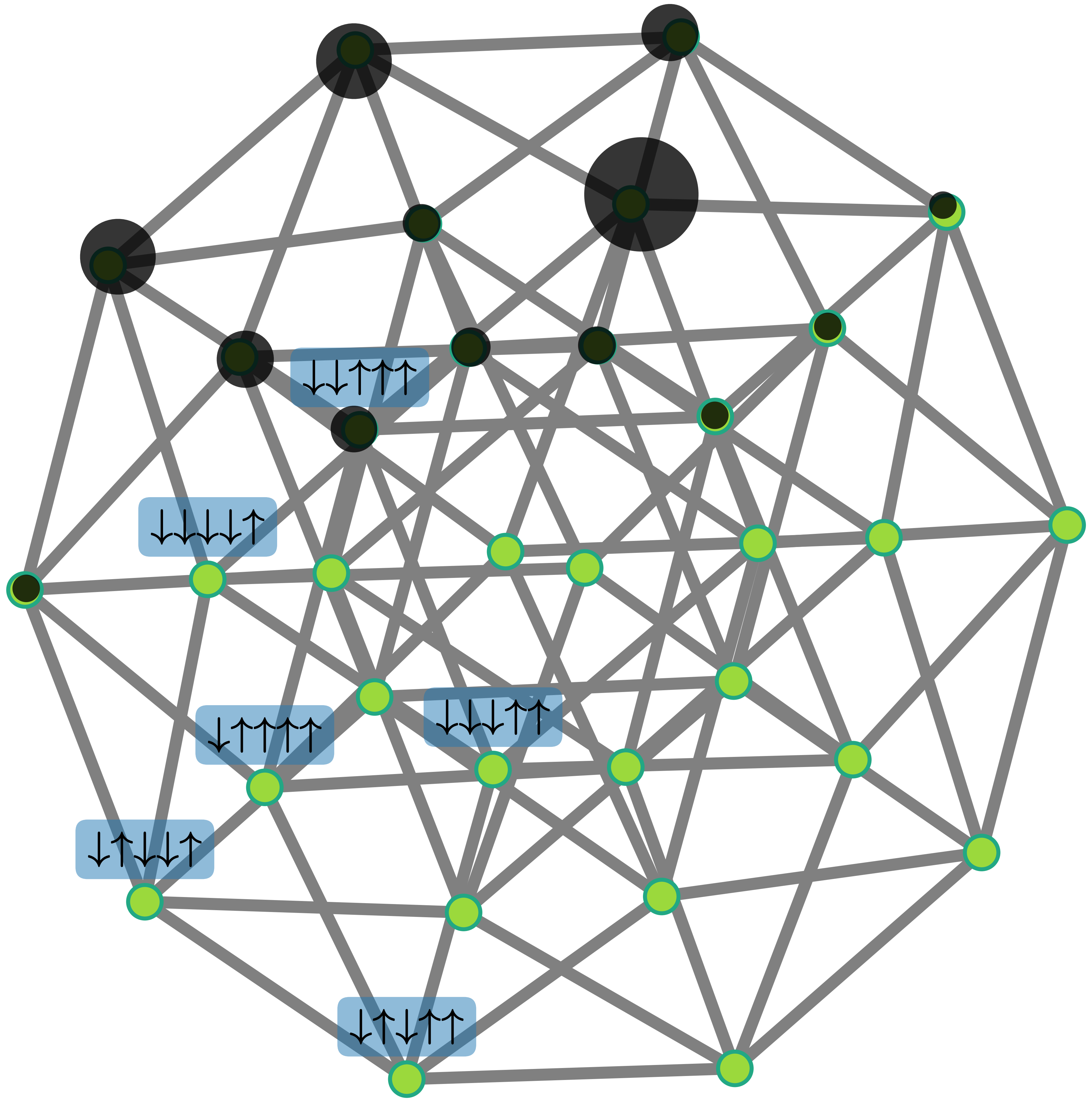
Encoded in the local Fock-space propagator: $G_I(t) = -i\Theta(t)\langle I|e^{-iHt}|I\rangle$

In the frequency domain: $G_I(\omega) = \langle I|(\omega + i\eta - H)^{-1}|I\rangle$

$$= [\omega^+ - \mathcal{E}_I - S_I(\omega)]^{-1}$$

Self-energy: $S_I(\omega) = X_I(\omega) - i\Delta_I(\omega)$

Rate of loss of probability from site I into states at energy ω



- — probabilistic order parameter for localisation-delocalisation transition
- — delocalised phase: $\Delta_I(\omega)$ is non-vanishing typically
- — localised phase: $\Delta_I(\omega)$ is vanishing ($\sim \eta$)

Self-consistent theory of MBL

How does a state initially localised on a Fock-space site spread out ?

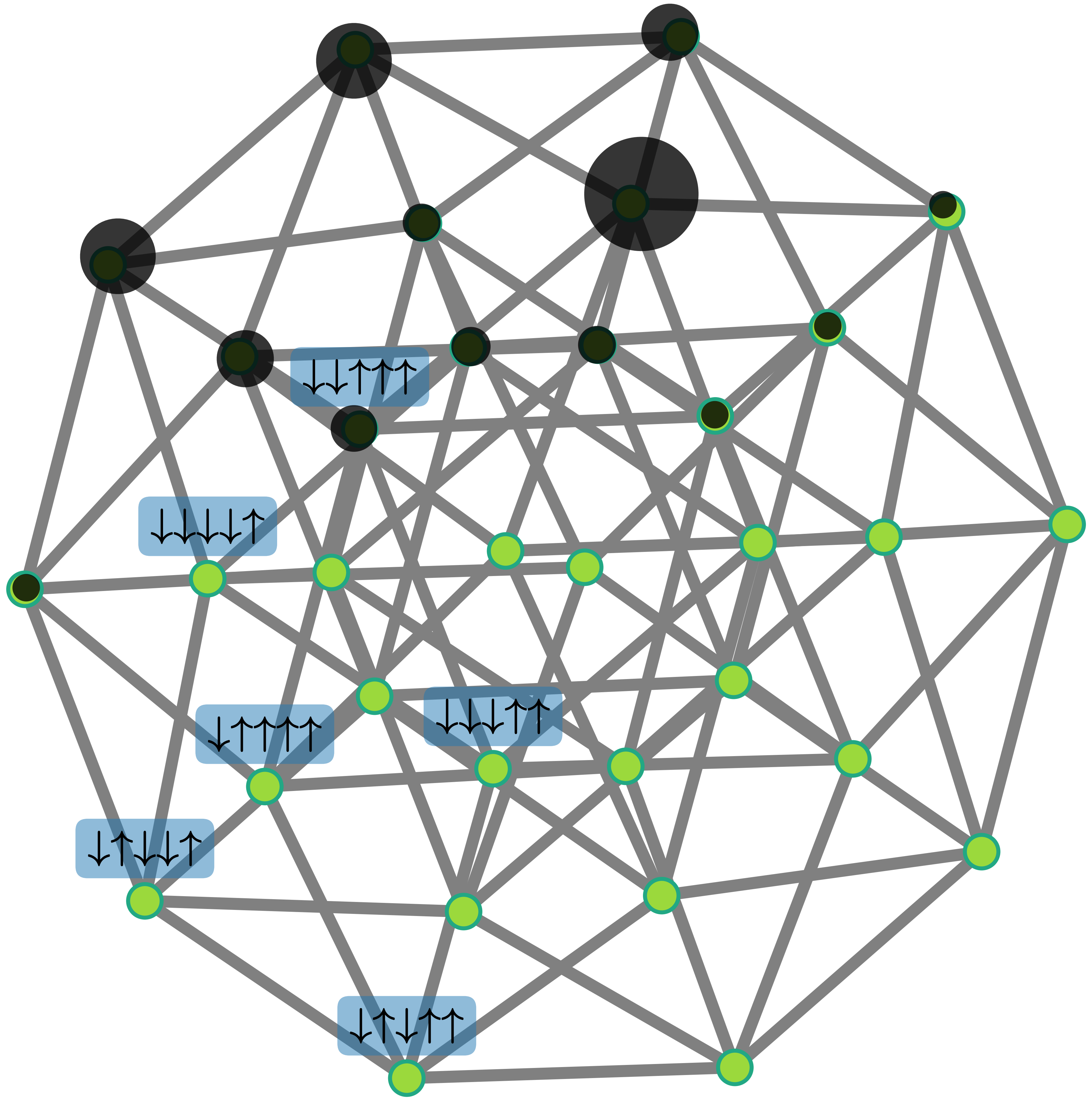
Encoded in the local Fock-space propagator: $G_I(t) = -i\Theta(t)\langle I | e^{-iHt} | I \rangle$

In the frequency domain: $G_I(\omega) = \langle I | (\omega + i\eta - H)^{-1} | I \rangle$

$$= [\omega^+ - \mathscr{E}_I - S_I(\omega)]^{-1}$$

Self-energy: $S_I(\omega) = X_I(\omega) - i\Delta_I(\omega)$

Rate of loss of probability from site I into states at energy ω



Deep in a delocalised phase:

$$\Delta(\omega) \sim \Gamma^2 \times \text{number of channels} \times \text{DoS}(\omega)$$

$$\sim \Gamma^2 \times N \times \frac{1}{\sqrt{N}}$$

$$\Delta(\omega) \sim \Gamma^2\sqrt{N}$$

For the problem to remain well defined in the $N \to \infty$ limit, the energy scales need to be rescaled by $\sqrt{N}$

Self-consistent theory of MBL

Renormalised perturbation series $$\Delta_I(\omega) = \mathrm{Im}[\sum_K \frac{\mathcal{T}^2_{IK}}{\omega^+ - \mathcal{E}_K - S_K(\omega)} + \cdots]$$

Feenberg, Phys. Rev. 1948

In terms of rescaled variables: $(\tilde{\cdots}) = (\cdots)/\sqrt{N}$

1. $$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathcal{T}}^2_{IK}}{\tilde{\omega}^+ - \tilde{\mathcal{E}}_K - \tilde{S}_K(\omega)} + \cdots]$$

replace by its typical value

2. $$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathcal{T}}^2_{IK}}{\tilde{\omega}^+ - \tilde{\mathcal{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

3. Obtain distribution of $\Delta_I(\omega)$ from the joint distributions of $\{\mathcal{E}_K\}$

information of correlations gets fed in !

4. Impose self-consistency: Δ_{typ} arising from distribution must coincide with *input* Δ_{typ}

Non-trivial correlations in Fock-space disorder the root of all complications

Self-consistent theory of MBL

Renormalised perturbation series

$$\Delta_I(\omega) = \mathrm{Im}[\sum_K \frac{\mathcal{T}_{IK}^2}{\omega^+ - \mathscr{E}_K - S_K(\omega)} + \cdots]$$

Feenberg, Phys. Rev. 1948

In terms of rescaled variables: $(\tilde{\cdots}) = (\cdots)/\sqrt{N}$

① $$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathcal{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_K(\omega)} + \cdots]$$

replace by its typical value

② $$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathcal{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

③ Obtain distribution of $\Delta_I(\omega)$ from the joint distributions of $\{\mathscr{E}_K\}$

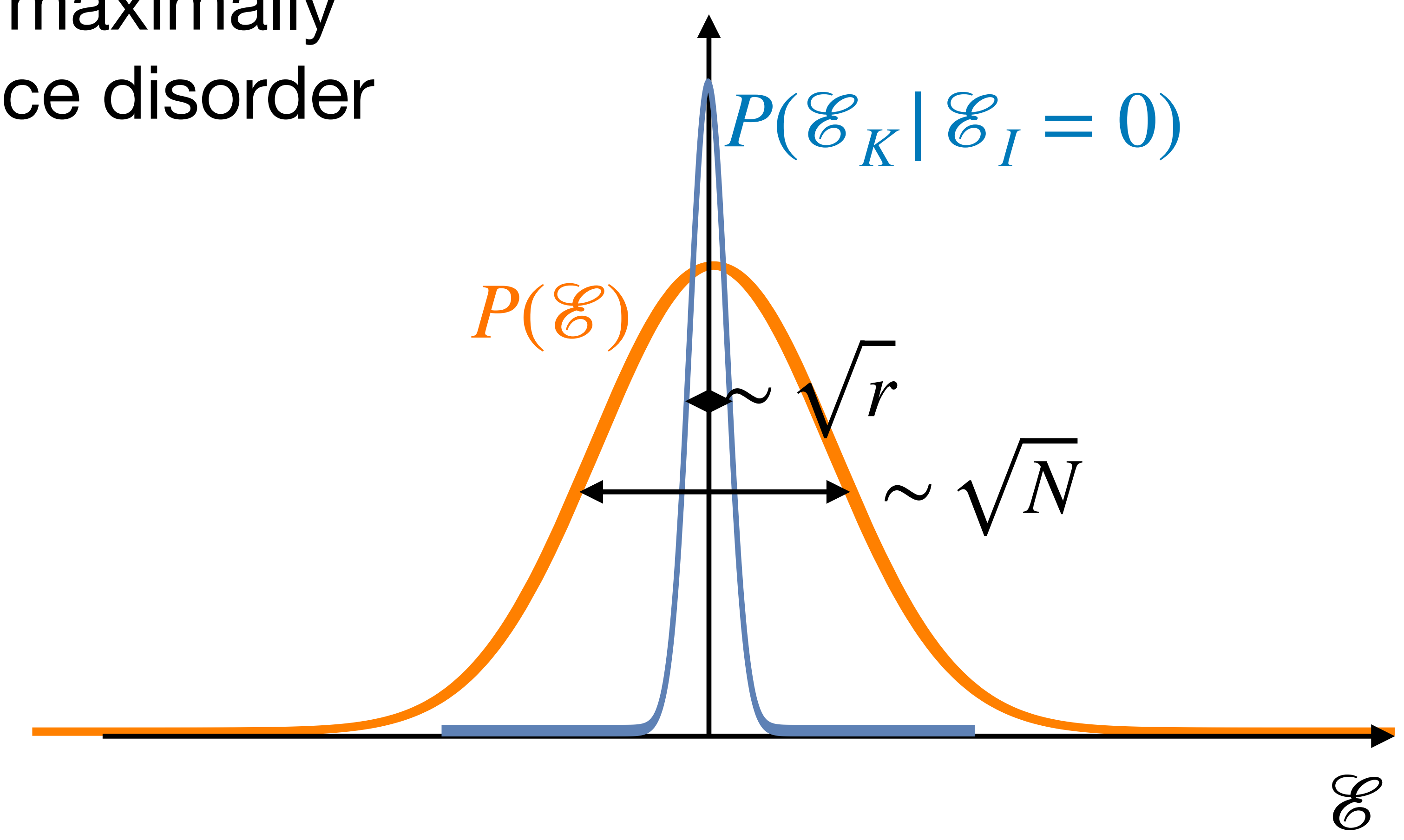
information of correlations gets fed in !

