A diagrammatic presentation of the category 3Cob

TQFT Club Seminar, Lisbon, December 9, 2022.

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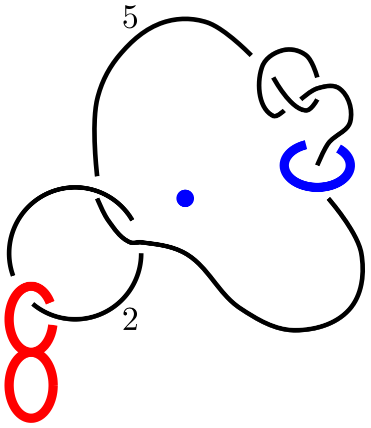
Joint work with Jovana Nikolić and Mladen Zekić



The speaker was supported by the Science Fund of the Republic of Serbia, Grant No. 7749891, Graphical Languages - GWORDS

1. The language is based on surgery description of closed manifolds introduced by Wallace (1960) and Lickorish (1962).

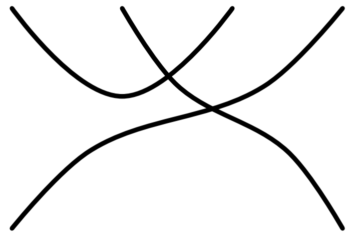
2. The calculus is based on the calculi introduced by Rolfsen (1976), Kirby (1978), Fenn and Rourke (1979) and Roberts (1997).

3. The composition of diagrams is new.

An n-dimensional TQFT is a symmetric monoidal functor from the category nCob to the category Vect.

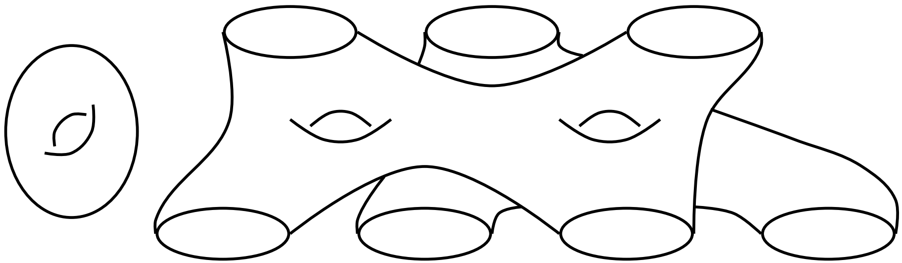
How faithful a 3-dimensional TQFT could be?

Diagrams for 1Cob $\leftrightsquigarrow$ Richard Brauer's representation



Under some minor provisos, every 1-dimensional TQFT is faithful.

Diagrams for 2Cob $\leftrightsquigarrow$ commutative Frobenius algebras



A 2-dimensional TQFT with respect to $\mathbb{Q}\mathbb{Z}_5 \otimes Z(\mathbb{Q}\mathbb{S}_3)$ is faithful.

Diagrams for 3Cob $\leftrightsquigarrow$?

? = Modular categories of Turaev or J-algebras of Juhasz or something else.

What are 3Cob diagrams and how to compose them?

S. SAWIN, *Three-dimensional 2-framed TQFTS and surgery*, ***Journal of Knot Theory and its Ramifications***, vol. 13 (2004), pp. 947-963

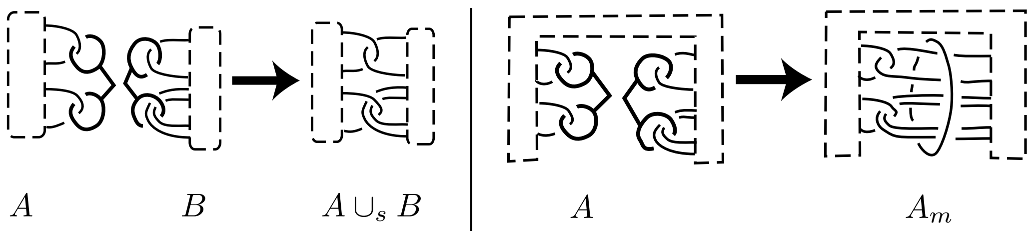
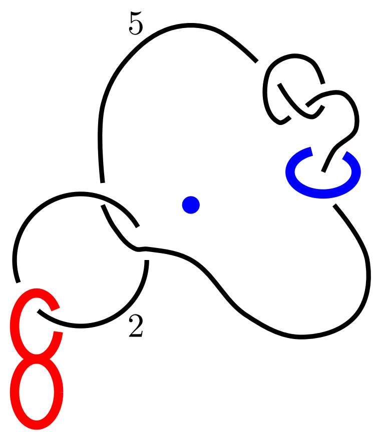


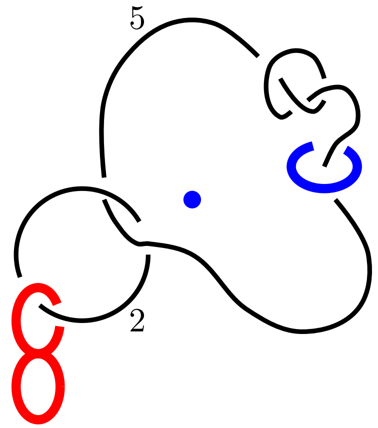
Fig. 9. Sewing and mending of three-manifolds.

