
# Model based control design combining Lyapunov and Optimization tools

## Examples in the area of motion control of Autonomous Robotic Vehicles

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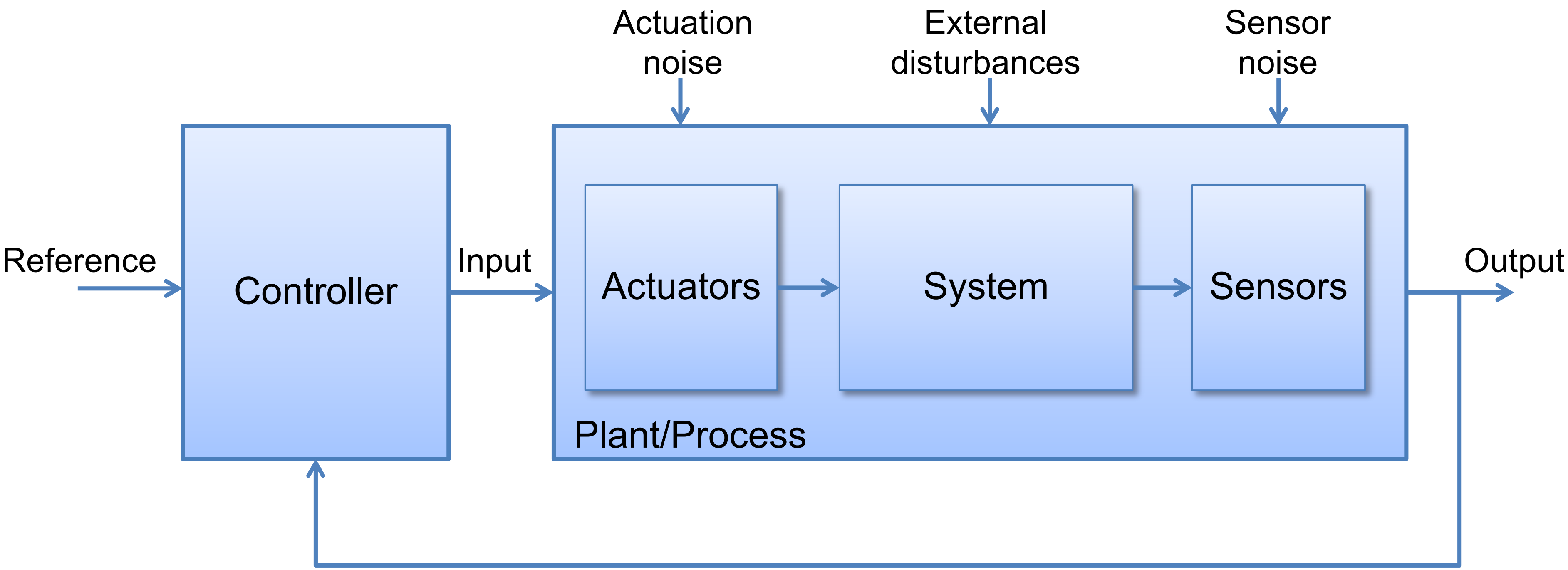
# Topics of the talk

- How can we design nonlinear feedback controllers using **Lyapunov based** techniques?

- Idem… using **optimization model-based** techniques...

- Several examples on motion control of single and multiple **autonomous robotic vehicles**

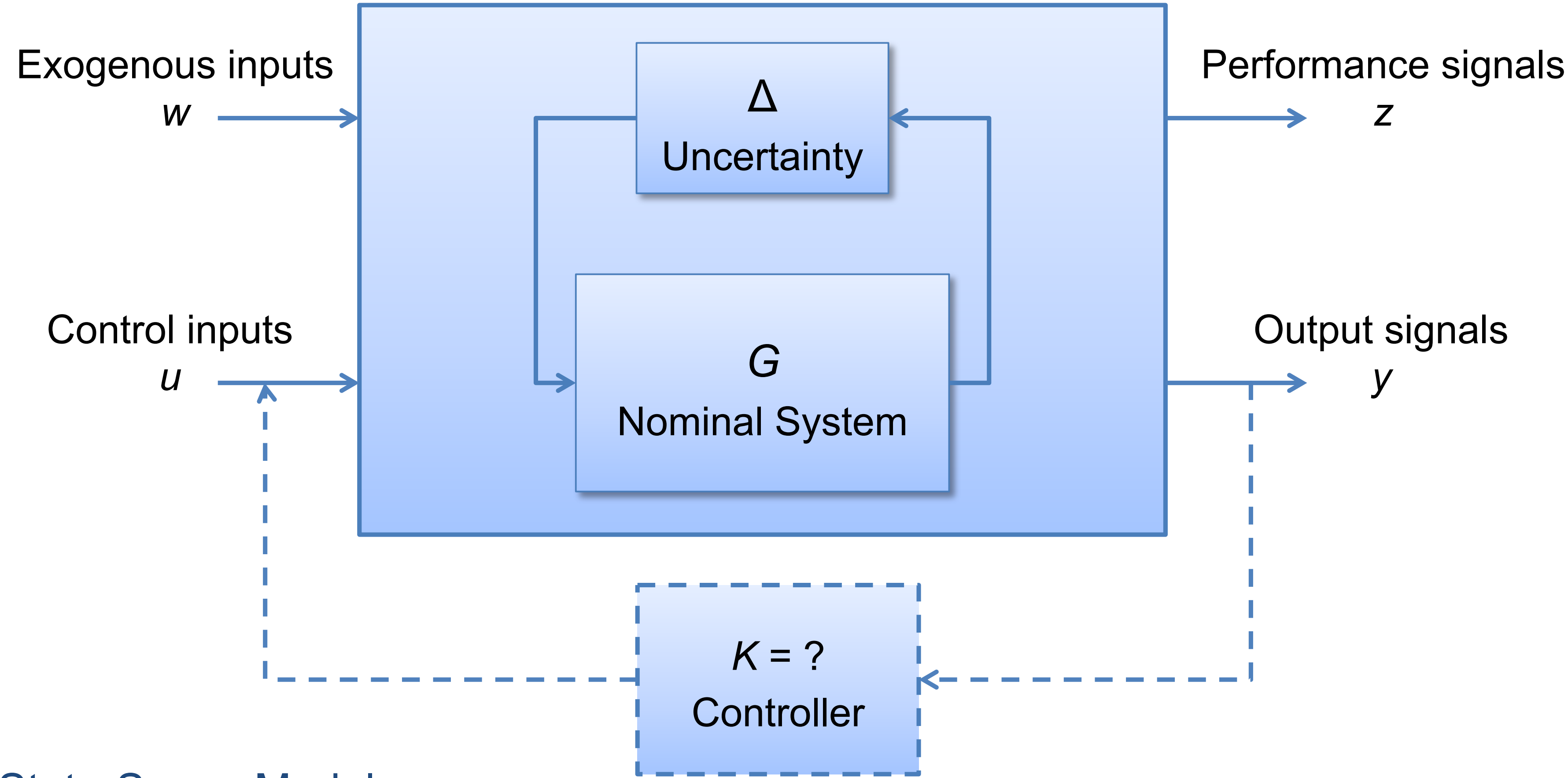
# What is a control system?




**Control Objectives:**

Design a controller that in **real-time** stabilizes the plant and the output signal track the reference despite external disturbances, noises, and plant parameter uncertainty (**robust stability and performance)**

# What is a model based control design?




State-Space Model

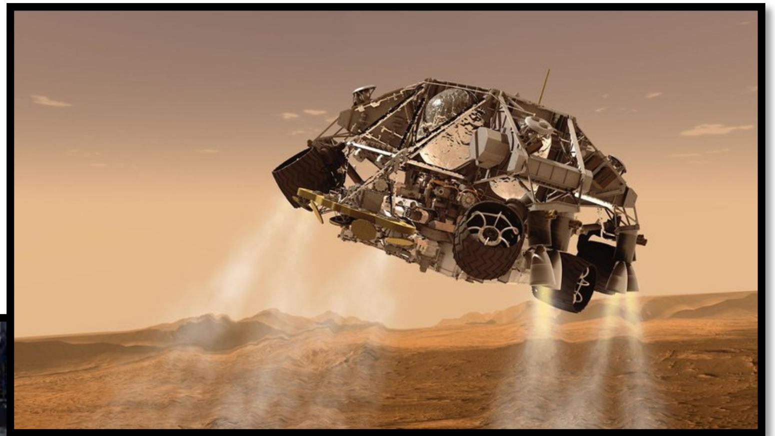
$$G: \quad \begin{aligned} \dot{x} &= f(x, u, w) \\ y &= h(x, u, w) \end{aligned}$$

$$x \in \mathcal{X} \quad u \in \mathcal{U}$$
$$y \in \mathcal{Y} \quad w \in \mathcal{W}$$

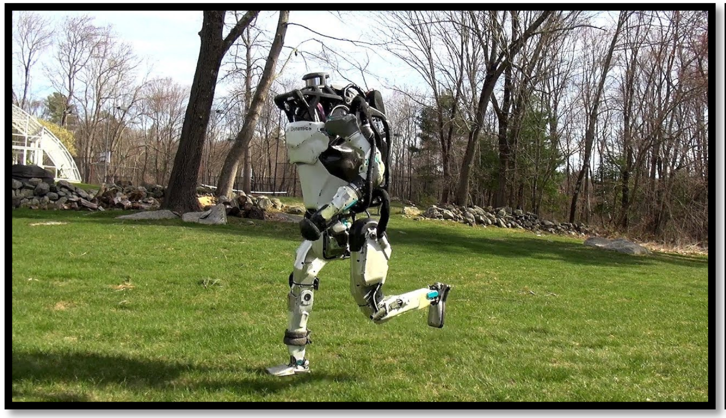
$$\Delta \in \mathcal{F}_\Delta$$

# Why do we care?

*We are interested in system methodological tools that are* ***provably*** *(mathematically)* ***certified by design guaranteeing the specifications*** *(e.g., stability, robustness, and performance) in the presence of challenging restrictions and uncertainties.*



**Safety-critical systems**

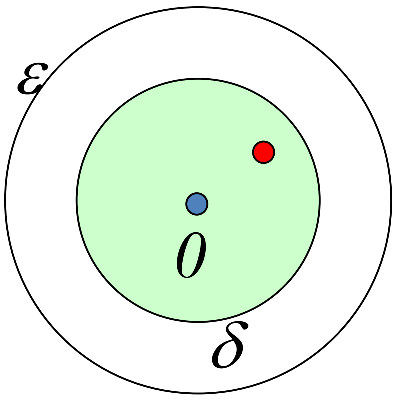


No matter what sequence of events occur (within a set of reasonable assumptions), the control system will always respond in a manner that satisfies the specification.

# Lyapunov Stability

## Stability definition

$$\dot{x} = f(x)$$






### Definition

The equilibrium point $x = 0$ is

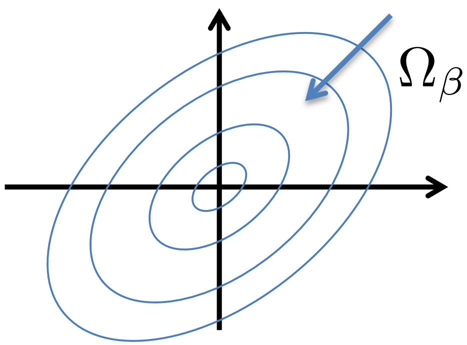
- stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon) > 0$ such that

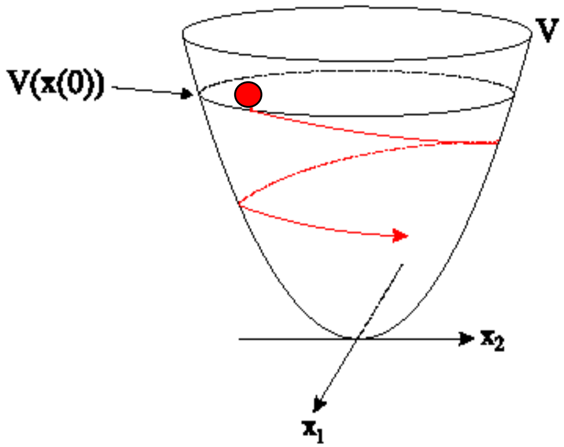
$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall t \geq 0$$

- unstable if it is not stable
- asymptotically stable if it is stable and $\delta$ can be chosen such that

$$\|x(0)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0$$

# Lyapunov Stability




***Lyapunov's stability theorem***

Let $V : D \to \mathbb{R}$ be a continuously differentiable function such that

- $V(0) = 0,\ V(x) > 0, \quad \forall x \in D \setminus \{0\}$
- $\dot{V}(x) \leq 0, \quad \forall x \in D$

Then, $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0, \forall x \in D \setminus \{0\}$$

then $x = 0$ is asymptotically stable.

Note that the set

$$\Omega_\beta = \{x \in B_r : V(x) \leq \beta\}$$

is an *invariant set*.

# Control Lyapunov functions

**Definition (CLFs)**

A positive definite function $V(x)$ is a **Control Lyapunov function (CLF)** for system $\dot{x} = f(x) + g(x)u$ if it satisfies (for every $x \neq 0$):

$$\inf_{u \in \mathcal{U}} \Big[ \underbrace{L_f V(x) + L_g V(x)\, u}_{\dot{V}(x) = \frac{\partial V}{\partial x}[f(x) + g(x)u]} \Big] < 0$$

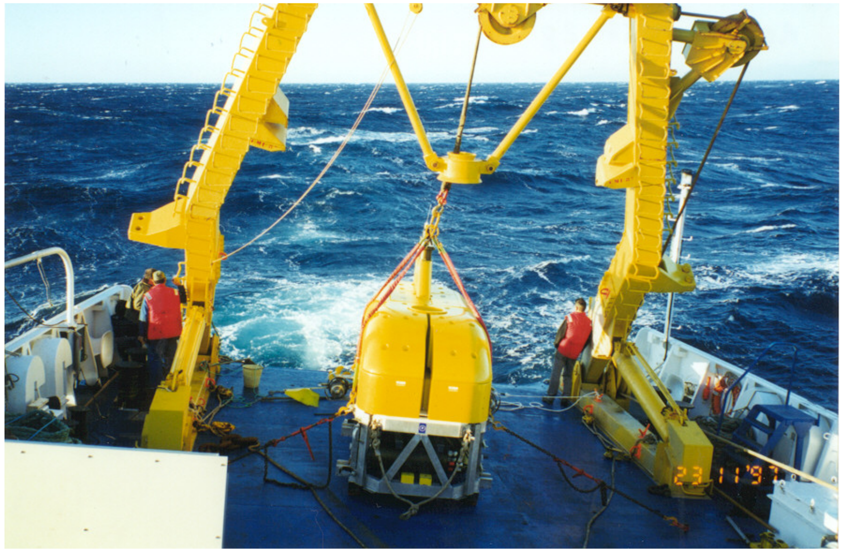
There is a feedback law such that the origin is asymptotically stable ⇔ There is a CLF

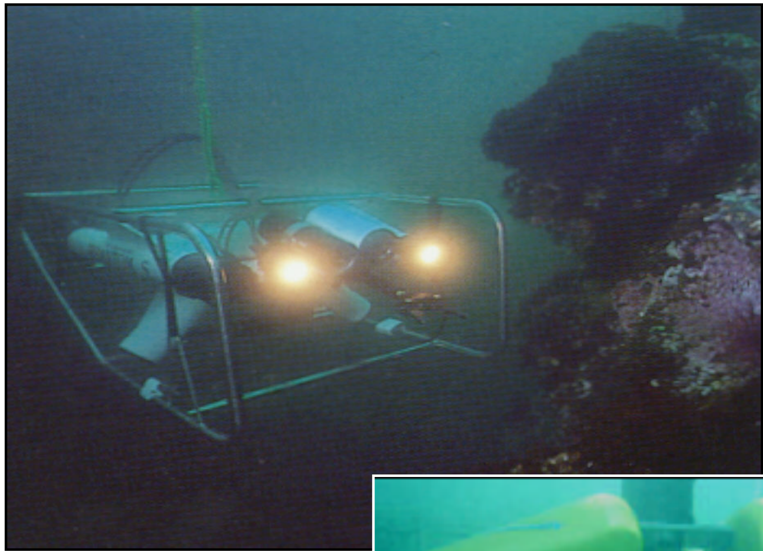
All these concepts can be extended to stability of trajectories, sets, global results, robustness to disturbances, discrete-time systems, etc.

