Lie-algebraic aspects of quantum control: gate realization and W-to-GHZ state conversion

Vladimir M. Stojanović

TU DARMSTADT

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TECHNISCHE UNIVERSITÄT DARMSTADT

Outline of the talk

- **Introduction to quantum (coherent) control**
  - Quantum state- vs operator (gate) control
  - Lie-algebraic (operator) controllability theorems

- **Local control of qubit arrays with Heisenberg-type interaction and realization of conditional three-qubit gates**
  - Interacting qubit arrays
  - Local control: application to qubit arrays with Heisenberg interaction
  - Control-based (single-shot) realization of Toffoli and Fredkin gates

  V. M. Stojanović, PRA 99, 012345 (2019).

- **W-to-GHZ state conversion in the Rydberg-blockade regime of neutral-atom systems**
  - Maximally-entangled three-qubit (multiqubit) states: W and GHZ
  - Symmetric sector of the three-qubit Hilbert space
  - Introduction to Rydberg-atom-based platform for QC
  - Dynamical-symmetry approach to W-to-GHZ state conversion

  T. Haase, G. Alber, and V. M. Stojanović, PRA 103, 032427 (2021).

- **Conclusions & Outlook**

Quantum control: generalities

- **State-to-state (state-selective) control**: How to steer a quantum system from a given initial- to a desired final state?
- **Operator (state-independent) control**: How to realize a pre-determined unitary transformation (target quantum gate)?

$$H(t) = H_0 + \sum_{j=1}^{p} f_j(t) H_j$$

$f_j(t)$ – control fields

The system is **completely controllable** if $H(t)$ can give rise to an arbitrary unitary transformation on its Hilbert space $\mathcal{H}$

i.e., the reachable set $\mathcal{R}$ is equal to $U(n)$ or $SU(n)$ ($n = \dim \mathcal{H}$)

General controllability theorems

$$\dot{U}(t) = -i[H_0 + \sum_{j=1}^{p} f_j(t) H_j] U(t) \quad , \quad U(0) = \mathbb{1}_{n\times n} \qquad (\#)$$

Lie-algebra rank condition

Theorem

The reachable set $\mathcal{R}$ of a quantum system described by Eq. $(\#)$ is the connected Lie group associated with the Lie algebra $\mathcal{L}_0$ generated by $-iH_0, -iH_1, \ldots, -iH_p$, i.e., $\mathcal{R} = e^{\mathcal{L}_0}$.

$\Rightarrow$ complete (operator) controllability

Theorem

A system described by Eq. $(\#)$ is completely (operator) controllable iff $\mathcal{L}_0 = u(n)$ [or $\mathcal{L}_0 = su(n)$], where $\mathcal{L}_0$ is the Lie algebra generated by $-iH_0, -iH_1, \ldots, -iH_p$. ($\mathcal{L}_0$ – **the dynamical Lie algebra**)

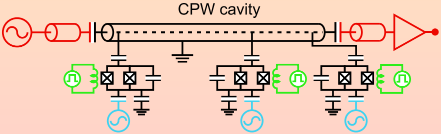
Interacting qubit arrays

general form: $$H_{\rm int} = \sum_{i<j} \sum_{\alpha,\beta} J_{ij}^{\alpha\beta} \sigma_{i,\alpha}\sigma_{j,\beta} \quad (\alpha,\beta = x,y,z)$$

qubit-qubit interaction	qubit system
Ising	Rydberg atom (g-r type)
XY	SC flux, phase, transmons
Heisenberg	spin, donor atom