The diagrams

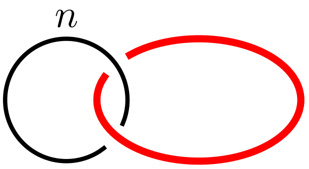
The diagrams are placed in R^3 in a narrow tubular neighbourhood of the xy-plane.

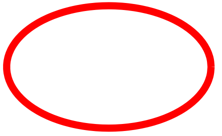


The interpretation (as 3-manifolds)



The interpretation of a diagram as a manifold includes something more than just the homeomorphism type of a manifold—it gives a canonical identification of the boundary.

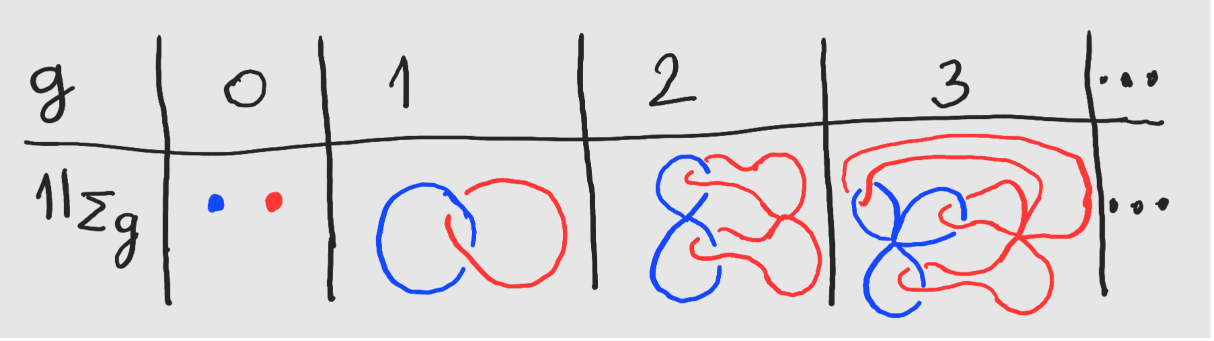




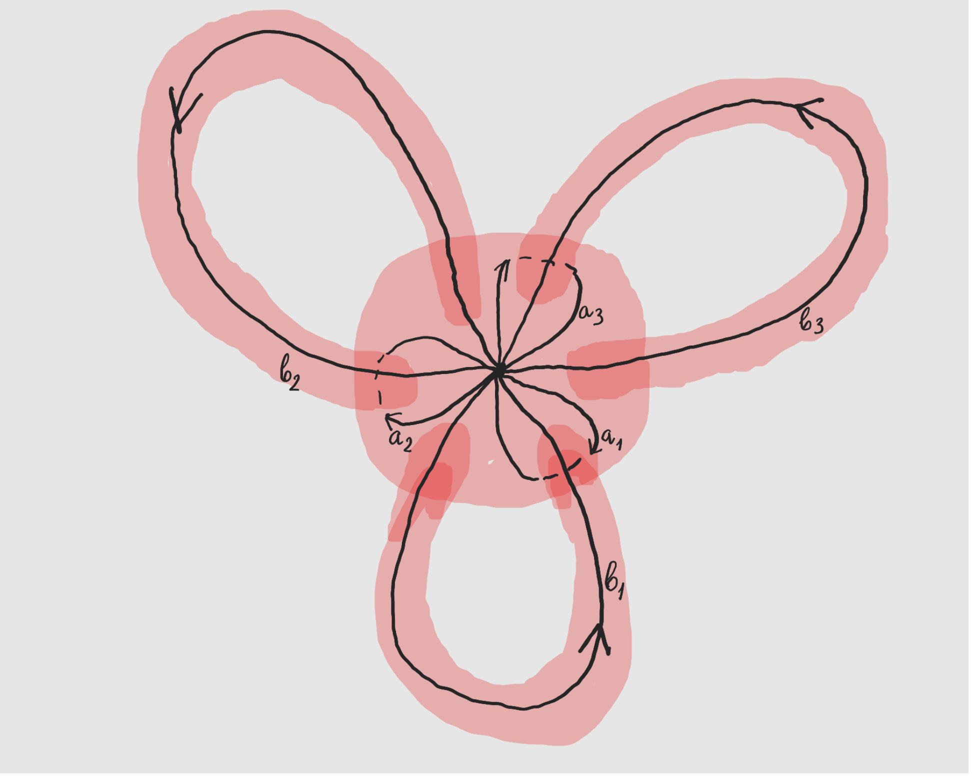
Both diagrams present the solid torus. However, if we consider the red circles to be identical, then there is no homeomorphism between the solid tori, which fixes the points of the common boundary.

The interpretation (as 3-cobordisms)

For every g a surface Σ_g of genus g is fixed and its embedding into a component of ∂M is defined. The goal is to interpret the following diagrams as identities.

