Multiple Gravitons and spectral sum rules in Fractional Quantum Hall systems

Dung Nguyen

Brown University

Phys. Rev. Research **3**, 023040 (2021)
Phys. Rev. Research **3**, 033217 (2021)
ArXiv: 2111.xxxxx

QM^3 ***Quantum Matter meets Maths, Lisbon***
22 Nov 2021

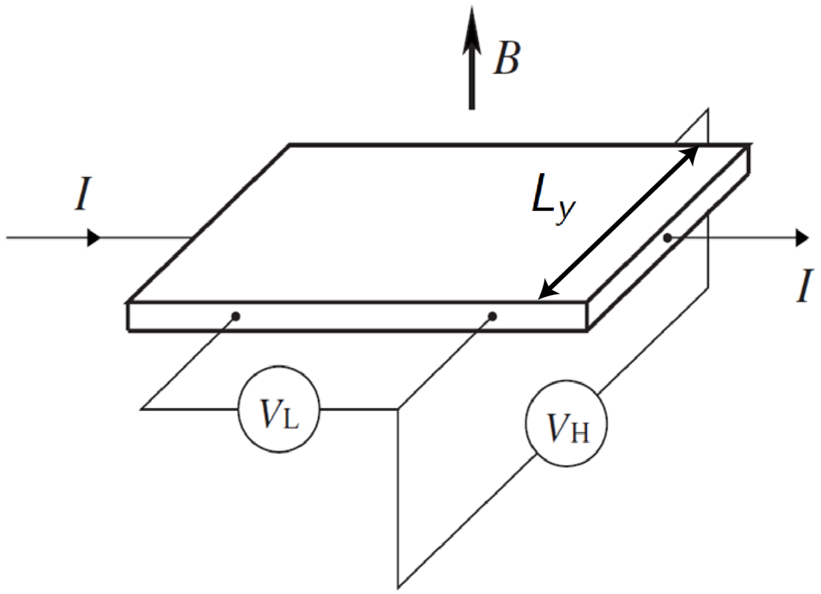
Plan

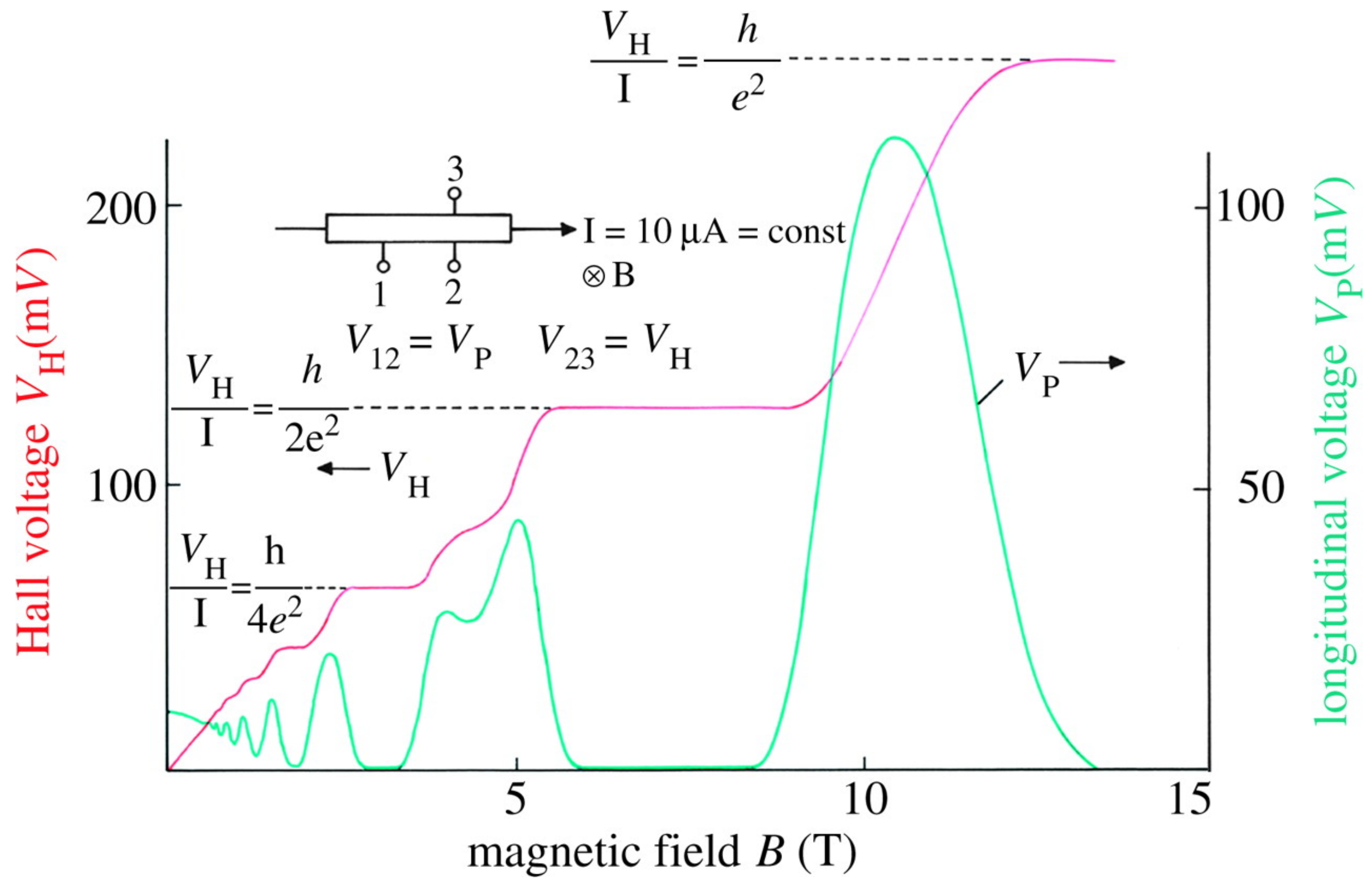
- (Brief) Introduction to Quantum Hall effect
- Magneto-roton is a massive graviton
- Gravitational spectral sum rules
- Graviton(s) in Dirac CF models
- Numerical results
- Raman scattering probes graviton(s)

Quantum Hall effect

Integer quantum Hall: A free electrons problem

- We consider 2D electrons gas in an applied magnetic field
- In a high magnetic field, we see "Integer plateaux" in the measurement of Hall resistance (conductance)





On plateaux

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

ν is an iteger

- Energy of a single electron is quantised to Landau levels

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c, \quad \omega_c = \frac{Be}{mc}$$

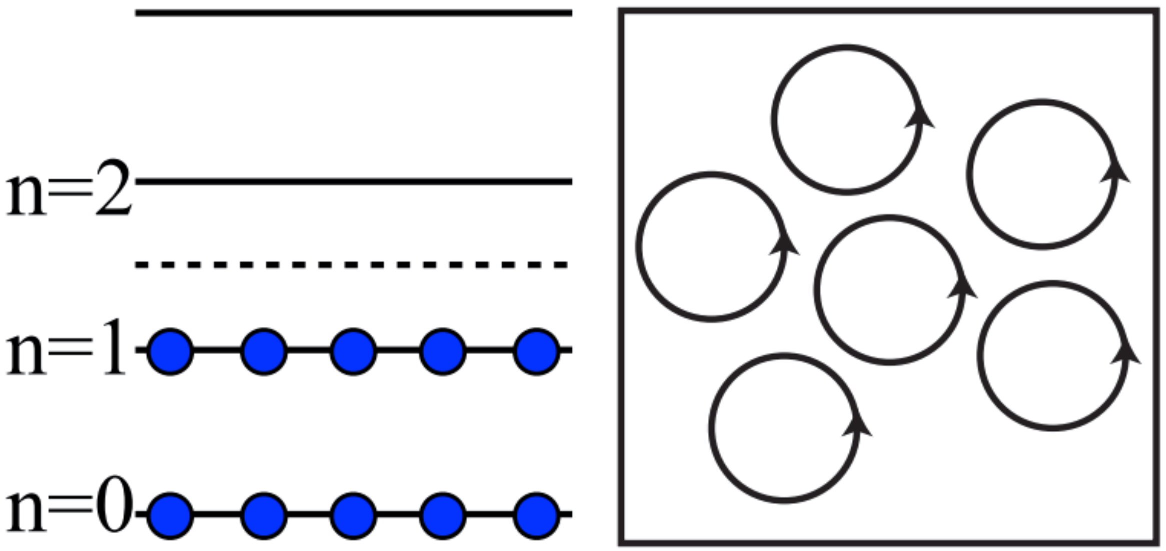
(m is electron's mass)

- In semi-classical picture: electrons in cyclotron orbits (orbitals)

Number of orbitals on each Landau level:

$$N_\phi = \frac{B \times \text{Area}}{\phi_0}, \quad \phi_0 = \frac{h}{e}$$

In applied electric field $\mathbf{E}$, each orbital has drift velocity $\mathbf{v} = \dfrac{\mathbf{E} \times \mathbf{B}}{B^2}$