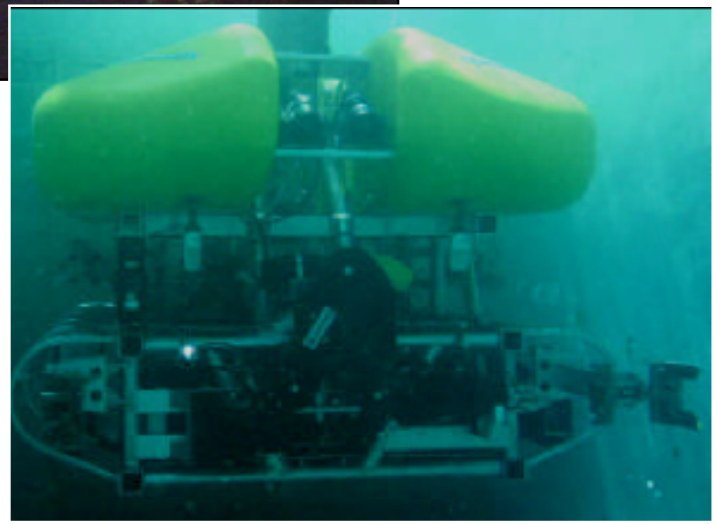
*How can we find a CLF / control law?*

# Example: Marine Robotic Vehicles

***Typical motion control problems***



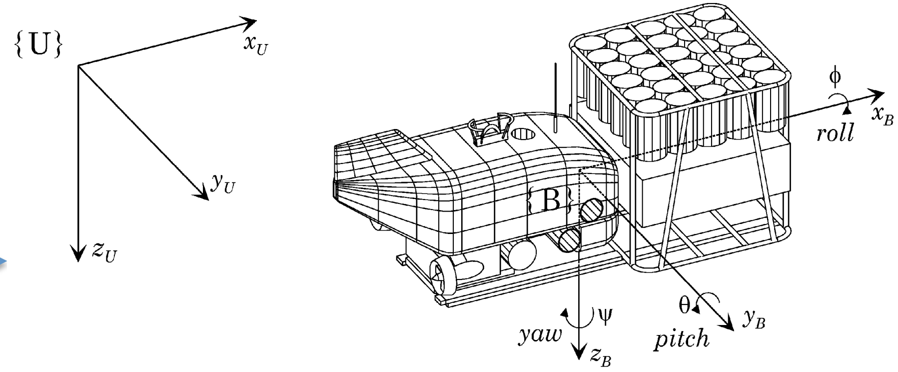




- Speed, Heading, and Depth Control
- Bottom Following (Terrain Contouring)
- Point Stabilization, Hovering, Manipulation
- Trajectory Tracking and Path Following
- Target Tracking…

# Point Stabilization

Fully actuated AUV/ROV


**<u>Goal</u>**: Design a state feedback control so that $\eta(t)$ converges to a desired position and attitude $\eta_d$ (Pose stabilization)

Model:

$$
\begin{aligned}
M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) &= \tau \\
\dot{\eta} &= J(\eta)\nu
\end{aligned}
$$

$$
M > 0 \qquad C(\nu)^T = -C(\nu) \qquad D(\nu) > 0
$$

$$\tau = (\tau_u, \tau_v, \tau_w, \tau_p, \tau_q, \tau_r)'$$

$$\nu = (u, v, w, p, q, r)'$$

$$\eta = (x, y, z, \phi, \theta, \psi)'$$

# Nonlinear Control Design

Model:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau$$
$$\dot{\eta} = J(\eta)\nu$$

Error dynamics:

$$e(t) = \eta(t) - \eta_d \longrightarrow \dot{e} = \dot{\eta} = J(\eta)\nu$$

Control Lyapunov function:

$$V(\nu, e) = \frac{1}{2}(\nu^T M\nu + e^T K_P e)$$

# Nonlinear Control Design

Computing the time derivative with respect to the trajectory of the system...

$$\dot{V} = \nu^T M \dot{\nu} + \dot{e}^T K_P e$$

$$= \nu^T \left( M\dot{\nu} + J^T(\eta) K_P e \right)$$

$$= \nu^T \left( \tau - D(\nu)\nu - g(\eta) + J^T(\eta) K_P e \right) - \underbrace{\nu^T C(\nu)\nu}_{0}$$

Assign a feedback law...

$$\tau = -J^T K_P e(t) - K_D \nu + g(\eta)$$

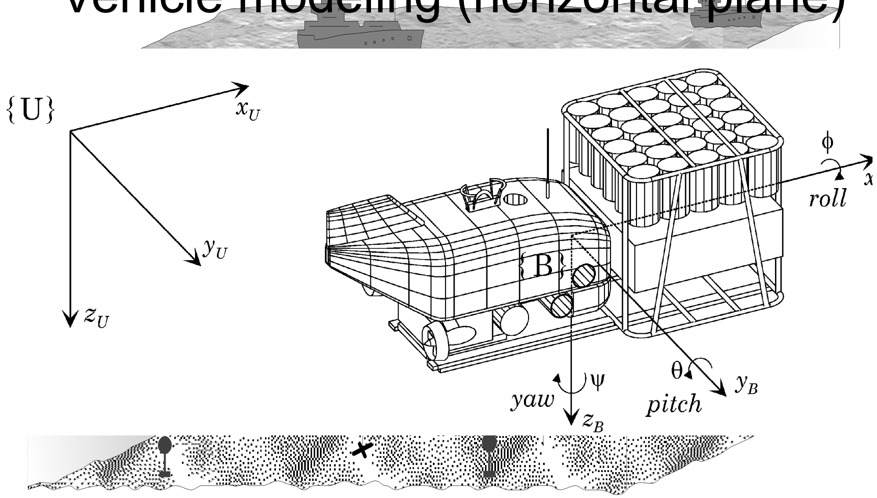
$$\dot{V} = -\nu^T \left( D(\nu) + K_D \right) \nu \leq 0 \longrightarrow$$ We have stability!

*Using now other tools (LaSalle's invariance principle) it is possible to conclude asymptotically stability!*

# Dynamic Positioning of an underactuated AUV

**Goal**: steer an underwater vehicle to a target point

Vehicle modeling (horizontal plane)




Strategy adopted:

## Main challenges

- Nonlinear dynamics
- Underactuated
- Parametric modeling uncertainty
- External disturbances

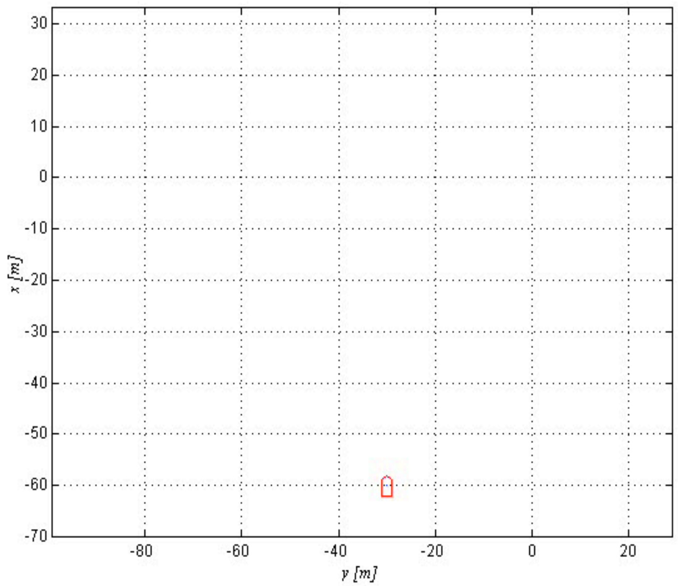
Adaptive controller

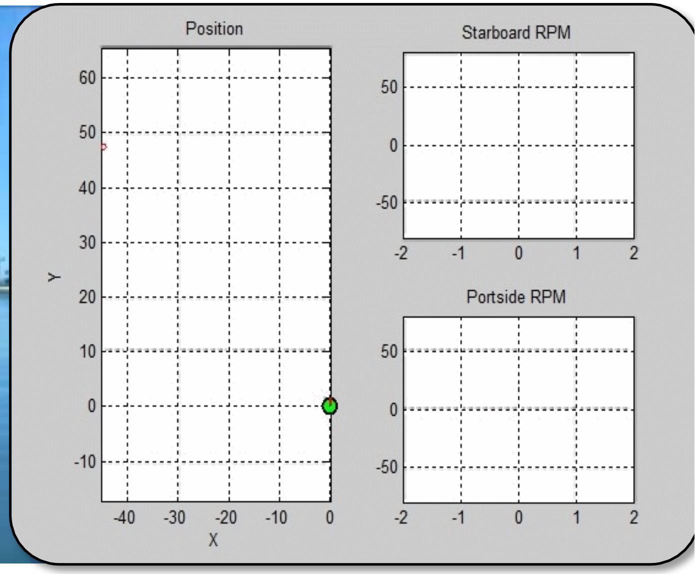
Dynamic controller

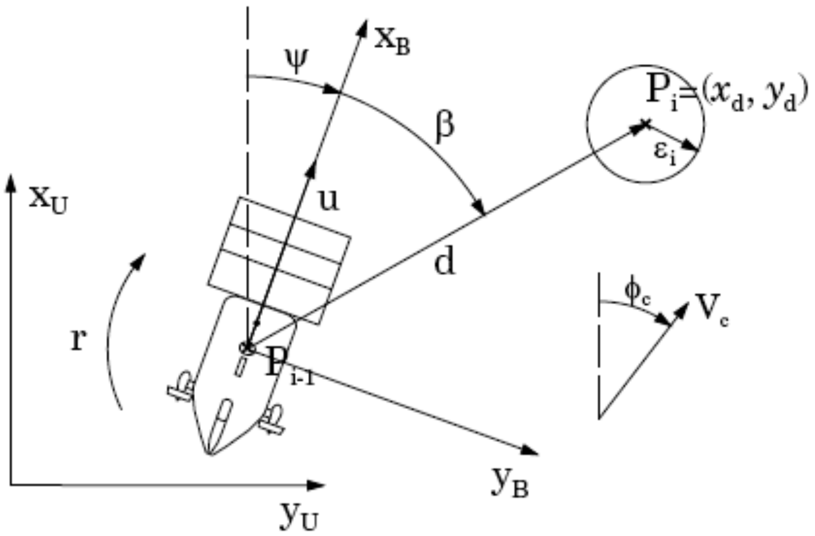
Observer

Kinematic controller

# Dynamic Positioning of an underactuated AUV




$$e = \sqrt{(x - x_d)^2 + (y - y_d)^2}$$

$$\dot{e} = -u_r \cos\beta - v_r \sin\beta + V_c \cos(\beta + \psi - \phi_c)$$

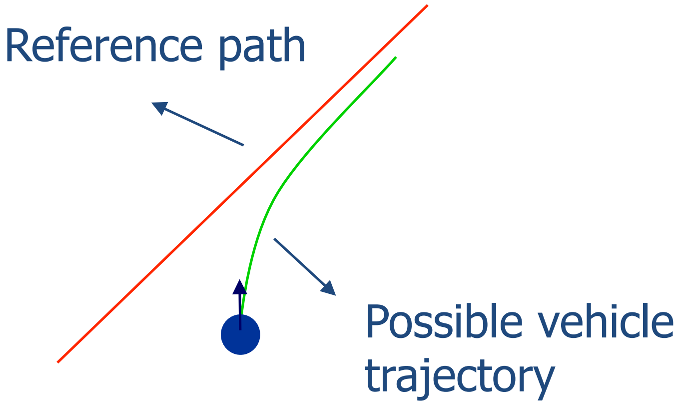
$$\dot{\beta} = \frac{\sin\beta}{e} u_r - \frac{\cos\beta}{e} v_r + \frac{V_c}{e} \sin(\beta + \psi - \phi_c)$$

$$\dot{\psi} = r \qquad \psi + \beta = \tan^{-1}\left(\frac{-(y - y_d)}{-(x - x_d)}\right)$$

# Trajectory tracking versus path following

## Path following

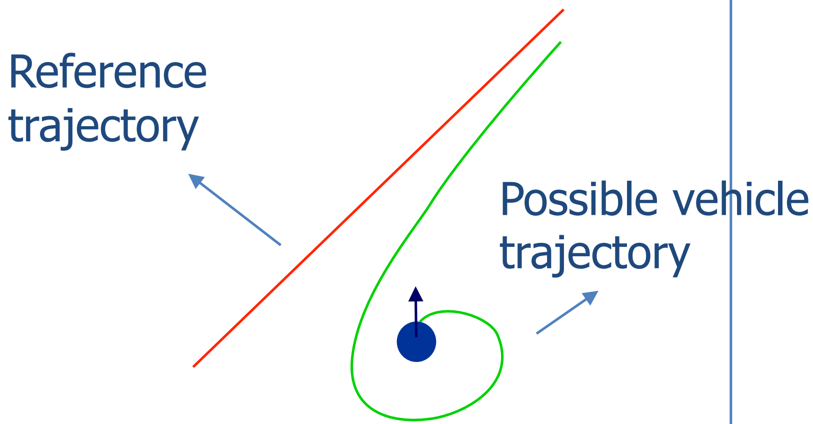
- Reference path given in a time-free parameterization




Space

## Trajectory Tracking

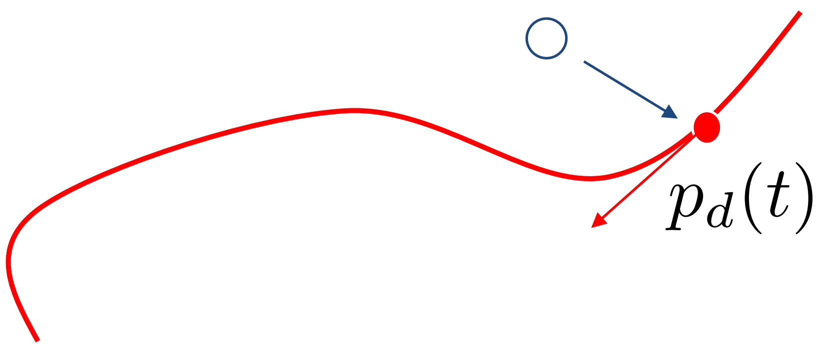
- Time and space reference trajectory

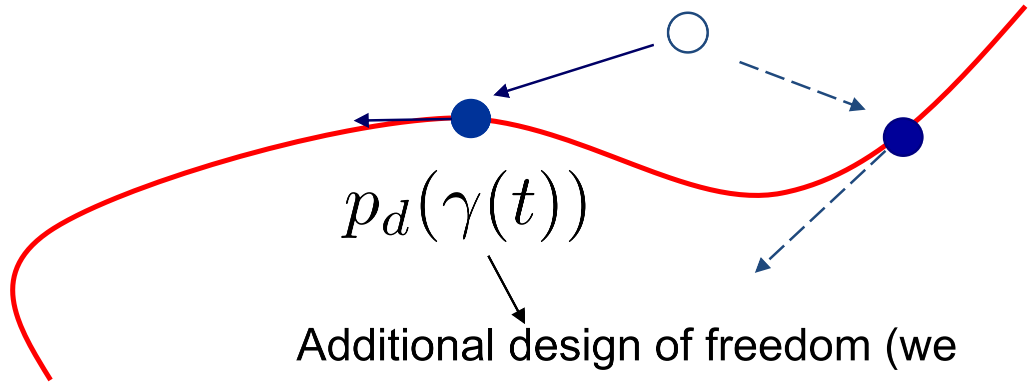



Space x Time

**Path-following is motivated by applications in which spatial errors are more critical than temporal errors**

# Trajectory tracking versus path following




Additional design of freedom (we control the virtual point!)

- Consider an underactuated vehicle modeled as a rigid body subject to external forces and torques
  - **Kinematics**

$$\dot{p} = Rv$$
$$\dot{R} = RS(\omega)$$

$$p, v, \omega \in \mathbb{R}^3,\ R \in SO(3)$$

$$S(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

  - **Dynamics**

$$\mathbf{M}\dot{v} = -S(\omega)\mathbf{M}v + f_v(v, \omega, R) + g_1 u_1$$
$$\mathbf{J}\dot{\omega} = -S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega + f_\omega(v, \omega, R) + G_2 u_2$$

$$u_1 \in \mathbb{R},\ u_2 \in \mathbb{R}^3$$

# Lyapunov based motion control of an underactuated vehicle

- **Step 1.** *Coordinate Transformation*

$$\boxed{e := R'(p - p_d)}$$

tracking error in body frame

$$\dot{e} = -S(\omega)e + v - R'\dot{p}_d$$

- **Step 2.** *Convergence of e*

error only in position!

$$V_1 := \frac{1}{2}e'e$$

$$\dot{V}_1 = \underbrace{-e'\mathrm{M}^{-1}e}_{<0} + e'z_1$$

linear velocity viewed as a virtual control input

$$\boxed{z_1 := v - R'\dot{p}_d + \mathrm{M}^{-1}e}$$

# Lyapunov based trajectory tracking of an underactuated vehicle

- **Step 3.** *Backstepping for $z_1$*

virtual control input

control input

$$\mathbf{M}\dot{z}_1 = S(\mathbf{M}z_1)\omega + \Gamma(R, v, \dot{p}_d)\omega + g_1 u_1 + h(\cdot)$$

It will not always be possible to drive $z_1$ to zero!
Instead, we will drive $z_1$ to a small constant $\delta$

$$V_2 := V_1 + \frac{1}{2}\varphi'\mathbf{M}^2\varphi = \frac{1}{2}e'e + \frac{1}{2}\varphi'\mathbf{M}^2\varphi \qquad \varphi := z_1 - \delta$$

$$\dot{V}_2 = \underbrace{-e'\mathbf{M}^{-1}e}_{<0} + e'\delta \underbrace{- \varphi' K_\varphi \varphi}_{<0} + \varphi'\mathbf{M}\mathcal{B}_b(\cdot)z_2$$

dominated by the first term

$$u_1 = \begin{bmatrix} 1 & 0_{1\times 3} \end{bmatrix} \alpha$$

1st control signal has been assigned

$$z_2 := \omega - \begin{bmatrix} 0_{3\times 1} & I_{3\times 3} \end{bmatrix} \alpha$$

angular velocity viewed as a virtual control input

# Lyapunov based trajectory tracking of an underactuated vehicle

- **Step 4.** *Backstepping for $z_2$*

$$V_3 := V_2 + \frac{1}{2} z_2' \mathbf{J} z_2 = \frac{1}{2} e'e + \frac{1}{2} \varphi' \mathrm{M}^2 \varphi + \frac{1}{2} z_2' \mathbf{J} z_2$$

$$\dot{V}_3 = -e'\mathrm{M}^{-1}e + e'\delta - \varphi' K_\varphi \varphi - z_2' K_{z_2} z_2$$