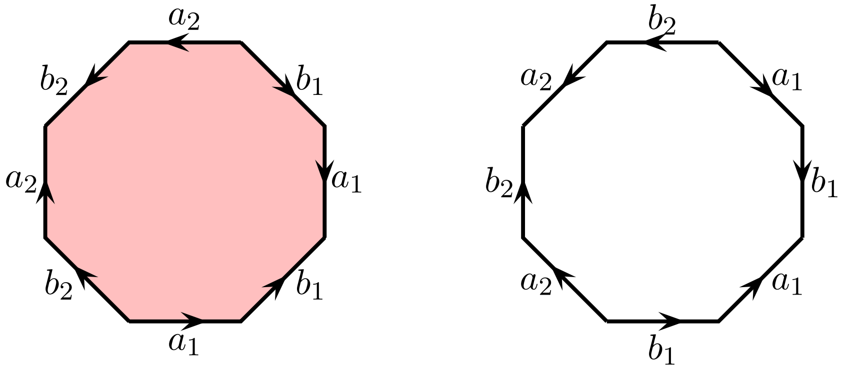


The handlebody H_g and its boundary Σ_g



Σ_g is equipped with a wedge of $2g$ circles, one pair, consisting of an a-circle and a b-circle, for each handle.

The reversion

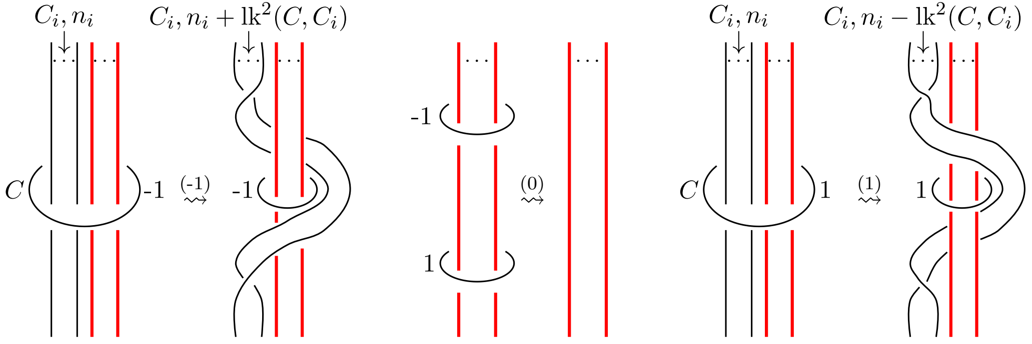


$$R = [b_1, a_1, b_g, a_g, \ldots, b_2, a_2]$$

Proposition 1. Every connected arrow of 3Cob is presentable by a diagram.

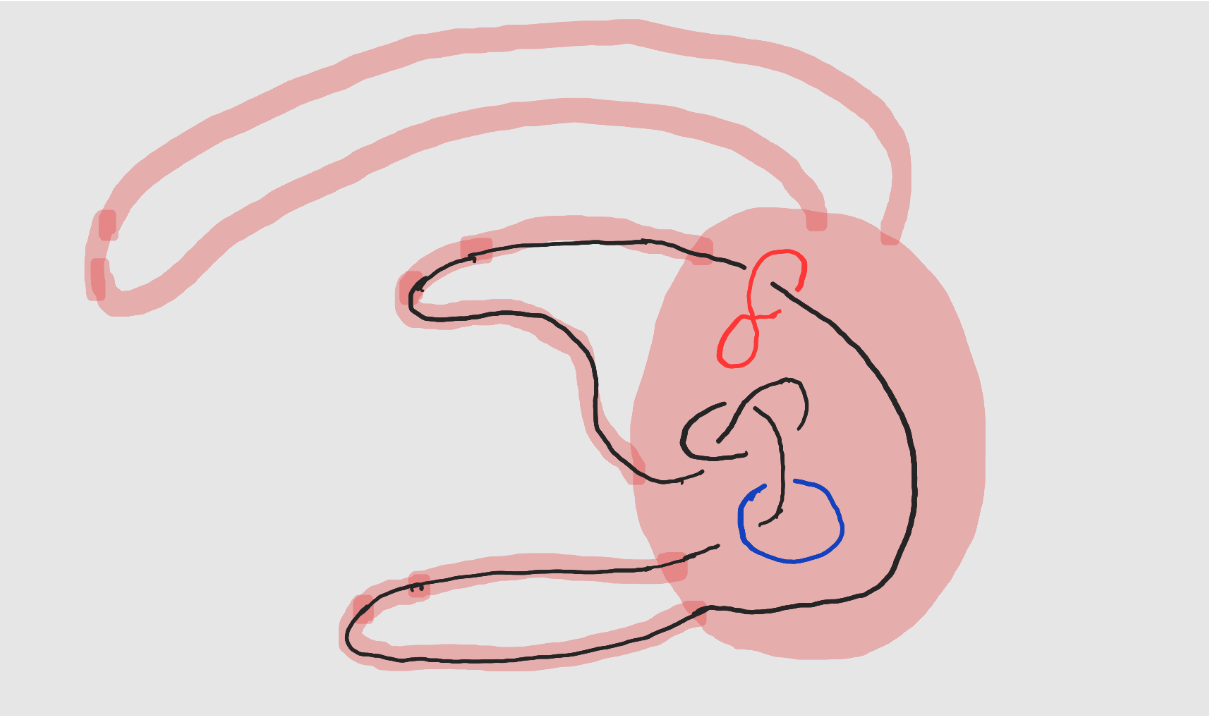
Corollary Every arrow of 3Cob is presentable by a sequence of diagrams and two permutations acting on the source and the target.