$\ell_B \sim \sqrt{1/B}$

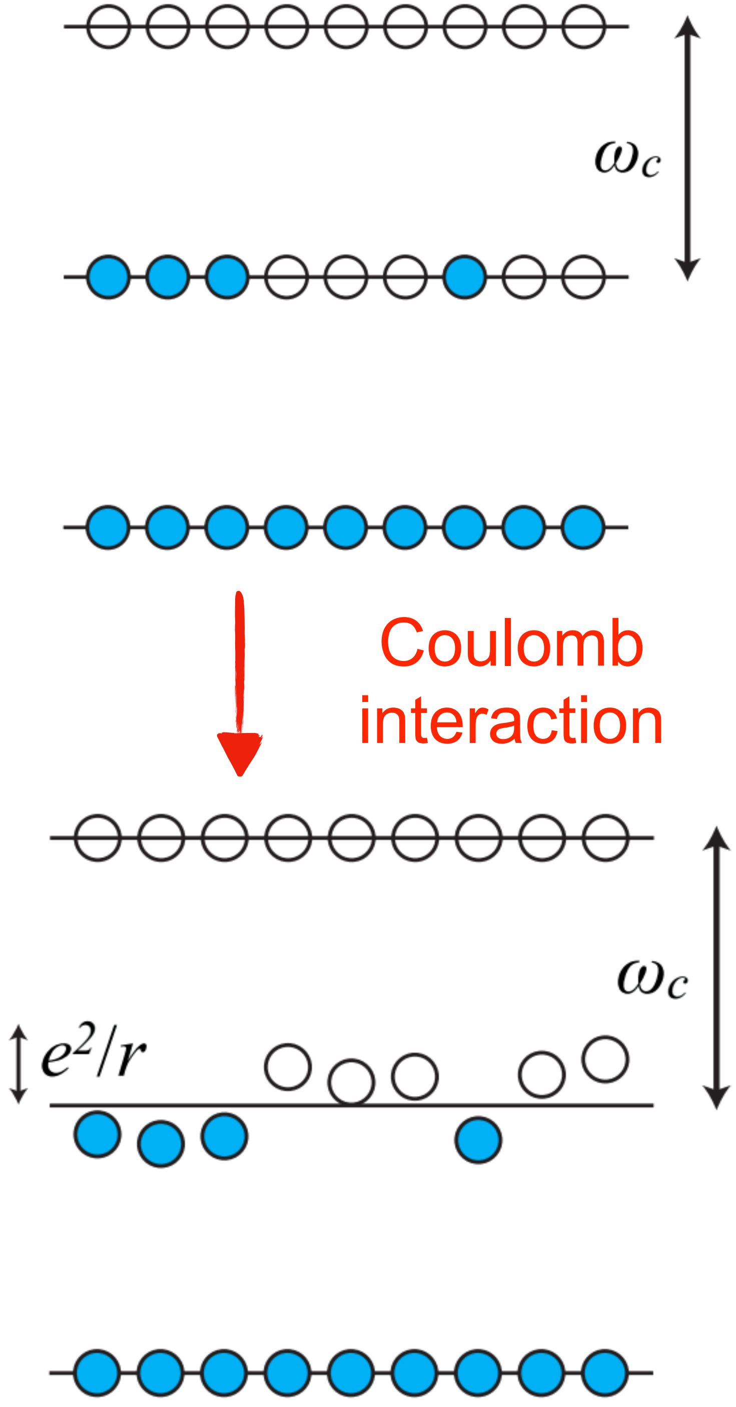
- Hall conductance if we have fully filled Landau levels

$$\sigma_{xy} = \nu\frac{e^2}{h} \qquad \nu = \frac{N_e}{N_\phi}$$

Filling fraction

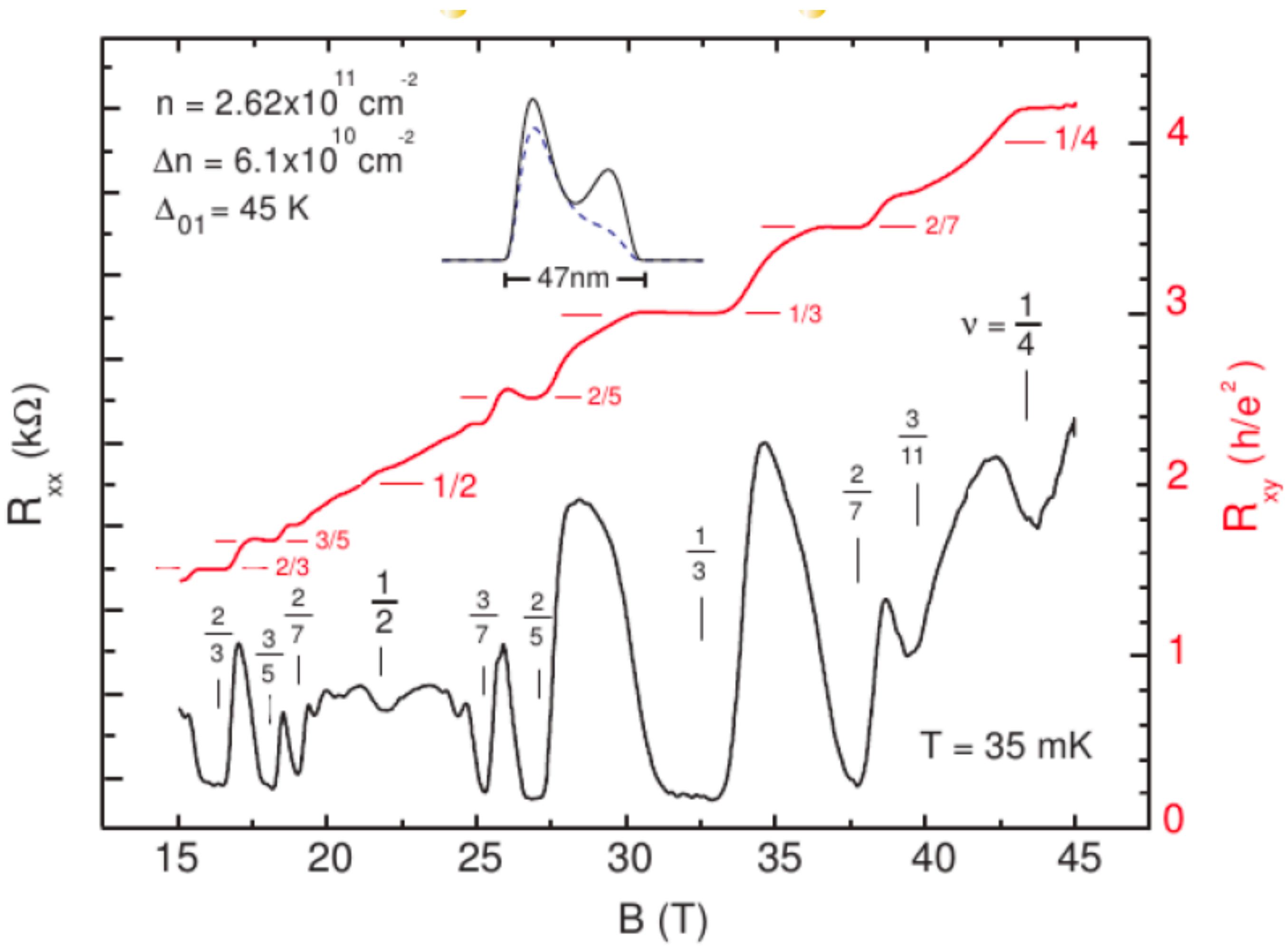
Fractional quantum Hall: A strongly interacting problem

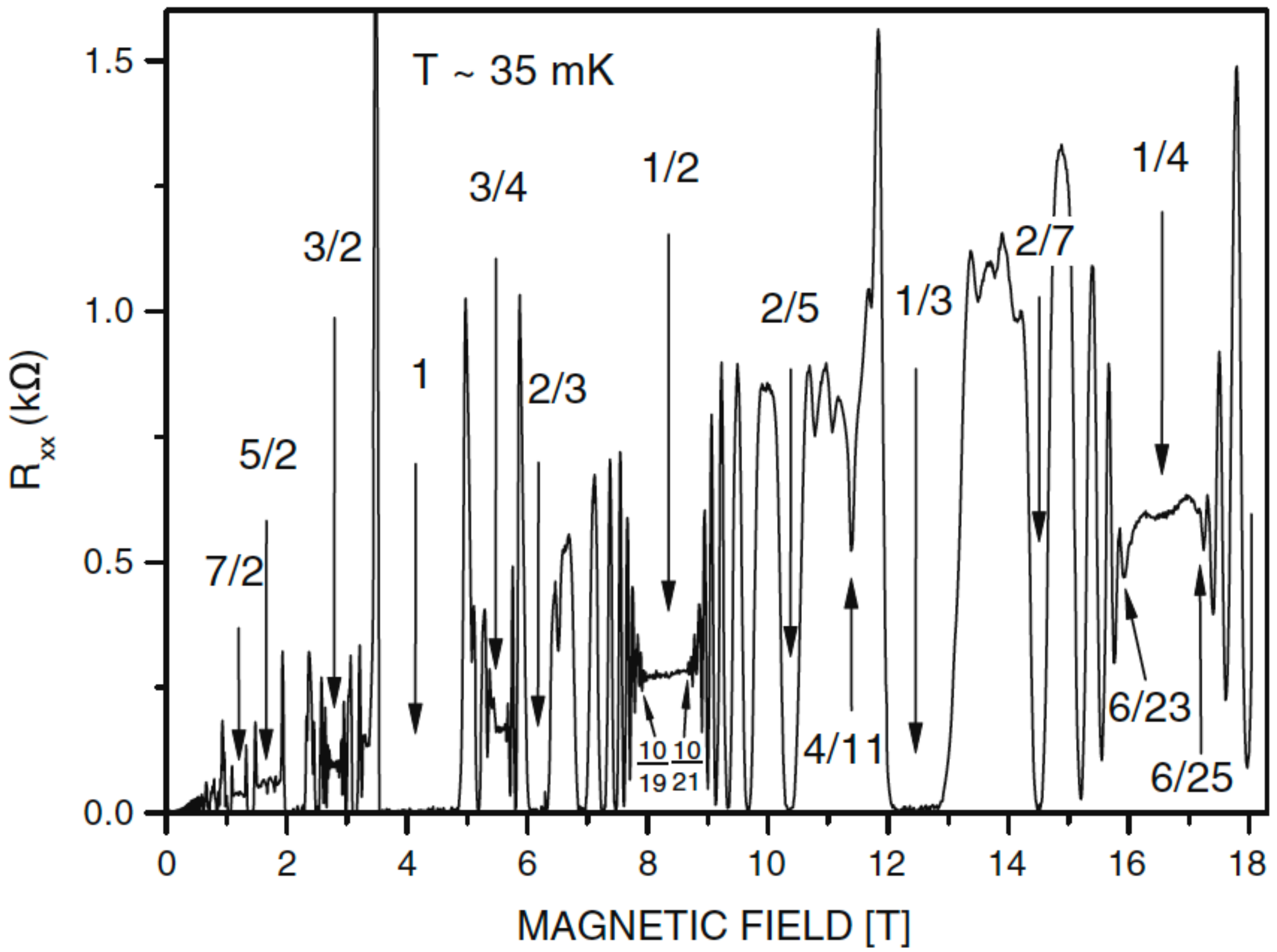
What happen if we only partially fill a Landau level?