$<0$    $<0$    $<0$

dominated by the first term

$$u_2 = G_2^{-1}\left(\cdots - K_{z_2} z_2\right)$$

2nd control signal has been assigned

Using Young's inequality for any $\gamma > 0$

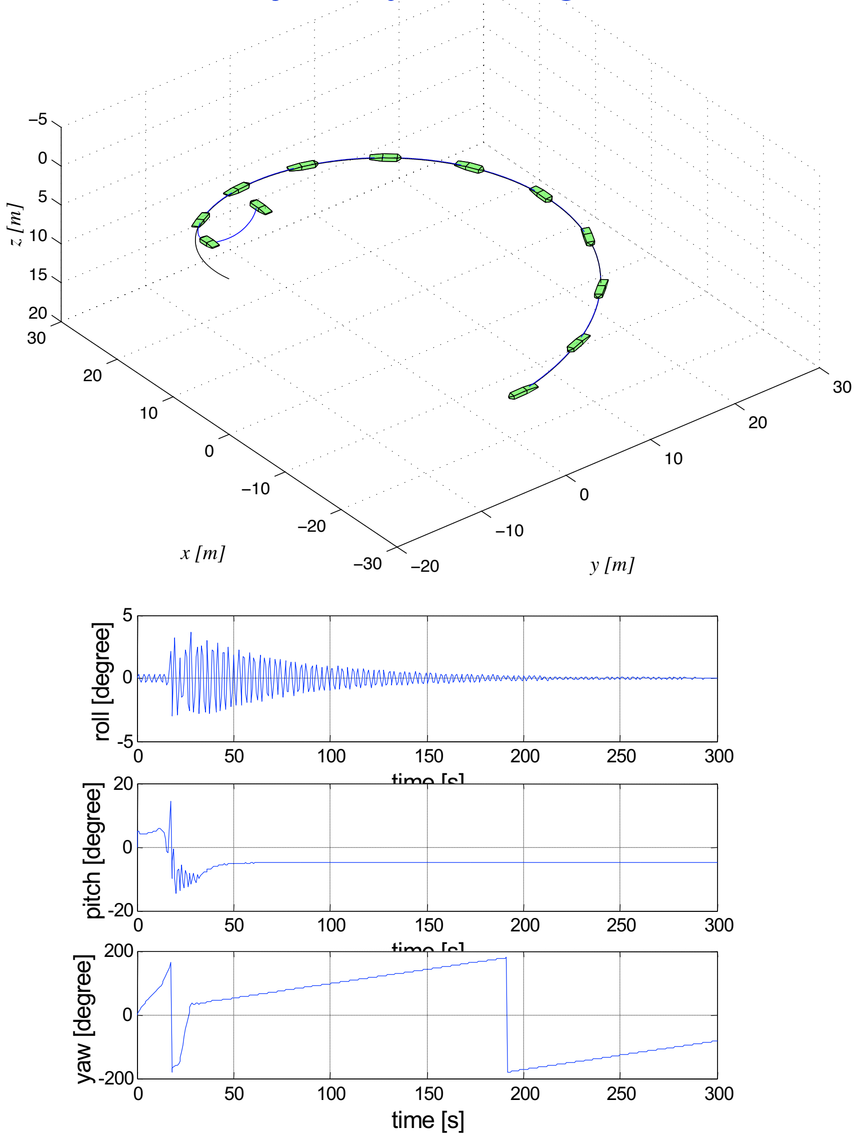
$$\dot{V}_3 \leq -\lambda V_3 + \frac{1}{2\gamma^2} \|\delta\|^2$$

can be made arbitrarily small

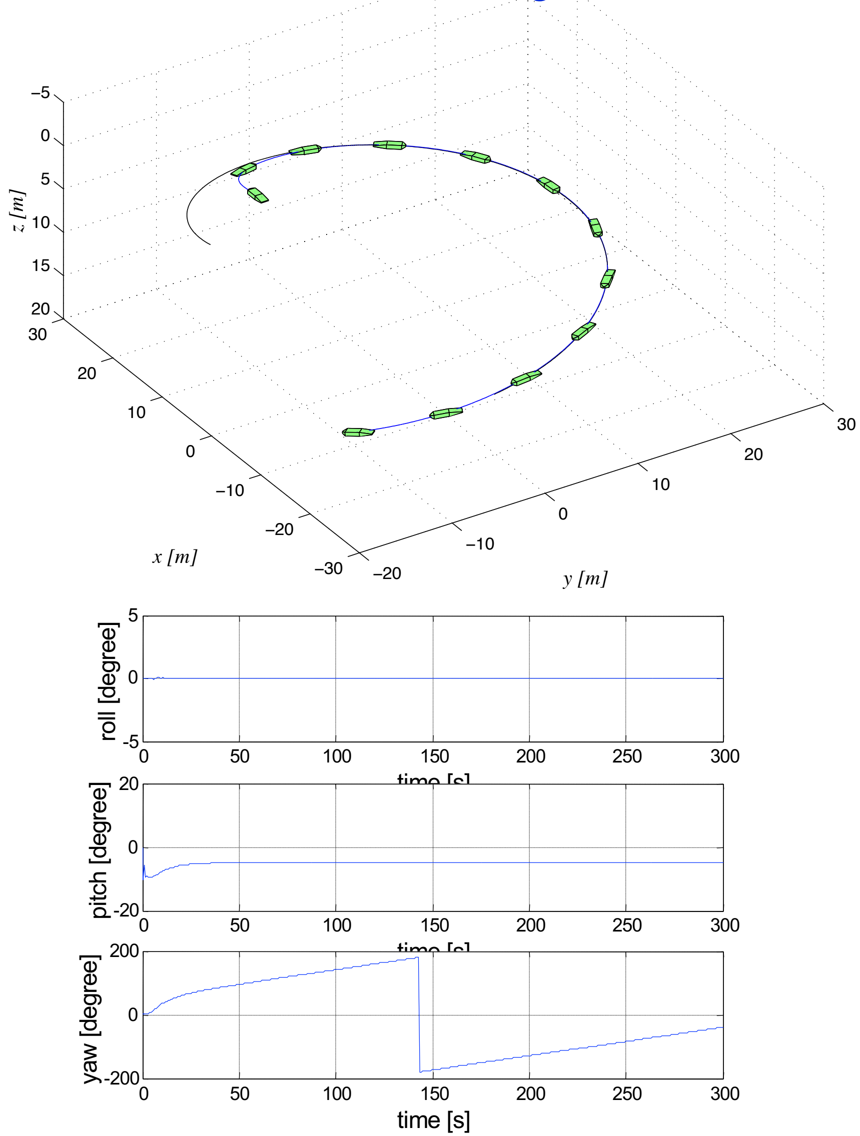
*All signals remain bounded and converges to ball of radius proportional to δ*

# Autonomous Underwater Vehicle (3D) (simulation results)

Trajectory tracking

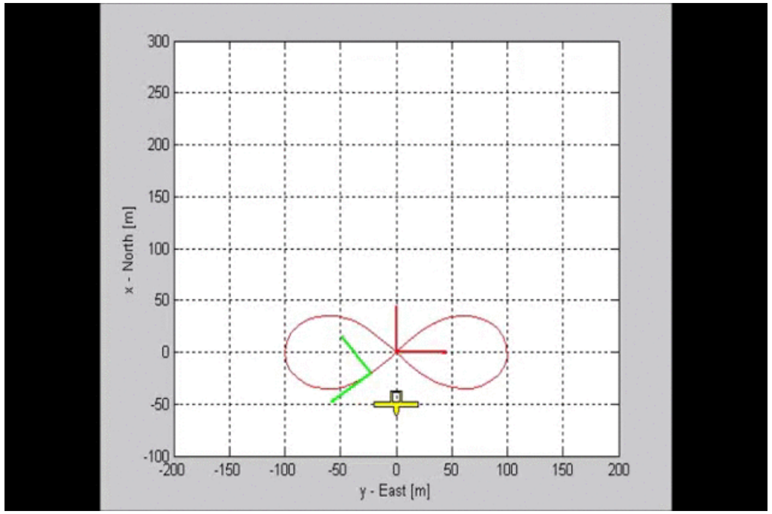



Path following

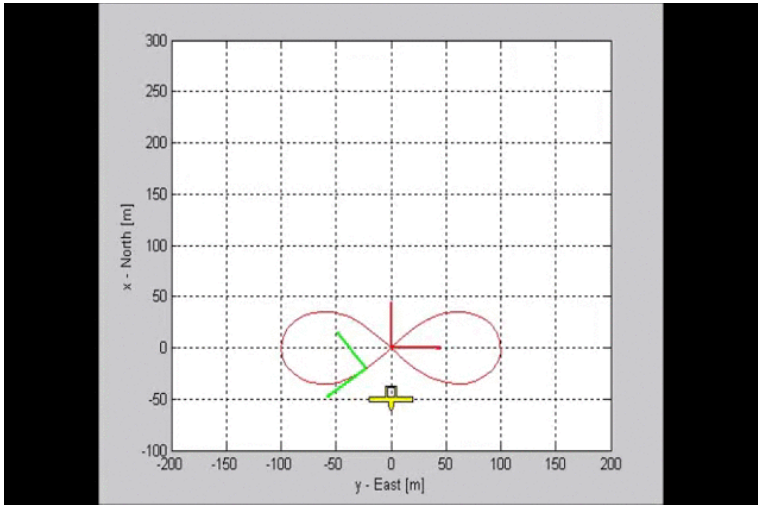


# Moving path following

- **Classic Path Following**

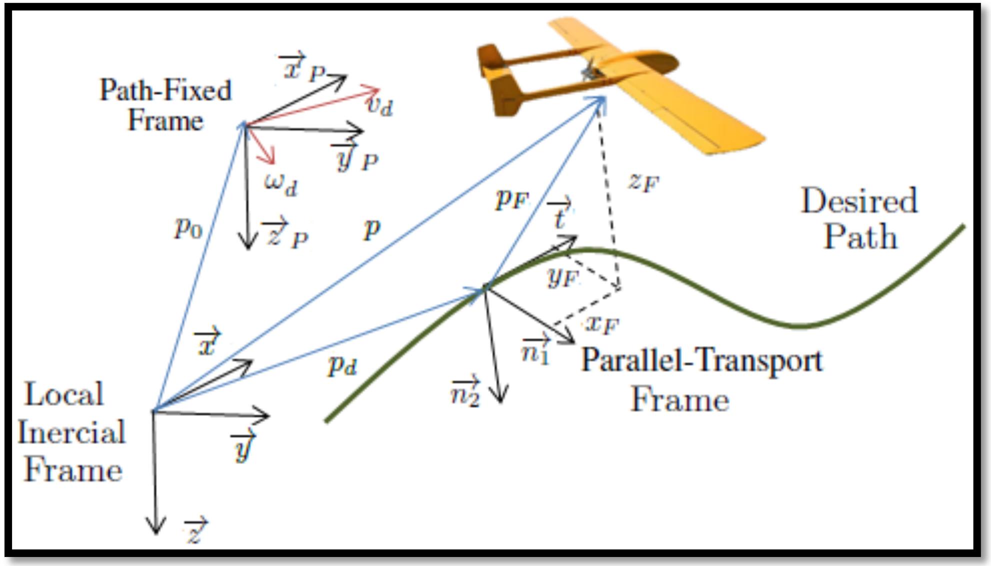



- **Moving Path Following**




Natural extension of the classical path following methods for stationary paths

# Moving path following

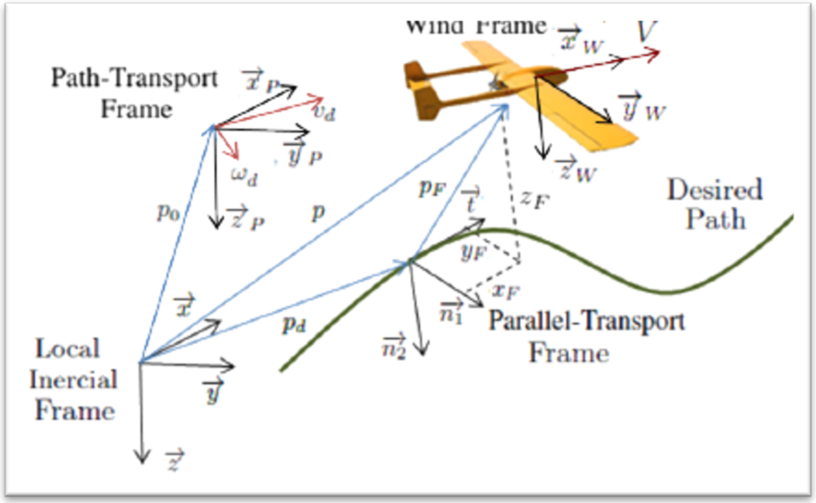


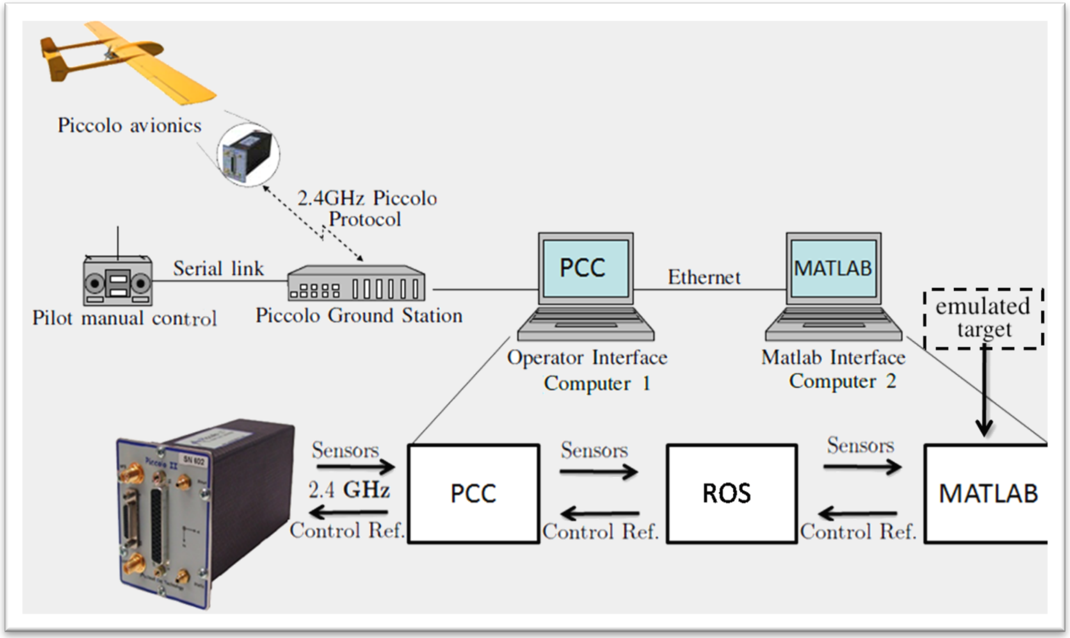


| Main features | |
|---|---|
| Maximum takeoff weight | 10 Kg |
| Wingspan | 2.415 m |
| Payload | 4 Kg |
| Maximum Speed | 150 Km/h |
| Autonomy | 1.5 h |

# Moving path following

**Three Dimensional Moving Path Following for Fixed-Wing Unmanned Aerial Vehicles**



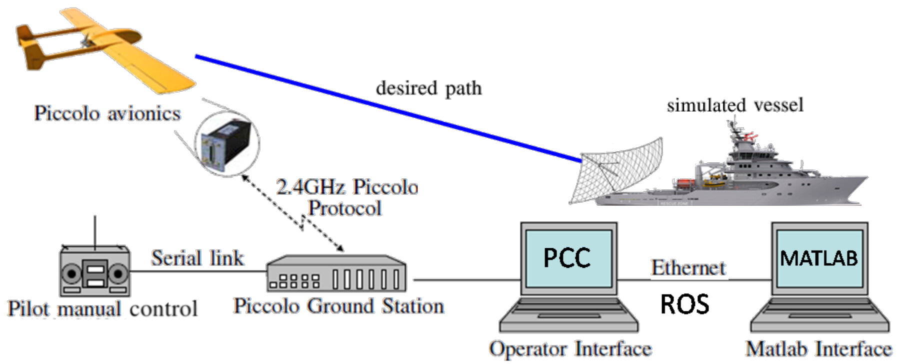

## Methodology:

Position error Orientation error ⇨ Error Kinematics ⇨ Lyapunov-based control design with stability guarantees ⇨ Numerical / hardware in the loop simulations ⇨ Flight tests