Proposition 2. Two diagrams present the same arrow of 3Cob if and only if there is a finite sequence of moves

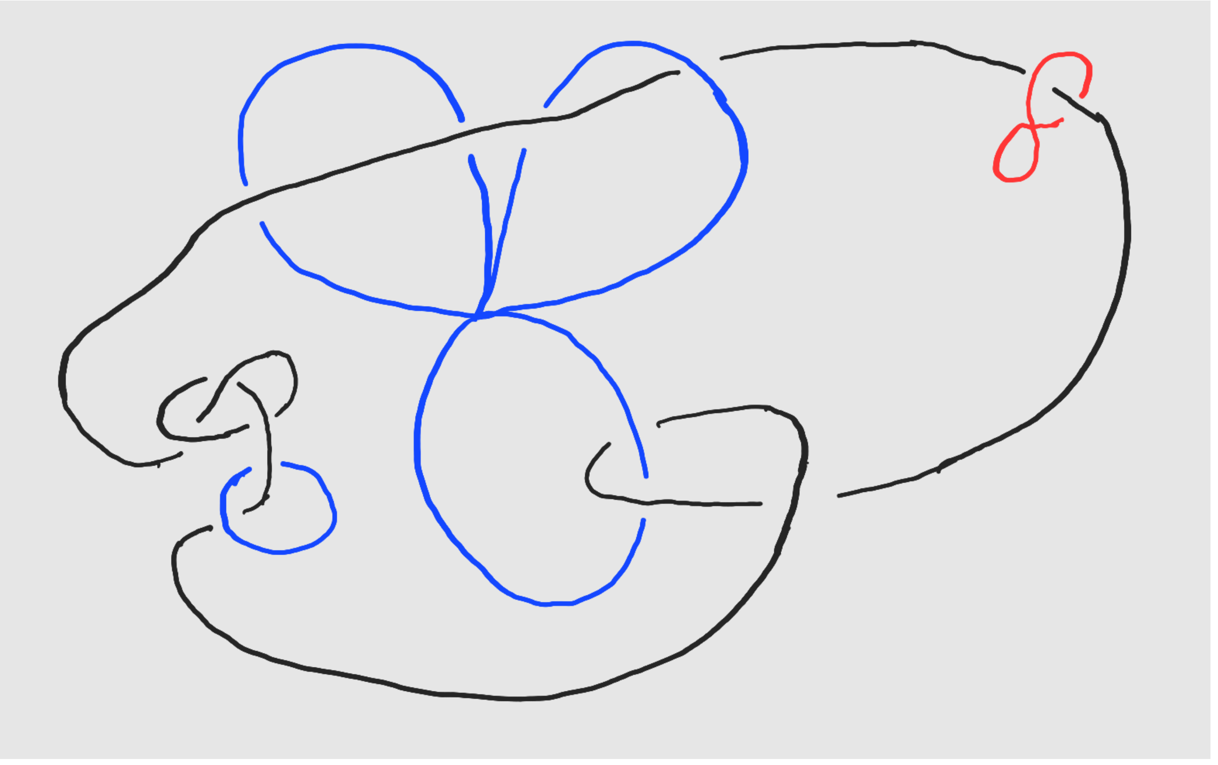


transforming one into the other.