transmons: XY coupling mediated by photons in a resonator

$$H_0 = \sum_{i<j} J_{ij}(X_iX_j + Y_iY_j)$$



V. M. Stojanović, A. Fedorov, A. Wallraff, and C. Bruder, PRB **85**, 054504 (2012)

Local control in interacting systems: general aspects

composite system $S = C \cup \bar{C}$ with controls acting only on C

total Hamiltonian: $H(t) = H_S + \sum_{j=1}^{p} f_j^C(t) H_j^C$

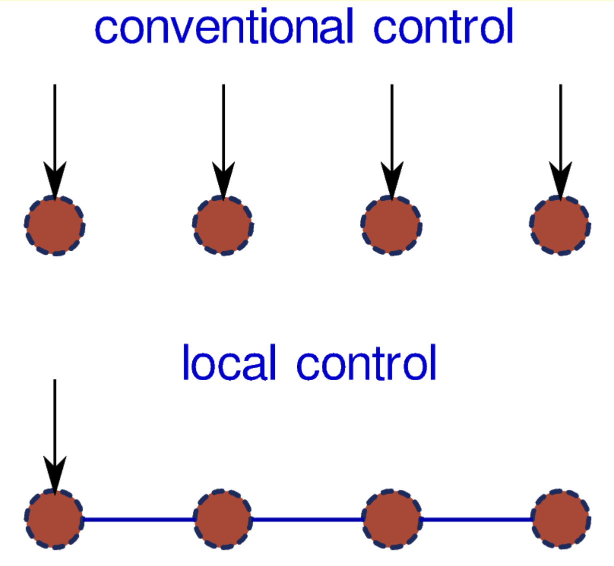
S is completely controllable iff $-iH_S$ and $-iH_j^C$ $(j = 1, \ldots, p)$ generate the Lie algebra $\mathcal{L}(S)$ of all skew-Hermitian operators on S

$$\langle iH_S, \mathcal{L}(C) \rangle = \mathcal{L}(S)$$

$\mathcal{L}(C) = \{-iH_1^C, \ldots, -iH_p^C\}_{\mathcal{L}}$

$\langle A, B \rangle$ – algebraic closure of the operator sets $\mathcal{A}$ and $\mathcal{B}$

Local control in qubit arrays with "always-on" interactions



Controllability of qubit arrays with Heisenberg interactions

$$H_0 = J \sum_{n=1}^{N-1} \Big(X_n X_{n+1} + Y_n Y_{n+1} + \Delta Z_n Z_{n+1} \Big)$$

$$H_c(t) = \underbrace{h_x(t)}_{f_1(t)} \underbrace{X_1}_{H_1} + \underbrace{h_y(t)}_{f_2(t)} \underbrace{Y_1}_{H_2}$$

$$H_{\text{total}}(t) = H_0 + H_c(t)$$

Acting on the x- and y-components of a **single qubit** in an XXZ- or Heisenberg-coupled qubit array renders the array **completely controllable**!

sufficient to show that the dimension of the dynamical Lie algebra $\mathcal{L}_{xy}$ generated by $\{-iH_0, -iX_1, -iY_1\}$ is $d^2 - 1 \;\; (d \equiv 2^N)$

$\Rightarrow \mathcal{L}_{xy} \cong su(d) \;\Rightarrow e^{\mathcal{L}_{xy}} \cong SU(d)$ **(complete controllability)**

$\Rightarrow$ any (multiqubit) gate can be realized through control of a single qubit

Control objectives (target gates)

controlled-NOT on the last two qubits of the array:

$$\mathbf{CNOT}_{N-1,N} \equiv \underbrace{\mathbf{I}\otimes\ldots\otimes\mathbf{I}}_{N-2}\otimes\Big(\underbrace{|0\rangle\langle 0|\otimes\mathbf{I}+|1\rangle\langle 1|\otimes\mathbf{X}}_{\mathbf{CNOT}}\Big)$$

$(X \equiv \sigma_x)$

flip (**NOT**) of the last qubit $\mathbf{X}_N \equiv \mathbb{1}\otimes\ldots\otimes\mathbb{1}\otimes\mathbf{X}$
requires only an x control!

$\sqrt{\mathrm{SWAP}}$ on the last two qubits: $\sqrt{\mathrm{SWAP}}_{N-1,N}$

reminder: $\sqrt{\mathrm{SWAP}} \equiv e^{i\frac{\pi}{8}}\, e^{-i\frac{\pi}{8}(X\otimes X+Y\otimes Y+Z\otimes Z)}$

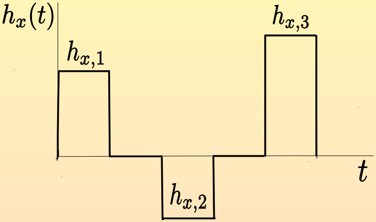
$N=3$, $\Delta=1$ case: $\dim\mathcal{L}_x = 18$, basis $\{-iH_0,\ldots,-iH_{17}\}$