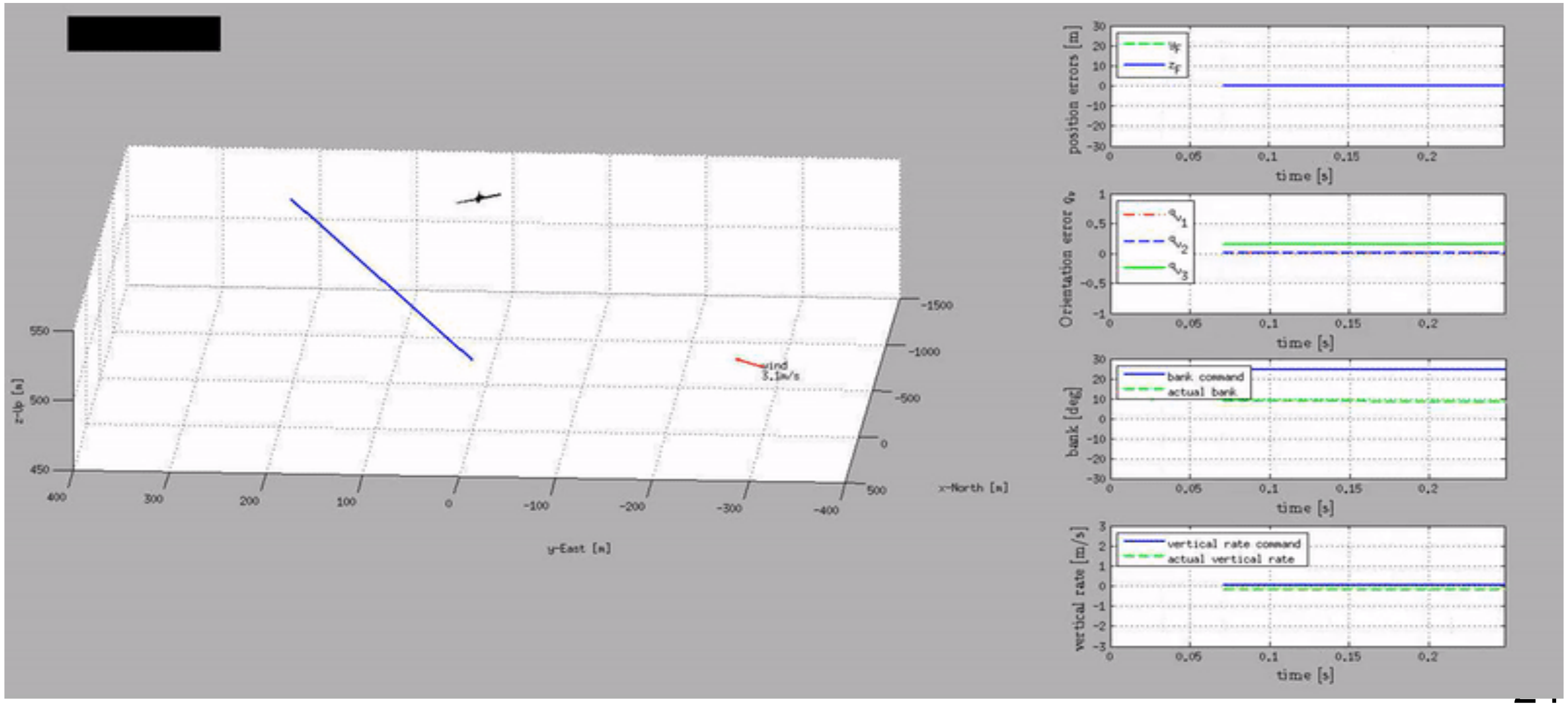
# Moving path following:

## Autonomous landing on a moving vessel

- Attach a desired landing pattern to the moving vessel and make the UAV converge to and track the moving landing pattern:

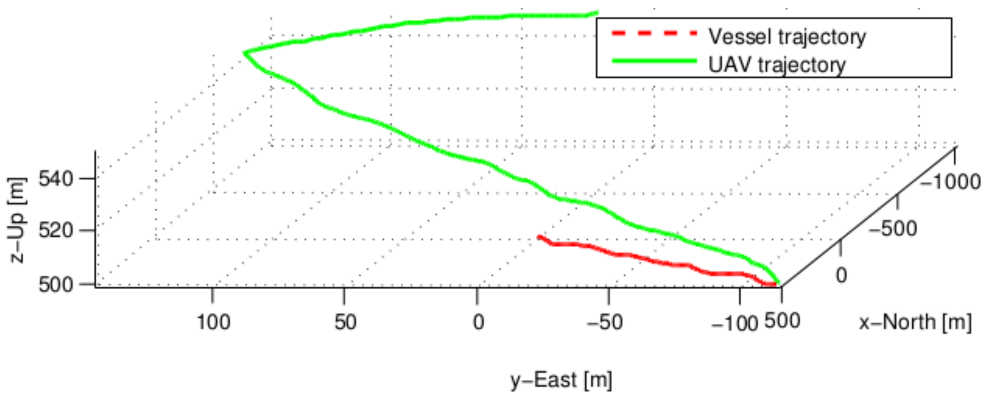


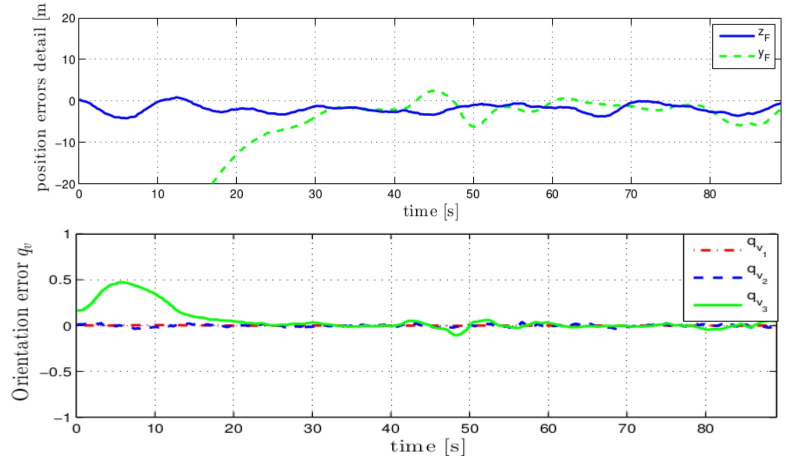

- Flight test results



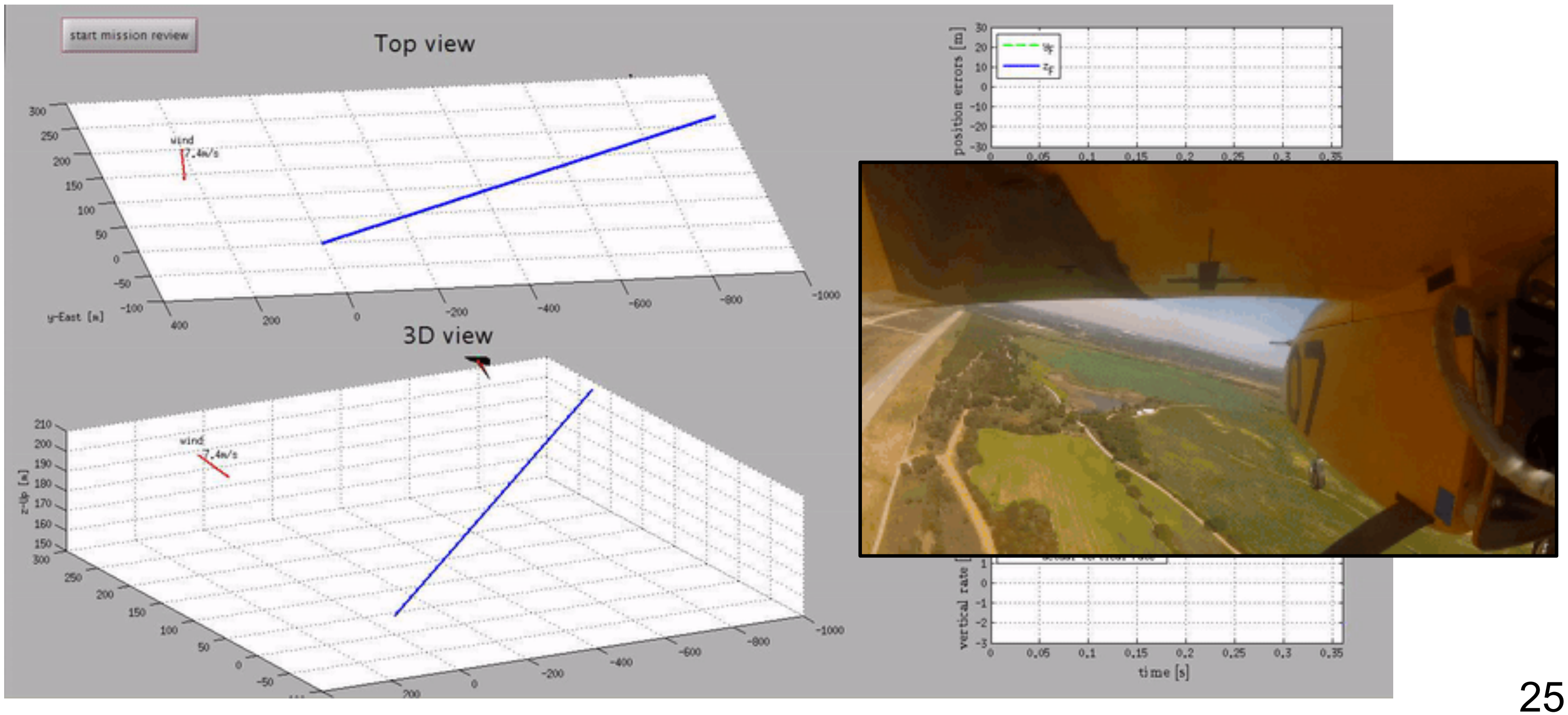
# Moving path following:

## Autonomous landing on a moving vessel



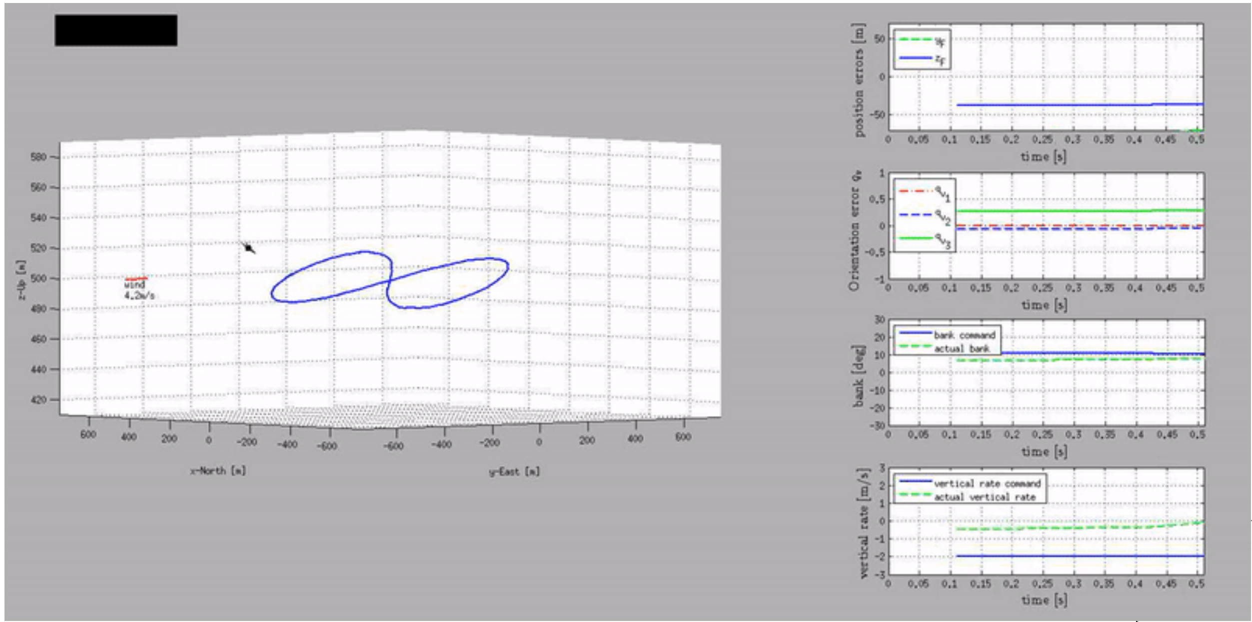




- Additional flight test results:



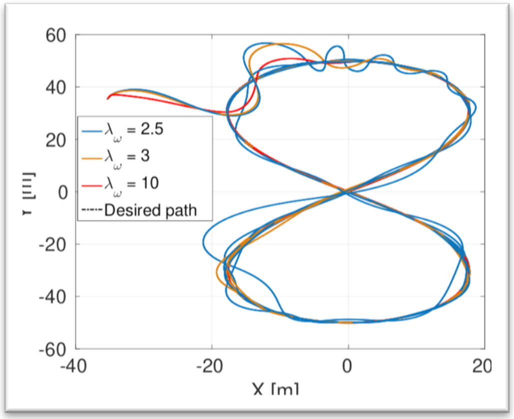
# Moving path following

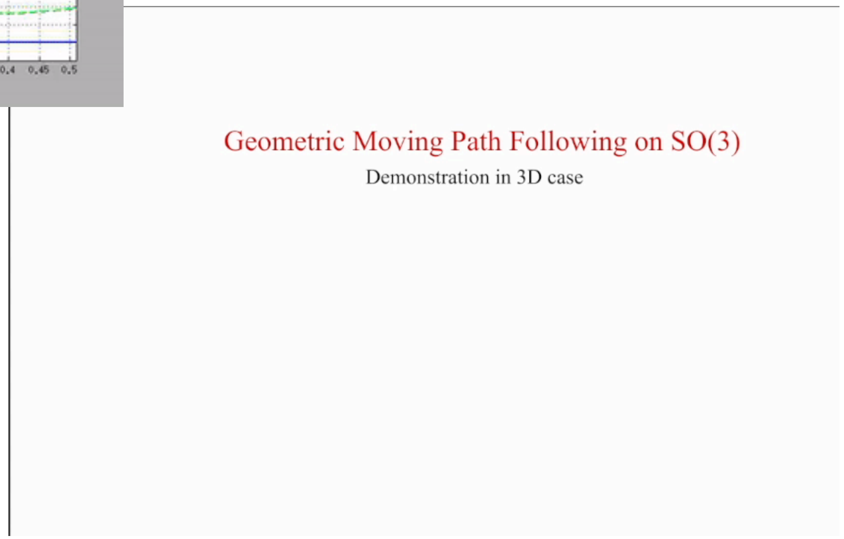
- A more general illustration of the MPF method for 3D reference paths




- Applications:
  - tracking a moving target
  - cloud monitoring,
  - thermal soaring, …

Relative UAV path with respect to the desired path under different autopilot bandwidths



# Optimization-based approach

**Definition (CLFs)**

A positive definite function $V(x)$ is a **Control Lyapunov function (CLF)** for system $\dot{x} = f(x) + g(x)u$

**Set of Stabilizing Controllers**

$$K_V(x) = \{u \in \mathcal{U} : L_f V(x) + L_g V(x) u \leq -\sigma(x)\}$$

Desired rate

**Pointwise Min-Norm**

$$\min_{u \in \mathcal{U}} \frac{1}{2}\|u\|^2$$

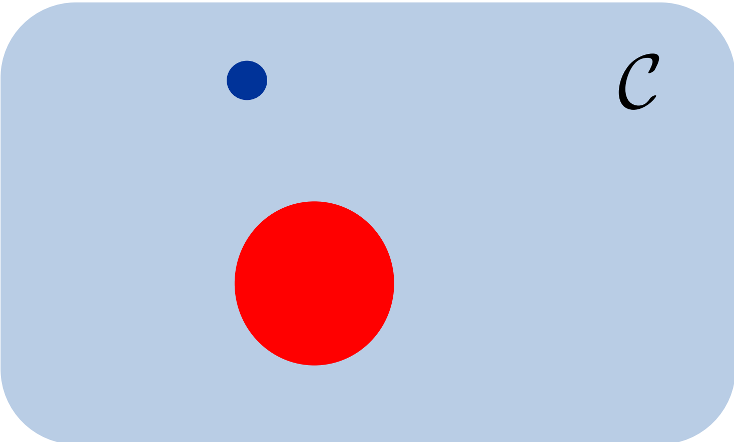
$$s.t. \quad L_f V(x) + L_g V(x) u \leq -\sigma(x) \qquad \textbf{(CLF)}$$

where $\sigma(x) \succ 0$.

*At every point, we select the smallest input which ensures that the CLF decays at the specified desired rate!*

# Set Invariance and Safety

*How can we address safety?*




$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$
$$\partial\mathcal{C} = \{x \in \mathbb{R}^n : h(x) = 0\}$$
$$\mathcal{O} = \{x \in \mathbb{R}^n : h(x) < 0\}$$

**Definition (Safety)**

The closed-loop system $\dot{x} = f(x) + g(x)u^\star$ is safe with respect to $\mathcal{C}$ if $\mathcal{C}$ is forward invariant.

# Control Barrier Functions

**Definition (CBF)**

The function $h(x)$ is a **Control Barrier Function (CBF)** if there exists a locally Lipschitz extended class $\mathcal{K}_\infty$ function $\alpha$ such that

$$\sup_{u \in \mathbb{R}^m} \left[ L_f h(x) + L_g h(x)\, u \right] \geq -\alpha(h(x))$$

Note that $h(x)$ is only allowed to decrease in the interior of the safe set $\mathrm{int}(\mathcal{C})$, but not on its boundary $\partial\mathcal{C}$, that is, $\mathcal{C}$ is forward invariant.