- Without interaction, a partially filled Landau level is gapless with huge degeneracy
- Coulomb interaction splits the degeneracy to have unique gapped group state
- The measurement of Hall resistance shows small plateaux with fractionally quantised σ_{xy}
- In the lowest Landau level limit $\omega_c \to \infty$ $(m \to 0)$, one can ignore higher Landau levels
- In this limit, the kinetic energy effectively vanishes
- We have to deal with a strongly interacting problem with **no direct solution**

Fractional quantum Hall in experiment





In this talk, we concentrate on states near $\nu = 1/2$ and $1/4$

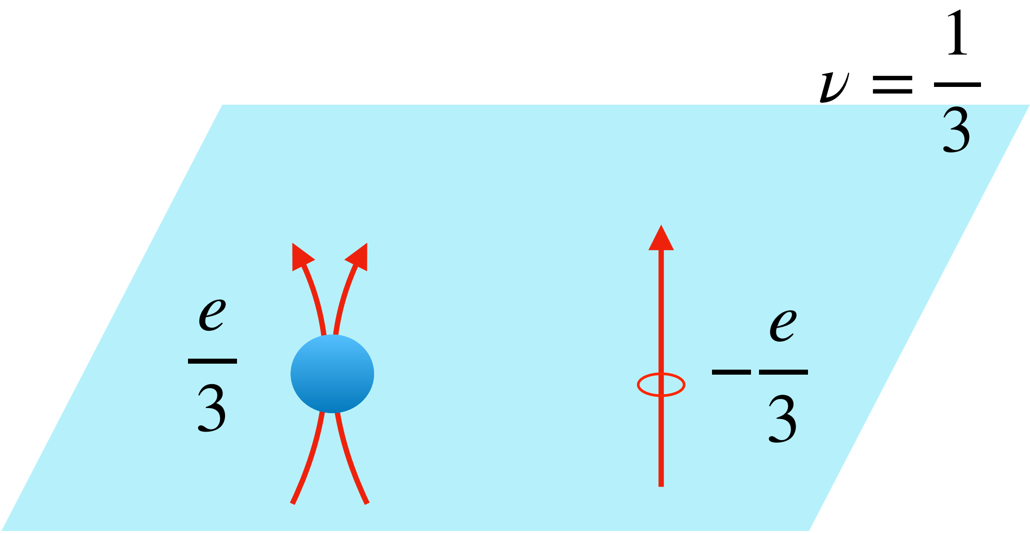
Excitations in the Lowest Landau level

- Charged excitations: quasi-particle and quasi-hole

  (1) Anyonic statistic

  (2) High energy gap

  (3) Restricted mobility (fracton)

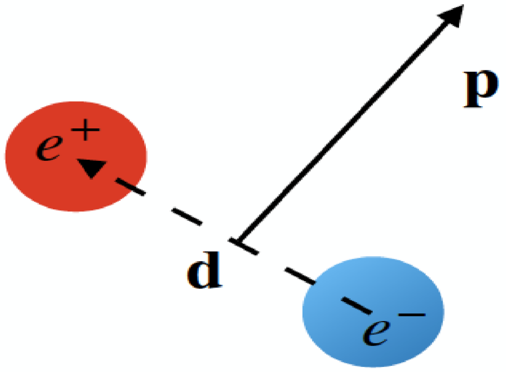


- Neutral excitations (Gapless states): Composite Fermion $\nu = \frac{1}{2n}$

  (1) Charged neutral

  (2) Dipole moment

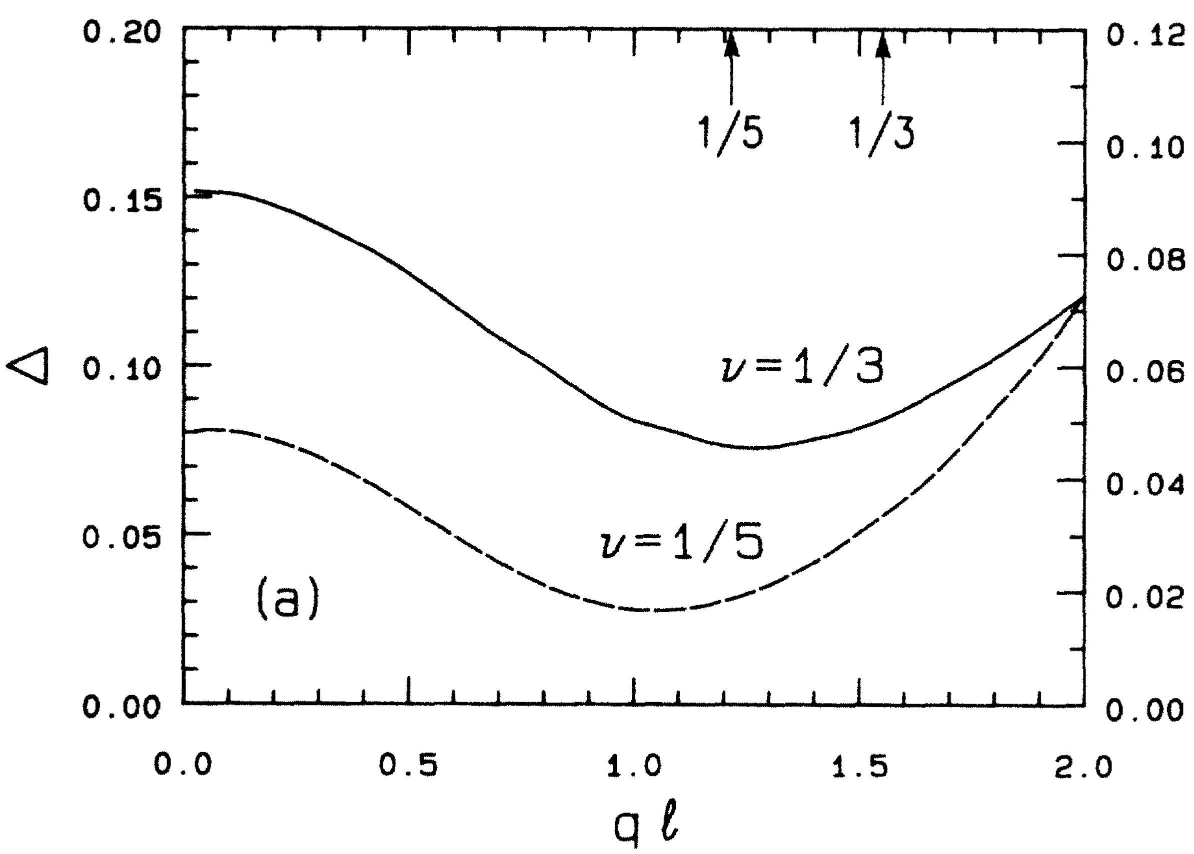
  (3) Move perpendicularly to their dipole moment (Fractonic behavior)