A diagram within a handlebody



Inside-out



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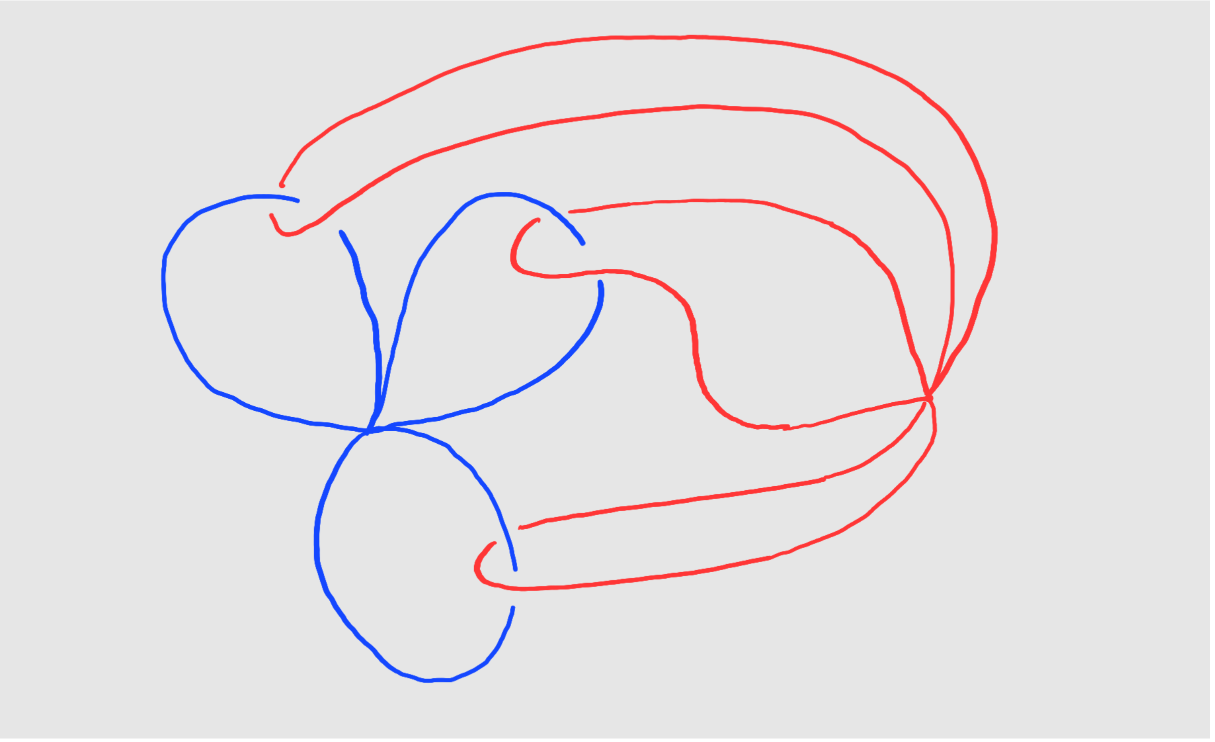

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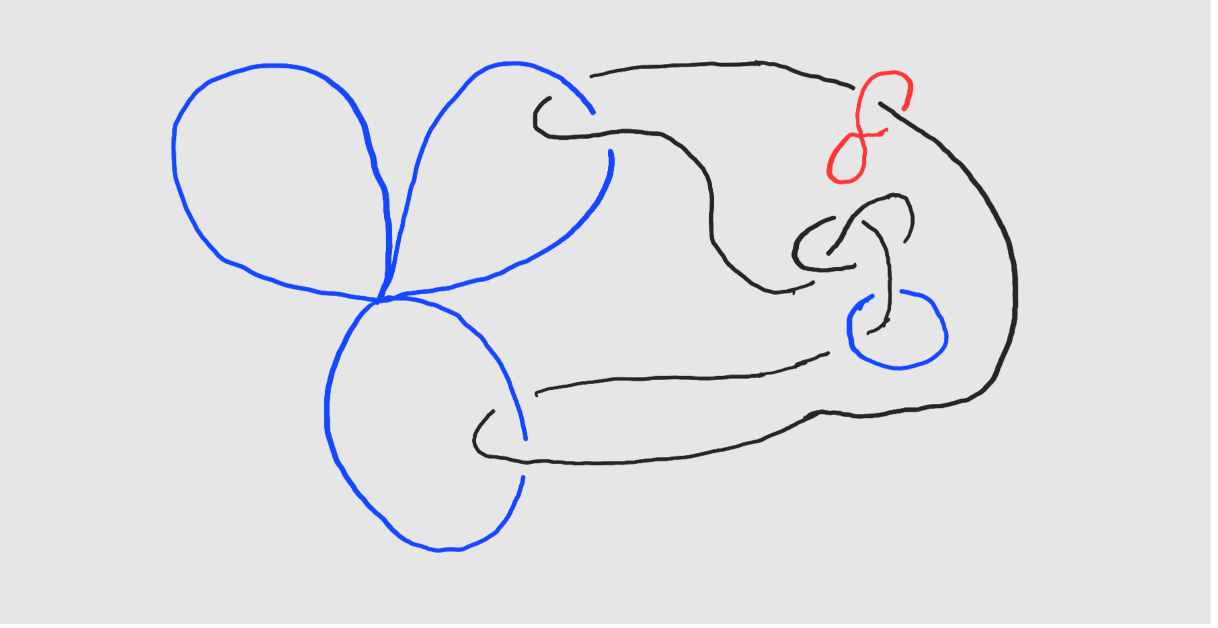