**Set of Safe Controllers**

$$K_{CBF}(x) = \{u \in \mathbb{R}^m : L_f h(x) + L_g h(x)\, u + \alpha(h(x)) \geq 0\}$$

**CBFs** can be used to design controllers enforcing **safety**.

# Stabilization and Safety using QPs

**Quadratic Program (QP) Formulation**

$$\min_{(u,\delta)\in\mathbb{R}^{m+1}} \frac{1}{2}\|u\|^2 + \frac{1}{2}\kappa\delta^2$$

$$s.t. \quad L_f V(x) + L_g V(x) u + \gamma(V(x)) \leq \delta \qquad \textbf{(CLF)}$$

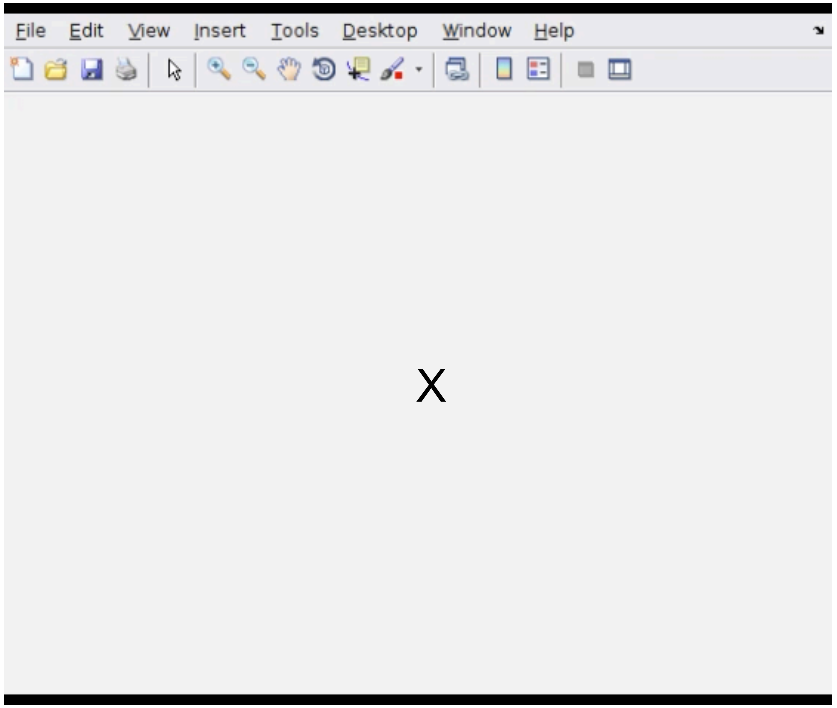
$$L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0 \qquad \textbf{(CBF)}$$

where $\kappa > 0$, $\gamma \in \mathcal{K}$, $\alpha \in \mathcal{K}_\infty$.

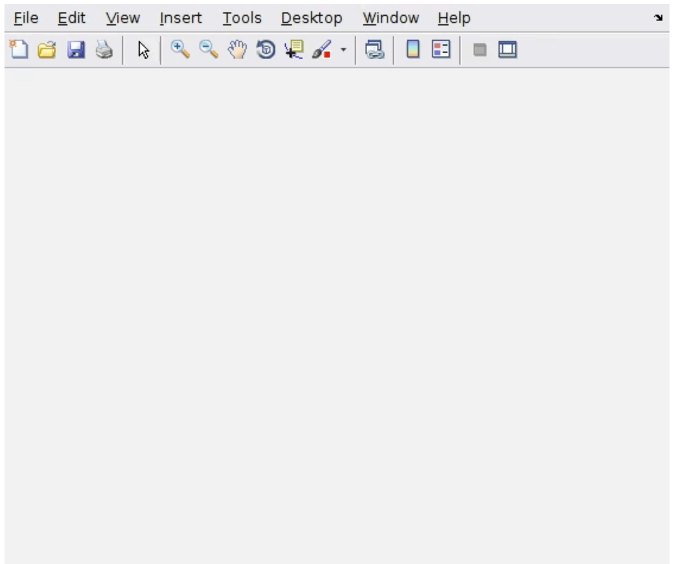
The CBF constraint guarantees that $u^\star \in K_{CBF}(x)$ keeps the system trajectories invariant with respect to the safe set $\mathcal{C}$.

The relaxation variable $\delta$ in the CLF constraint *softens* the stabilization objective, maintaining the feasibility of the QP.

# Stabilization and Safety using QPs



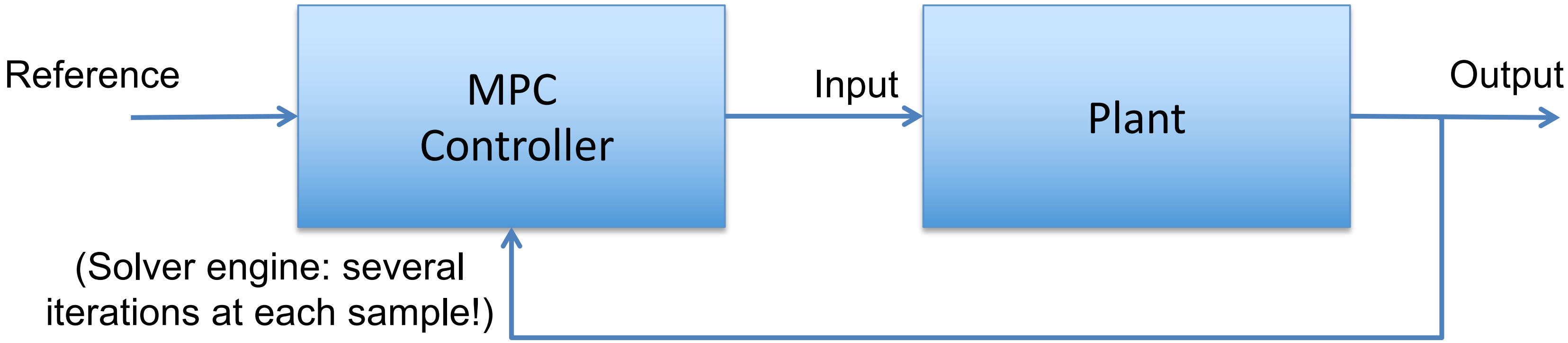

The QP-based approach can introduce **asymptotically** **stable** equilibrium points on the boundary $\partial\mathcal{C}$.




***Solution***: *Include the freedom of rotating and scaling the initial proposed CLF V by augmenting the state of the control system.*

# Optimization-based approach

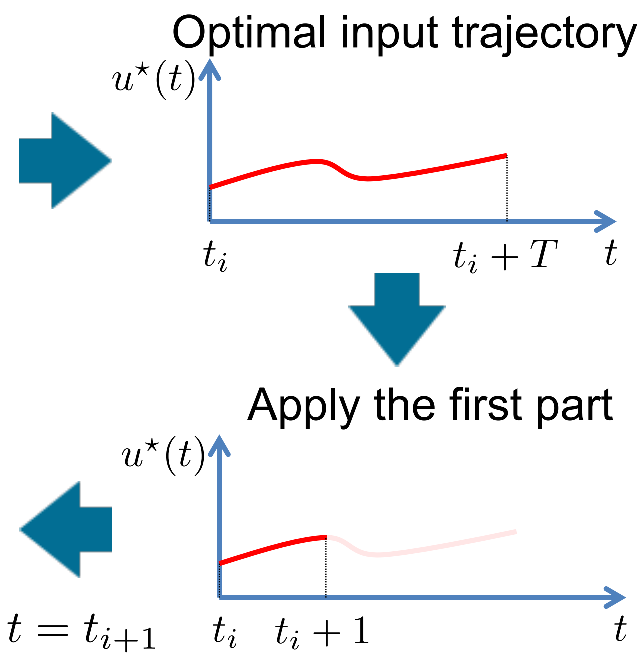
## Receding horizon approach




**Open loop finite horizon problem**

$$\min_{u(\cdot)} \int_{t_i}^{t_i+T} \ell(x(\tau), u(\tau))\, d\tau + \varphi(x(t_i+T))$$

$$\begin{aligned} s.t. \quad & \dot{x} = f(x,u) \\ & (x,u) \in \mathcal{X} \times \mathcal{U} \\ & x(t_i) = x_{t_i} \\ & x(t_i+T) \in \mathcal{X}_{aux} \end{aligned}$$



# Model Predictive Control

**Performance Index**

$$J_T(x,u) := \int_{t_i}^{t_i+T} \ell(x(\tau),u(\tau))\,d\tau + \varphi(x(t_i+T))$$

*Stage Cost* *Terminal Cost*

**Auxiliary elements are crucial for stability!**

**Open-Loop finite horizon problem**

$$J_T^\star(x,u) = \min_{u(\cdot)} J_T(x,u)$$

$$s.t. \quad \dot{x} = f(x,u)$$ *Dynamical Model*

$$(x(\tau),u(\tau)) \in \mathcal{X} \times \mathcal{U}$$ *State and Input Constraints*

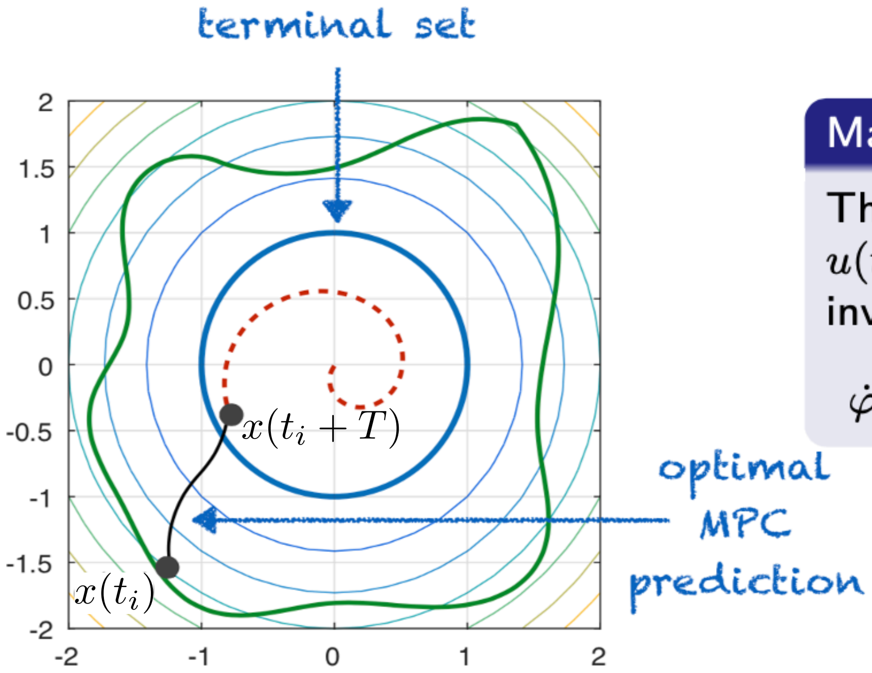
$$x(t_i) = x_{t_i}$$

$$x(t_i+T) \in \mathcal{X}_{aux}$$ *Terminal Constraint*

**Control Policy**

$$u(t) = u^\star(t), \quad t \in [t_i, t_{i+1})$$

# Model Predictive Control




**Main (sufficient) condition for closed-loop stability**

There exists a feasible **auxiliary control law** $u(t) = K_{aux}(\cdot)$ that renders $\mathcal{X}_{aux}$ positively invariant, and such that

$$\dot{\varphi}(x) \leq -\ell(x, K_{aux}(x)), \quad t \geq t_i + T,\ x \in \mathcal{X}_{aux}$$

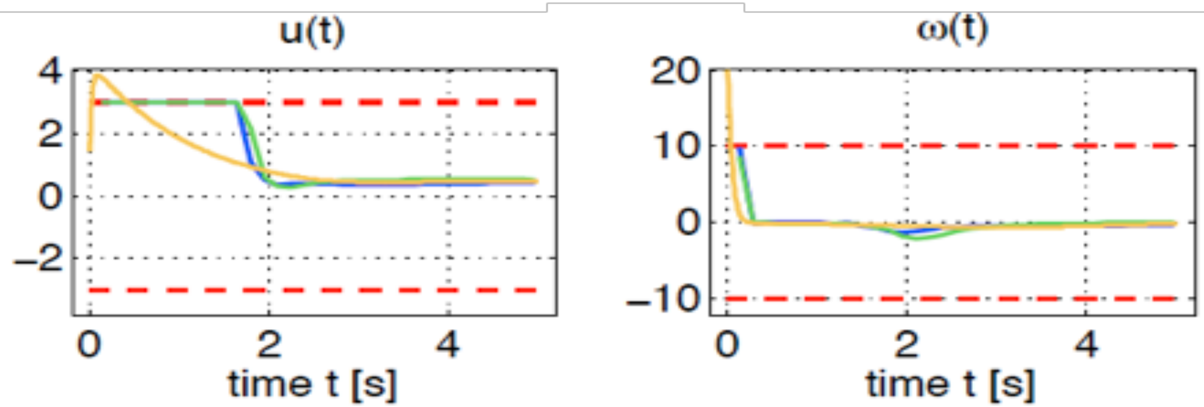
- **Key idea:**

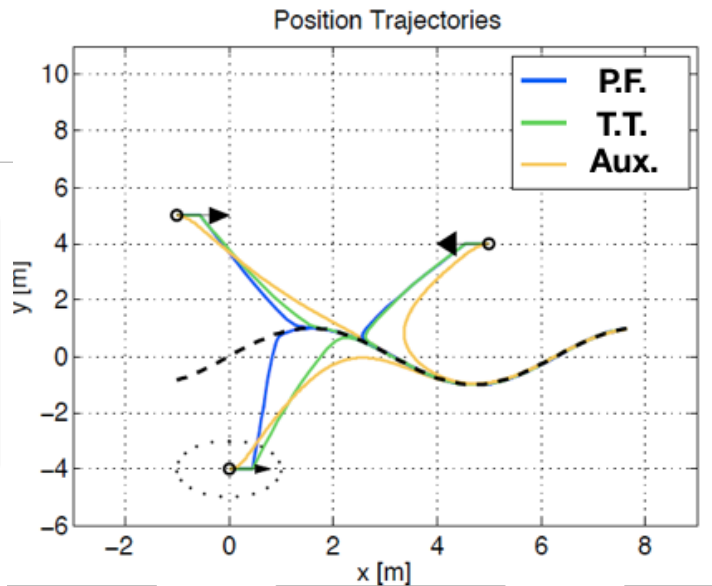
  Design an **auxiliary Lyapunov based control law** and use it to compute the

  - **Terminal set:** usually a level set of the Lyapunov function
  - **Terminal cost:** to approximately recover the infinite horizon control solution $\varphi(x(t_i+T)) = \int_{t_i+T}^{\infty} \ell(x(\tau), K_{aux}(x))\, d\tau$

**Bonus:** If the auxiliary control law yields global asymptotical stability (under the constraints) then the terminal set is the all space, and the Region of Attraction (R.O.A) of the MPC is all the state space! (MPC improves the designed auxiliary controller)

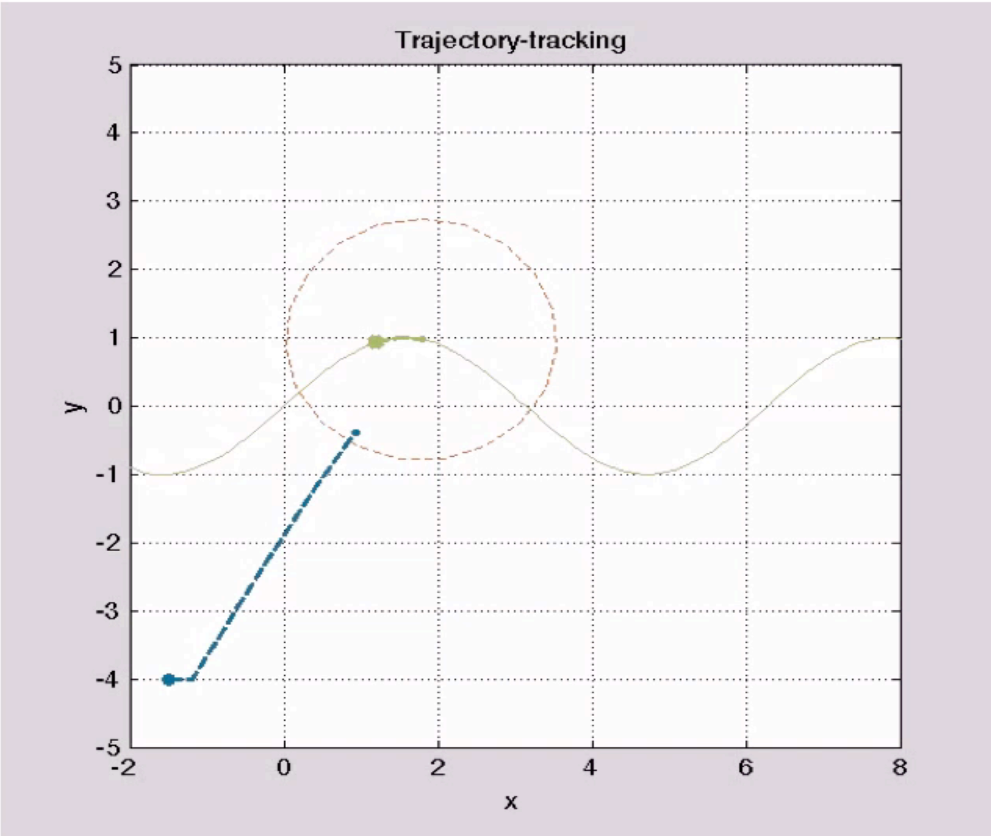
# Simulation results

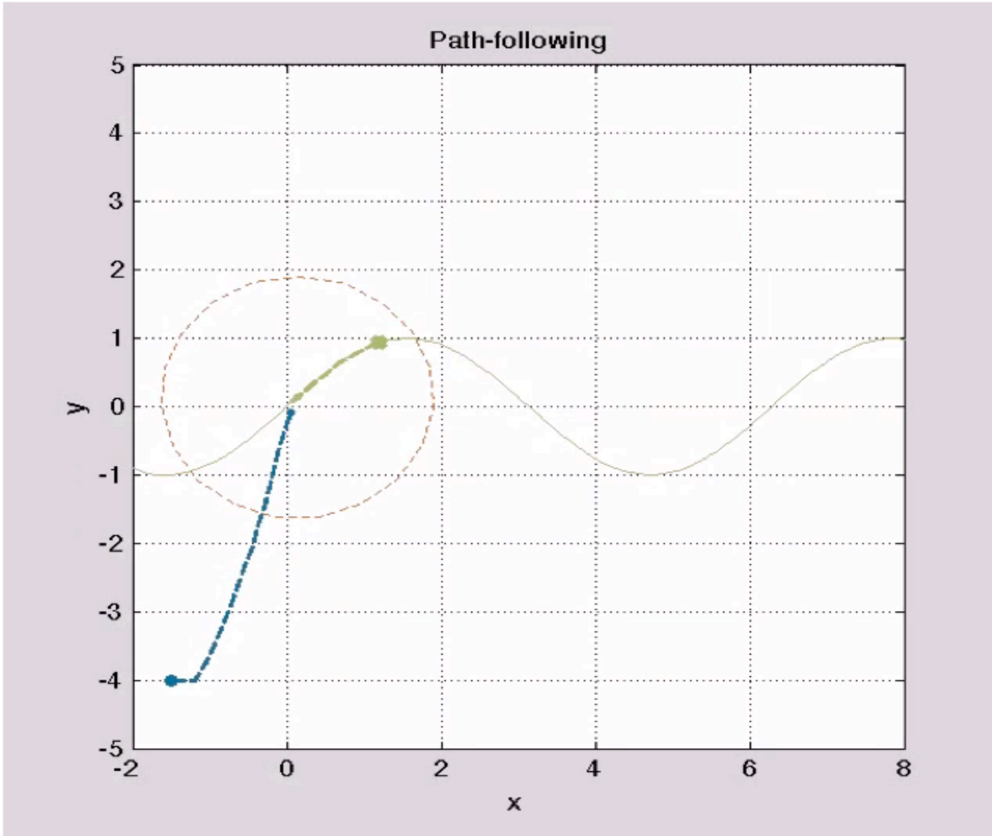



Trajectory tracking

Path following



# Dual-Objective MPC for Economic Optimization

- ***Application scenarios:***
  - When there is margin to reduce tracking accuracy to perform economic optimization