Is there $A\in\mathcal{L}_x$ such that $\sqrt{\mathrm{SWAP}}_{2,3} = e^A$?

$$\mathbb{1}XX+\mathbb{1}YY+\mathbb{1}ZZ = \tfrac{1}{2}(H_0 - H_3 + H_6 - H_{16} + H_{17})$$

Control pulses and fidelity maximization

alternate x and y (or x only !) piecewise-constant controls:



full time evolution (total time $t_f \equiv N_t T$):

$$U(t_f) = U_{y,N_t/2} U_{x,N_t/2} \ldots U_{y,1} U_{x,1}$$

$$\left[\, U_{j,n} \equiv e^{-iH_{j,n}T} \;\; (j = x, y) \,\right]$$

gate fidelity: $$F(t_f) = \frac{1}{d} \left| \mathrm{tr}\left[U^\dagger(t_f) U_{\text{target}} \right] \right|$$ $$\left[\, 0 \le F(t_f) \le 1 \,\right]$$

maximize $F = F(\{h_{x,n}; h_{y,n}\})$ numerically

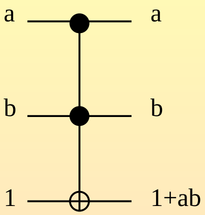
frequency-filtered control fields:

$$\widetilde{h}_j(t) = \mathcal{F}^{-1}\left[f(\omega) \mathcal{F}[h_j(t)] \right]$$

ideal low-pass filter:

$$f(\omega) = \theta(\omega + \omega_0) - \theta(\omega - \omega_0)$$

Three-qubit Toffoli- and Fredkin gates

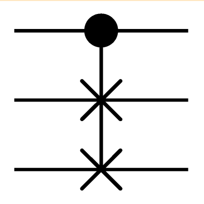


TOFFOLI ≡ controlled-controlled-NOT

two-qubit-gate counterpart: controlled-NOT (CNOT)

conventional realization:

6 CNOTs + 10 single-qubit operations



FREDKIN ≡ controlled-SWAP

related two-qubit gate: exponential SWAP (eSWAP)

$$\exp(i\theta_c \mathrm{SWAP}) \equiv \cos\theta_c \mathbb{1}_{4\times 4} + i \sin\theta_c \,\mathrm{SWAP}$$

conventional realization:

at least 5 entangling two-qubit gates; at least 8 CNOTs

Q: Can single-shot Toffoli and Fredkin gates efficiently be realized in Heisenberg(XXZ)-coupled qubit arrays with local control?

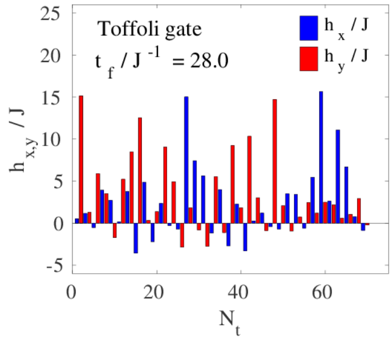
Single-shot realizations of Toffoli and Fredkin gates

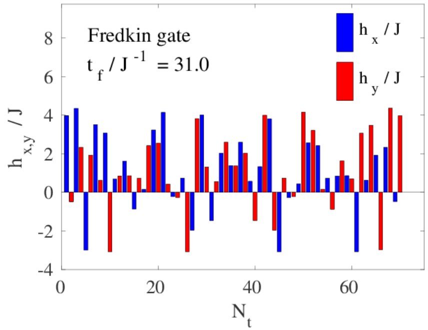
Gate times only slightly longer than for CNOT and eSWAP!

e.g. for $F = 1 - 10^{-4}$:

$t_f \approx 28\ J^{-1}$ (Toffoli) vs. $t_f \approx 25\ J^{-1}$ (CNOT)

$t_f \approx 31\ J^{-1}$ (Fredkin) vs. $t_f \approx 29\ J^{-1}$ (eSWAP)