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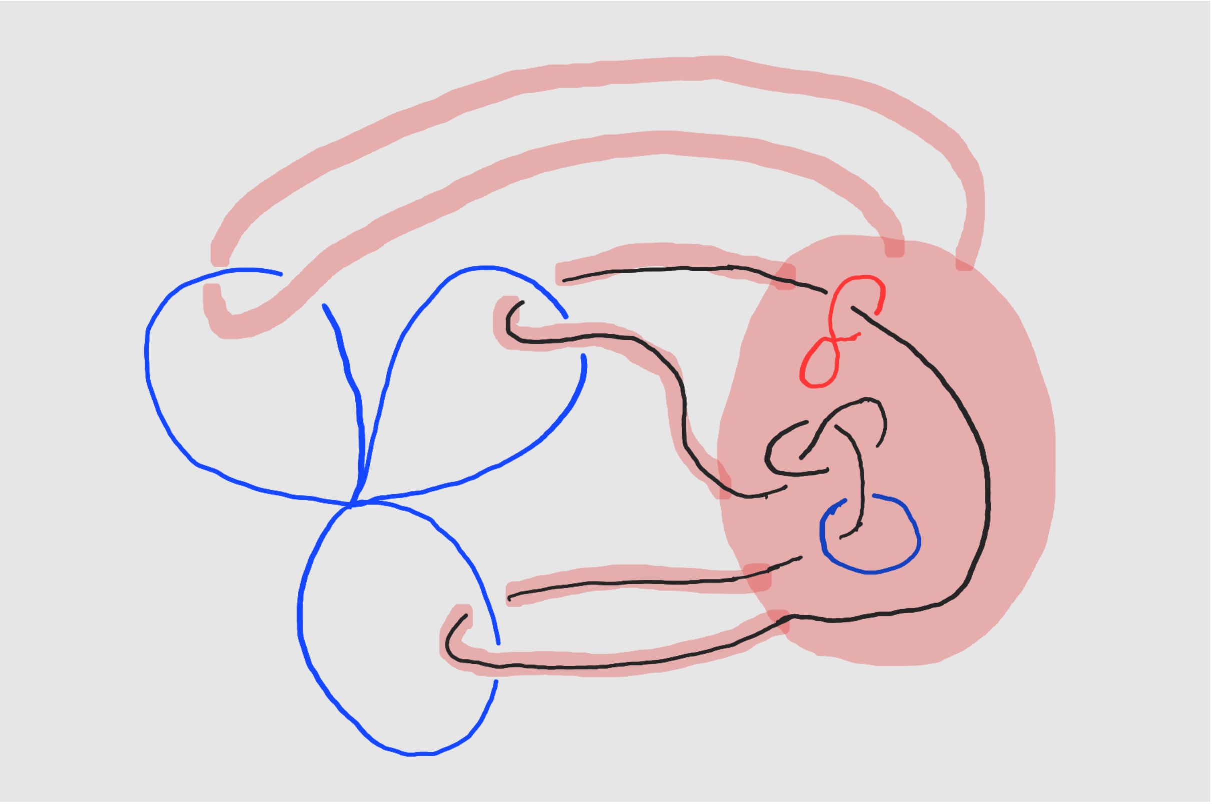