④ Impose self-consistency: Δ_{typ} arising from distribution must coincide with *input* Δ_{typ}

Non-trivial correlations in Fock-space disorder the root of all complications

Self-consistent theory of MBL

Local Hamiltonians: maximally correlated Fock-space disorder

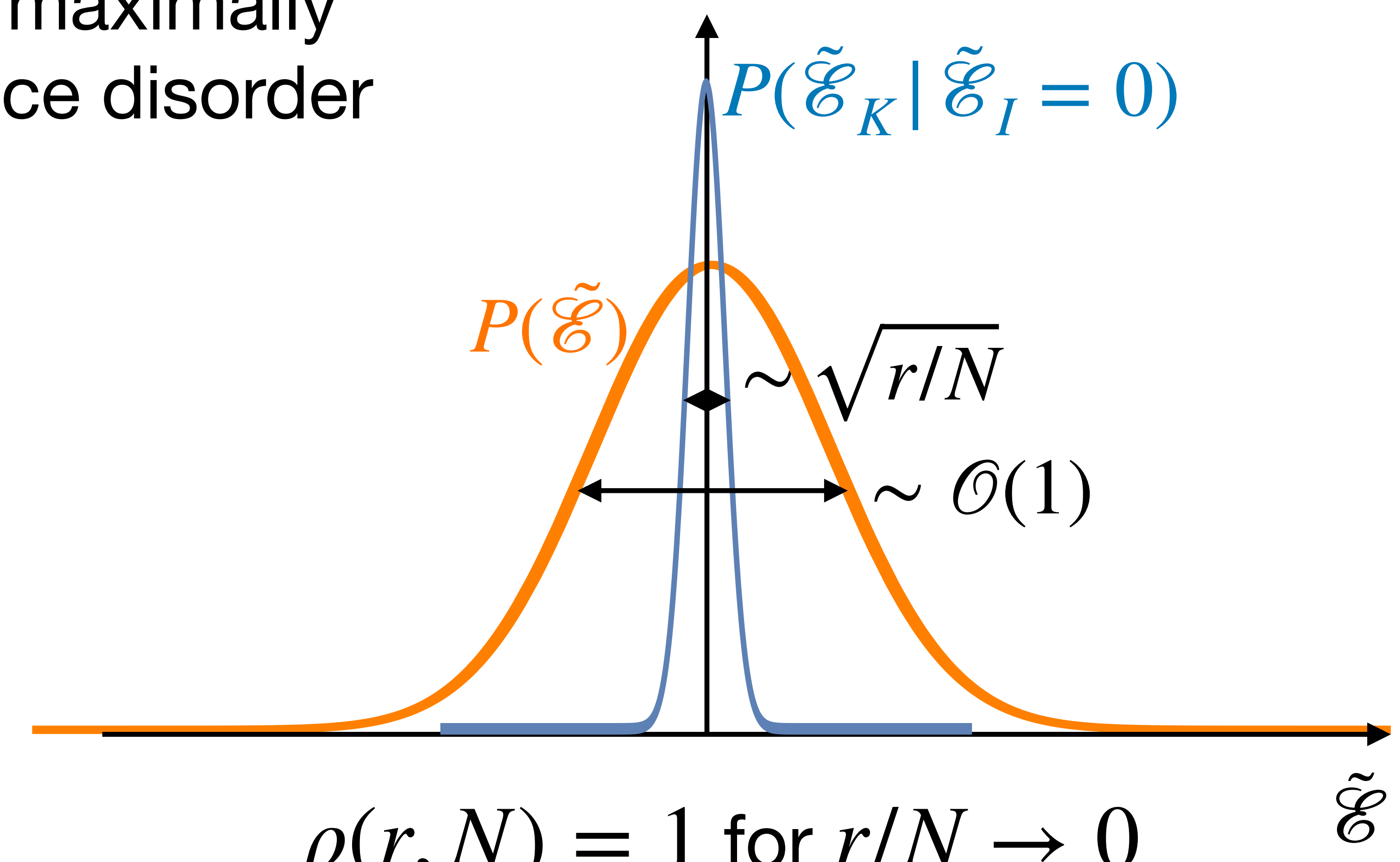


$$\boxed{\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathscr{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]}$$

—sum over states K that are a finite distance away from I on Fock space

Self-consistent theory of MBL

Local Hamiltonians: maximally correlated Fock-space disorder



$\rho(r, N) = 1$ for $r/N \to 0$

$$\boxed{\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathscr{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]}$$

—sum over states K that are a finite distance away from I on Fock space

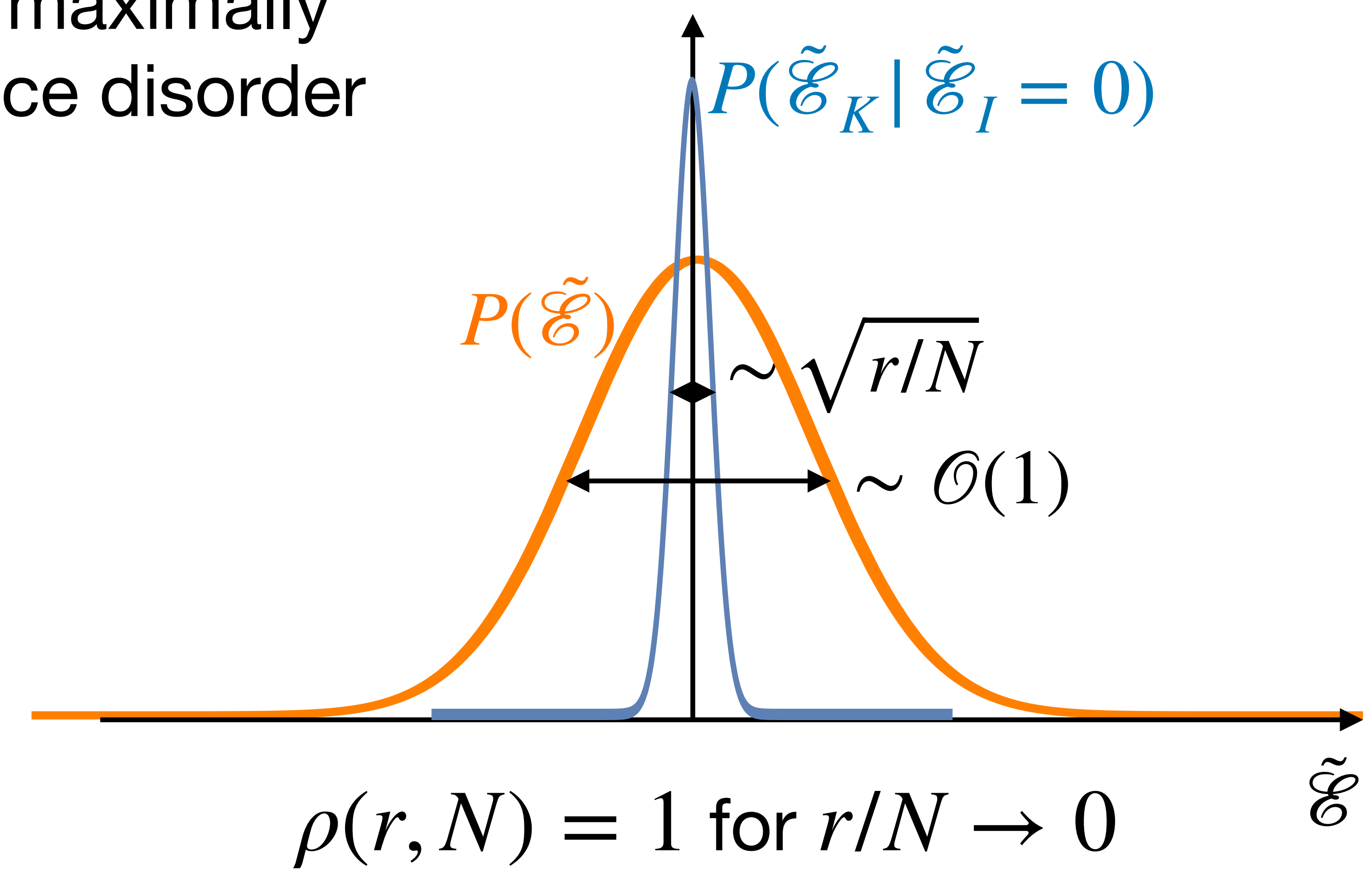
Self-consistent theory of MBL

$$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\frac{\Gamma^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_I - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

—Upshot: the self-energy is just a single term

—Break down of self-consistency of either phase indicates the MBL transition

Local Hamiltonians: maximally correlated Fock-space disorder



$\rho(r, N) = 1$ for $r/N \to 0$

$$\boxed{\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathscr{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]}$$

—sum over states K that are a finite distance away from I on Fock space

Self-consistent theory of MBL

$$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\frac{\Gamma^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_I - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

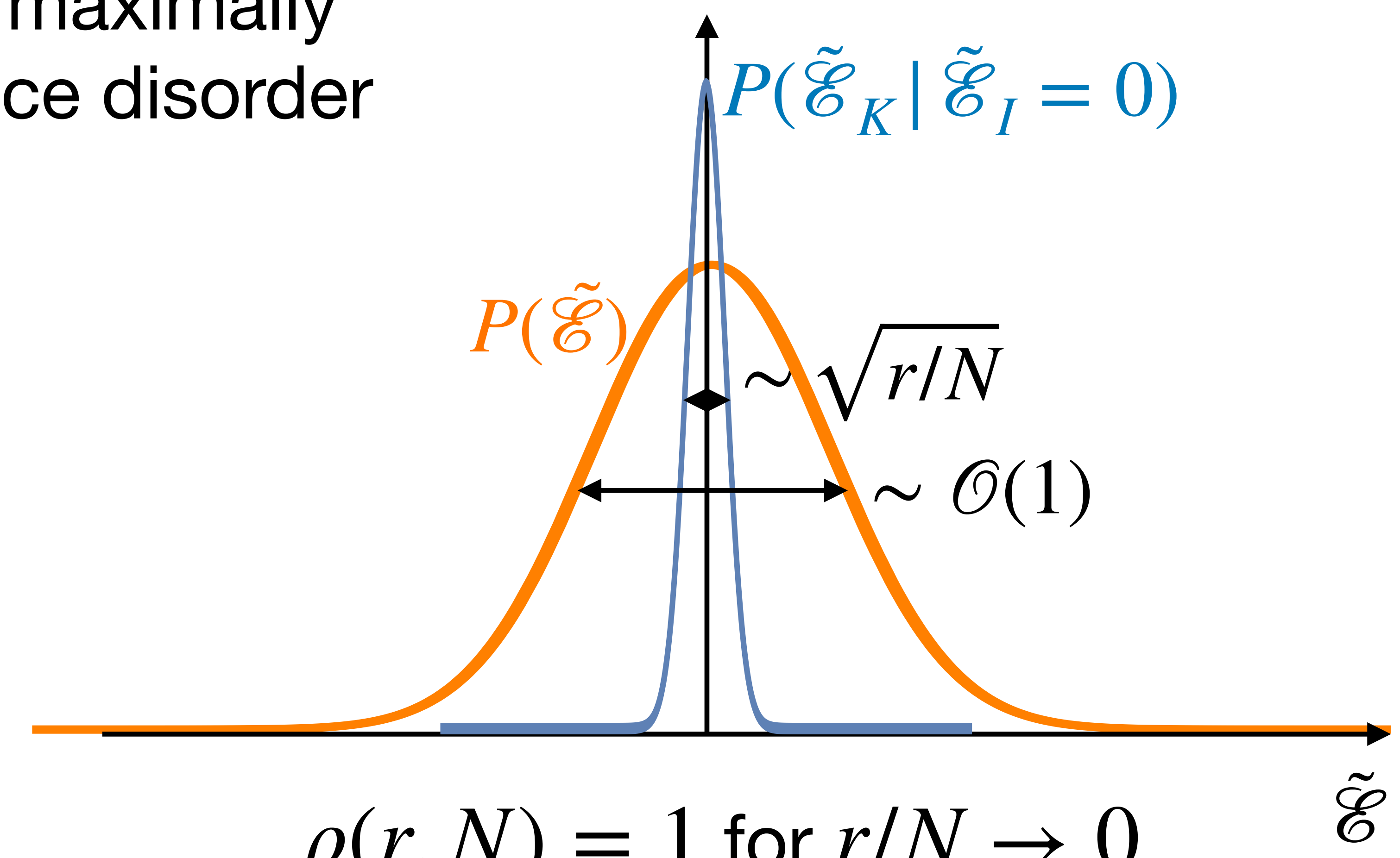
—Upshot: the self-energy is just a single term

—Break down of self-consistency of either phase indicates the MBL transition

$$\boxed{\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathscr{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]}$$

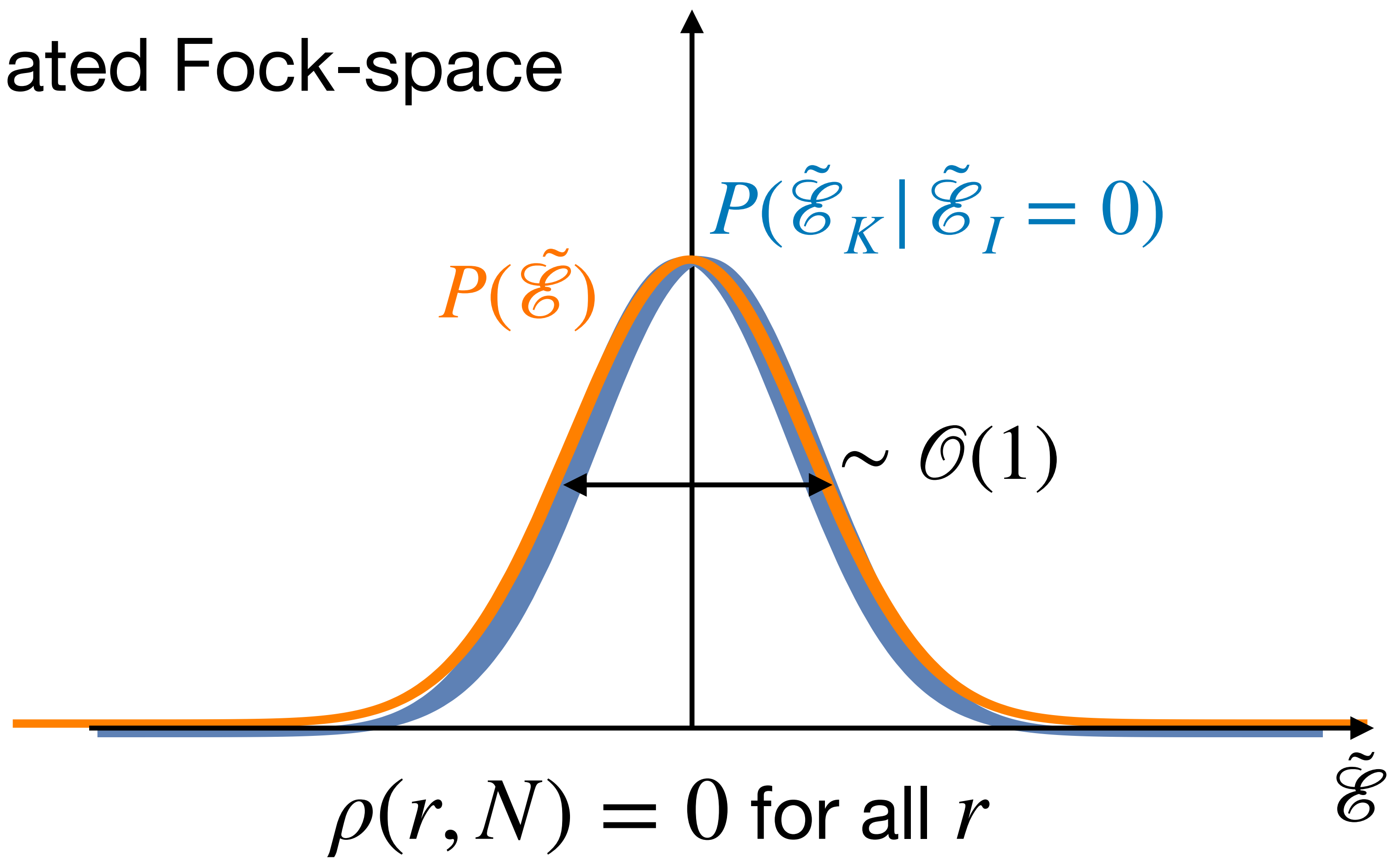
—sum over states K that are a finite distance away from I on Fock space

Local Hamiltonians: maximally correlated Fock-space disorder



$\rho(r, N) = 1$ for $r/N \to 0$

QREM: uncorrelated Fock-space disorder



$\rho(r, N) = 0$ for all r

Self-consistent theory of MBL

$$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\frac{\Gamma^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_I - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

—Upshot: the self-energy is just a single term

—Break down of self-consistency of either phase indicates the MBL transition

$$\boxed{\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\tilde{\mathscr{T}}_{IK}^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]}$$

—sum over states K that are a finite distance away from I on Fock space

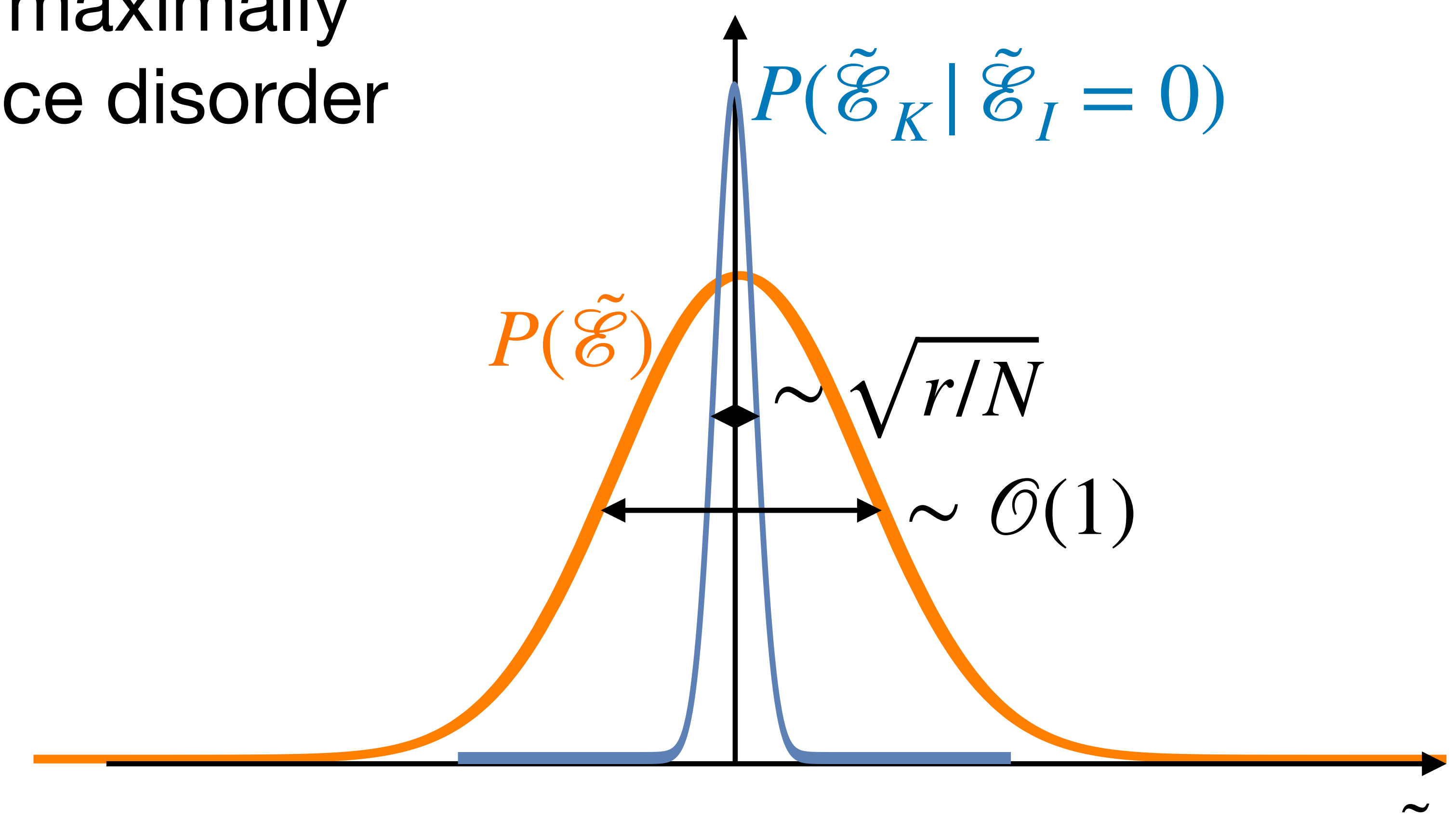
$$\tilde{\Delta}_I(\omega) = \mathrm{Im}[\sum_K \frac{\Gamma^2}{\tilde{\omega}^+ - \tilde{\mathscr{E}}_K - \tilde{S}_{\mathrm{typ}}(\omega)} + \cdots]$$