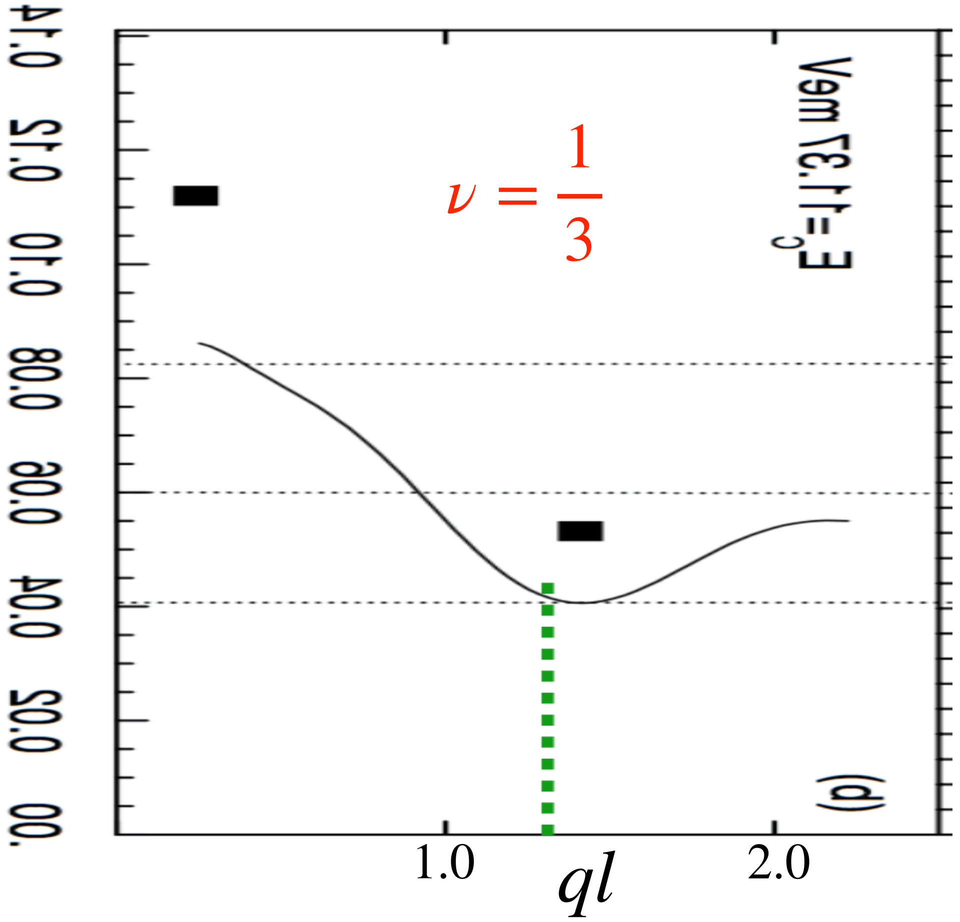
Du, Mehta, DXN, Son arXiv:2103.09826

Magneto-roton excitations

- Neutral excitation appears in Gapped FQH states
- Proposed by Girvin, MacDonald, Platzman (GMP 1986) as the charge density wave $\rho(\mathbf{q})|0\rangle$ in the lowest Landau level (LLL)
- The energy spectrum of magneto-roton (in GMP paper) was derived using the single mode approximation (similar as Feymann's model of roton in superfluid ^4He).

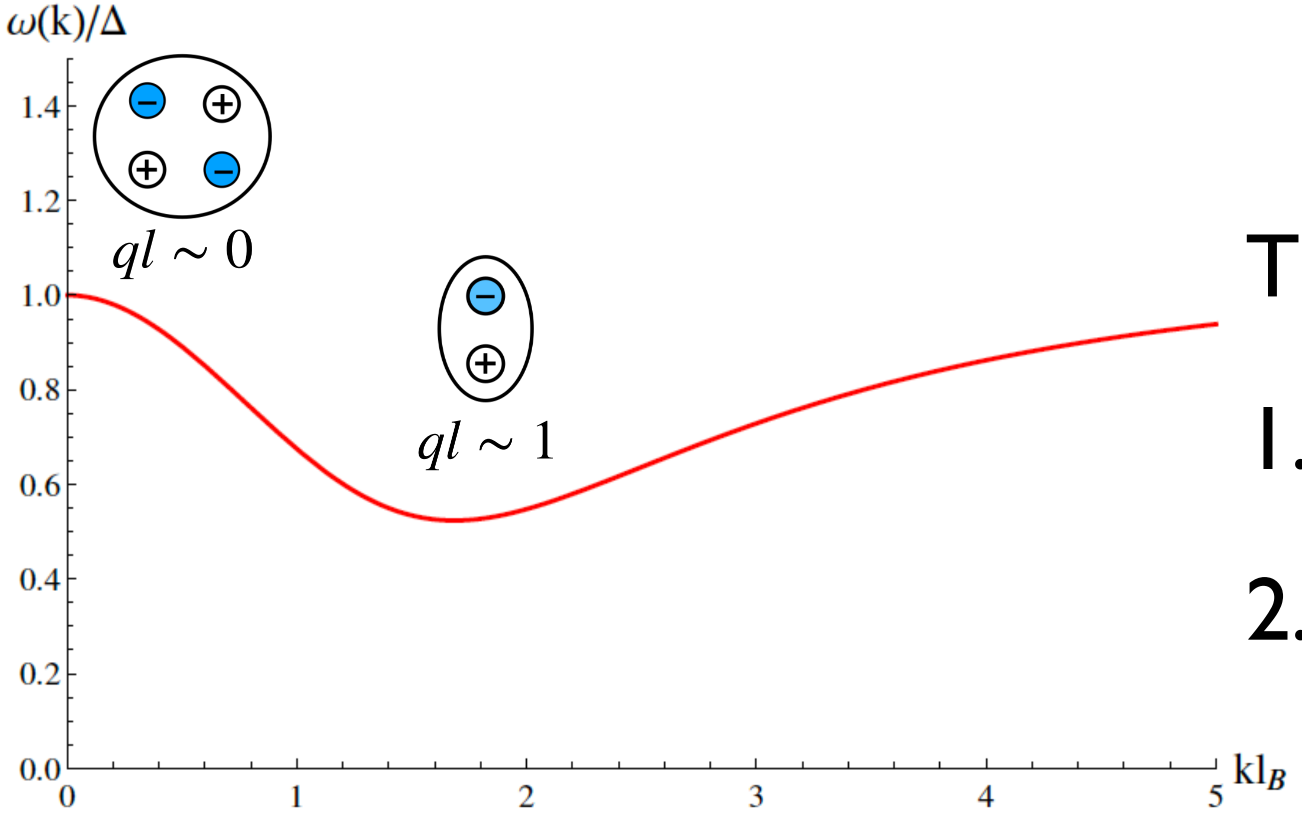


GMP 1986



Observed in Pinczuk et al. 1993

Magneto-roton: Microscopic structure



Shou-Cheng Zhang 1992

The magneto-roton is

1. charge quadrupole at $q\ell_B \sim 0$
2. charge dipole at the roton minimum

- The idea of quantum Hall "graviton" was initiated by **Haldane** as the intrinsic "guiding center metric" in arXiv:0906.1854 and Phys. Rev. Lett. **107**, 116801 (2011)
- Magneto-roton is a spin-2 excitation generated by $T_{zz}(\mathbf{q})|0\rangle$ (Golkar, DXN, Son 2013)

$(z = x + iy)$

Lowest Landau level conservation laws and magneto-roton spin

- Symmetries lead to the Ward identities

Charge conservation

Momentum Conservation

$$\left.\begin{array}{l} \partial_t\rho + \nabla\cdot\mathbf{j} = 0 \\ \partial_t(\cancel{m}j_i) + \partial_k T_{ki} = (\mathbf{j}\times\mathbf{B})_i \end{array}\right\} \quad \begin{array}{l} \mathbf{j} \sim \frac{1}{B}\partial T \\ \dot{\rho} + \frac{1}{B}\partial^2 T = 0 \end{array} \qquad \langle\rho\rho\rangle \sim q^4$$

LLL limit

- $\langle\rho\rho\rangle = \sum_n \langle 0|\rho|n\rangle\langle n|\rho|0\rangle$
- Excitations at $q \approx 0$ can be classified by spin
- For a spin-s excitation (rotational symmetry)
  - $\langle q,s|\rho|0\rangle \sim q^s$ (leading order)
  - $\langle\rho\rho\rangle \sim q^4 \Rightarrow s = 2$

$$\Big\} \quad \langle q=0,s|T|0\rangle \neq 0$$

Magnero-roton at $q = 0$ is induced by T

Magneto-roton as graviton excitations

- Due to its spin-2, magneto-roton can be considered as the ***graviton*** mode in FQH.
- Gromov and Son (2017) proposed the bi-metric theory of FQH, in which the "*intrinsic metric*" is identified with a massive graviton excitation.

Spectral sum rules and the Haldane bound

Stress tensor spectral functions

$(z = x + iy)$

- We define the stress tensor spectral functions

$$I_-(\omega) = \frac{1}{N_e} \sum_n |\langle n| \int d\mathbf{x}\, T_{zz} |0\rangle|^2 \delta(\omega - \omega_n),$$

$$I_+(\omega) = \frac{1}{N_e} \sum_n |\langle n| \int d\mathbf{x}\, T_{\bar{z}\bar{z}} |0\rangle|^2 \delta(\omega - \omega_n),$$

N_e : Total number of electrons

ω_n : Energy of n^{th} state

- We define the projected static structure factor (SSF)

(Equal time density-density correlation function)

$$\bar{S} = \frac{1}{\bar{\rho}_e} \langle \delta\rho(-\mathbf{q}) \delta\rho(\mathbf{q}) \rangle$$

Wen-Zee shift

- Wen-Zee shift is the topological quantum number of a Quantum Hall state.
- It determines how the FQH system couples with the background curvature.
- In the effective action, the WZ term has the form

$$S_{WZ} = \frac{\nu \mathcal{S}}{2\pi} \int_{\mathcal{M}} A d\omega \longleftarrow \text{Spin connection}$$

- Total electrons on a Riemann surface with Euler characteristic χ

$$N_e = \nu \left(N_\phi + \mathcal{S} \chi \right)$$

N_ϕ Total number of quantum fluxes

- From the lowest Landau level Ward's identities

$$\partial_t \rho = \frac{4i}{B}\left(\partial_{\bar z}\partial_{\bar z}T_{zz} - \partial_z\partial_z T_{\bar z\bar z}\right)$$

- We obtain the first sum rule

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I_-(\omega) + I_+(\omega)\right] = S_4,$$

S_4 is the coefficient of $q^4\ell_B^4$ of projected SSF.