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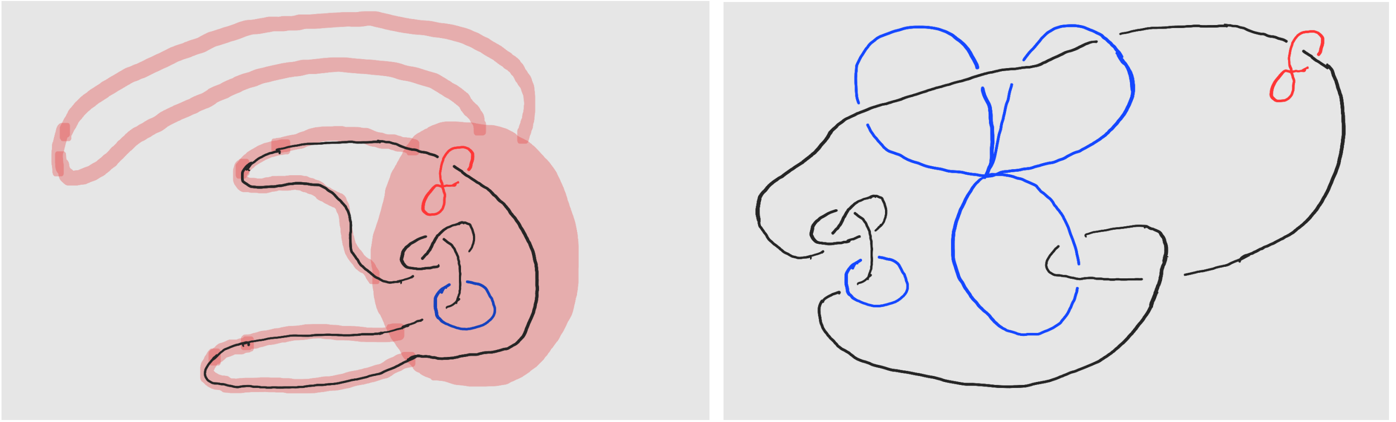
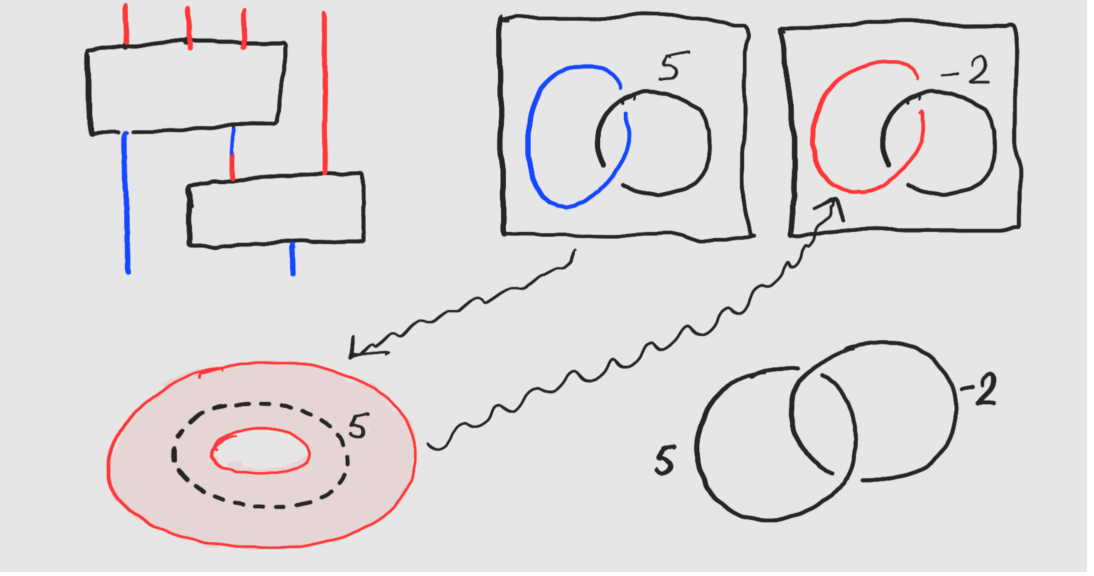
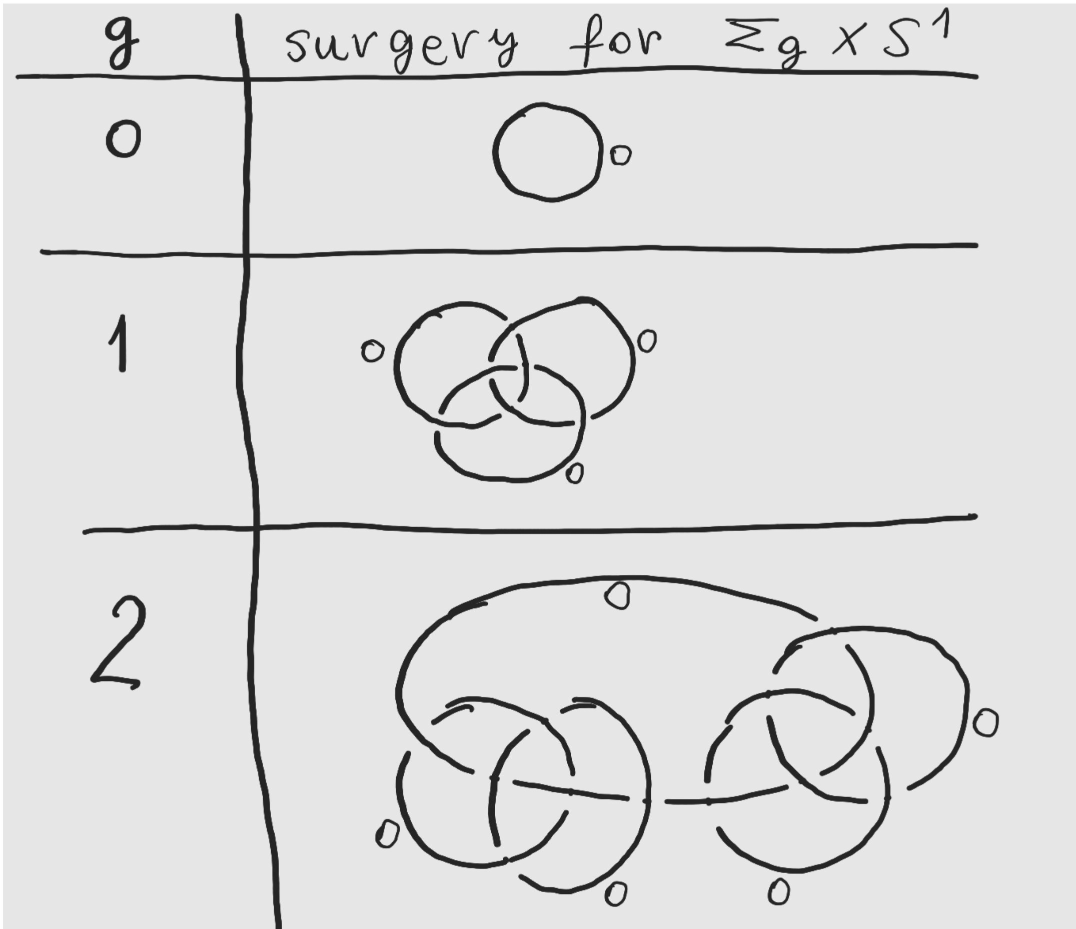