**Performance Index**

$$J_T(x,u) := \int_{t_i}^{t_i+T} \ell(x(\tau),u(\tau))\,d\tau + \varphi(x(t_i+T))$$

***Stage Cost:***

$$\ell(x,u) = \ell_s(x,u) + \ell_e(x,u)$$

$\ell_s(x,u)$ → ***Stabilizing Cost***

$\ell_e(x,u)$ → ***Economic Cost***

***Under some suitable design of the auxiliary elements…*** $|\ell_e(x,u)| \leq b(t)$

… it is possible to show that

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t-t_0) + \gamma\Big(\sup_{t\geq t_0} \|\ell(x(t),u(t))\|\Big)$$

***Transient Economic optimization:*** $\|\ell(x(t),u(t))\| \to 0 \implies \|x(t)\| \to 0$

# Dual-Objective MPC for Economic Optimization

**Motivation** Reduce energy consumption.

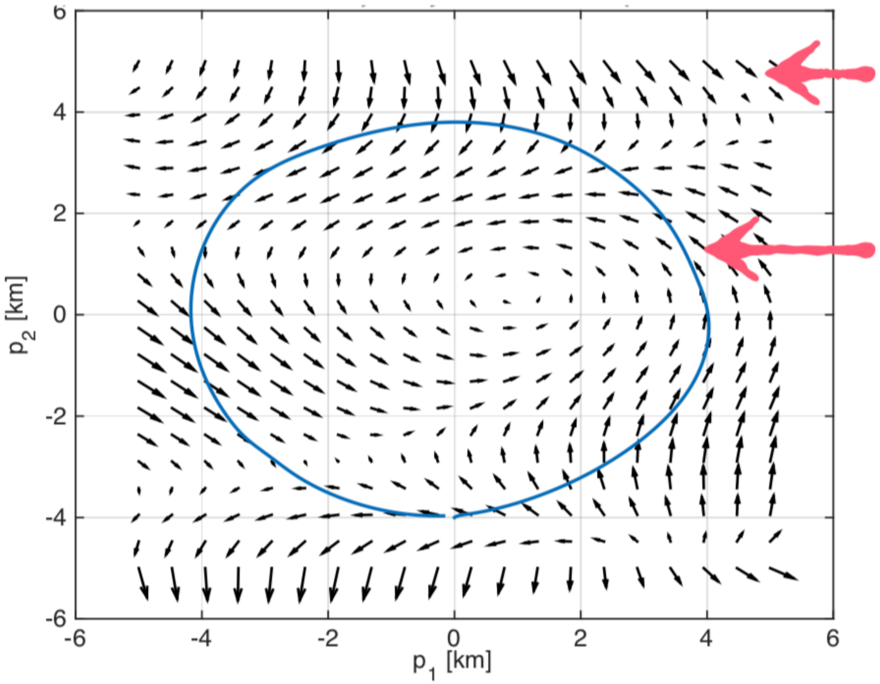
**Control objectives**

1) Track a predefined position trajectory
2) Save energy

***ISS-based EMPC***

$l(t, x, u) := l_s(t, x, u) + l_e(t, x, u)$

Trajectory-Tracking Stage Cost — Consumption Index




*Water currents velocity vectors*

*Desired position trajectory*

$$l_e(t, x, u) = k_c \operatorname{atan}\left(\frac{1.5}{k_c}\|u(t)\|^2\right)$$

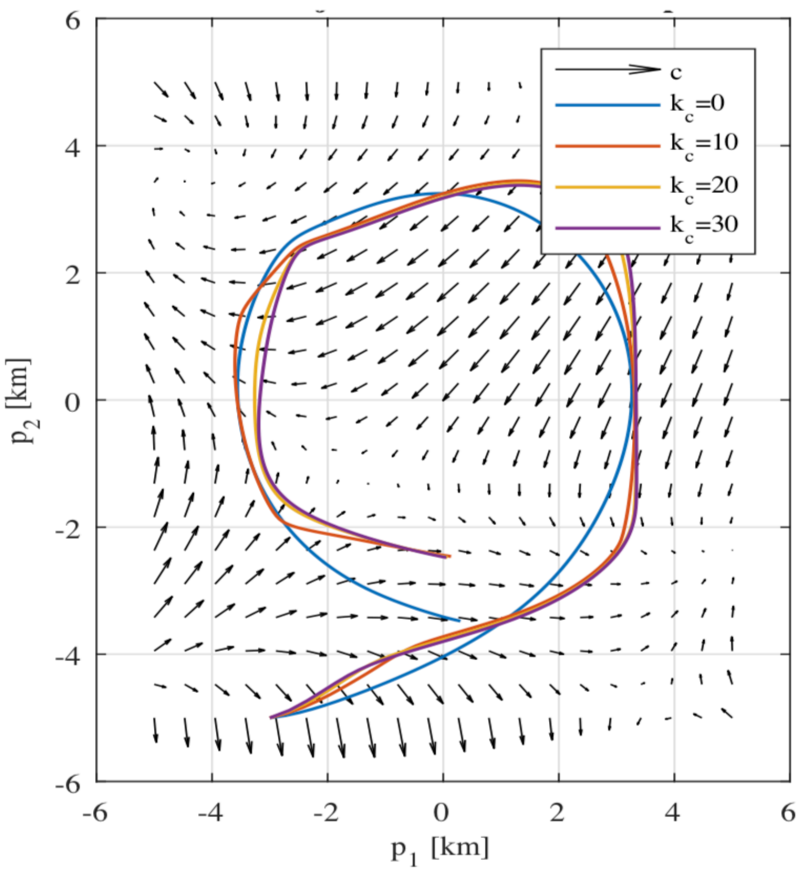
# Dual-Objective MPC for Economic Optimization

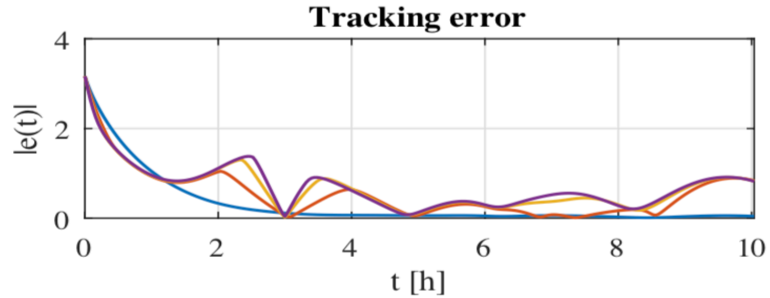
**Motivation** Reduce energy consumption.

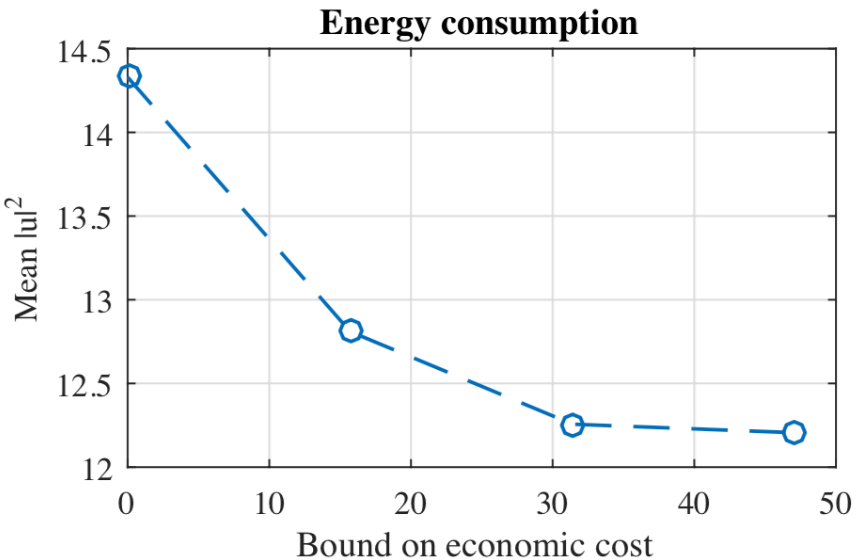
**Control objectives**

$$l_e(t,x,u) = k_c \operatorname{atan}\left(\frac{1.5}{k_c}\|u(t)\|^2\right)$$

1) Track a predefined position trajectory
2) Save energy



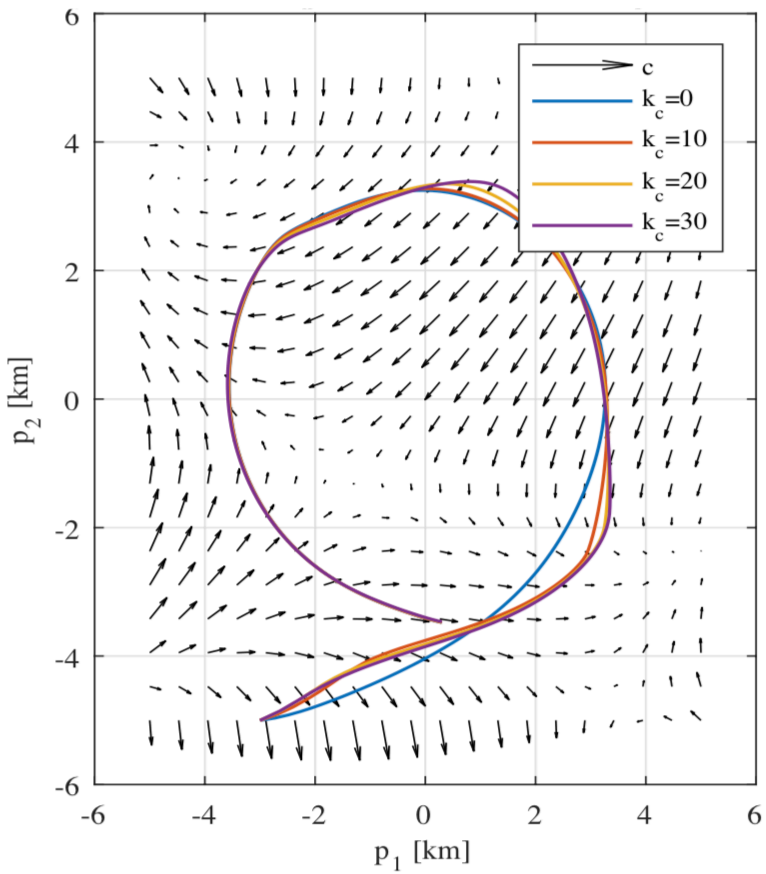
# Dual-Objective MPC for Economic Optimization

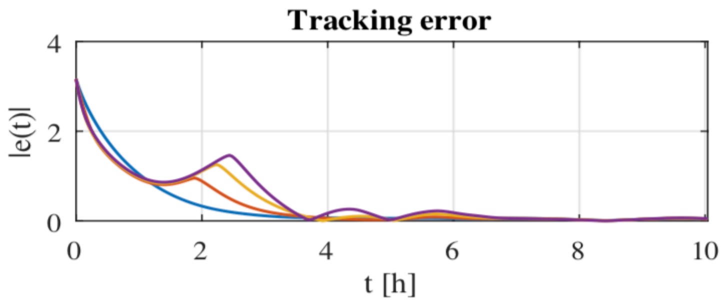
**Motivation** Reduce energy consumption.

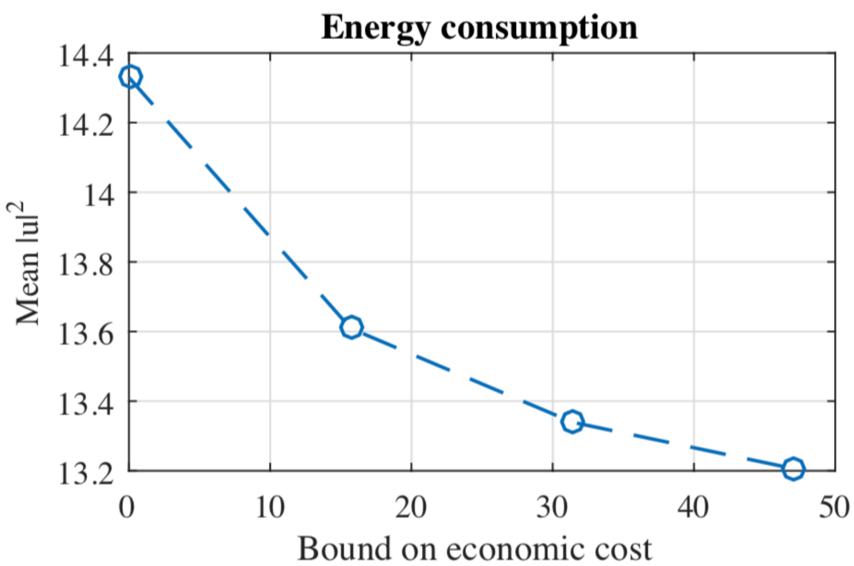
**Control objectives**

$$l_e(t, x, u) = k_c \operatorname{atan}\left(\frac{1.5}{k_c}\|u(t)\|^2\right) e^{-\frac{1}{2}t}$$