- We use the relation of Hall viscosity and stress tensor correlation functions (retarded Green's fuction)

$$\omega\eta_H(\omega) = \langle T_{zz}T_{\bar{z}\bar{z}}\rangle_\omega - \langle T_{\bar{z}\bar{z}}T_{zz}\rangle_\omega$$

- With the definition of retard Green's functions

$$\langle AB\rangle_\omega = -i\int dtd^2\mathbf{x}\, e^{i\omega t}\Theta(t)\langle A(t,\mathbf{x})B(0,\mathbf{0})\rangle$$

- We obtain the second sum rule including the Wen-Zee shift $\mathscr{S}$

$$\int_0^\infty \frac{d\omega}{\omega^2}\left(I_-(\omega) - I_+(\omega)\right) = \frac{\eta_H(0) - \eta_H(\infty)}{2\bar{\rho}_e} = \frac{\mathscr{S} - 1}{8}$$

The value low-frequency and high-frequency Hall viscosity

$$\eta_H(0) = \bar{\rho}_e\frac{\mathscr{S}}{4}, \quad \eta_H(\infty) = \frac{\bar{\rho}_e}{4}.$$

Consequence of the WZ term

Universal for all FQH states

The Haldane bound

- Both $I_-(\omega)$ and $I_+(\omega)$ are non-negative by definition
- Two spectral sum rules:

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I_-(\omega)+I_+(\omega)\right] = S_4,$$

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I_-(\omega)-I_+(\omega)\right] = \frac{\mathcal{S}-1}{8} = \frac{\bar{s}}{4},$$

Guiding center spin

- Imply an inequality, the Haldane bound

$$S_4 \geq \frac{|\mathcal{S}-1|}{8},$$

- Haldane bound is the consequence of the Lowest Landau level
- If the FQH state is chiral with either $I_-(\omega)$ or $I_+(\omega)$ vanishes, then the state saturates the Haldane bound $S_4 = |\mathcal{S}-1|/8$.

Graviton modes in Composite Fermion models

Composite Fermion model of general FQH Jain’s sates

- We are dealing with a strongly interacting many bodies problem with no perturbative solution since we have no small parameter.
- Direct approach is numerical calculation with limited in system size (no way to get the thermal dynamics limit)
- It is the place for **effective field theory models**
- **Composite fermions were observed in experiments!**

Dirac composite fermion models

- Dirac composite fermion models near $\nu = \dfrac{1}{2n}$

Goldman and Fradkin (2018)

$$\mathscr{L}_{CF} = \frac{i}{2}\left(\psi^\dagger \overleftrightarrow{D}_t \psi + \psi^\dagger \sigma^i \overleftrightarrow{D}_i \psi + v^i \psi^\dagger \overleftrightarrow{D}_i \psi\right)$$

Extra term for Galilean invariant

$$-\frac{1}{8\pi}\left(1 - \frac{1}{n}\right) ada + \frac{1}{4\pi n}\tilde{A}da + \frac{1}{8\pi n}\tilde{A}d\tilde{A},$$

- $D_\mu = \partial_\mu - ia_\mu$ and $v^i = \epsilon^{ij}E_j/B$ is the **drift velocity**

$$\tilde{A}_0 = A_0 - \frac{1}{2}\epsilon^{ij}\partial_i v_j + \frac{1}{2}\omega_0,$$

$$\tilde{A}_i = A_i + \frac{1}{2}\omega_i\,.$$

Son 2013
Newton Cartan

Dirac composite fermion models near 1/2

$n = 1 \rightarrow$ no ada term

$$\mathscr{L}_{CF} = \frac{i}{2}\left(\psi^\dagger \overleftrightarrow{D}_t \psi + \psi^\dagger \sigma^i \overleftrightarrow{D}_i \psi + v^i \psi^\dagger \overleftrightarrow{D}_i \psi\right) - \frac{1}{4\pi}\tilde{A}da + \frac{1}{8\pi}\tilde{A}d\tilde{A},$$

Son - Dirac Composite Fermion (2015)

- Field equation of a_μ gives constraints

$$\rho_{\text{CF}} = \psi^\dagger \psi = \frac{B}{4\pi} \qquad J^i_{\text{CF}} = \psi^\dagger \sigma^i \psi = 0$$

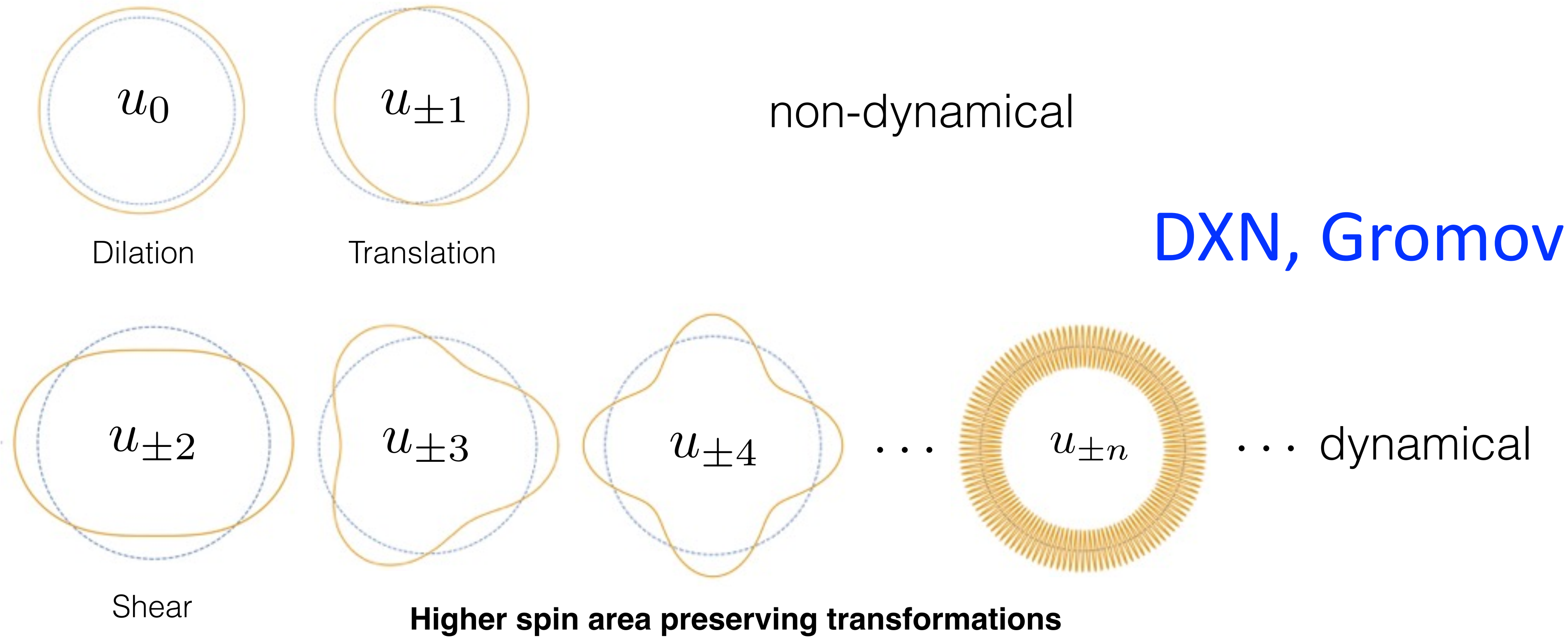
- DCF satisfies PH symmetry and reproduced expected results of electromagnetic responses.

DXN., S. Golkar, M.M. Robert and D. T. Son (2018)

Graviton (Magneto-roton) as deformation of DCF Fermi surface

- Low energy degrees of freedom are multipolar deformation of Fermi surface

$$\rho_{\mathrm{CF}} = \psi^\dagger \psi = \frac{B}{4\pi} \qquad J^i_{\mathrm{CF}} = \psi^\dagger \sigma^i \psi = 0$$



DXN, Gromov and Son (2018)

The shear mode is spin-2, and it is the graviton excitation (Magneto-roton)

Dirac composite fermion models near $1/2n$

$$\mathscr{L}_{CF} = \frac{i}{2}\left(\psi^\dagger \overleftrightarrow{D}_t \psi + \psi^\dagger \sigma^i \overleftrightarrow{D}_i \psi + v^i \psi^\dagger \overleftrightarrow{D}_i \psi\right)$$

$$-\frac{1}{8\pi}\left(1-\frac{1}{n}\right) ada - \frac{1}{4\pi n}\tilde{A}da + \frac{1}{8\pi n}\tilde{A}d\tilde{A},$$

- We consider the general Jain's states (p is large)

$$\nu_\pm = \frac{p}{2np \pm 1}$$

- Wen-Zee shift: (Gromov, Cho, You, Abanov, and Fradkin 2015)

$$\mathcal{S}_{\nu_+} = p + 2n, \qquad \mathcal{S}_{\nu_-} = -p + 2n$$

Static Structure factor and Haldane bound

DXN, and Son PRR **3**, 033217 (2021)

- We calculate the projected SSF of the DCF model

$$\bar{S} = \frac{1}{\bar{\rho}_e} \langle \delta\rho(-\mathbf{q})\delta\rho(\mathbf{q}) \rangle$$

- and found the value of S_4

$$S_4^{\nu_+} = \frac{1}{8}\left(p+1\right) \qquad S_4^{\nu_-} = \frac{1}{8}\left(p-1\right).$$

- One of them violates the Haldane bound for $n > 1$

$$(S_4 \geq \frac{|\mathcal{S} - 1|}{8}) \qquad S_4^{\nu_+} < \frac{|\mathcal{S}_{\nu_+} - 1|}{8} = \frac{p + 2n - 1}{8}$$

$$\mathcal{S}_{\nu_+} = p + 2n$$

$$\mathcal{S}_{\nu_-} = -p + 2n$$

Extra graviton(s) (spin-2)