V. M. Stojanović, PRA 99, 012345 (2019).

W states: a reminder

$$|W_{\mathrm{N}}\rangle \equiv \frac{1}{\sqrt{N}}\left(|10\ldots0\rangle + |01\ldots0\rangle + \ldots + |00\ldots1\rangle\right)$$

- **generalizations:**

  "twisted" W states: $|W_{\mathrm{N}}(k)\rangle \equiv \frac{1}{\sqrt{N}}\sum_{n=1}^{N} e^{-ikn}|0\ldots1_n\ldots0\rangle$

  W-like states:

  $$|W_N(\{A_n\})\rangle = \left(\sum_{n=1}^{N}|A_n|^2\right)^{-1/2}\sum_{n=1}^{N}A_n|0\ldots1_n\ldots0\rangle$$

- **robustness:** remains entangled even if any 2 parties are removed; the most robust N-qubit state to particle loss!

- **applications:** quantum teleportation, superdense coding, etc.

GHZ states: a reminder

$$|\mathrm{GHZ_N}\rangle \equiv \frac{1}{\sqrt{2}}\left(|00\ldots0\rangle + e^{i\phi}|11\ldots1\rangle\right)$$

- **generalization:** $\alpha_j + \beta_j = 1 \quad (\ \alpha_j, \beta_j \in \{0,1\}\ )$

  $$|\mathrm{GHZ_N}\rangle \equiv \frac{1}{\sqrt{2}}\left(|\alpha_1\alpha_2\ldots\alpha_\mathrm{N}\rangle + e^{i\phi}|\beta_1\beta_2\ldots\beta_\mathrm{N}\rangle\right)$$

- **robustness:** extremely fragile to particle loss; $|\mathrm{GHZ_3}\rangle$ is not entangled at all any more if one of the three qubits is traced out!

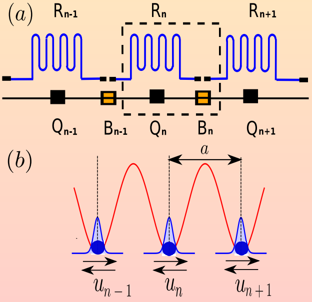
- **applications:** high-precision spectroscopy, concatenated error-correcting codes, etc.

$|\mathrm{GHZ_N}\rangle$ and $|W_\mathrm{N}\rangle$ are **LOCC-inequivalent**

Physical realizations of W and GHZ states

record experimental realizations of multipartite entanglement:

- $\mathbf{20}$ trapped ions with $\boldsymbol{F = 63.2\%}$
- $\mathbf{20}$ Rydberg atoms with $\boldsymbol{F = 54.2\%}$
- $\mathbf{12}$ photons with $\boldsymbol{F = 59.8\%}$
- $\mathbf{12}$ SC qubits with $\boldsymbol{F = 55.6\%}$;
  $\mathbf{18}$ SC qubits with $\boldsymbol{F = 53.0\%}$



proposals for W-state engineering based on a solid-state-physics analogy

SC qubits:

V. M. Stojanović, PRL **124**, 190504 (2020)

Rydberg-dressed qubits:

V. M. Stojanović, PRA **103**, 022410 (2021)

Three-qubit W vs. GHZ states and W-to-GHZ conversion

GHZ and W states: the only two inequivalent kinds of tripartite entanglement in a three-qubit system!

- **GHZ :** maximal essential three-way entanglement ($\tau_{\mathrm{ABC}} = 1$), while pairwise entanglements vanish ($C_{\alpha\beta} = 0\,;\ \alpha, \beta \in \{\mathrm{A,B,C}\}$)
- **W :** no essential three-way entanglement ($\tau_{\mathrm{ABC}} = 0$), but a strong pairwise entanglement ($C^2_{\mathrm{AB}} + C^2_{\mathrm{BC}} + C^2_{\mathrm{AC}} = 4/3$)

$\Rightarrow$ **Q: How about an interconversion between W and GHZ states?**

proposals: photons (nondeterministic), Rydberg atoms (STA / LRI), etc.

important observation:

both $|W_3\rangle$ and $|\mathrm{GHZ}_3\rangle$ are fully symmetric w.r.t. permutations of qubits, i.e. under S_3!