—Upshot: the self-energy is a sum of extensive number of independent terms

— localised phase never self-consistently stable

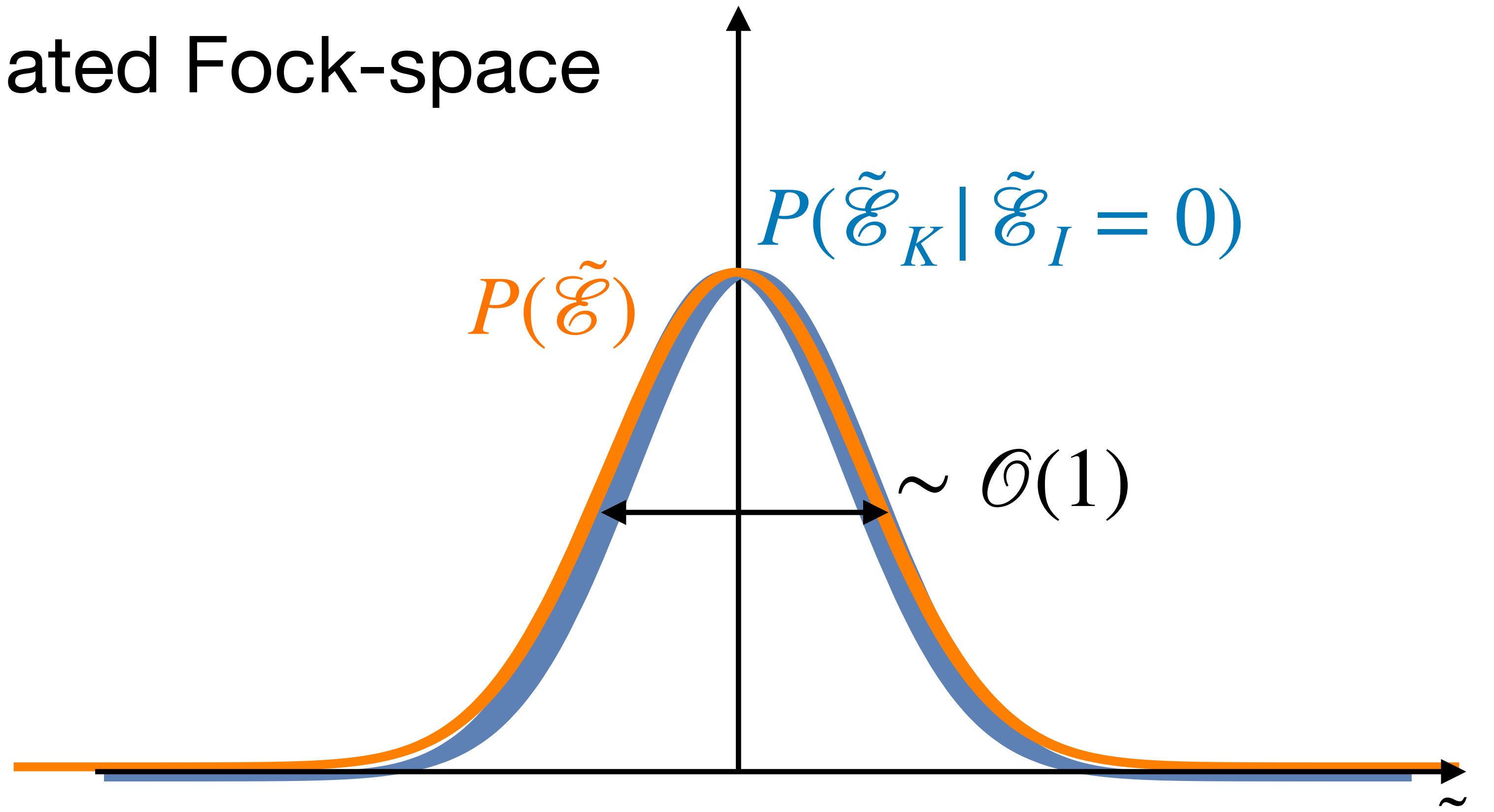
~ standard Anderson localisation on infinite-dimensional graph.

Local Hamiltonians: maximally correlated Fock-space disorder



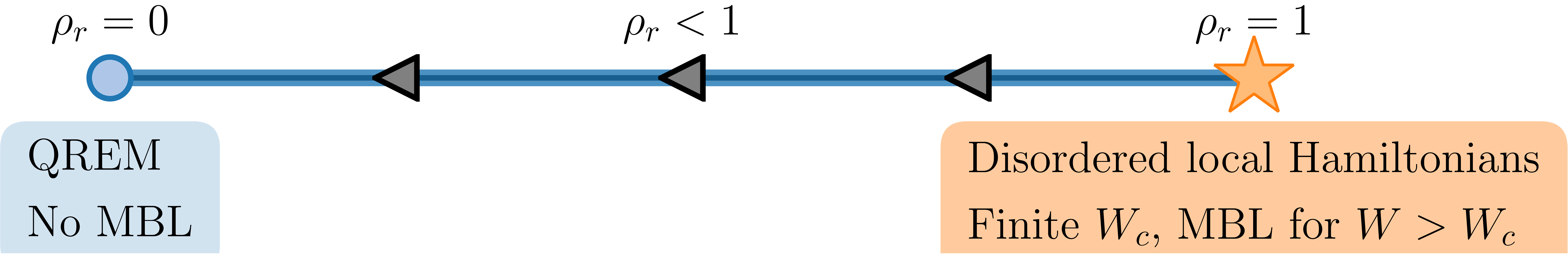
$\rho(r, N) = 1$ for $r/N \to 0$

QREM: uncorrelated Fock-space disorder



$\rho(r, N) = 0$ for all r

Central result



$\rho_r = 0$

$\rho_r < 1$

$\rho_r = 1$

QREM

No MBL

Disordered local Hamiltonians

Finite W_c, MBL for $W > W_c$

★ MBL possible only if Fock-space site energies at finite distances maximally correlated; minimum requirement for MBL to be stable

★ Any randomness/independence in them leads to delocalisation

SR, D. E. Logan, Phys. Rev. B 101, 134202 (2020)

Classical Percolation and MBL on Fock space

- Lightning review of many-body localisation

- Fock-space correlations and origins of MBL
  - MBL on Fock-space — how and why ?
  - Why MBL on Fock space $\neq$ Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

- Classical percolation in Fock space as a proxy for MBL
  - Fock-space fragmentation due to local frozen degrees of freedom
  - Heuristic picture for the effect of correlations

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

- Anderson localisation on graphs with strongly correlated disorder
  - Disorder correlations analogous to Fock-space correlations
  - Arguably a more controlled setting

Phys. Rev. Lett. 125, 250402 (2020)

Classical percolation on Fock space

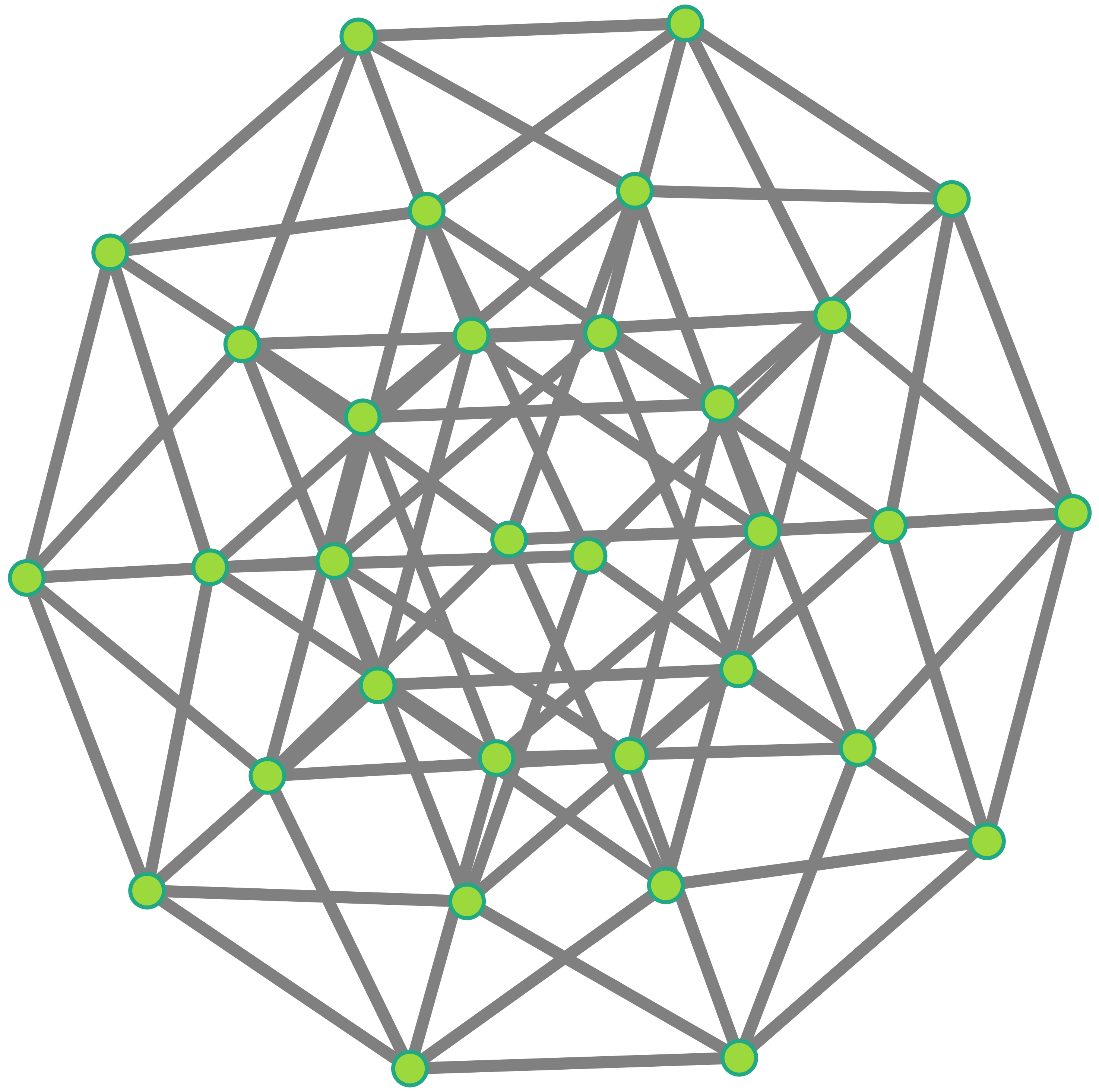
Define a correlated bond percolation problem

A link between I and K present if $\left|\dfrac{\mathcal{T}_{IK}}{\mathcal{E}_I - \mathcal{E}_K}\right| > 1$

$$H = \sum_I \mathcal{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathcal{T}_{IK} |I\rangle\langle K|$$

Statistics of connected clusters in Fock space ?

$S_{\text{typ}} \sim N_{\mathscr{H}}^{\alpha}$ How does α vary with disorder strength ?



$$H = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

$|I\rangle \equiv \sigma^z$- product state

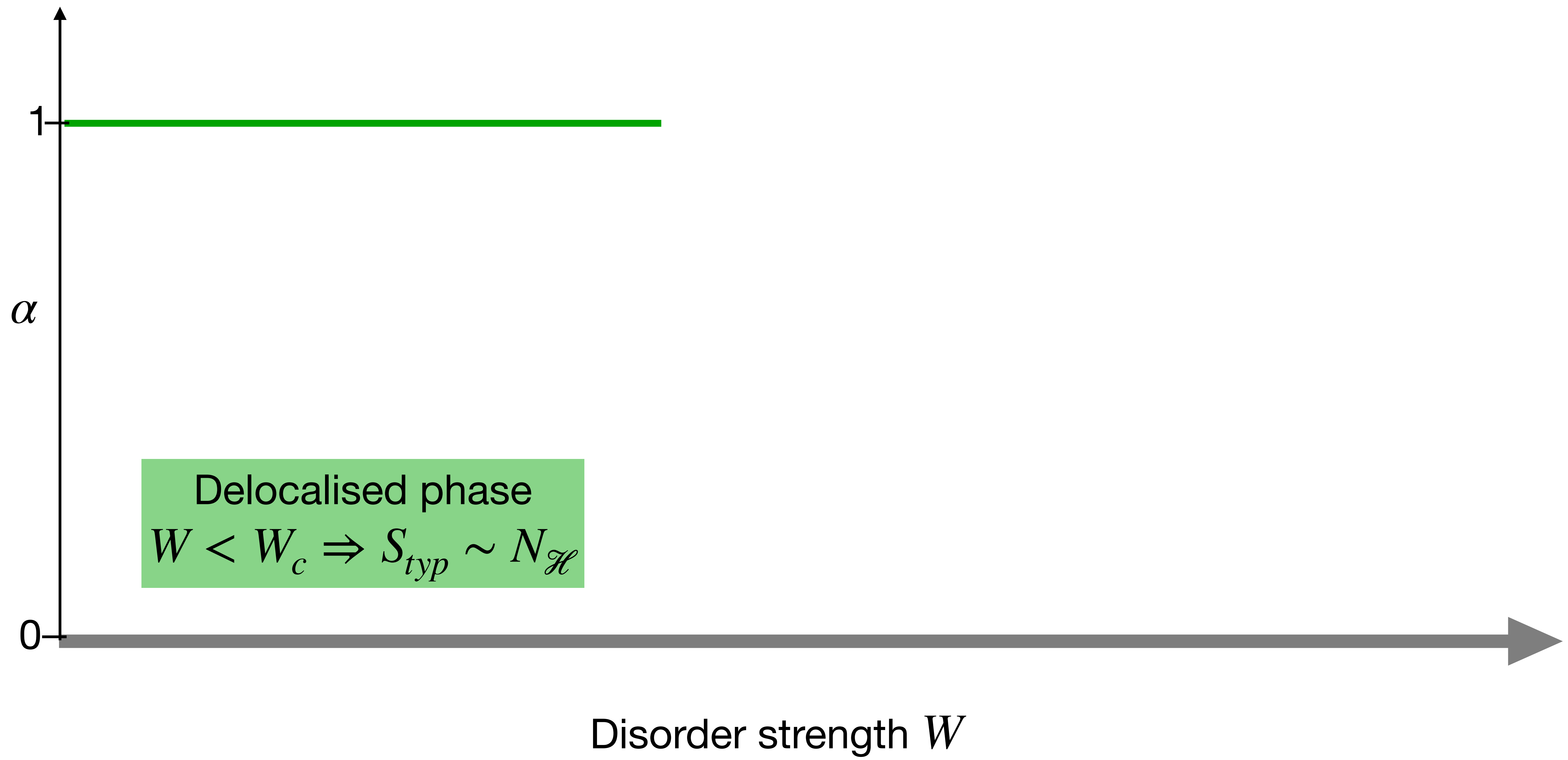
Classical percolation on Fock space

Define a correlated bond percolation problem

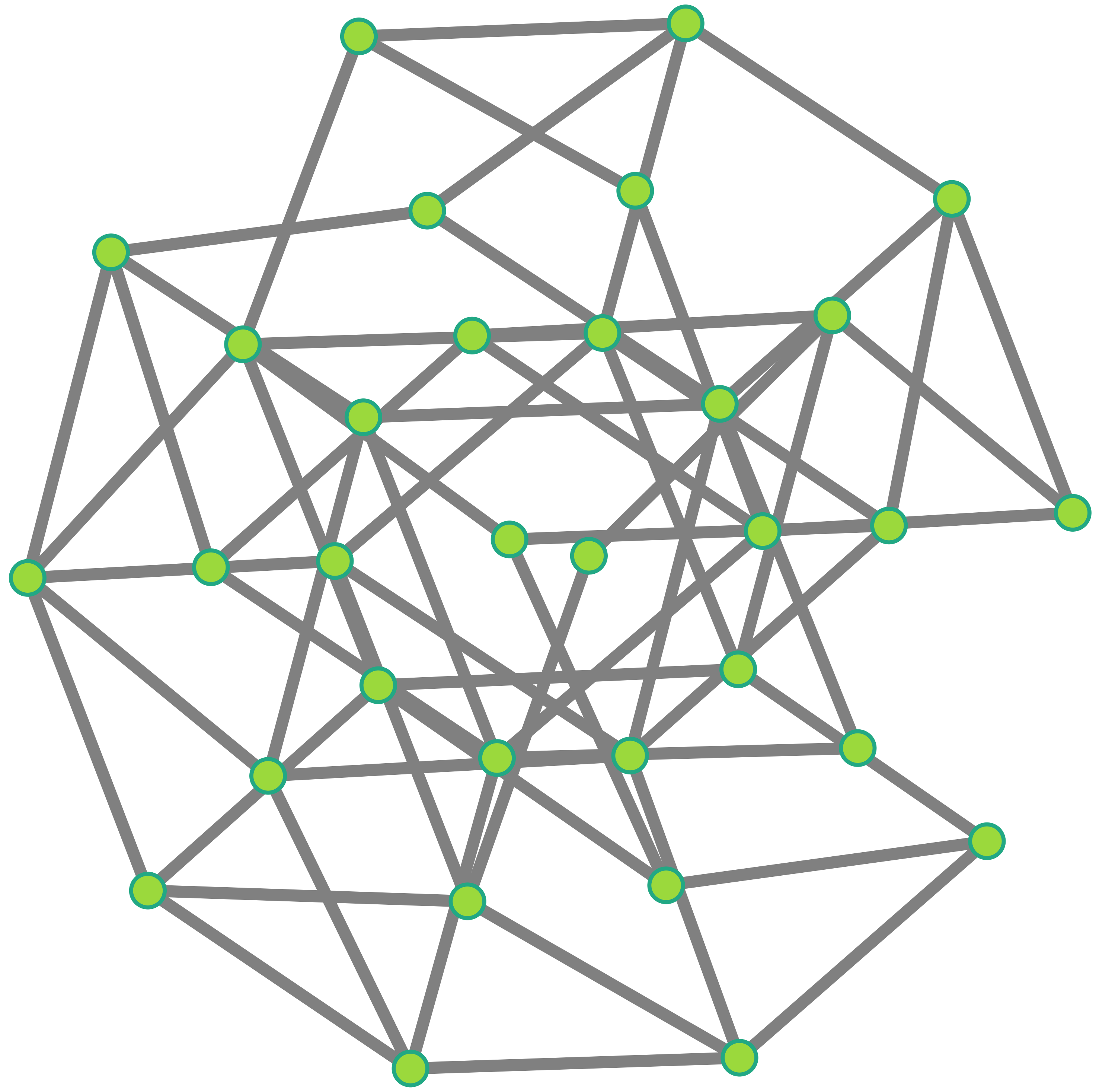
A link between I and K present if $\left| \frac{\mathcal{T}_{IK}}{\mathcal{E}_I - \mathcal{E}_K} \right| > 1$

Statistics of connected clusters in Fock space ?