DXN, and Son PRR **3**, 033217 (2021)

- We need more contribution to the q^4 term of the density-density correlation function to satisfy the Haldane bound.
- From the previous argument, we need more spin-2 mode(s) $\rightarrow$ More graviton(s)
- We add more spin-2 mode(s) to the effective action. We call this Haldane sector (due to Haldane's initial of the area-preserving deformation of elementary droplets)
- The theoretical model of the extra spin-2 mode(s) follows closely the bi-metric theory by Gromov and Son 2017

Bi-metric theory formalism

Gromov and Son 2017

- Consider the emergent vielbein $\hat{e}_i^\alpha$ that defines the emergent metric

$$\hat{g}_{ij} = \hat{e}_i^\alpha \hat{e}_j^\beta \delta_{\alpha\beta}$$

- With the inversion $\hat{E}_\alpha^i \hat{e}_i^\beta = \delta_\alpha^\beta$

- The emergent spin connection

$$\hat{\omega}_0 = \frac{1}{2}\epsilon^{\alpha\beta}\hat{E}_\beta^i \partial_0 \hat{e}_i^\alpha,$$

$$\hat{\omega}_j = \frac{1}{2}\epsilon^{\alpha\beta}\hat{E}_\beta^i \hat{\nabla}_i \hat{e}_i^\alpha,$$

- The covariant derivative $\hat{\nabla}_i$ is defined in the same manner as in GR with the metric $\hat{g}_{ij}$.

Haldane sector

DXN, and Son PRR **3**, 033217 (2021)

- With the definition of emergent metric and vielbein, we define the Lagrangian of Haldane sector (leading order in derivative)

Single unknown ζ

$$\mathscr{B} = \frac{\partial_1 A_2 - \partial_2 A_1}{\sqrt{g}}$$

$$S_{\text{Haldane}} = \int \frac{\zeta}{4n\pi} A d\hat{\omega} - \int d^3x \sqrt{g} \left[\frac{\tilde{m}}{2} \left(\frac{1}{2} \hat{g}_{ij} g^{ij} - \gamma \right)^2 + \frac{\zeta}{8n\pi B} g^{ij} \left(\partial_i E_j \right) \mathscr{B} \right],$$

- $\gamma < 1$ and $\tilde{m}$ are constant
  - $\gamma > 1$ corresponds to the nematic phase in FQH
  - $\tilde{m}$ determines the energy gap of the extra spin-2 mode
- The last term appears because the time component of spin connection goes with the combination $\hat{\omega}_0 + \frac{1}{2}\epsilon^{ij}\partial_i v_j$

Geracie, Prabhu, Roberts 2016

Static structure factor and WZ shift

$$S = S_{\mathrm{CF}} + S_{\mathrm{Haldane}}$$

$$\mathcal{S}_{\nu_+} = p + 2n$$

$$\mathcal{S}_{\nu_-} = -p + 2n$$

- One can calculate the Wen-Zee shift directly from the effective model

$$\mathcal{S}_{\nu_+} = p + 2 + 2\zeta, \quad \mathcal{S}_{\nu_-} = -p + 2 + 2\zeta$$

- If we chose $\zeta = n - 1$, we get the expected value of WZ shift
- We also calculated the pSSF

$$S_4^{\nu_+} = \frac{1}{8}\left(p + 1 + 2\zeta\right), \quad S_4^{\nu_-} = \frac{1}{8}\left(p - 1 + 2\zeta\right).$$

- Again, with $\zeta = n - 1$, the SSF satisfies the Haldane bound

$$S_4^{\nu_+} = \frac{|\mathcal{S}_{\nu_+} - 1|}{8}, \quad S_4^{\nu_-} > \frac{|\mathcal{S}_{\nu_-} - 1|}{8} \qquad \left(S_4 \geq \frac{|\mathcal{S} - 1|}{8}\right)$$

DXN, and Son PRR **3**, 033217 (2021)

Spectral densities of gravitons in the Dirac composite fermion model

Graviton spectral densities

- The combination of results of S_4 and $\mathscr{S}$ in the general Dirac composite fermion model and the gravitation spectral sum rules

$$\nu_+ = \frac{p}{2np+1} \longrightarrow \frac{I_-(\omega)}{\omega^2} = \frac{p+1}{8}\delta(\omega - \omega_L) + \frac{n-1}{4}\delta(\omega - \omega_H),$$

$$I_+(\omega) = 0$$

$$\nu_- = \frac{p}{2np-1} \longrightarrow \frac{I_-(\omega)}{\omega^2} = \frac{n-1}{4}\delta(\omega - \omega_H),$$

$$\frac{I_+(\omega)}{\omega^2} = \frac{p-1}{8}\delta(\omega - \omega_L)$$

- ω_L is the energy of low-energy graviton (the deformation of DCF fermi surface).
- ω_H is the energy of high-energy graviton (the Haldane mode)
- The same contribution of Haldane mode to graviton spectral densities of ν_+ and ν_-→ **Universal**

Spectral sum rule of Fermi-liquid state $\nu = \frac{1}{2n}$

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I\_(\omega) - I_+(\omega)\right] = \frac{\mathcal{S}-1}{8} = \frac{\mathcal{S}_{\text{Haldane}}}{8} = \frac{n-1}{8}$$

- The difference between positive and negative chiral gravitons' spectra density due to the Haldane mode.
- With $n = 1$, Fermi-liquid state $\nu = 1/2$ is particle hole symmetric, $I_+(\omega)$ and $I\_(\omega)$ are identical.
- With $n > 1$, the Haldane mode appears with the negative chirality.

$n = 1$

$$\nu_+ = \frac{p}{2p+1} \quad \rightarrow \quad \frac{I_-(\omega)}{\omega^2} = \frac{p+1}{8}\delta(\omega - \omega_L), \qquad I_+(\omega) = 0$$

$$\nu_- = \frac{p}{2p-1} \quad \rightarrow \quad \frac{I_-(\omega)}{\omega^2} = 0, \qquad \frac{I_+(\omega)}{\omega^2} = \frac{p-1}{8}\delta(\omega - \omega_L)$$

- No high-energy mode.
- Jain states are chiral with only one graviton chirality

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I_-(\omega) - I_+(\omega)\right] = \frac{\mathcal{S}-1}{8} = \frac{\bar{s}}{4} = \begin{cases} \dfrac{p+1}{8} & (\nu_+) \\ -\dfrac{p-1}{8} & (\nu_-) \end{cases}$$

$n = 2$

$$\nu_+ = \frac{p}{4p+1} \quad \rightarrow \quad \frac{I_-(\omega)}{\omega^2} = \frac{p+1}{8}\delta(\omega - \omega_L) + \frac{1}{4}\delta(\omega - \omega_H),$$

$$I_+(\omega) = 0$$

$$\nu_- = \frac{p}{4p-1} \quad \rightarrow \quad \frac{I_-(\omega)}{\omega^2} = \frac{1}{4}\delta(\omega - \omega_H),$$

$$\frac{I_+(\omega)}{\omega^2} = \frac{p-1}{8}\delta(\omega - \omega_L)$$

- ν_+ is still chiral with both gravitons have negative chirality.
- ν_- is non-chiral with high-energy mode in negative chirality and low-energy mode in positive chirality.

$$\int_0^\infty \frac{d\omega}{\omega^2}\left[I_-(\omega) - I_+(\omega)\right] = \frac{\mathcal{S} - 1}{8} = \frac{\bar{s}}{4} = \begin{cases} \dfrac{p+3}{8} & (\nu_+) \\ \dfrac{3-p}{8} & (\nu_-) \end{cases}$$

Numerical results

DXN, Haldane, Rezayi, Son, Yang (ArXiv:2111.xxxxx)

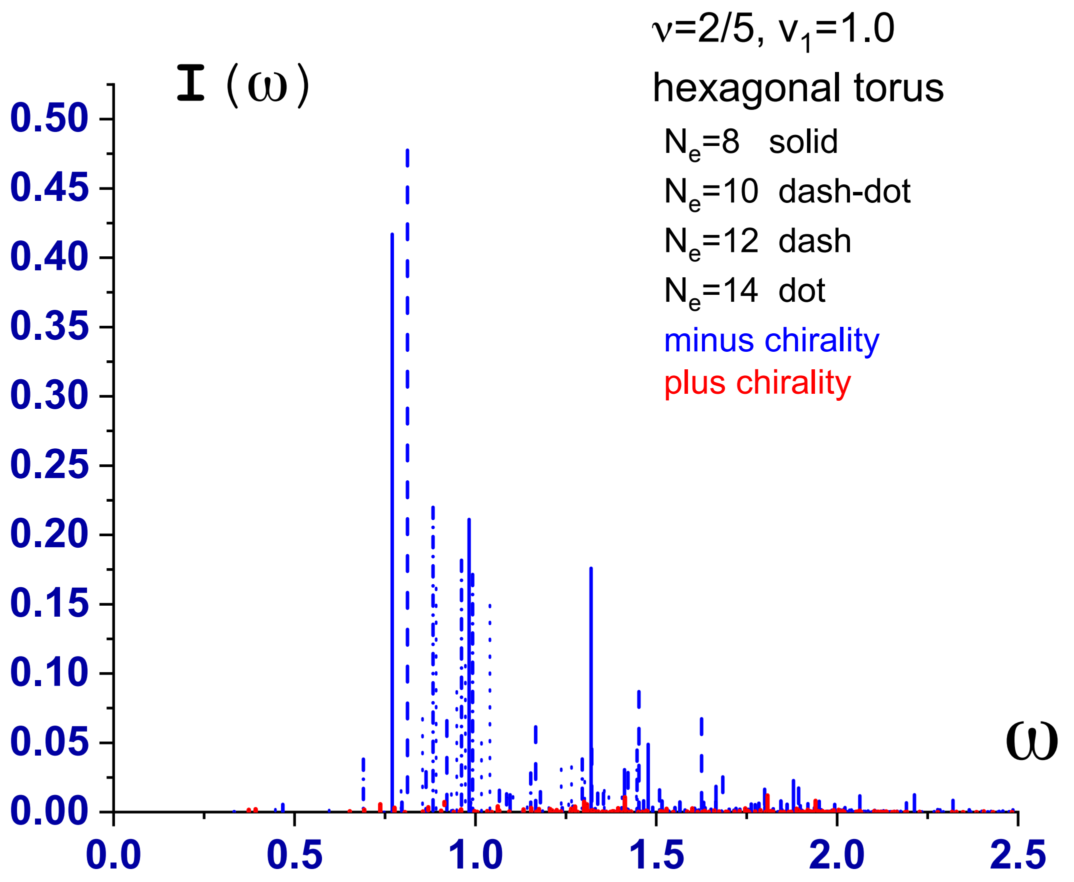
Balram, Liu, Gromov, Papic (ArXiv:2111.xxxxx)