$\Rightarrow$ if $|W_3\rangle \xrightarrow{U} |\mathrm{GHZ}_3\rangle$, the unitary U should be symmetric as well!

Symmetric sector of the three-qubit Hilbert space

unitaries invariant under S_3: Lie subgroup $U^{S_3}(8)$ of $U(8)$

$$\dim U^{S_3}(8) = 20$$

$\Longrightarrow$ Lie algebra: $u^{S_3}(8) = \mathrm{span}\{i\Pi(\sigma_1 \otimes \sigma_2 \otimes \sigma_3)\}$

$$\Pi = \frac{1}{3!} \sum_{P \in S_3} P \quad \text{(symmetrization operator)}$$

$$\sigma_n \in \{\mathbb{1}_n, X_n, Y_n, Z_n\} \quad (\, n = 1, 2, 3 \,)$$

invariant subspaces of $\mathcal{H} \equiv (\mathbb{C}^2)^{\otimes 3}$ under the action of $U^{S_3}(8)$:

$\mathcal{H}$ splits into 3 invariant subspaces with dimensions 2, 2, and 4 !

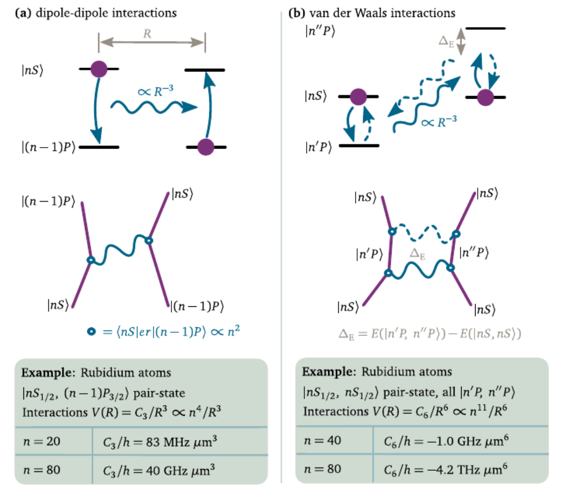
basis of the 4-dimensional subspace (**symmetric sector**):

$$|\phi_0\rangle = |000\rangle \qquad |\phi_1\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

$$|\phi_2\rangle = \frac{1}{\sqrt{3}}(|101\rangle + |010\rangle + |011\rangle) \qquad |\phi_3\rangle = |111\rangle$$

Rydberg atoms: basic properties and interactions

- long lifetimes $\tau_r \propto n^3$ (e.g. $\tau_r \sim 100\ \mu$s for $n \sim 50$)
- large dipole moments $d \propto n^2$ between states n and $n-1$



resonant dipole-dipole interaction (RDDI)

off-resonant dipole-dipole interaction (van der Waals)

Sibalic & Adams, IOP (2018)

vdW interaction between Rydberg atoms:

$$H_{\rm int} = \sum_{i<j} \frac{C_6}{R_{ij}^6}\, n_i n_j$$

$$|g\rangle \equiv |0\rangle\ ,\ |r\rangle \equiv |1\rangle$$

$$n_i \equiv (1 + Z_i)/2$$

$\Rightarrow$ Ising-type interaction:

$$H_{\rm int} = \sum_{i<j} J_{ij}\, Z_i Z_j$$

Rydberg blockade (RB) and its implications

coherent coupling of ground and Rydberg states: $|g\rangle \xrightarrow{\Omega} |r\rangle$

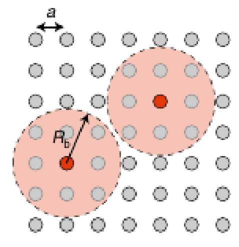
blockade condition: two atoms interacting through vdW, such that

$$\boxed{C_6 R^{-6} \gg \hbar\Omega} \rightarrow R \ll R_b \equiv \left(\frac{C_6}{\hbar\Omega}\right)^{1/6} \quad (\textbf{ blockade radius })$$

simultaneous excitation of both atoms not possible, i.e. $|gg\rangle \overset{\Omega}{\nrightarrow} |rr\rangle$