$S_{\text{typ}} \sim N_{\mathscr{H}}^{\alpha}$ How does α vary with disorder strength ?



$$H = \sum_I \mathcal{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathcal{T}_{IK} |I\rangle\langle K|$$



$$H = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

$|I\rangle \equiv \sigma^z$- product state

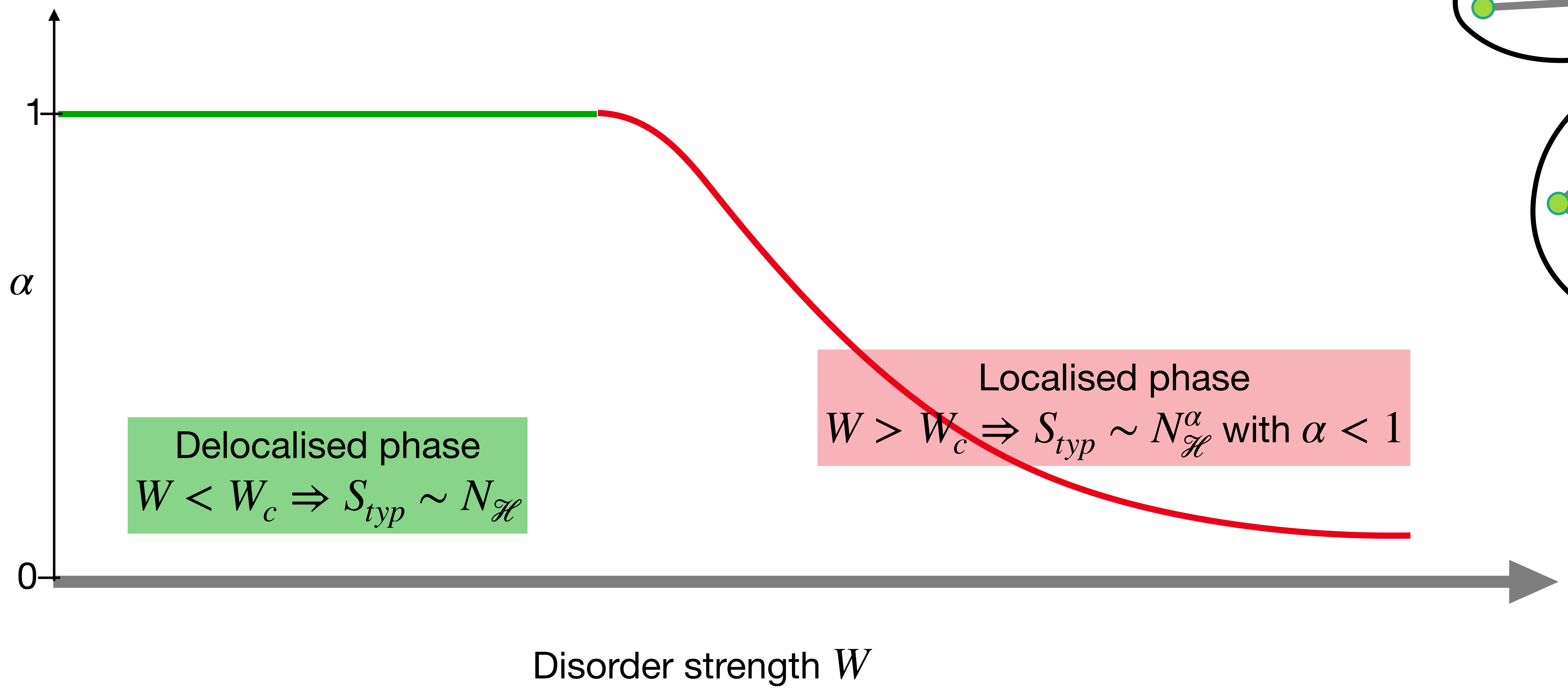
Classical percolation on Fock space

Define a correlated bond percolation problem

A link between I and K present if $\left|\frac{\mathcal{T}_{IK}}{\mathcal{E}_I - \mathcal{E}_K}\right| > 1$

Statistics of connected clusters in Fock space ?

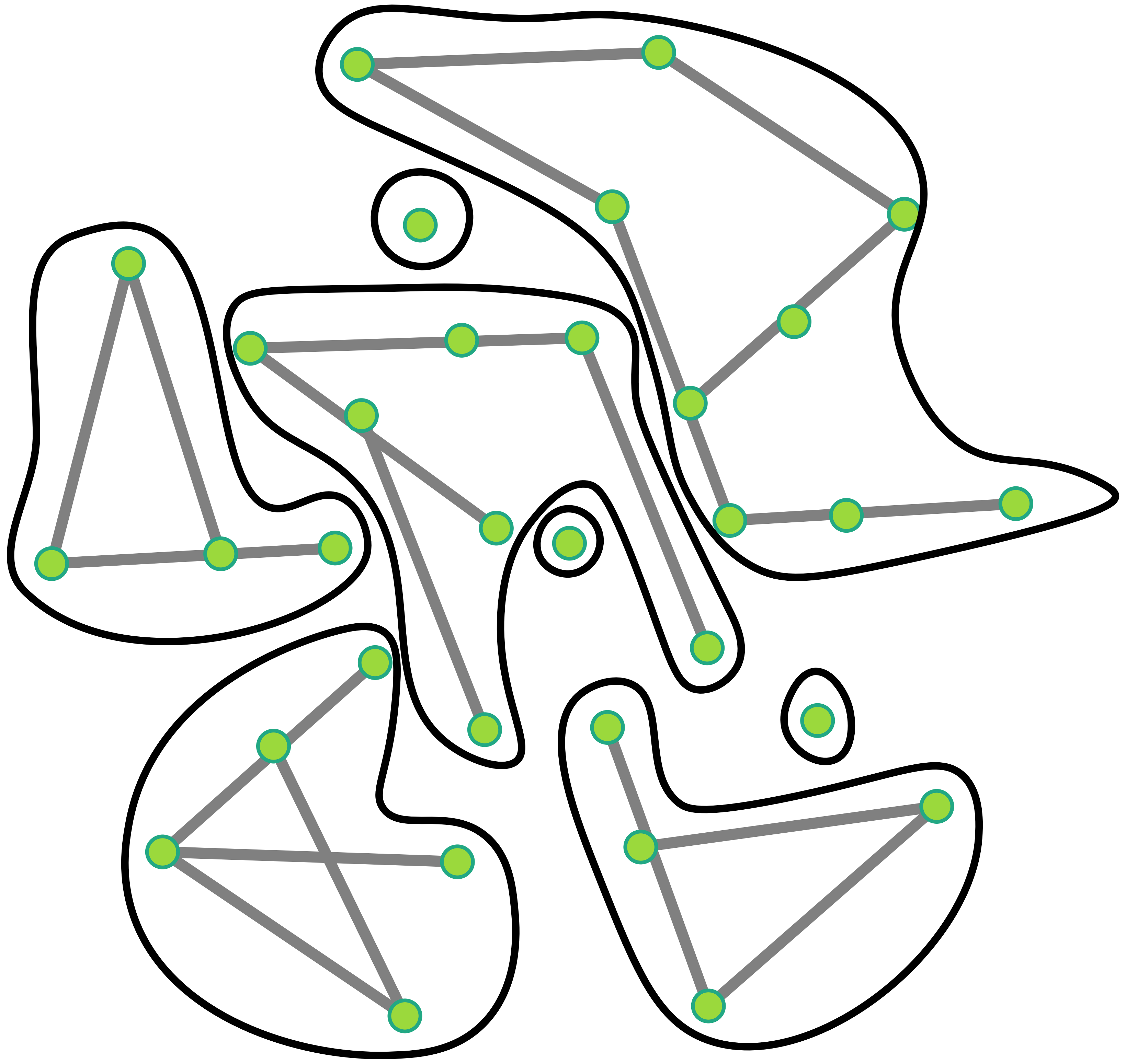
$S_{\text{typ}} \sim N_{\mathcal{H}}^{\alpha}$ How does α vary with disorder strength ?



Delocalised phase
$W < W_c \Rightarrow S_{typ} \sim N_{\mathcal{H}}$

Localised phase
$W > W_c \Rightarrow S_{typ} \sim N_{\mathcal{H}}^{\alpha}$ with $\alpha < 1$

$$H = \sum_I \mathcal{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathcal{T}_{IK} |I\rangle\langle K|$$



$$H = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

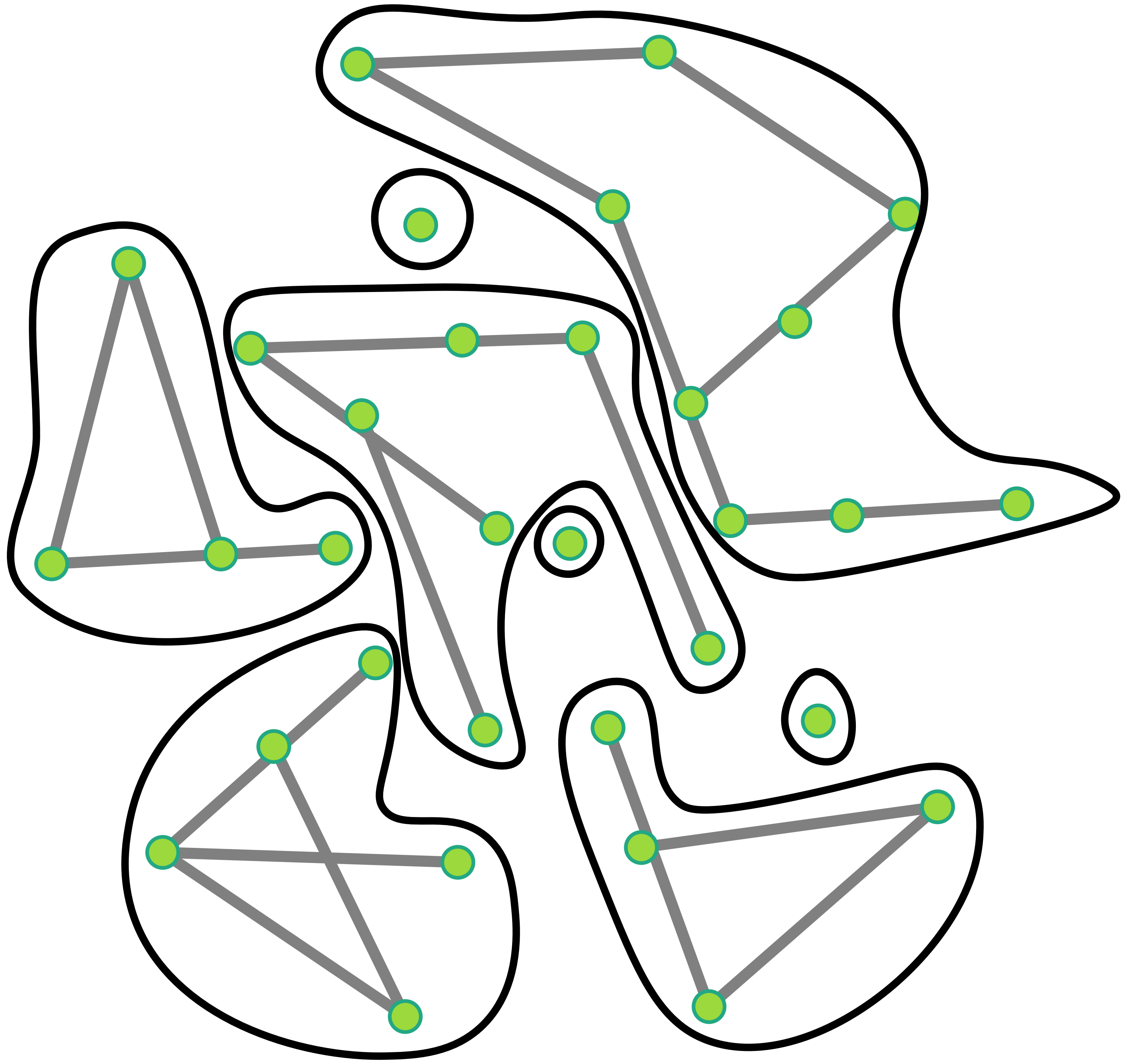
$|I\rangle \equiv \sigma^z$- product state

Classical percolation on Fock space

Define a correlated bond percolation problem

A link between I and K present if $\left|\dfrac{\mathcal{T}_{IK}}{\mathcal{E}_I - \mathcal{E}_K}\right| > 1$

$$H = \sum_I \mathcal{E}_I |I\rangle\langle I| + \sum_{I \neq K} \mathcal{T}_{IK} |I\rangle\langle K|$$

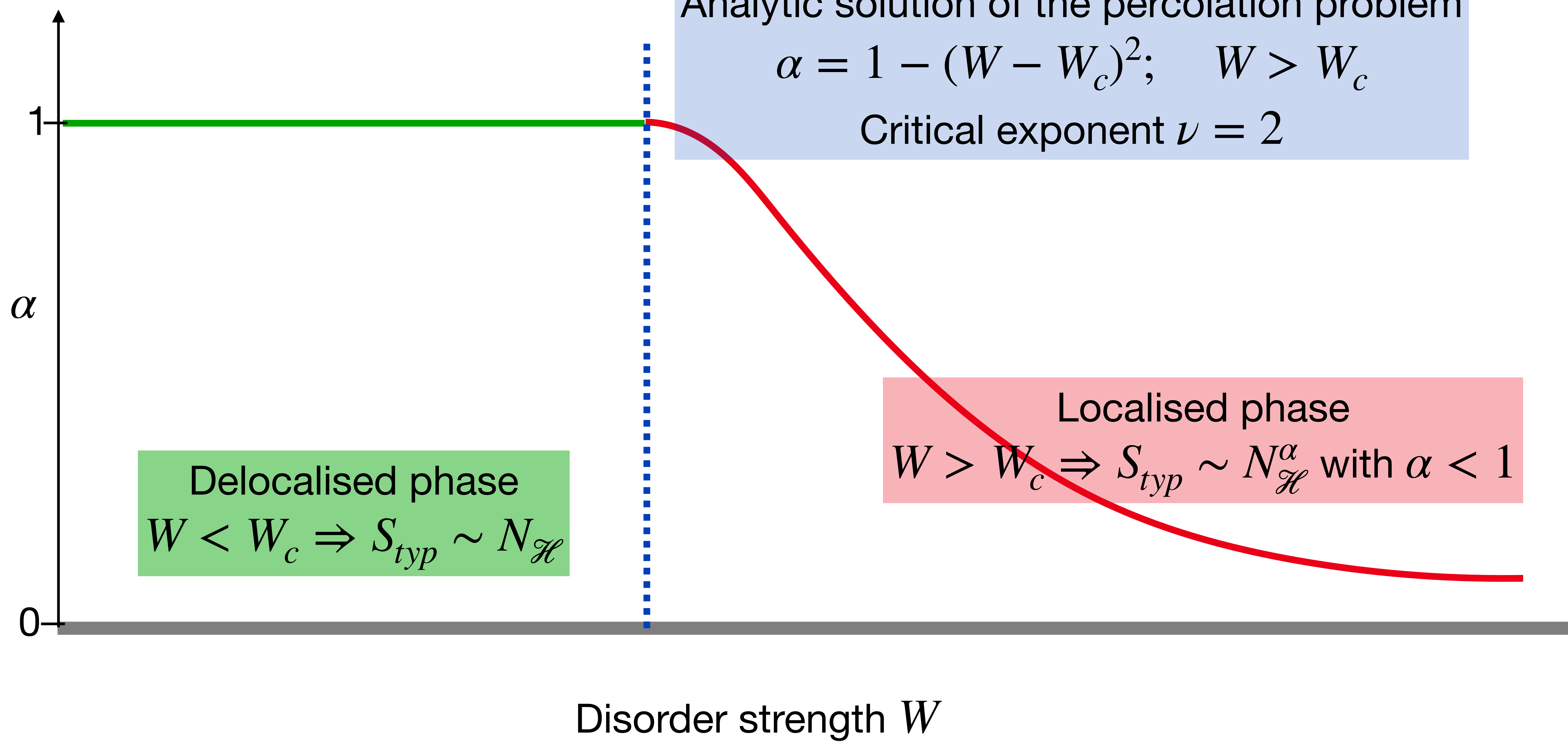


$$H = \sum_i [J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + \Gamma \sigma_i^x]$$

$|I\rangle \equiv \sigma^z$- product state

Statistics of connected clusters in Fock space ?