1) Track a predefined position trajectory
2) Save energy

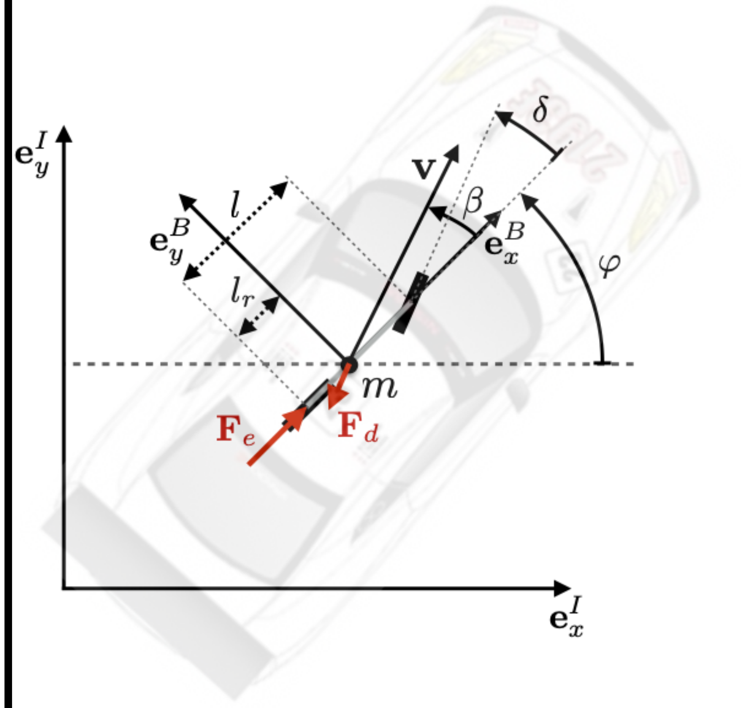


# Dual-Objective MPC for Economic Optimization

**Example - Car Control**




**Kinematic**

$$\dot{p} = R(\phi + \beta)\begin{pmatrix} v_f \\ 0 \end{pmatrix}$$

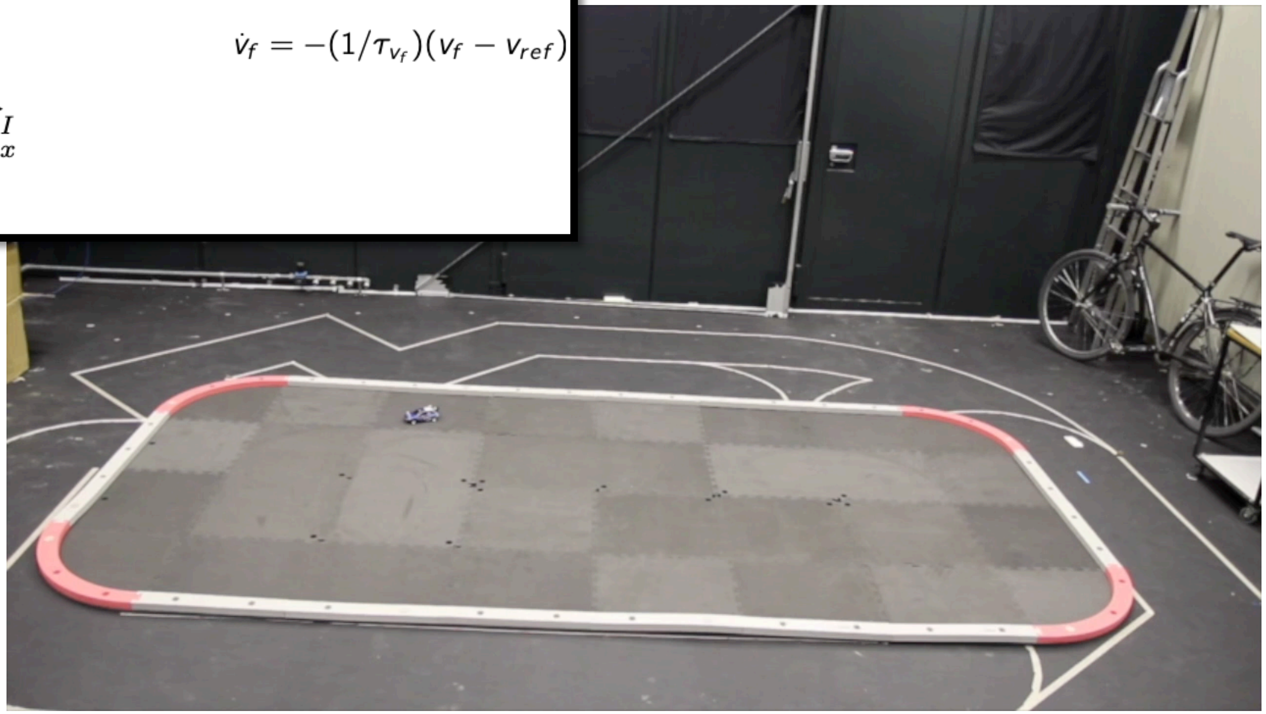
$$\dot{\phi} = \frac{v_f}{l_r}\sin\beta$$

$$\beta := \tan^{-1}\left(\frac{l_r}{l}\tan(\delta)\right)$$

**Inner Loops (Servo & E**

$$\dot{\delta} = -(1/\tau_\delta)(\delta - \delta_{ref})$$

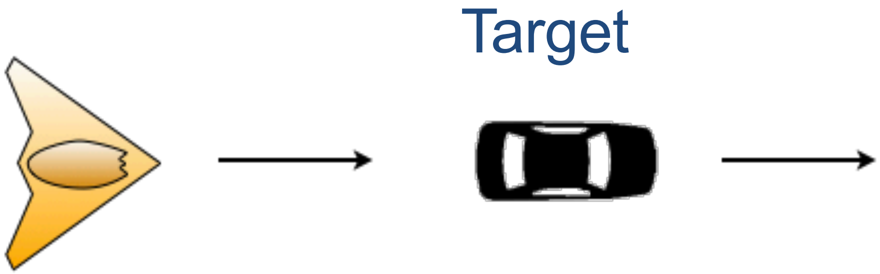
$$\dot{v}_f = -(1/\tau_{v_f})(v_f - v_{ref})$$



# Dual-Objective MPC for Economic Optimization

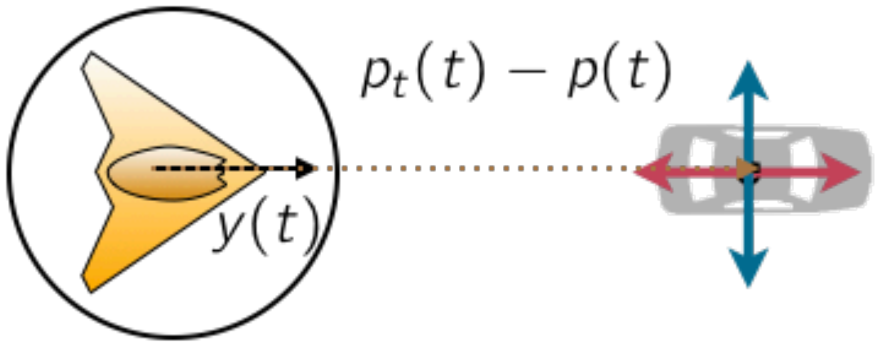
**Target monitoring:**

1) Estimate the position of the target
2) Follow the target




*Attention!!! The two objectives might conflict*

# Dual-Objective MPC for Economic Optimization




Unobservable motion

"Well" observable motion

**State equations:**

$x := (x_f', x_t')'$

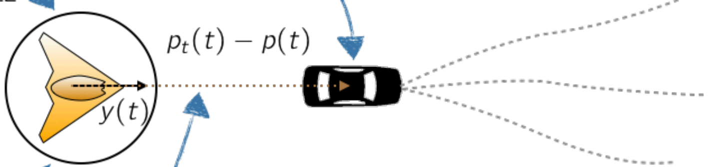
$\dot{x} = f(x, u_f, u_t, w_f, w_t) := \begin{pmatrix} f_f(x_f, u_f, w_f) \\ f_t(x_t, u_t, w_t) \end{pmatrix}$

FOLLOWER MOTION MODEL

TARGET MOTION MODEL (GUESS)

**Target input parametrization:**

$\bar{u}_t(\cdot; p_u) \quad \dot{p}_u = f_u(p_u)$




LOCAL SENSOR MODEL (E.G. IMU, GPS, ...)

**Output equations:**

$y = h(x, v) := \begin{pmatrix} h_f(x_f, v_f) \\ h_t(x_t, v_t) \end{pmatrix}$

$y := (y_f', y_t')' \quad v := (v_f', v_t')$

Observability matrix: $\mathcal{O}(x, u_f, u_t) = \frac{\partial}{\partial x} \begin{pmatrix} y \\ \dot{y} \\ \vdots \end{pmatrix}$

Main property: $\mathcal{O}(x, u_f, u_t)$ full rank => state x locally observable at (x,u)

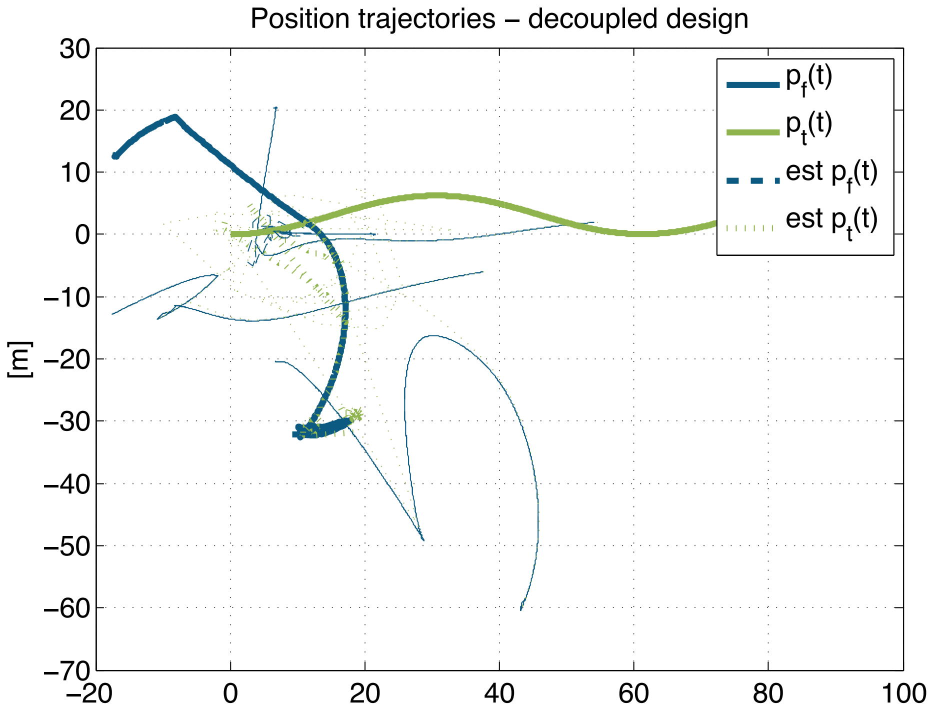
Note:

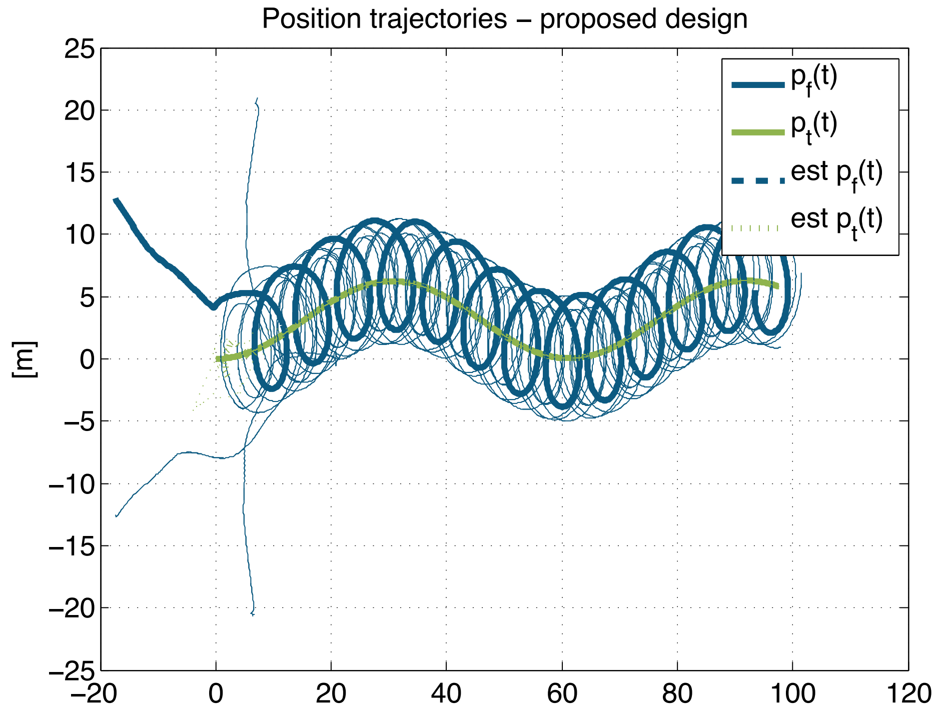
- Observability is function of the state and input
- Large minimum singular value => high "Quantity"
- Condition number close to 1 => high "Quality"

**Index of observability** (saturated):

$$I_o(x, u_f, u_t) = k \arctan\left(\frac{1}{k}\left(\frac{\alpha_1}{\sigma_{min}\mathcal{O}(x, u_f, u_t)} + \alpha_2(\kappa(\mathcal{O}(x, u_f, u_t)) - 1)^2\right)\right)$$

# Dual-Objective MPC for Economic Optimization

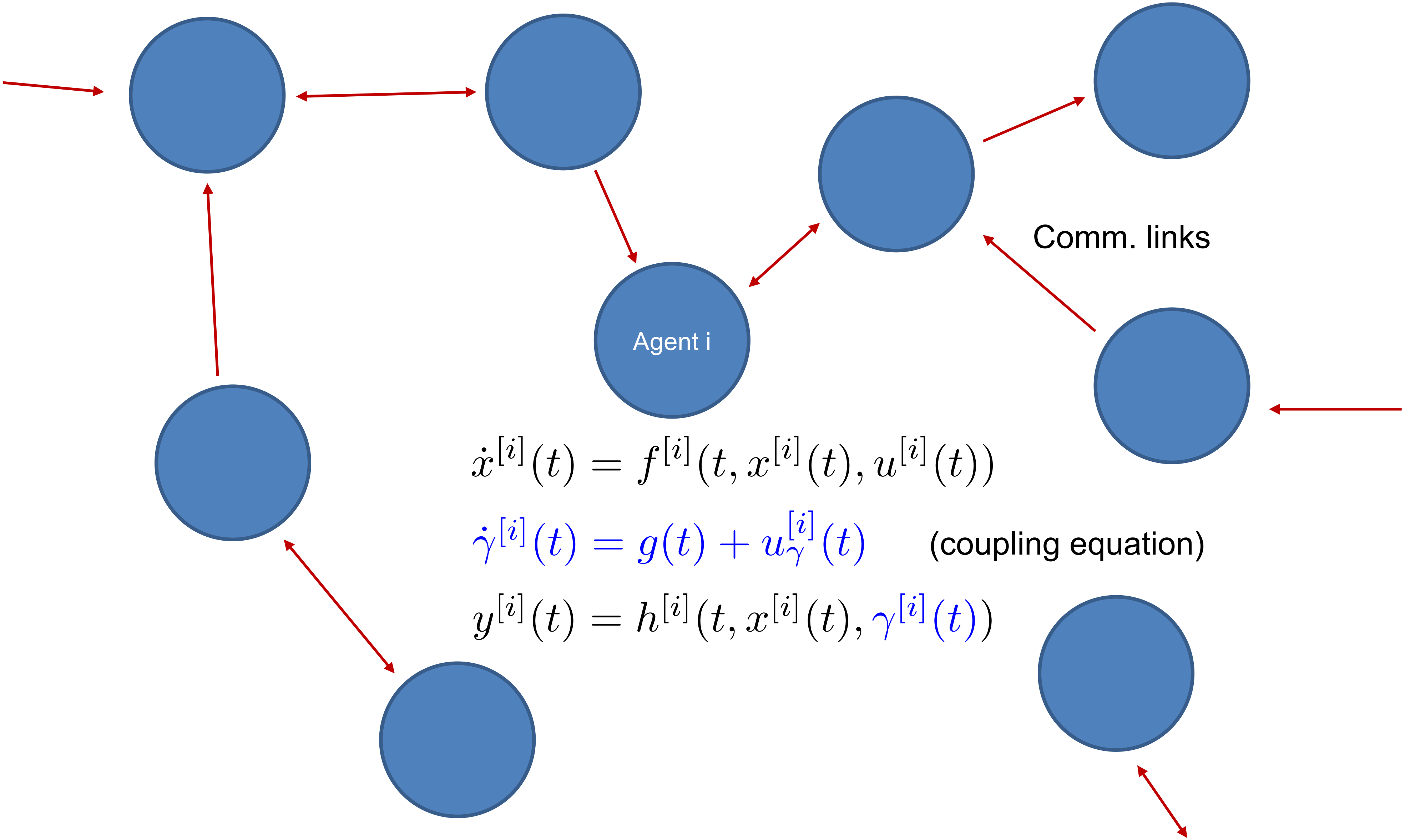



$$J_T(x,u) = \int_{t_i}^{t_i+T} \left(\ell(x,u) + \ell_o(x,u)\right) d\tau + \varphi(x(t_i+T))$$

***Stabilizing Cost*** ***Observability Cost***

# Problem Statement and Motivation




$$\dot{x}^{[i]}(t) = f^{[i]}(t, x^{[i]}(t), u^{[i]}(t))$$

$$\dot{\gamma}^{[i]}(t) = g(t) + u_{\gamma}^{[i]}(t) \qquad \text{(coupling equation)}$$

$$y^{[i]}(t) = h^{[i]}(t, x^{[i]}(t), \gamma^{[i]}(t))$$

# Problem Statement and Motivation

## Control objectives

**Communication graph** $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$

$$\mathcal{G} := (\mathcal{V}, \mathcal{E})$$

$(i,j) \in \mathcal{E} \iff$ system i reads $\gamma^{[j]}(t)$

### Output regulation

$$y^{[i]}(t) \to 0$$

### Coordination

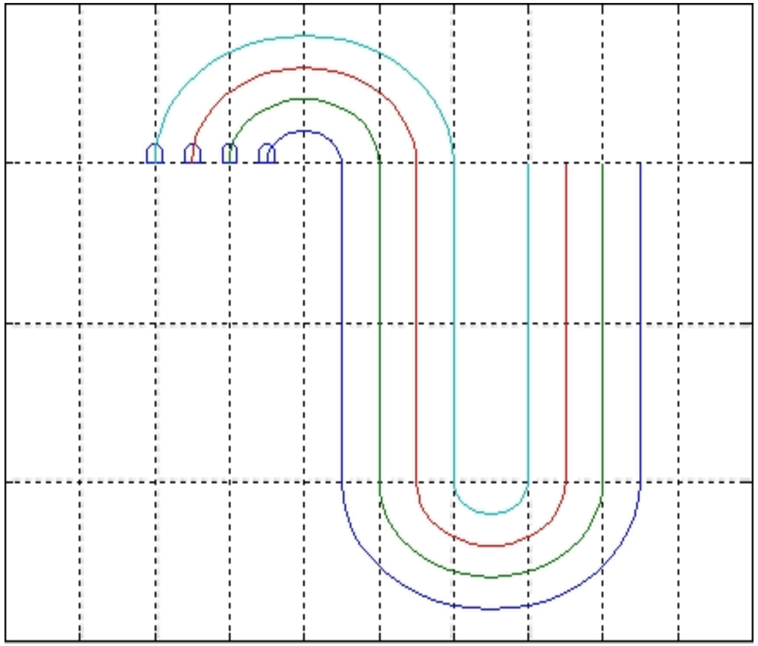
$$\sum_{(i,j)\in\mathcal{E}} (\gamma^{[i]}(t) - \gamma^{[j]}(t))^2 \to 0$$