$\sqrt{2}$ Rabi-enhancement and entanglement:

$$\boxed{|gg\rangle \xrightarrow{\Omega\sqrt{2}} \frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)}$$

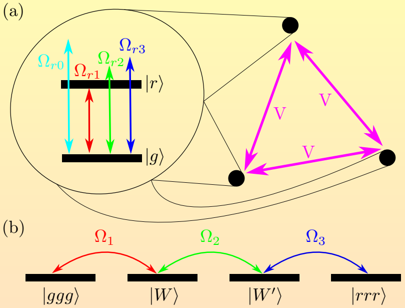


Browaeys & Lahaye, Nat. Phys. (2020)

generalization to N atoms:

$$\boxed{|g\ldots g\rangle \xrightarrow{\Omega\sqrt{N}} \frac{1}{\sqrt{N}}(|rg\ldots g\rangle + \ldots + |gg\ldots r\rangle)}$$

W-to-GHZ state conversion in Rydberg-atom trimers



- 3 equidistant neutral atoms subject to 4 external lasers:

$$\omega_j,\ \Omega_{r0},\ \Omega_{rj}(t)\ (j=1,2,3)$$

- atom – effective two-level system ($g-r$ type qubits):

$$|g\rangle \equiv |0\rangle\ ,\ |r\rangle \equiv |1\rangle\ .$$

task: deterministic W-to-GHZ state conversion

$$\frac{1}{\sqrt{3}}\left(|rgg\rangle + |grg\rangle + |ggr\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(|ggg\rangle + e^{i\phi}|rrr\rangle\right)$$

interaction-picture Hamiltonian: $H_{\mathrm{I}}(t)/\hbar$

$$= \sum_{k=1}^{3}\sum_{j=0}^{3}[\Omega_{rj}(t)e^{-i(\delta_j+\Delta_j)t}|r\rangle_{kk}\langle g| + \text{H.c.}] + \sum_{p<q} V|rr\rangle_{pq}\langle rr|$$

Effective Hamiltonian on a 4-state manifold

An effective Hamiltonian on the **symmetric sector**?

conditions of validity:

$|\boldsymbol{\Delta_0}|T_{\rm int} \gg 1$, $|V|T_{\rm int} \gg 1$ (RB regime), $|\boldsymbol{\Delta_0}|, |V| \gg |\boldsymbol{\Omega_{r0}}|$

⇒ **effective Hamiltonian on a 4-state manifold:** $H_{\rm eff}(t)/\hbar$

$$= \boldsymbol{\Omega_1}(t)|ggg\rangle\langle W| + \boldsymbol{\Omega_2}(t)|W\rangle\langle W'| + \boldsymbol{\Omega_3}(t)|W'\rangle\langle rrr| + \text{H.c.}$$

$\boldsymbol{\Omega_1}(t) = \sqrt{3}\,\boldsymbol{\Omega_{r1}}(t)$, $\boldsymbol{\Omega_2}(t) = 2\,\boldsymbol{\Omega_{r2}}(t)$, $\boldsymbol{\Omega_3}(t) = \sqrt{3}\,\boldsymbol{\Omega_{r3}}(t)$

basis of the **symmetric sector**:

$$|ggg\rangle \quad , \quad |W\rangle = \frac{1}{\sqrt{3}}\left(|rgg\rangle + |grg\rangle + |ggr\rangle\right)$$

$$|W'\rangle = \frac{1}{\sqrt{3}}\left(|rrg\rangle + |rgr\rangle + |grr\rangle\right) \quad , \quad |rrr\rangle$$

Dynamical symmetry of $H_{\text{eff}}(t)$

dynamical symmetry of $H_{\text{eff}}(t)$: $su(2) \oplus su(2) \cong so(4)$

six-dimensional maximal subalgebra of $su(4)$

a generic property of four-level systems
with adjacent level couplings

$a, b, c \rightarrow \ \Omega_1(t), \Omega_2(t), \Omega_3(t)$

$$H = \begin{pmatrix} 0 & a & 0 & 0 \\ a & 0 & b & 0 \\ 0 & b & 0 & c \\ 0 & 0 & c & 0 \end{pmatrix}$$

generalization to n-level systems: dynamical symmetry $so(n)$

angular-momentum operators: $\{S_i | i = 1, 2, 3\}$, $\{T_i \, | i = 1, 2, 3\}$

$$[S_i, S_j] = i\epsilon_{ijk} S_k \ , \quad [T_i, T_j] = i\epsilon_{ijk} T_k \ , \quad [S_i, T_j] = 0$$

$$\frac{H_{\text{eff}}(t)}{\hbar} = \Omega_1(t)\,(S_1 + T_1) + \Omega_2(t)\,(S_2 + T_2) + \Omega_3(t)\,(S_1 - T_1)$$