$S_{\text{typ}} \sim N_{\mathscr{H}}^{\alpha}$ How does α vary with disorder strength ?



Analytic solution of the percolation problem

$$\alpha = 1 - (W - W_c)^2; \quad W > W_c$$

Critical exponent $\nu = 2$

Localised phase

$W > W_c \Rightarrow S_{typ} \sim N_{\mathscr{H}}^{\alpha}$ with $\alpha < 1$

Delocalised phase

$W < W_c \Rightarrow S_{typ} \sim N_{\mathscr{H}}$

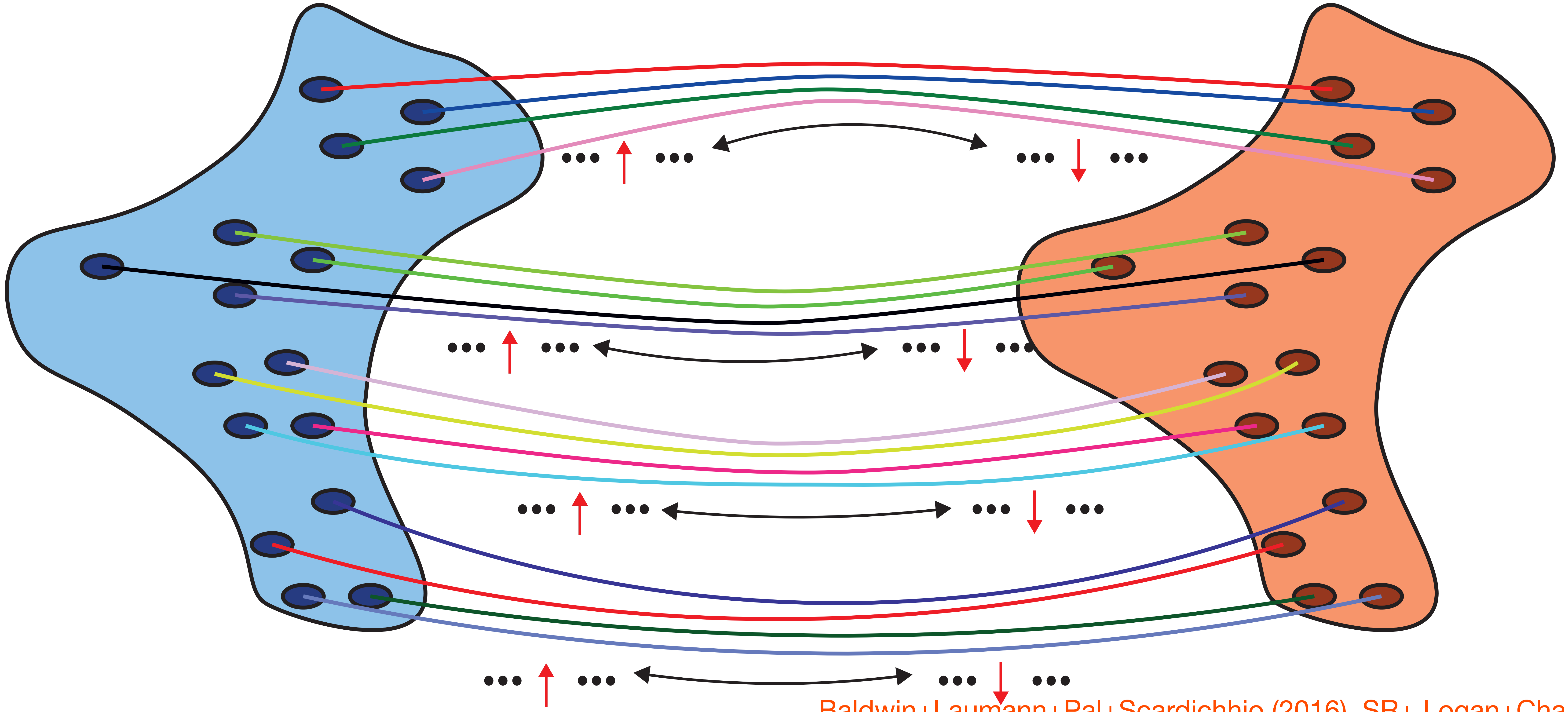
SR, D. E. Logan, J. T. Chalker, Phys. Rev. B 99, 220201(R) (2019)

Classical percolation on Fock space: cartoon for the effect of correlations

$$H_{\text{QREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \Gamma \sum_i \sigma_i^x$$

A link between I and K present if $\left|\dfrac{\mathscr{T}_{IK}}{\mathscr{E}_I - \mathscr{E}_K}\right| > 1$

- Exponentially large number of possible energy scales for any spin
- Impossible to avoid at least one resonance in the thermodynamic limit; enough to force delocalisation



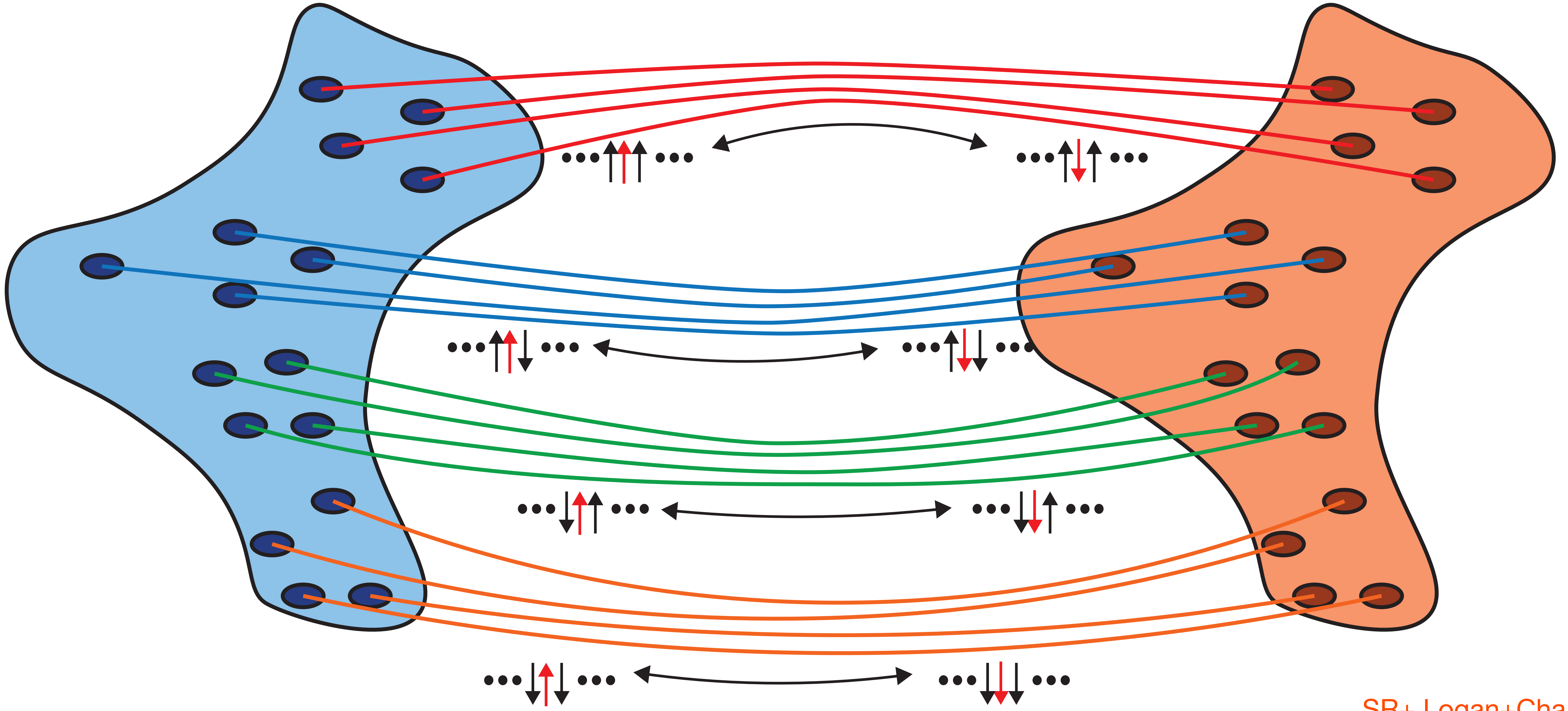
Baldwin+Laumann+Pal+Scardichhio (2016), SR+ Logan+Chalker(2019), SR+Logan (2020)

Classical percolation on Fock space: cartoon for the effect of correlations

$$H_{\mathrm{TFI}} = \sum_i [J_i\sigma_i^z\sigma_{i+1}^z + h_i\sigma_i^z] + \Gamma \sum_i \sigma_i^x$$

A link between I and K present if $\left|\frac{\mathscr{T}_{IK}}{\mathscr{E}_I - \mathscr{E}_K}\right| > 1$

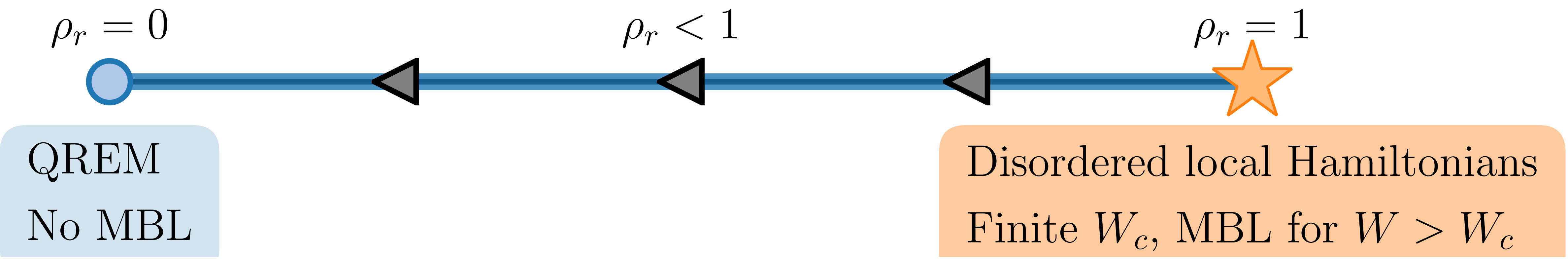
- Only a few energy scales control the flipping of a local spin
- A few energy scales becoming off-resonant can kill exponentially large number of links on the Fock space
- Correlations help in defeating the high-connectivity of the graph



SR+ Logan+Chalker(2019), SR+Logan (2020)

Fock-space correlations and origins of MBL

- MBL possible only when the correlations are maximal at finite distances on the Fock-space graph
- Generally the situation for local disordered Hamiltonians
- Any randomness/independence enough to delocalise the system
- Classical percolation picture on the Fock space