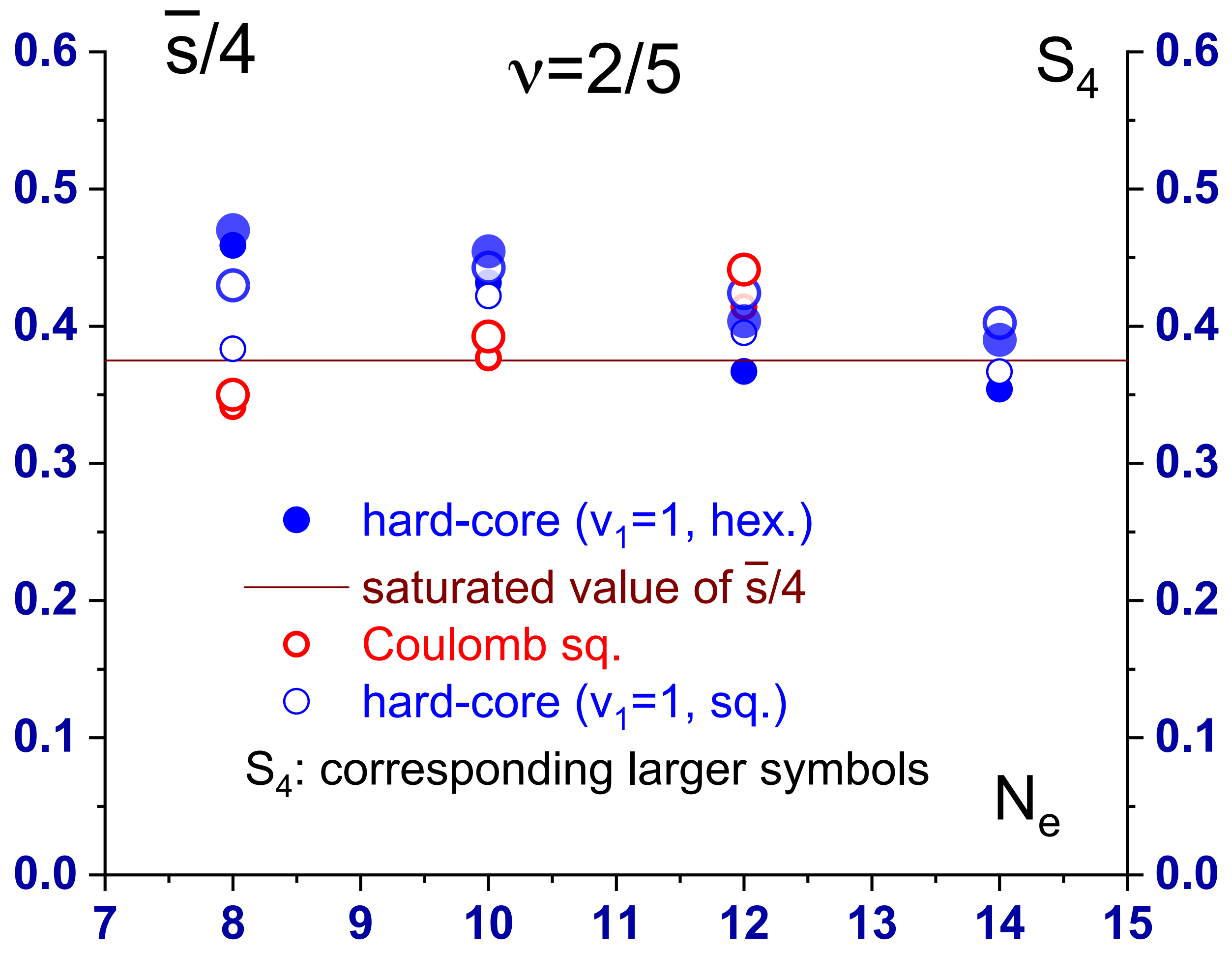
To appear tomorrow

- In numerical calculations, the delta functions $\delta(\omega - \omega_L)$ and $\delta(\omega - \omega_H)$ can be broaden due to the finite size effect.

$\nu_+, n = 1$

$$I_{-}(\omega) = \frac{p+1}{8}\omega^2\delta(\omega-\omega_L), \quad I_{+}(\omega) = 0$$