$\Omega_j(t) \in \mathbb{R} \rightarrow H_{\text{eff}}(t)$ describes the dynamics of two constrained pseudospin-$1/2$ degrees of freedom!

Dynamical-symmetry approach

dynamical Lie group of the system: $\mathrm{SU}(2) \times \mathrm{SU}(2) \subset \mathrm{SU}(4)$

unitary transformations: $U(\boldsymbol{\alpha}, \boldsymbol{\beta}) = e^{-i\boldsymbol{\alpha}\cdot\mathbf{S}} e^{-i\boldsymbol{\beta}\cdot\mathbf{T}} \equiv e^{-i(\boldsymbol{\alpha}\cdot\mathbf{S}+\boldsymbol{\beta}\cdot\mathbf{T})}$

time evolution of the system:

six-dimensional differentiable curve

$\gamma : t \longrightarrow \{\boldsymbol{\alpha}(t), \boldsymbol{\beta}(t)\} \quad t \in [0, T_{\mathrm{conv}}]$

TDSE: $$i\hbar \frac{d}{dt} U[\boldsymbol{\alpha}(t), \boldsymbol{\beta}(t)] = H(t) U[\boldsymbol{\alpha}(t), \boldsymbol{\beta}(t)]$$

task: find $U[\boldsymbol{\alpha}(t), \boldsymbol{\beta}(t)]$ such that

$$U[\boldsymbol{\alpha}(t = T_{\mathrm{conv}}), \boldsymbol{\beta}(t = T_{\mathrm{conv}})] |W\rangle = e^{i\Phi} |\mathrm{GHZ}\rangle$$

W-to-GHZ conversion

the most general Hamiltonian satisfying the last TDSE:

$$\frac{H(t)}{\hbar} = \omega[\alpha(t), \dot{\alpha}(t)] \cdot \mathbf{S} + \omega[\beta(t), \dot{\beta}(t)] \cdot \mathbf{T}$$

time-dependent vectorial frequency-like quantity (Richtmyer, Vol. II):

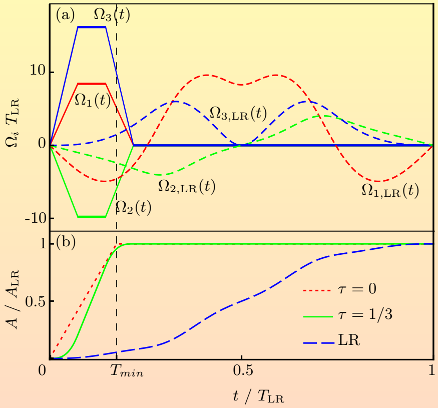
$$\begin{aligned} \omega[\alpha(t), \dot{\alpha}(t)] &= \frac{\sin|\alpha|}{|\alpha(t)|}\,\dot{\alpha}(t) + \frac{2\sin^2\frac{|\alpha|}{2}}{|\alpha|^2}\,[\alpha(t)\times\dot{\alpha}(t)] \\ &\quad + \frac{|\alpha| - \sin|\alpha(t)|}{|\alpha(t)|^3}\,[\alpha(t)\cdot\dot{\alpha}(t)]\alpha(t) \end{aligned}$$