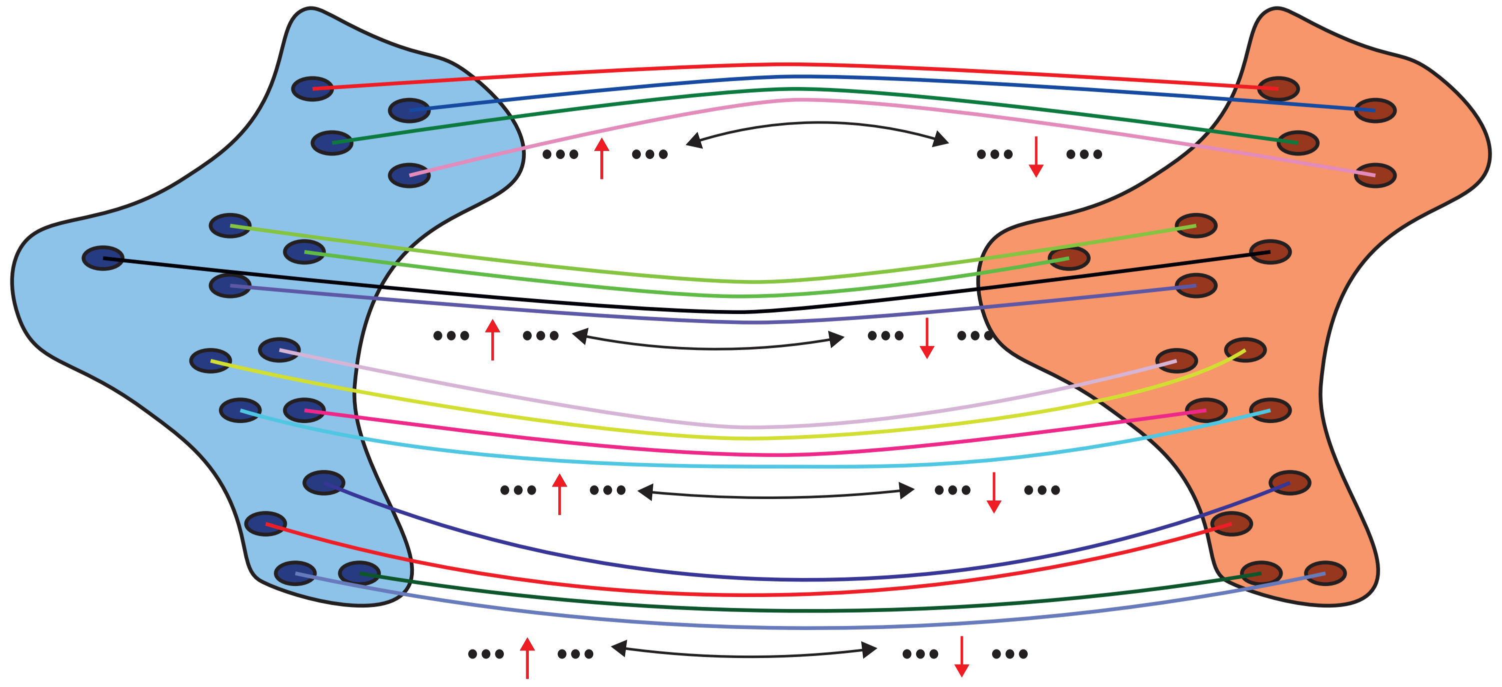

$\rho_r = 0$

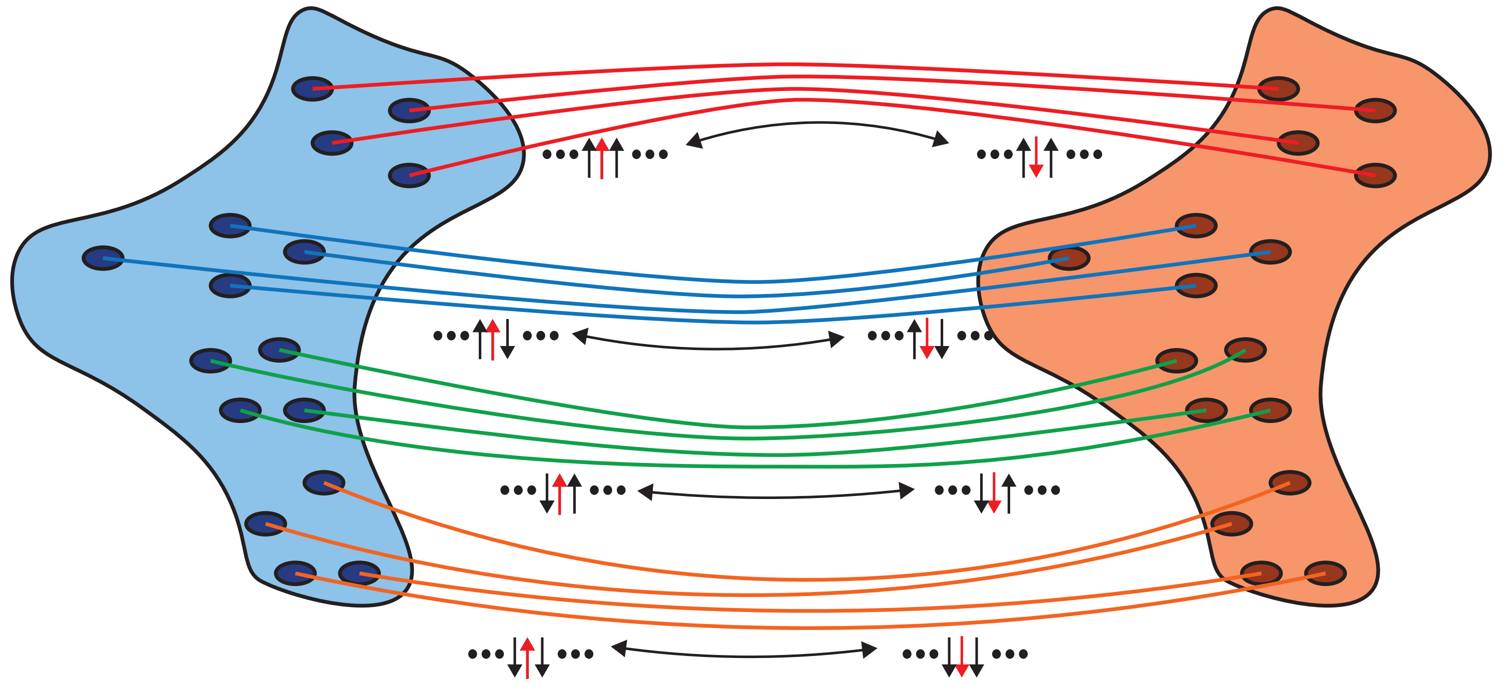
$\rho_r < 1$

$\rho_r = 1$

QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

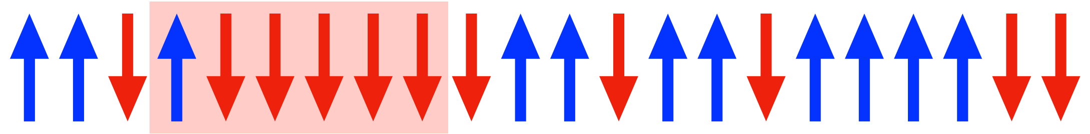




QREM + constrained dynamics = MBL

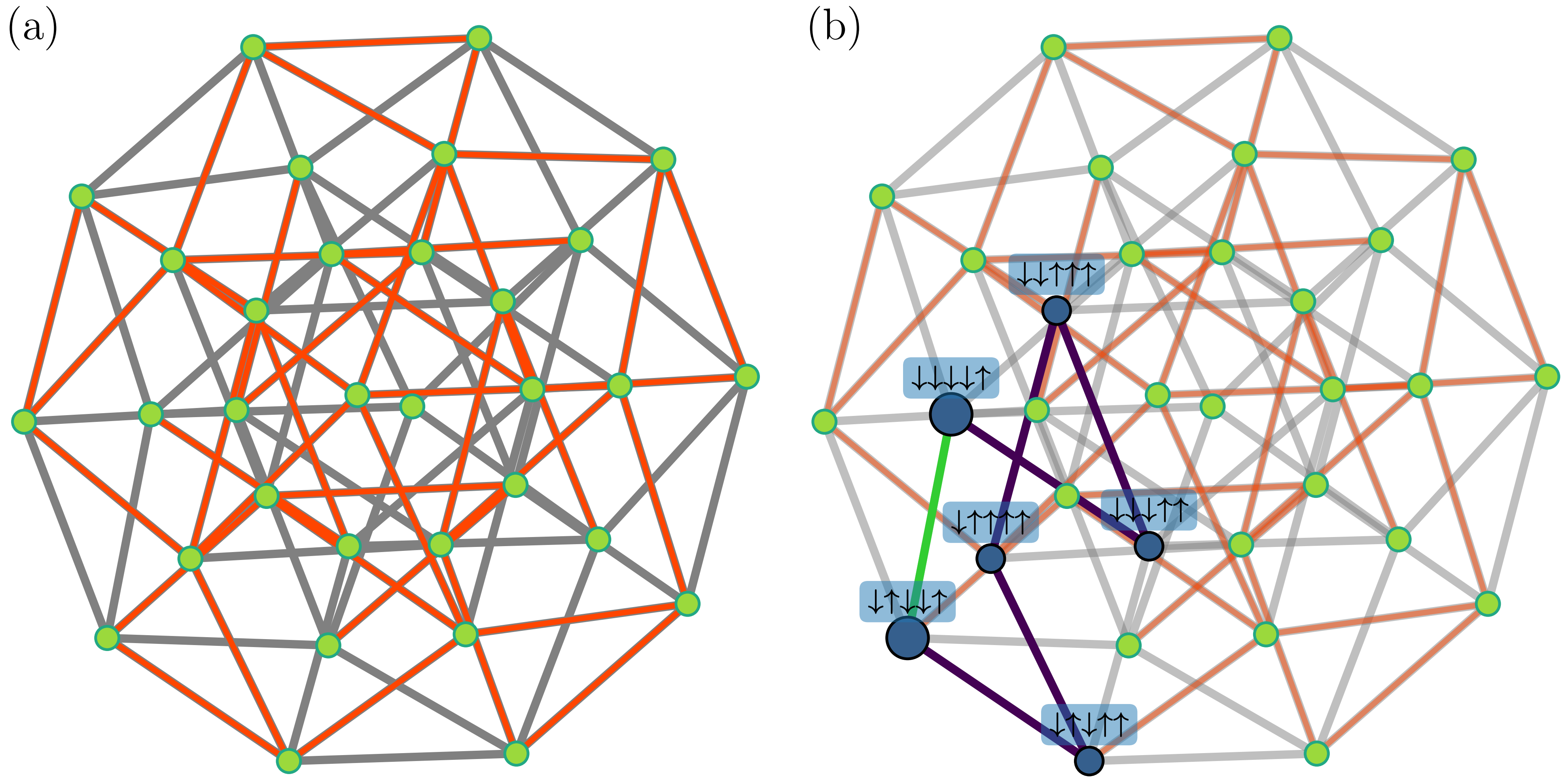
$$H_{\mathrm{EastREM}} = \sum_I \mathscr{E}_I |I\rangle\langle I| + \frac{\Gamma}{2}\sum_i \sigma_i^x(1+\sigma_{i+1}^z)$$

a spin can flip only if the one to its right is up

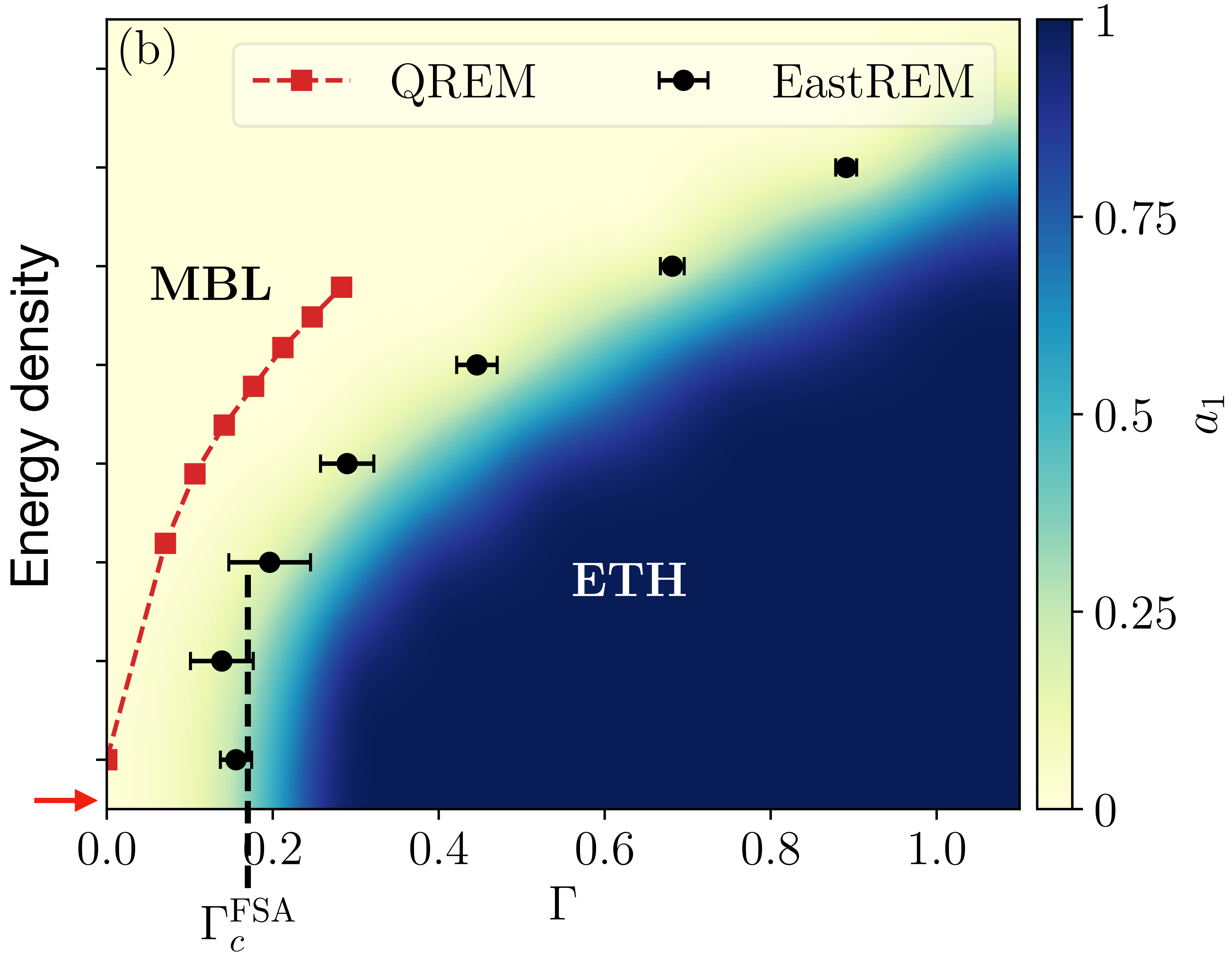


Frozen block of spins; can melt only from the right; arrested dynamics !!

On Fock space



- Constraints switch off some of the links
- Increase the typical distance between two nodes
- Decrease the number of paths between two nodes



Middle of the spectrum

SR + A. Lazarides, Phys. Rev. Research 2, 023159 (2020)

Anderson localisation with strongly correlated disorder

- Lightning review of many-body localisation

- Fock-space correlations and origins of MBL
  - MBL on Fock-space — how and why ?
  - Why MBL on Fock space $\neq$ Anderson localisation on high dimensional graph ?
  - Fock-space correlations as a necessary requirement for MBL

Phys. Rev. B 101, 134202 (2020)

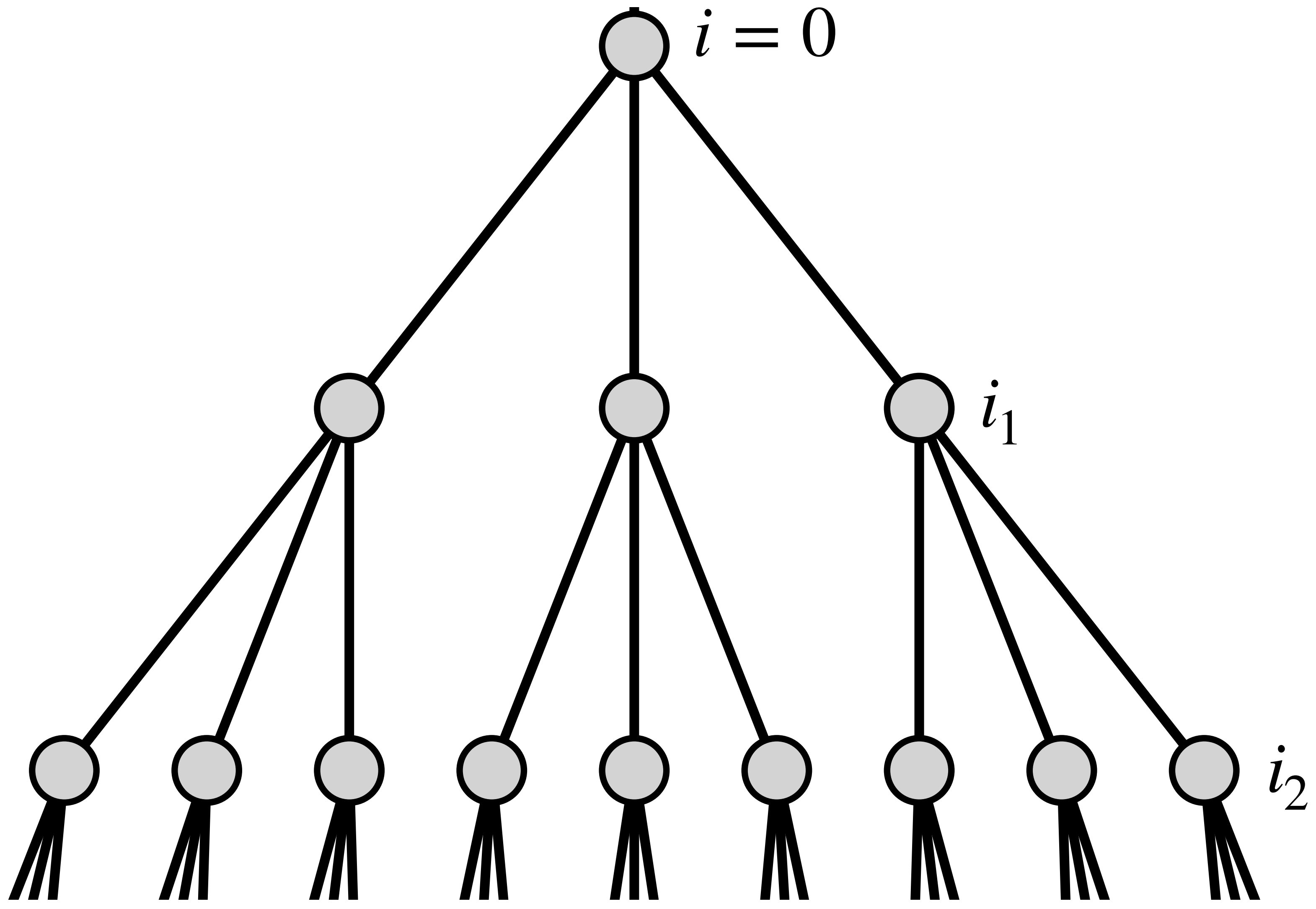
- Classical percolation in Fock space as a proxy for MBL
  - Fock-space fragmentation due to local frozen degrees of freedom
  - Heuristic picture for the effect of correlations

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

- Anderson localisation on trees with strongly correlated disorder
  - Disorder correlations analogous to Fock-space correlations
  - Arguably a more controlled setting

Phys. Rev. Lett. 125, 250402 (2020)

Anderson localisation with strongly correlated disorder



Disordered tight-binding problem on a tree

$$H = \Gamma \sum_{\langle i,j \rangle} [\,|i\rangle\langle j| + \mathrm{h.c.}] + \sum_i \epsilon_i |i\rangle\langle i|$$

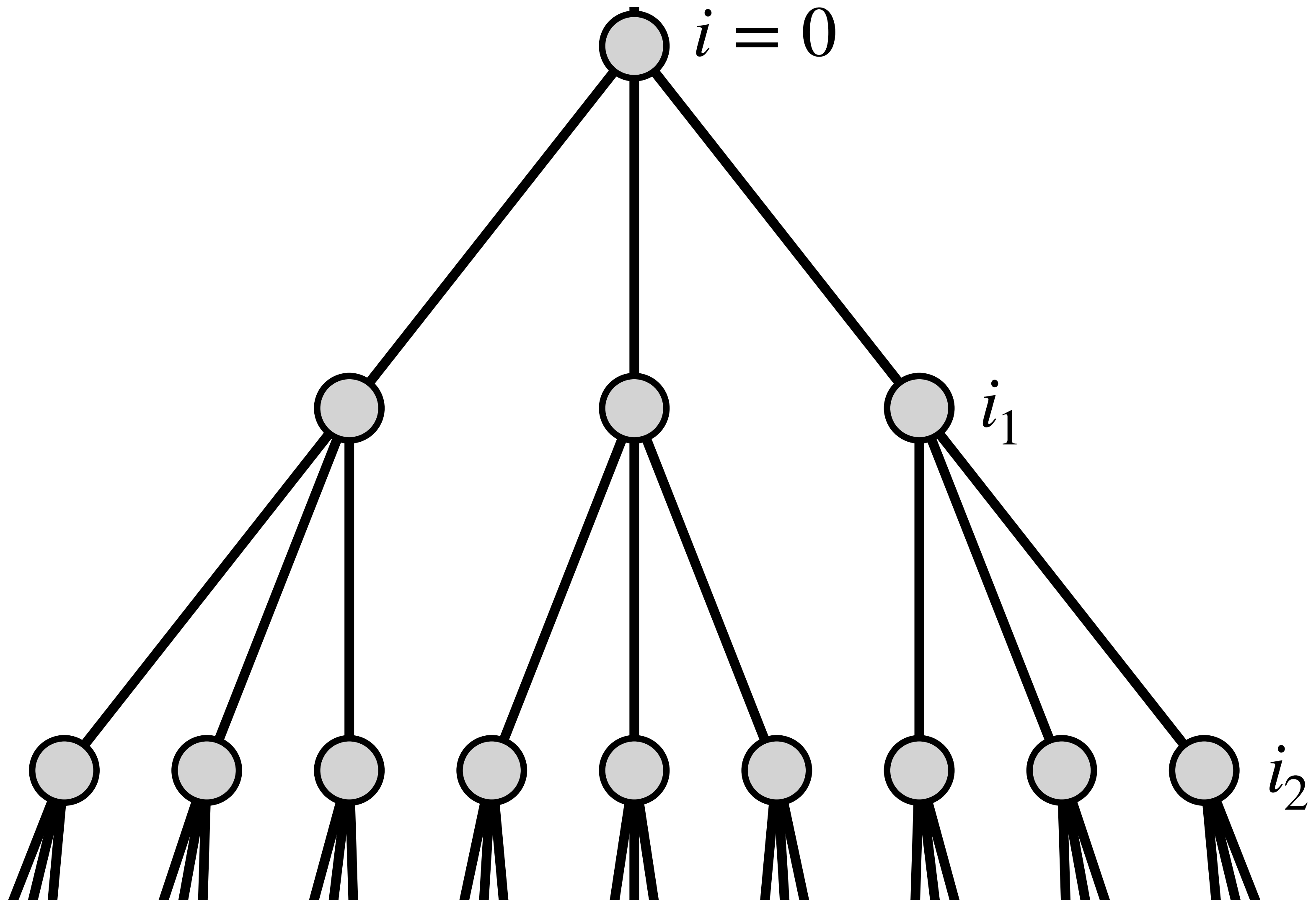
Branching number $K = 2$ Disorder strength W