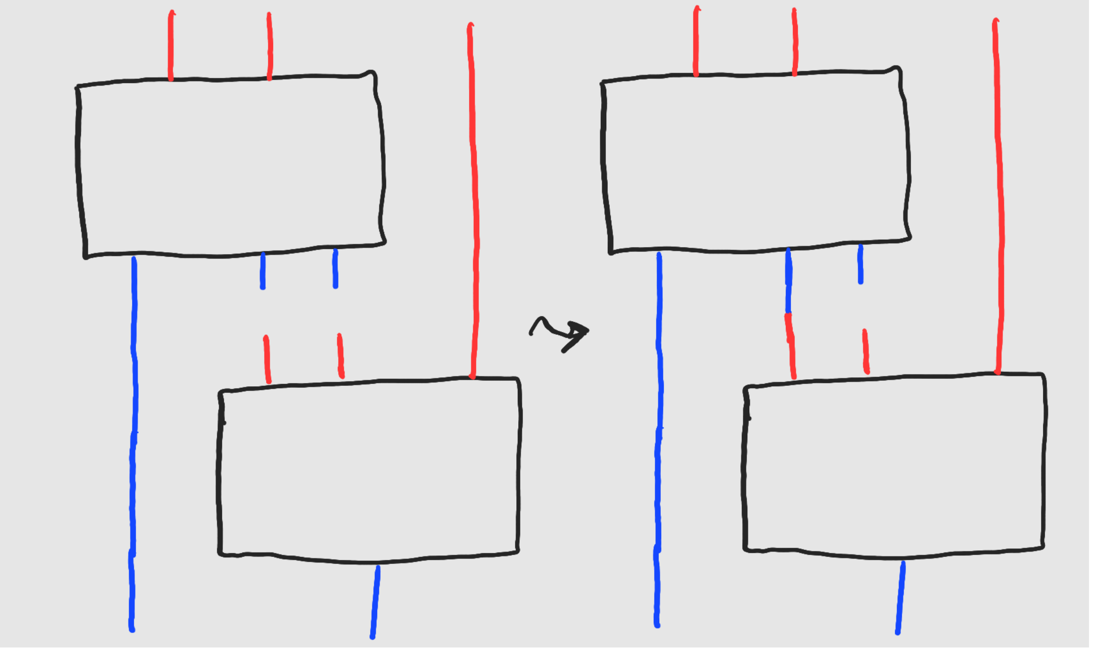
Gluing along a connected boundary



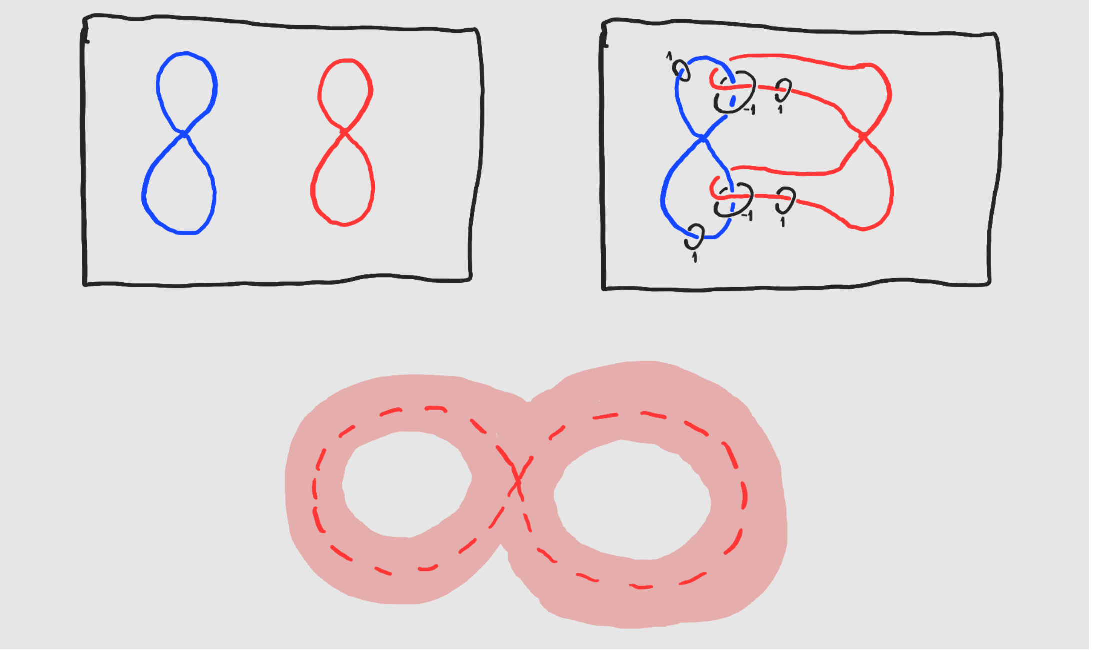
Surgery for $\Sigma_g \times S^1$.

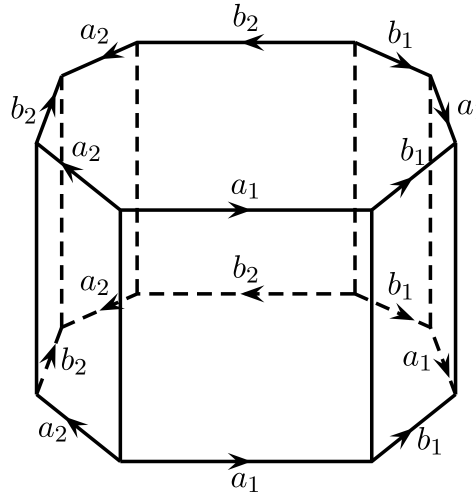


Gluing along a disconnected boundary



Gluing along a disconnected boundary





$\Sigma_g \times I$