Quantum cohomology = intersection th. on moduli spaces of stable maps

$\overline{\mathcal{M}}_{g,n}(X, d)$ degree of $\Sigma \to X$

Kähler target space

Quantum K-theory = K-theoretic analogue

$\mathrm{str}_h \, \check{H}^\bullet\left(\overline{\mathcal{M}}_{g,n}(X,d); \mathcal{O}^{\mathrm{vir}} \bigotimes_{i=1}^{n} \text{interesting v.bundle associated to } i\text{th marking}\right)$

renumbering of n markings

virtual structure sheaf (Y.-P. Lee)

Interesting vector bundle $= \sum_m ev_i^*(\phi_m) \, L_i^{\otimes m}$

evaluation at the ith marking

elements of $K^0(X)$

universal cotangent lines

Generating functions $\mathcal{F}_g(t_1, t_2, t_3, \ldots)$

$g \leftarrow$ genus

$t_r = \sum_m \phi_m q^m \in K^0(X)[q, q^{-1}]$

one Laurent polyn. for each cycle length r

$\mathcal{D} \neq e^{\sum_g \hbar^{g-1} \mathcal{F}_g}$ ← connected curves

in K-theory ← disconnected curves

$$\sum_{n=0}^{\infty} \frac{1}{n!} \sum_{h \in S_n} \mathrm{tr}_h \, V^{\otimes n} = e^{\sum_{k>0} \Psi^k(V)/k}$$

Adams $\Psi^k(V) = \mathrm{tr}_{(1,2,\ldots,k)} V^{\otimes k}$

$$\mathcal{D} = e^{\sum_{k>0} \frac{\Psi^k}{k} \sum_g \hbar^{g-1} \mathcal{F}_g(t_k, t_{2k}, t_{3k}, \ldots)}$$

$\Psi^k(\hbar) = \hbar^k$

Quantum state in the quantization of some symplectic space (to be explained)

Lefschetz - Kawasaki - RR Formula

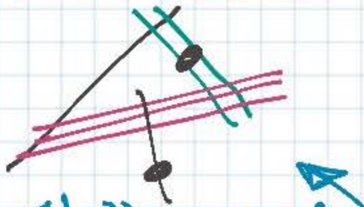
$$\mathrm{str}_{\underline{h}}\, \check{H}^{\bullet}(\underline{\mathcal{M}}; \underline{V}) = \chi^{fake}\left(\underline{I\mathcal{M}^h}; \frac{\mathrm{tr}_h\, V|_{I\mathcal{M}^h}}{\mathrm{str}_h\, \Lambda^{\bullet} N^{\vee}_{I\mathcal{M}^h}}\right)$$

automorph. of orbifold / orbibundle

inertia orbifold

$$\chi^{fake}(\mathcal{M}; V) := \int_{\mathcal{M}} ch(V)\, td(T_{\mathcal{M}})$$

Lefschetz-Kawasaki strata of $\overline{\mathcal{M}}_{g,n}(X,d)$



Stable maps with prescribed symmetry h

$\Rightarrow$ $\mathcal{D}$ = Wick's summation over graphs

$= e^{\text{"edge propagators"}} \prod$ vertex contributions

Vertices: Fake (twisted) Quantum K-theory of $X \times B\mathbb{Z}/M$ in Chen-Ruan sense

$M = 1, 2, 3, \ldots$

$\mathcal{D}^{fake, twisted}_{X \times B\mathbb{Z}/M}$ = [Jarvis/Kimura, Tonita, Tseng, Coates] some cohomolog. GW. invariants of X

Theorem (Quantum Hirzebruch-RR)

$$\mathcal{D}_X = \bigotimes_{M=1}^{\infty} \mathcal{D}^{fake, twisted}_{X \times B\mathbb{Z}/M}$$

as quantum states

Symplectic loop spaces

$$\bigoplus_{r=1,2,\ldots} K^0(X)(q, q^{-1}) \to \bigoplus_{M=1,2,\ldots} \bigoplus_{\xi : \xi^M = 1} K^0(X)((q-1))$$

global $\qquad$ adelic

$$(f_1, f_2, \ldots, f_r, \ldots) \mapsto \{ f_r(q^{1/m(\xi)} \xi) \}$$

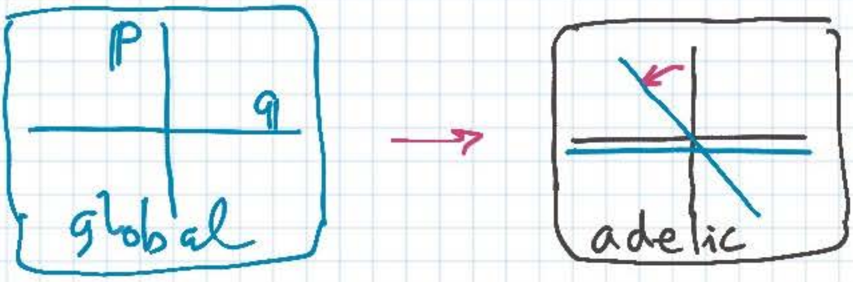
$M = r\, m(\xi)$, primitive order

Symplectic forms.

$$-\mathrm{Res}_{q=0,\infty} (f(q^{-1}), g(q)) \frac{dq}{q} \mapsto \mathrm{Res}_{q=1} (f(q^{-1}), g(q)) \frac{dq}{q}$$

Lagrangian polarizations:

$$\{ f_r \mid f_r(0) \neq \infty,\ f_r(\infty) = 0 \} \not\mapsto \text{principal parts of Laurent series}$$



$e^{\widehat{\mathbb{P}}^2/2}$

$$\mathcal{D} = e^{\widehat{\mathbb{P}}^2/2} \bigotimes_{M=1,2,\dots} \mathcal{D}_M^{\text{local}}$$

① Quantum measurement problem

Is it possible to reconstruct $\mathcal{D}_M^{\text{local}}$ from $\mathcal{D}$?

Conjecture: Yes (the image of the adelic map is dense)

② Is interaction an illusion of observer?

③ Quasi-classical limit

$$e^{\mathcal{F}_0^{\text{fake}}/\hbar + \dots} \rightsquigarrow \mathcal{L}^{\text{fake}} = \{(p,q) \mid p = d_q \mathcal{F}_0^{\text{fake}}\}$$

↑ Lagrangian submanifold

$$e^{\sum_{k>0} \frac{1}{\hbar^k} \frac{\Psi^k}{k} \mathcal{F}_0(t_k, t_{2k}, \dots, t_{rk}, \dots)}$$

$$\mathcal{L} = \{(p,q) \mid p_k = (d_{t_1} \mathcal{F}_0)(q_k, q_{2k}, \dots, q_{rk}, \dots)\}$$

$\{p_1 = d_{q_1} \mathcal{F}_0(q_1, \underbrace{q_2, \dots}_{\text{parameters}})\}$ - family of Lagrangian mfds

Theorem (adelic characterization of $\mathcal{L}$)

$f = (f_1, f_2 \dots) \in \bigoplus_{r=1,2,\dots} K^0(X)(q, q')$ lies in $\mathcal{L}$

iff (i) $f_k^{\text{local}} \in \mathcal{L}^{\text{fake}}$, $k = 1, 2, \dots$

(ii) $f_k(q^{1/m}\zeta)^{\text{local}} \in \Delta_\zeta \Psi^m T_{f_{mk}}^{\text{local}} \mathcal{L}^{\text{fake}}$

primitive m-th root of unity

some explicit linear transformation

④ Question:

What is symplectic geometry formalism behind this structure?