For uncorrelated randomness $\langle \epsilon_i \epsilon_j \rangle = \delta_{ij} W^2$

$$W_c \sim \Gamma K \ln K$$

Abou-Chacra, Anderson, Thouless (1983) and many others…

Anderson localisation with strongly correlated disorder



Disordered tight-binding problem on a tree

$$H = \Gamma \sum_{\langle i,j \rangle} [\,|i\rangle\langle j| + \mathrm{h.c.}] + \sum_i \epsilon_i |i\rangle\langle i|$$

Branching number $K = 2$ Disorder strength W

For uncorrelated randomness $\langle \epsilon_i \epsilon_j \rangle = \delta_{ij} W^2$

$$W_c \sim \Gamma K \ln K$$

Fate of localisation in the presence of maximal disorder correlations

$$\langle \epsilon_i \epsilon_j \rangle = f(r_{ij}/L); \qquad \lim_{x \to 0} f(x) \to 1$$

Distance between sites i and j

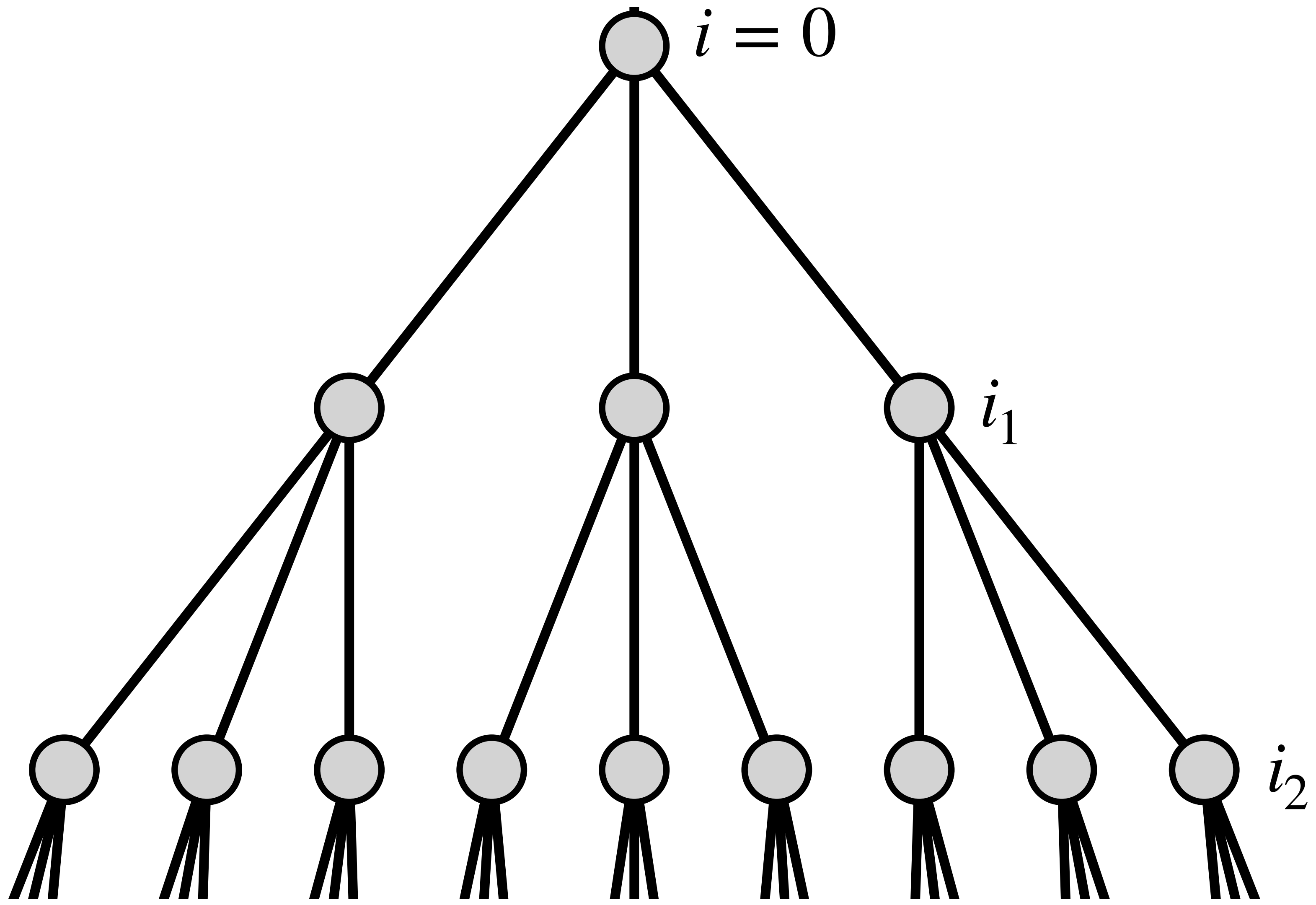
Number of generations in the tree

On Fock space for a many-body problem:
$\rho(r, N)$ generally a p-order polynomial of r/N for a p-spin system
$\Rightarrow \rho(r, N) \to 1$ for $r/N \to 0$

- With increasing system size, energies of nearby sites on the tree become more and more uniform
- The presence of a localised phase itself *a priori* not remotely obvious

Abou-Chacra, Anderson, Thouless (1983) and many others…

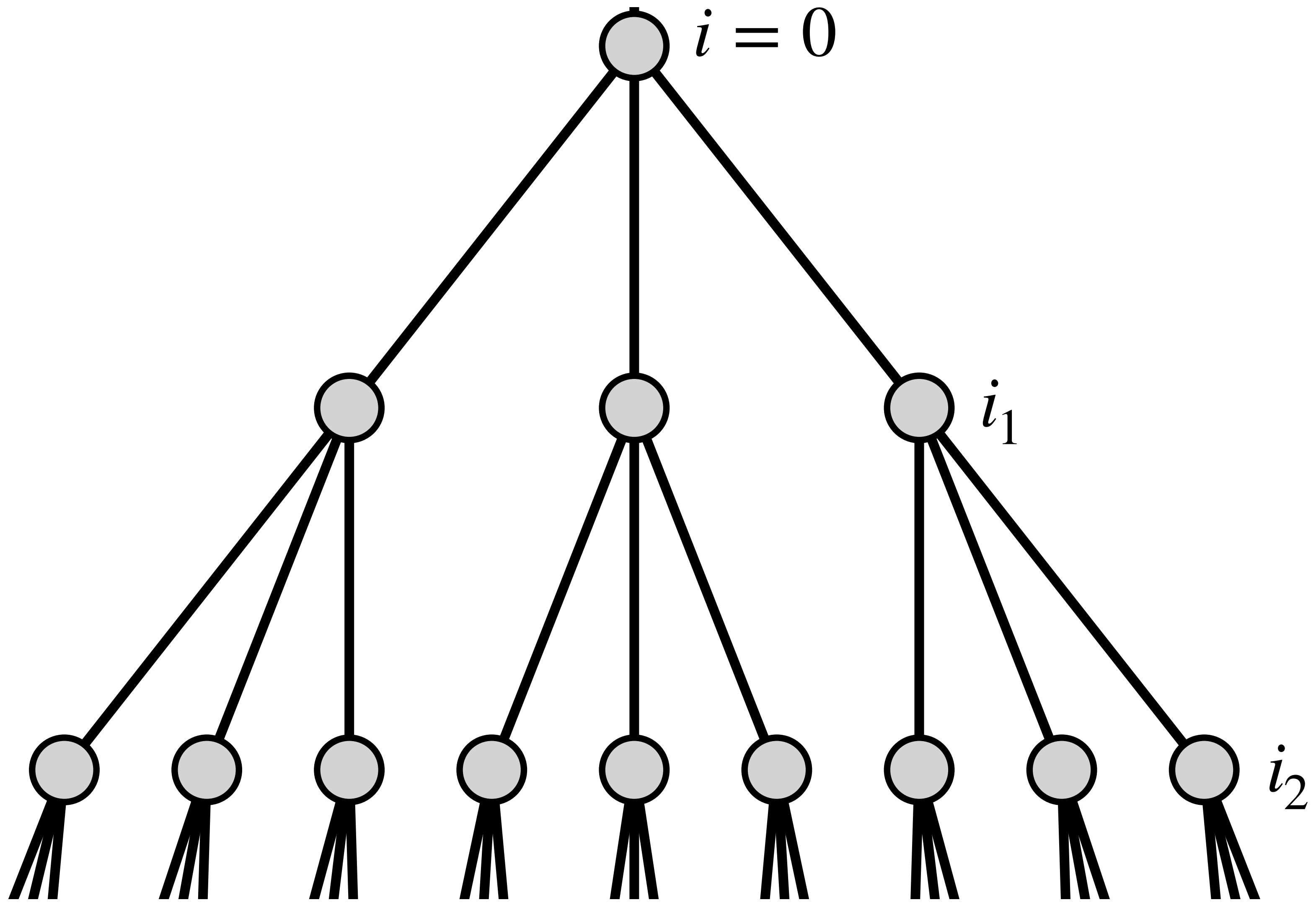
Anderson localisation with strongly correlated disorder



$$S_0(\omega) = \Gamma^2 \sum_{i_1} [\omega^+ - \epsilon_{i_1} - S_{i_1}^{(0)}]^{-1}$$ (exact!!)

Abou-Chacra, Anderson, Thouless (1983) and many others…

Anderson localisation with strongly correlated disorder



$$S_0(\omega) = \Gamma^2 \sum_{i_1} [\omega^+ - \epsilon_{i_1} - S_{i_1}^{(0)}]^{-1}$$ (exact!!)

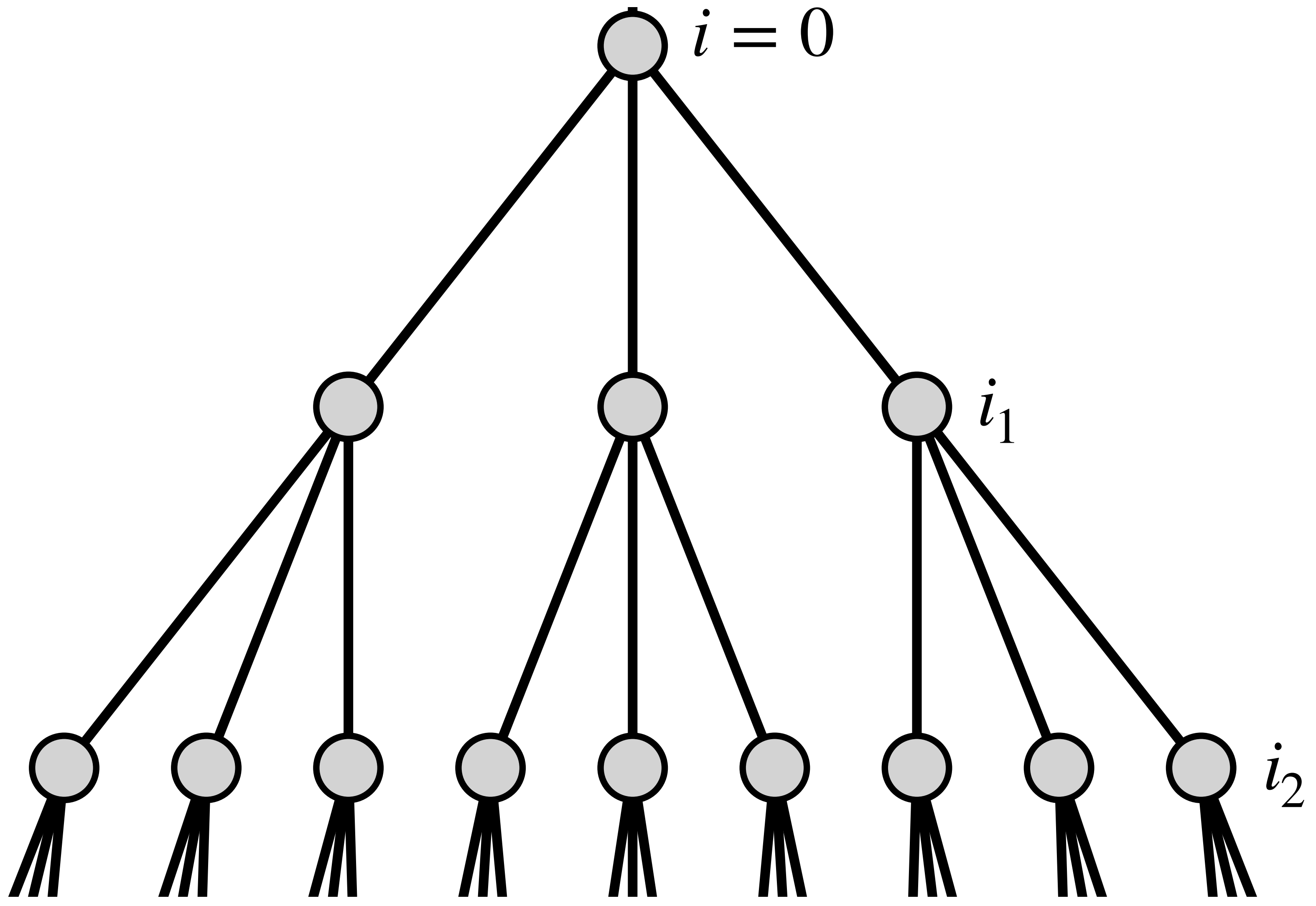
$$S_0(\omega) = \sum_{i_1} \cfrac{\Gamma^2}{\omega^+ - \epsilon_{i_1} - \sum_{i_2} \cfrac{\Gamma^2}{\omega^+ - \epsilon_{i_2} - \sum_{i_3} \cfrac{\Gamma^2}{\ddots}}}$$

- The continued fraction to all orders takes into account all the correlated energies
- Instead of a self-consistent theory, one looks for convergence properties of the continued fraction
- The treatment is formally exact

Abou-Chacra, Anderson, Thouless (1983) and many others…

Phys. Rev. Lett. 125, 250402 (2020)

Anderson localisation with strongly correlated disorder

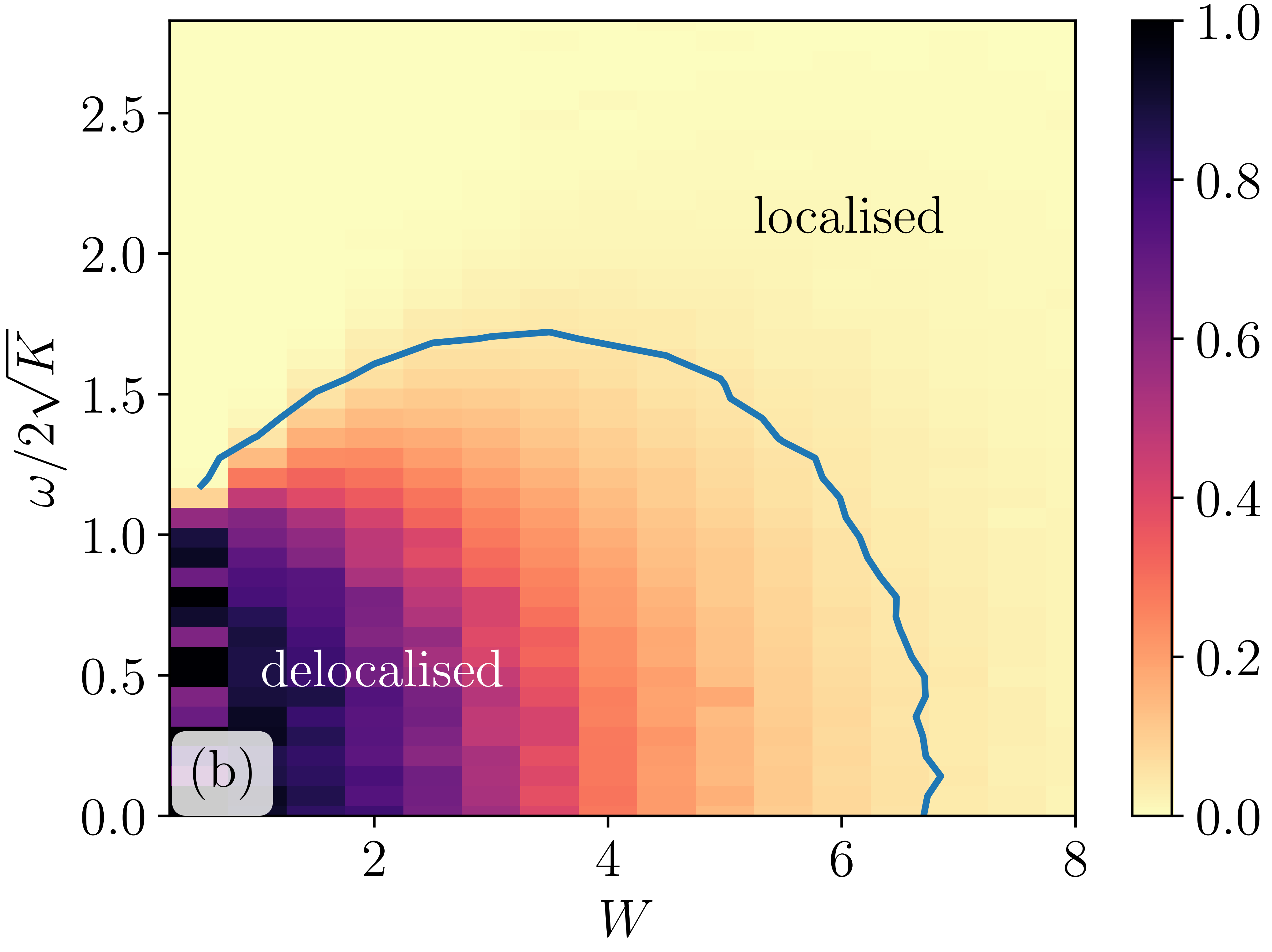


$$S_0(\omega) = \Gamma^2 \sum_{i_1} [\omega^+ - \epsilon_{i_1} - S_{i_1}^{(0)}]^{-1}$$