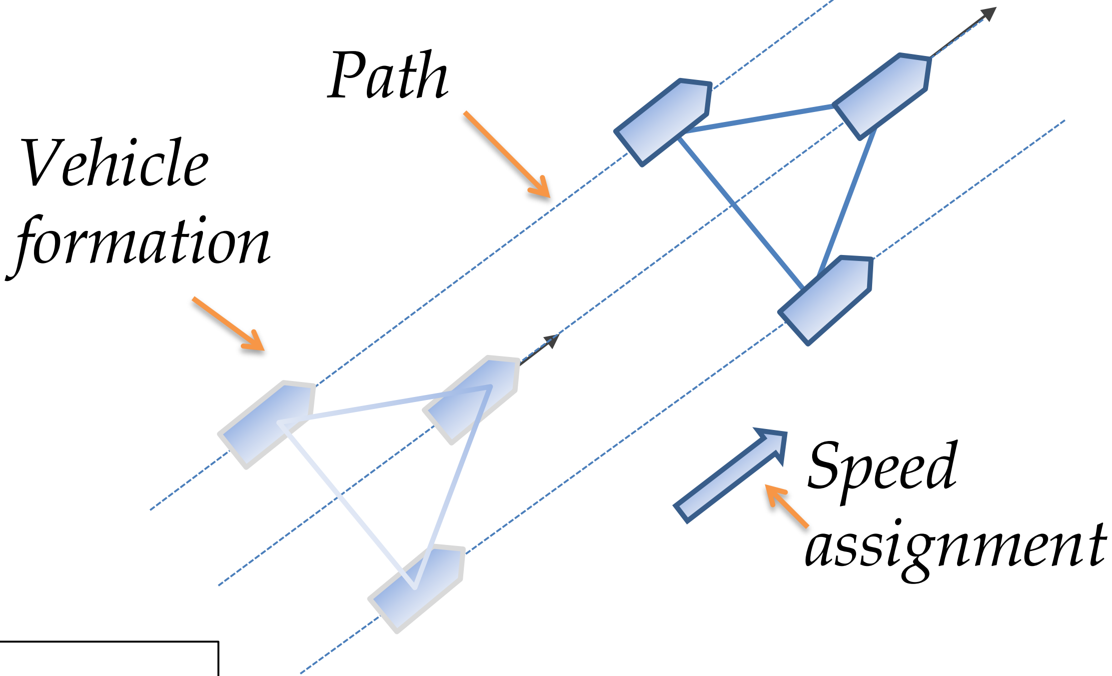
### Asymptotic assignment

$$\dot{\gamma}^{[i]}(t) \to v_d$$

**How to combine the output regulation objective with the consensus objective?**

# Cooperative Path Following



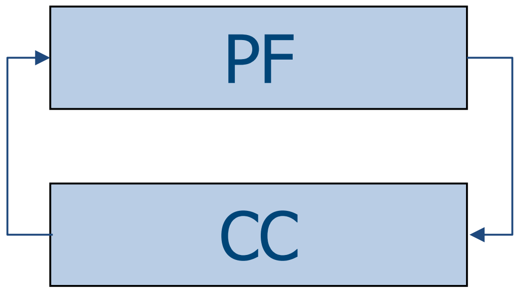



**Key ingredients:**

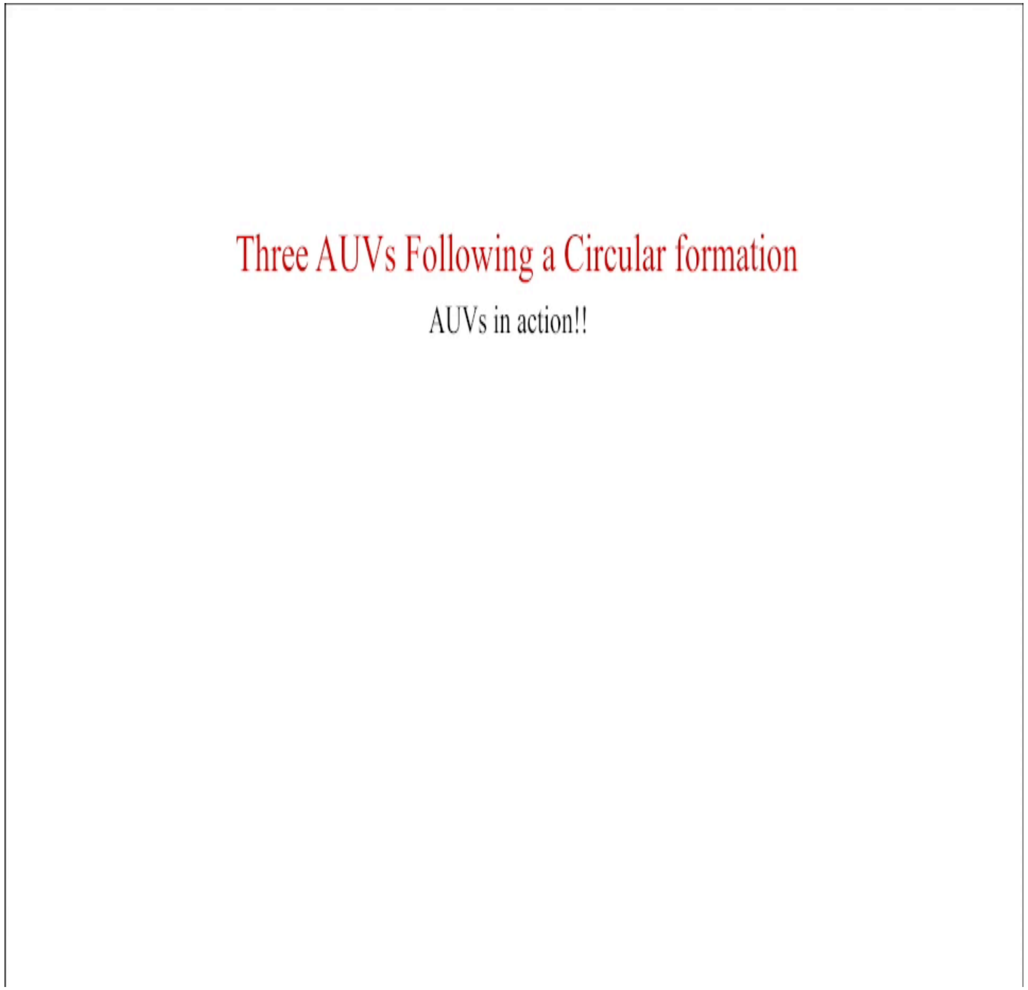
- Path following for each vehicle
- Inter vehicle coordination

  speed adjustments based on
  VERY LITTLE INFO EXCHANGED)
  (space-time decoupling)

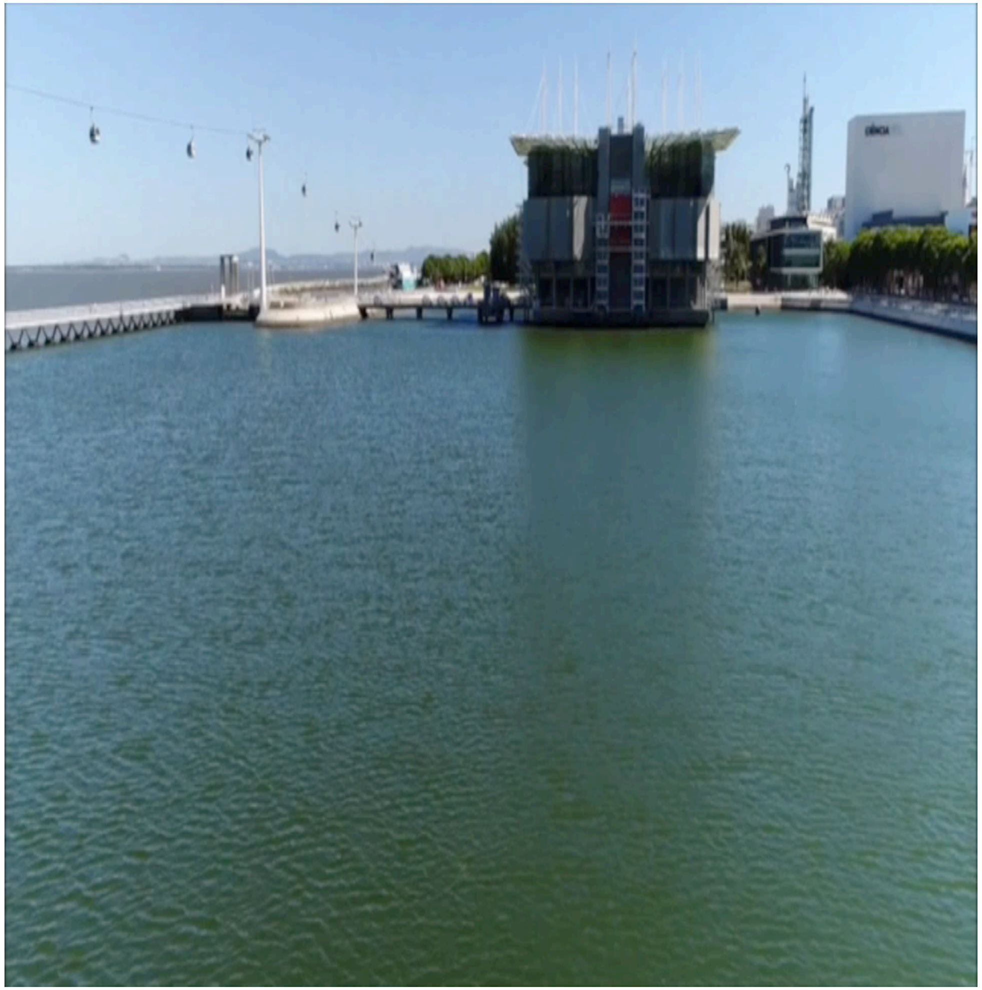
PF and CC interconnection



# Logic/event-based communications: Field tests




Event-based, Porto, 2018



Logic-based, Lisbon, 2018

# CPF and CMPF (numerical results)

**MPC goal:** Optimize over a pre-existing auxiliary consensus control law to balance coordination and regulation !

## Cooperative Moving Path Following


Cooperative Moving Path Following using Dynamic Event-triggered Communication

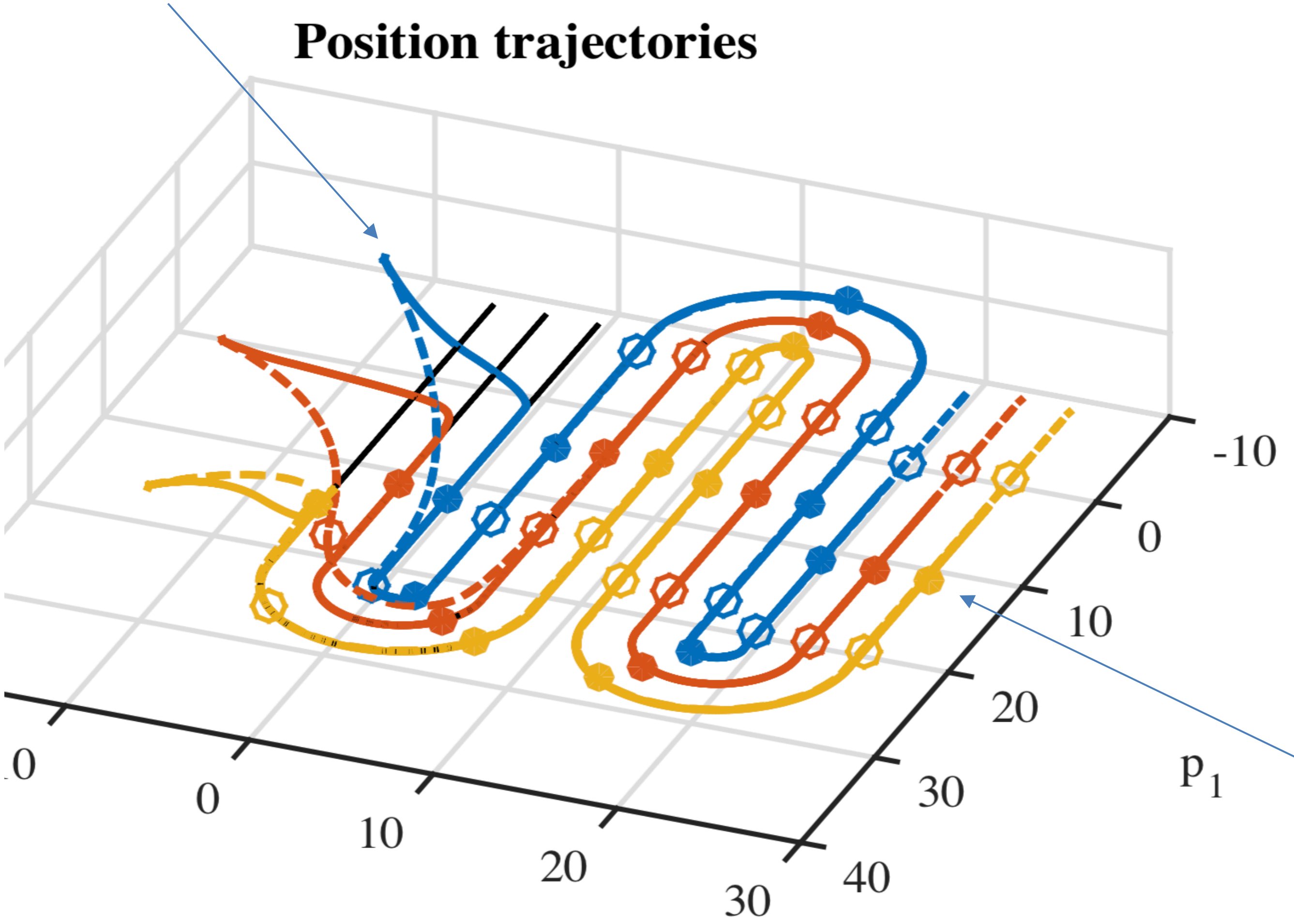
een Jain, A. Pedro Aguiar, and João Borges de Sousa

*Electrical and Computer Engineering, University of Porto, Portugal*



e European Union's Horizon 2020 research and innovation programme MarineUAS 642153, and project IMPROVE - POCI- 01-0145- FEDER-031823 - funded by FEDE funds (PIDDAC).

Strong penalty on the distance to the path (solid line)




Asymptotic convergence to consensus

# Conclusions

- Brief overview of Lyapunov model based and optimization model based control design
- Applications for motion control of single and multiple autonomous robotic vehicles
- Control architectures that accounts for vehicle dynamics, external disturbances, sensor noise, inter-vehicle time-varying communication topologies and communication losses

> *On-going and Future Research: How to better exploit (and **LEARN** through) **DATA** (off-line and **real-time**) and how to combine with (potentially poor) **nominal dynamic models** to improve performance and robustness in the presence of challenging restrictions and uncertainties, but **guaranteeing key specifications** (e.g., safety, interpretability, …)?*

- Implementation, and proof-of-concept of the algorithms on specific high impact applications

# Main References from C2SR Lab

Cyber-Physical Control Systems and Robotics Lab

**Check my web-page: https://paginas.fe.up.pt/~apra**

**Selected on Motion Control (Lyapunov-based):**


- R. Praveen Jain, João Sousa, A. Pedro Aguiar, *Three-Dimensional Moving Path Following Control for Robotic Vehicles with Minimum Positive Forward Speed.* IEEE Control Systems Letters (L-CSS), 2021.
- Francisco C. Rego, Nguyen T. Hung, Colin N. Jones, Antonio M. Pascoal, A. Pedro Aguiar, *Cooperative Path-Following Control with Logic-Based Communications: Theory and Practice.* In "Navigation and Control of Autonomous Marine Vehicles", IET, pp. 187-224, 2019.
- Tiago Oliveira, A. Pedro Aguiar, and Pedro Encarnação, *Moving Path Following for Unmanned Aerial Vehicles With Applications to Single and Multiple Target Tracking Problems.* IEEE Transactions on Robotics, Vol. 32, No. 5, pp. 1062-1078, Oct. 2016.
- A. Pedro Aguiar and João P. Hespanha, *Trajectory-Tracking and Path-Following of Underactuated Autonomous Vehicles with Parametric Modeling Uncertainty.* IEEE Transactions on Automatic Control, Vol. 52, No. 8, pp. 1362-1379, Aug. 2007.


**Selected on Safety:**


- Matheus Reis, A. Pedro Aguiar, Paulo Tabuada, *Control Barrier Function based Quadratic Programs Introduce Undesirable Asymptotically Stable Equilibria.* IEEE Control Systems Letters (L-CSS), vol. 5, no. 2, pp. 731-736, April 2021.


**Selected on Motion Control (MPC):**


- Andrea Alessandretti, A. Pedro Aguiar, *An optimization-based cooperative path-following framework for multiple robotic vehicles.* IEEE Transactions on Control of Network Systems, Vol. 7, No. 2, pp. 1002-1014, 2020.


**Selected on Dual-Objective for Economic Optimization:**


- Andrea Alessandretti, A. Pedro Aguiar, and Colin N. Jones, *An Input-to-State-Stability approach to Economic Optimization in Model Predictive Control.* IEEE Transactions on Automatic Control, Vol. 62, No. 12, pp. 6081-6093, Dec. 2017.