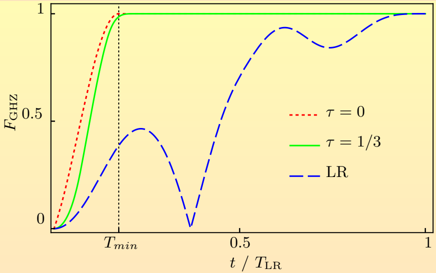
conditions resulting from the actual form of $H_{\text{eff}}(t)$:

$$\begin{aligned} \omega_3[\alpha(t)] &= \omega_3[\beta(t)] = 0 \\ \omega_2[\alpha(t)] &= \omega_2[\beta(t)] \end{aligned}$$

$$\begin{aligned} \Omega_1(t) &= \frac{\omega_1[\alpha(t)] + \omega_1[\beta(t)]}{2} \\ \Omega_3(t) &= \frac{\omega_1[\alpha(t)] - \omega_1[\beta(t)]}{2} \\ \Omega_2(t) &= \frac{\omega_2[\alpha(t)] + \omega_2[\beta(t)]}{2} \end{aligned}$$

Resulting W-to-GHZ conversion protocol





$$F_{\mathrm{GHZ}}(t) = |\langle \mathrm{GHZ}|\psi(t)\rangle|$$

$$A = \int_0^{T_{\mathrm{conv}}} \sum_{j=3}^{3} \Omega_j^2(t)dt$$

Both simpler and 5 times faster protocol than the one based on Lewis-Riesenfeld (LR) invariants!

R. -H. Zheng et. al, PRA 101, 012345 (2020).

Conclusions & Outlook

- Arbitrary multiqubit gate is reachable through single-qubit control in qubit arrays with XXZ (Heisenberg) type interactions; efficient single-shot realizations of Toffoli and Fredkin gates are possible!

  V. M. Stojanović, PRA 99, 012345 (2019).

- W-to-GHZ conversion in the RB regime of neutral-atom system possible with time-independent Rabi frequencies of external lasers; much faster than STA-based protocol with time-dependent ones.

  T. Haase, G. Alber, and V. M. Stojanović, PRA 103, 032427 (2021); generalization: twisted-W to GHZ – arXiv:2111.09718.

- Further applications of Lie-algebraic concepts in quantum-state control (W-to-GHZ conversion for other types of qubit-qubit interactions and for $N > 3$; engineering of entangled states, etc.).

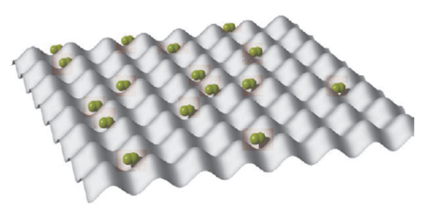
  V. M. Stojanović (in preparation).

Acknowledgment: support by DFG – SFB 1119 – 236615297.

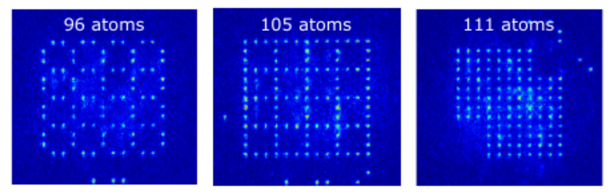
Optical lattices vs. arrays of individual dipole traps

common schemes for optical trapping of neutral-atom ensembles:

- optical lattices (period $a = \lambda_\mathrm{L}/2 < 1\ \mu$m)



- arrays of individual optical dipole traps (tweezers)
  (period $3\ \mu\text{m} \lesssim a \lesssim 25\ \mu\text{m}$)



D. Ohl de Mello *et al.*, PRL (2019)

Generalization to graphs

Local controllability on a graph $G = (S, E)$ by acting on $C \subseteq S$

$$H = H_S + \sum_{j=1}^{p} f_j^C(t) H_j^C \qquad \boxed{H_S = \sum_{n,m \in E} H_{nm}}$$

graph criterion of controllability (sufficient condition):
algebraic property of H_{nm} + topological property of G

H_S is **algebraically propagating** if for all $n \in S$ and $(n, m) \in E$

$$\boxed{\langle [iH_{nm}, \mathcal{L}(n)], \mathcal{L}(n) \rangle = \mathcal{L}(n, m)}$$

Heisenberg and Affleck-Kennedy-Lieb-Tasaki (AKLT) couplings are A.P. !

S is controllable by acting on C if H_S is A.P. and C is infecting

D. Burgarth *et. al.*, PRA **79**, 060305(R) (2009)