(exact!!)

$$S_0(\omega) = \sum_{i_1} \cfrac{\Gamma^2}{\omega^+ - \epsilon_{i_1} - \sum_{i_2} \cfrac{\Gamma^2}{\omega^+ - \epsilon_{i_2} - \sum_{i_3} \cfrac{\Gamma^2}{\ddots}}}$$

Colour-map of Δ_0^{typ}



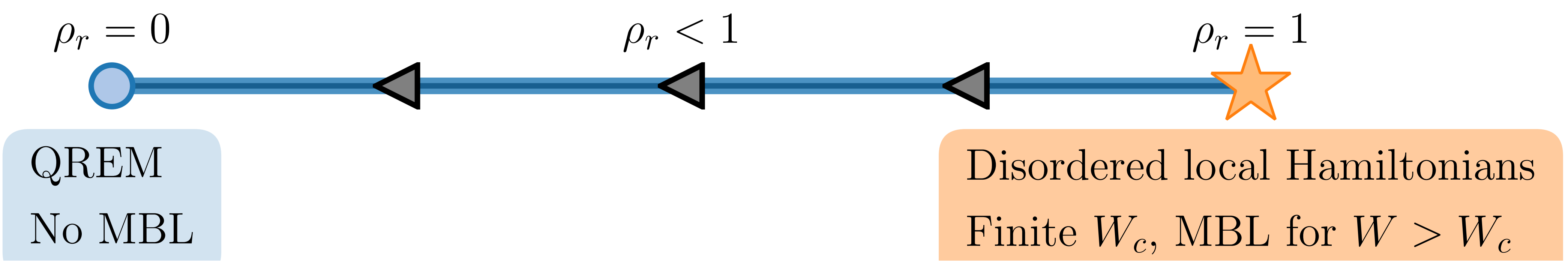
Localised phase indeed present

- The continued fraction to all orders takes into account all the correlated energies
- Instead of a self-consistent theory, one looks for convergence properties of the continued fraction
- The treatment is formally exact

Abou-Chacra, Anderson, Thouless (1983) and many others…

Phys. Rev. Lett. 125, 250402 (2020)

Summary and Outlook



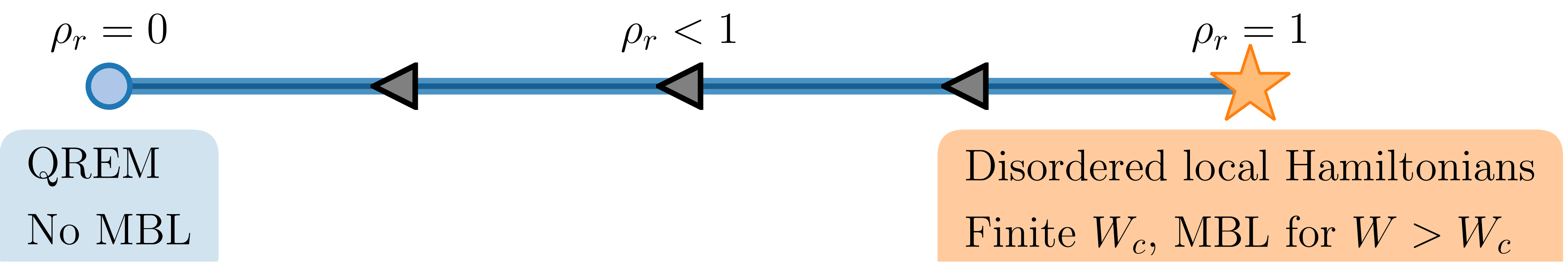
QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

MBL possible only if Fock-space site energies at finite distances maximally correlated; minimum requirement for MBL to be stable

Phys. Rev. B 101, 134202 (2020)

Summary and Outlook



$\rho_r = 0$

$\rho_r < 1$

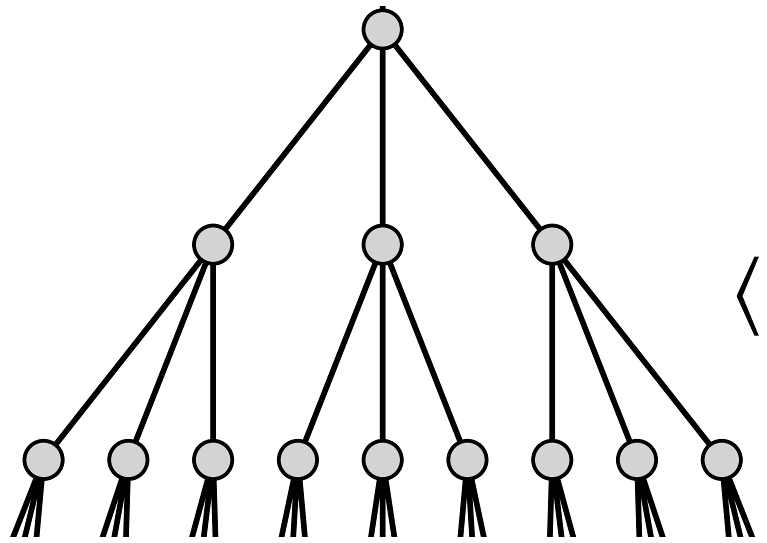
$\rho_r = 1$

QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

MBL possible only if Fock-space site energies at finite distances maximally correlated; minimum requirement for MBL to be stable

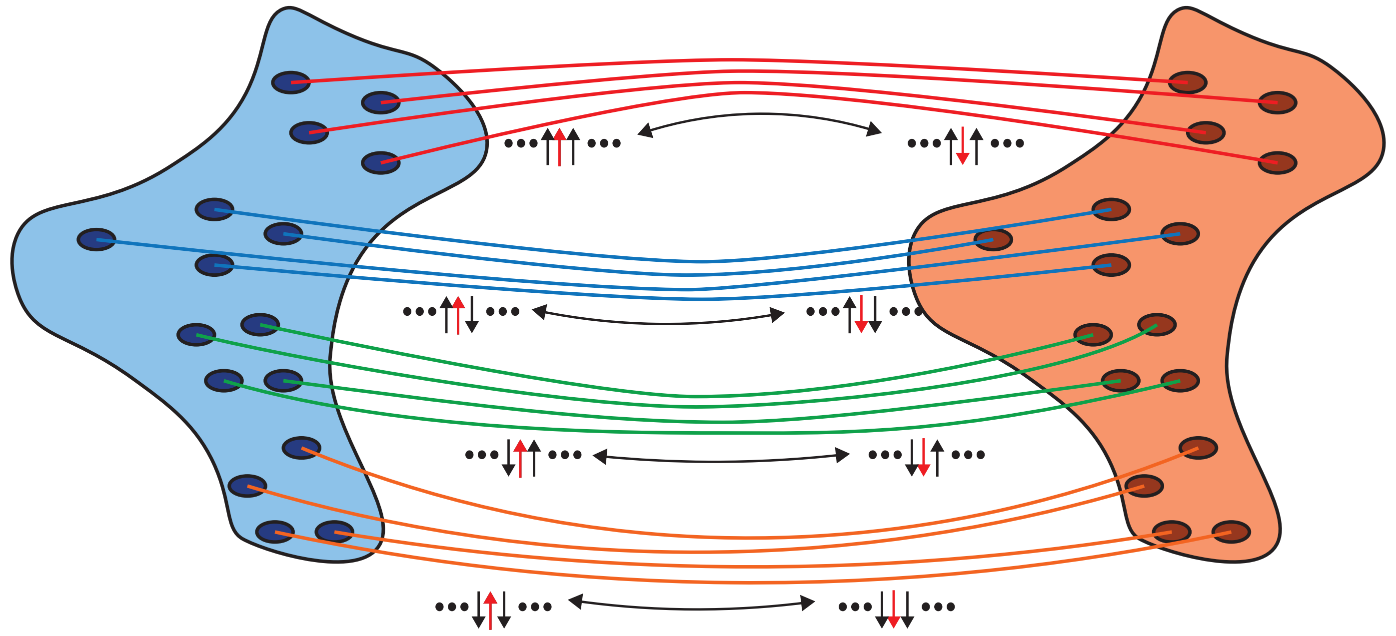
Phys. Rev. B 101, 134202 (2020)



$$\langle \epsilon_i \epsilon_j \rangle = f(r_{ij}/L); \qquad \lim_{x\to 0} f(x) \to 1$$

Anderson localisation on trees with maximally correlated disorder as controlled settings for understanding the effect of correlations

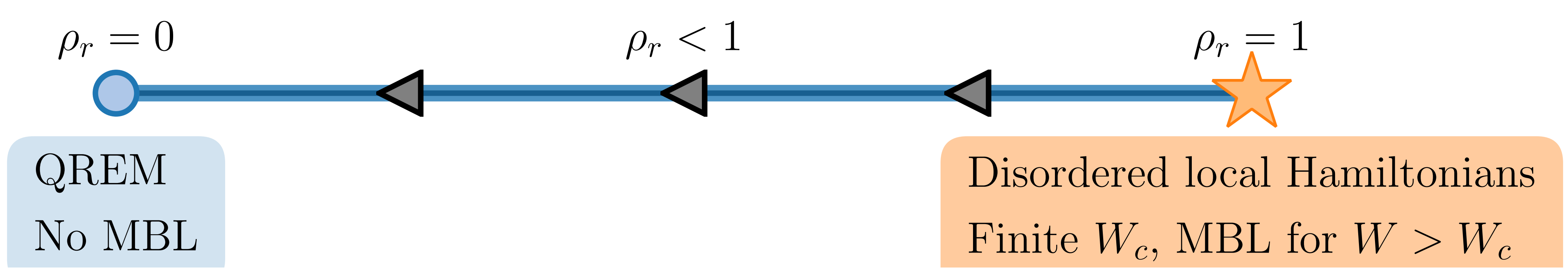
Phys. Rev. Lett. 125, 250402 (2020)



Classical percolation proxy to highlight how the correlations stop local degrees of freedom from thermalising

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

Summary and Outlook



$\rho_r = 0$

$\rho_r < 1$

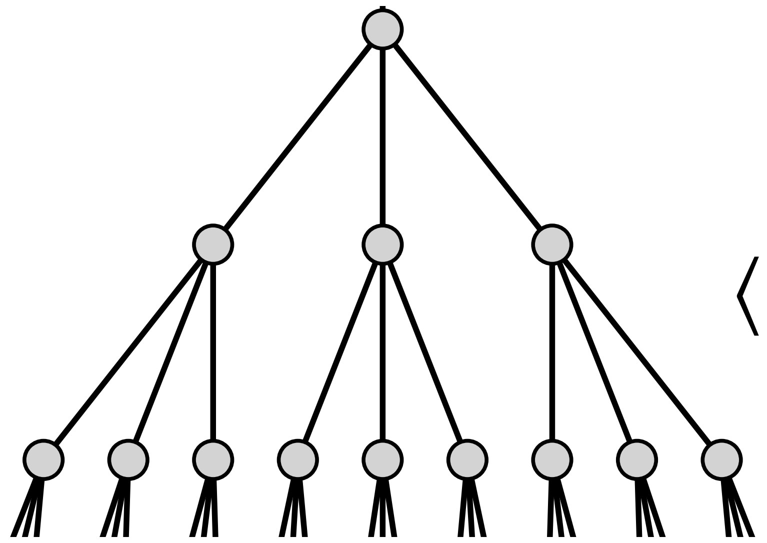
$\rho_r = 1$

QREM
No MBL

Disordered local Hamiltonians
Finite W_c, MBL for $W > W_c$

MBL possible only if Fock-space site energies at finite distances maximally correlated; minimum requirement for MBL to be stable

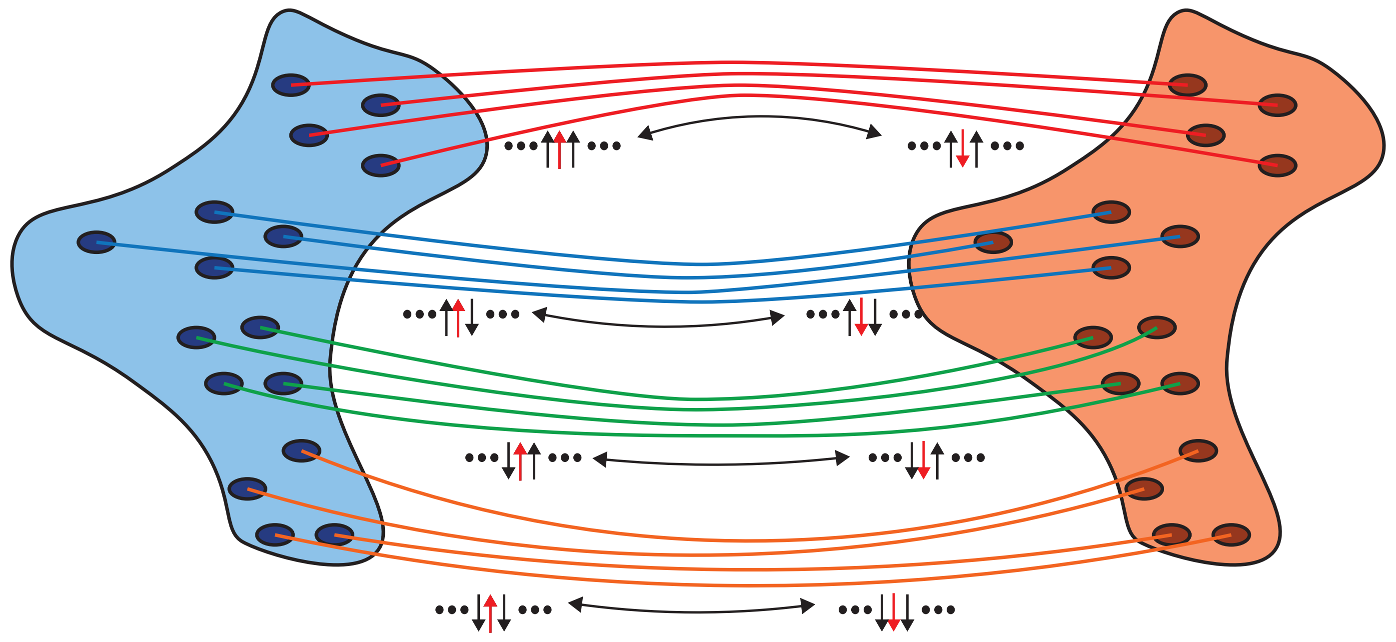
Phys. Rev. B 101, 134202 (2020)



$$\langle \epsilon_i \epsilon_j \rangle = f(r_{ij}/L); \qquad \lim_{x \to 0} f(x) \to 1$$

Anderson localisation on trees with maximally correlated disorder as controlled settings for understanding the effect of correlations

Phys. Rev. Lett. 125, 250402 (2020)



Classical percolation proxy to highlight how the correlations stop local degrees of freedom from thermalising

Phys. Rev. B 99, 220201(R) (2019)
Phys. Rev. B 99, 104206 (2019)

Important outstanding questions

Possible connections between the Fock-space approach and results from phenomenological treatments ?

Connecting the microscopic theory on Fock-space to real-space pictures ?

Acknowledgements and References

J. T. Chalker

D. E. Logan

EPSRC

- SR, D. E. Logan, Phys. Rev. B 101, 134202 (2020)
- SR, D. E. Logan, J. T. Chalker, Phys. Rev. B 99, 220201(R) (2019)
- SR, J. T. Chalker, D. E. Logan, Phys. Rev. B 99, 104206 (2019)
- SR, D. E. Logan, Phys. Rev. Lett. 125, 250402 (2020)