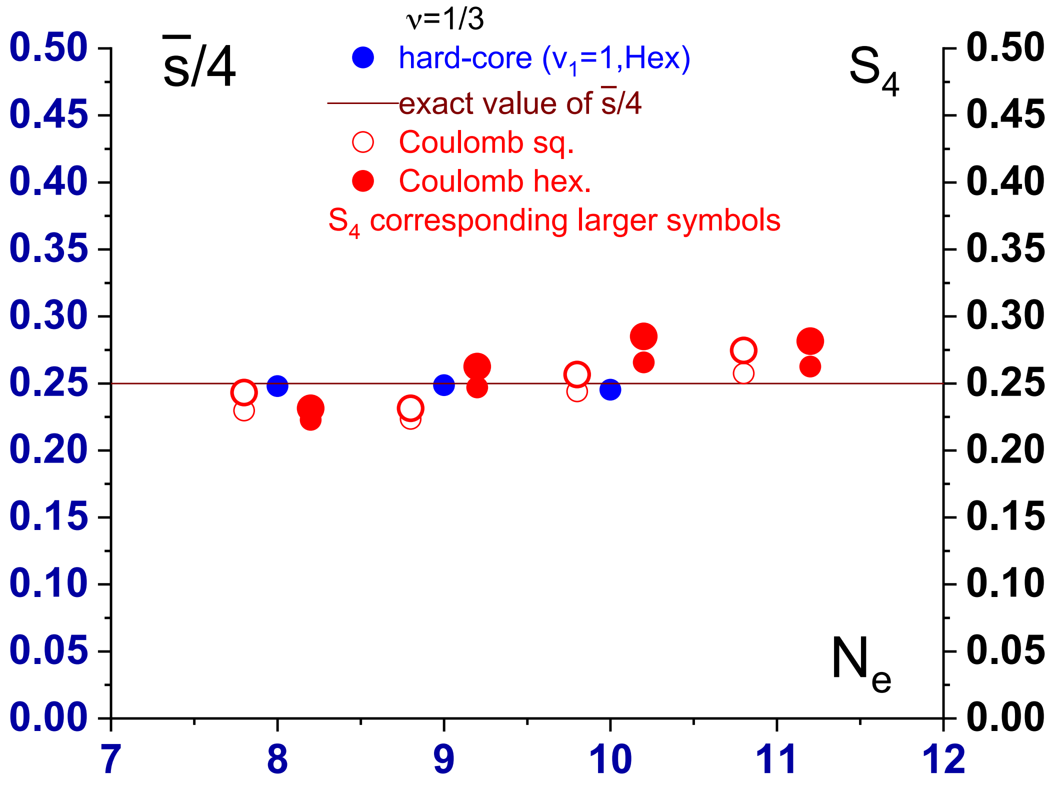




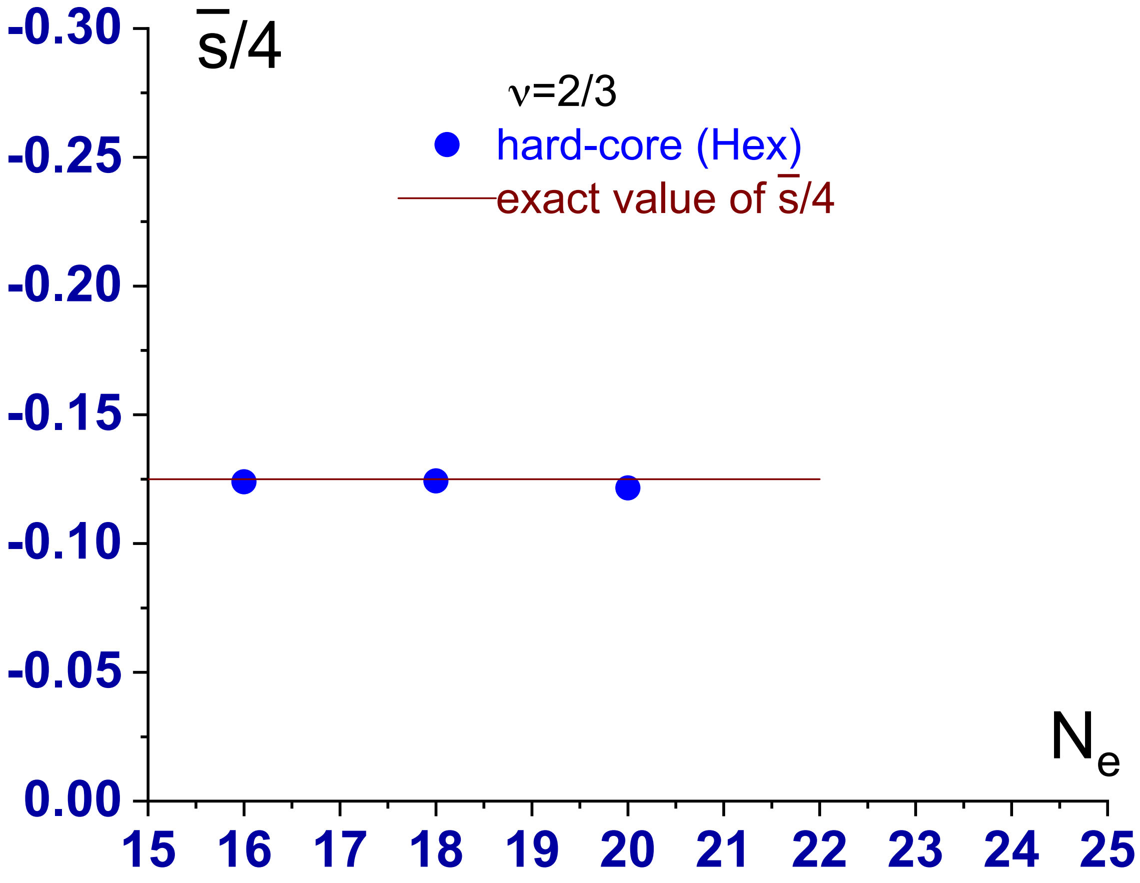
- State $\nu = 2/5$.
- No high-energy mode.
- WZ shift sum rule: $\frac{\bar{s}}{4} = \frac{3}{8} = 0.375$

$\bar{s}$ is the average guiding center spin per electron

$n = 1$



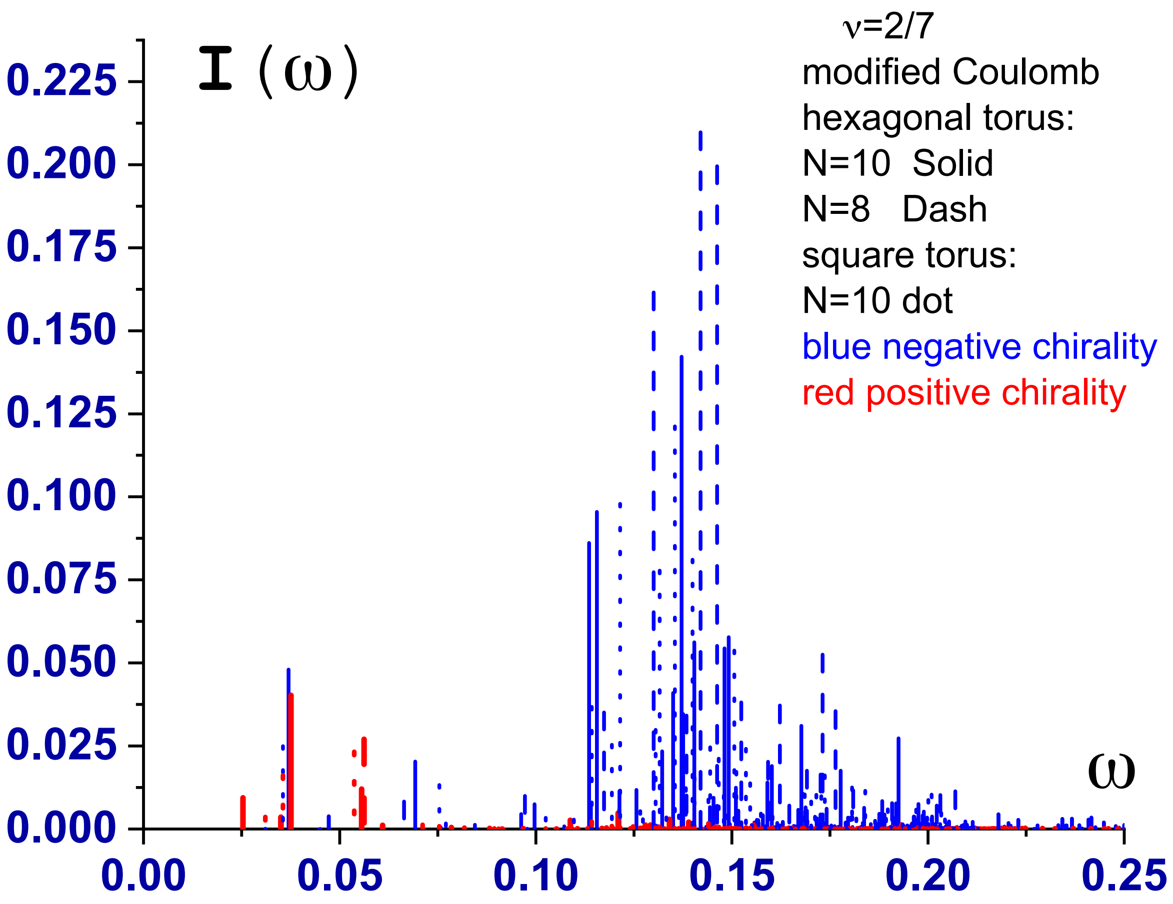
- State $\nu = 1/3$.
- Graviton with negative chirality
- No high-energy mode.
- WZ shift sum rule: $\frac{\bar{s}}{4} = \frac{1}{4} = 0.25$

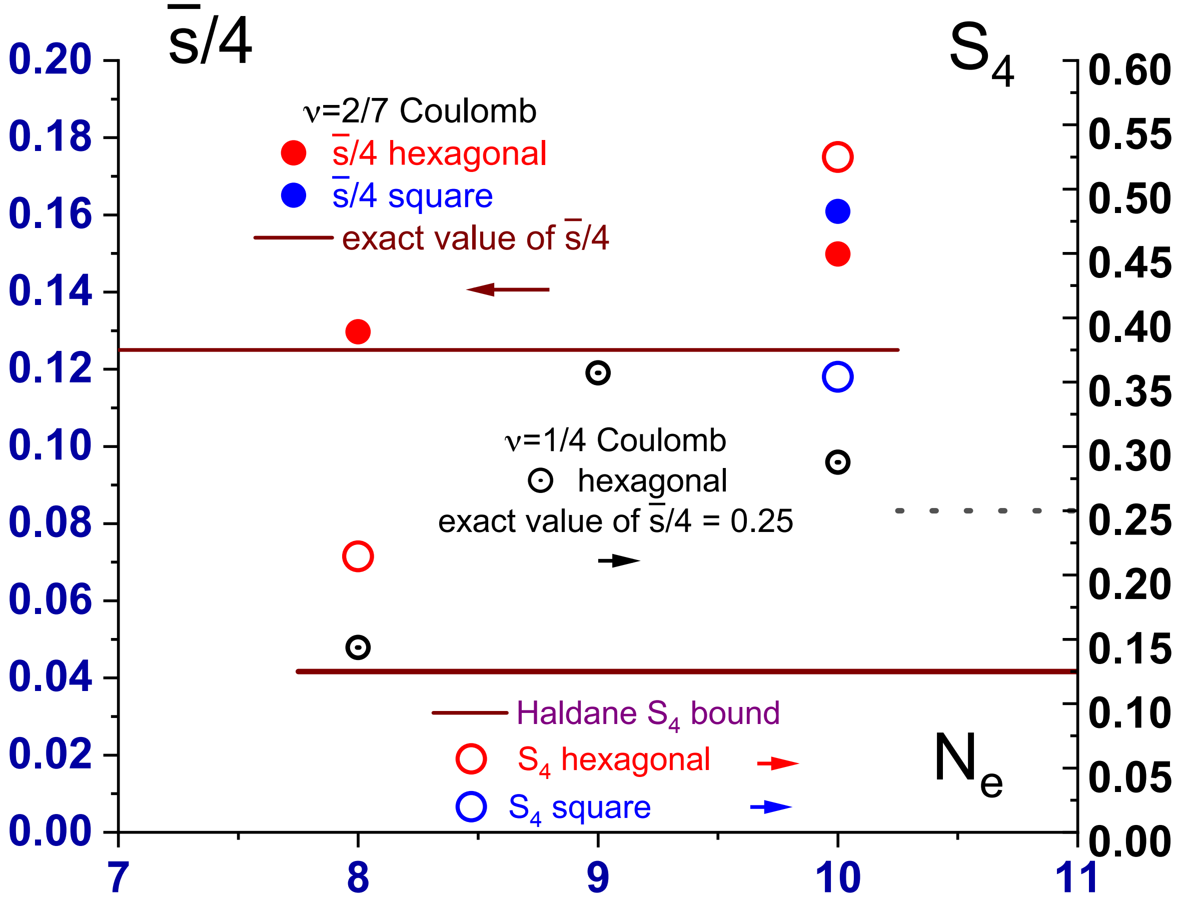


- State $\nu = 2/3$.
- Graviton with positive chirality
- No high-energy mode.
- WZ shift sum rule: $\frac{\bar{s}}{4} = -\frac{1}{8} = -0.125$

$\nu\_, n = 2$

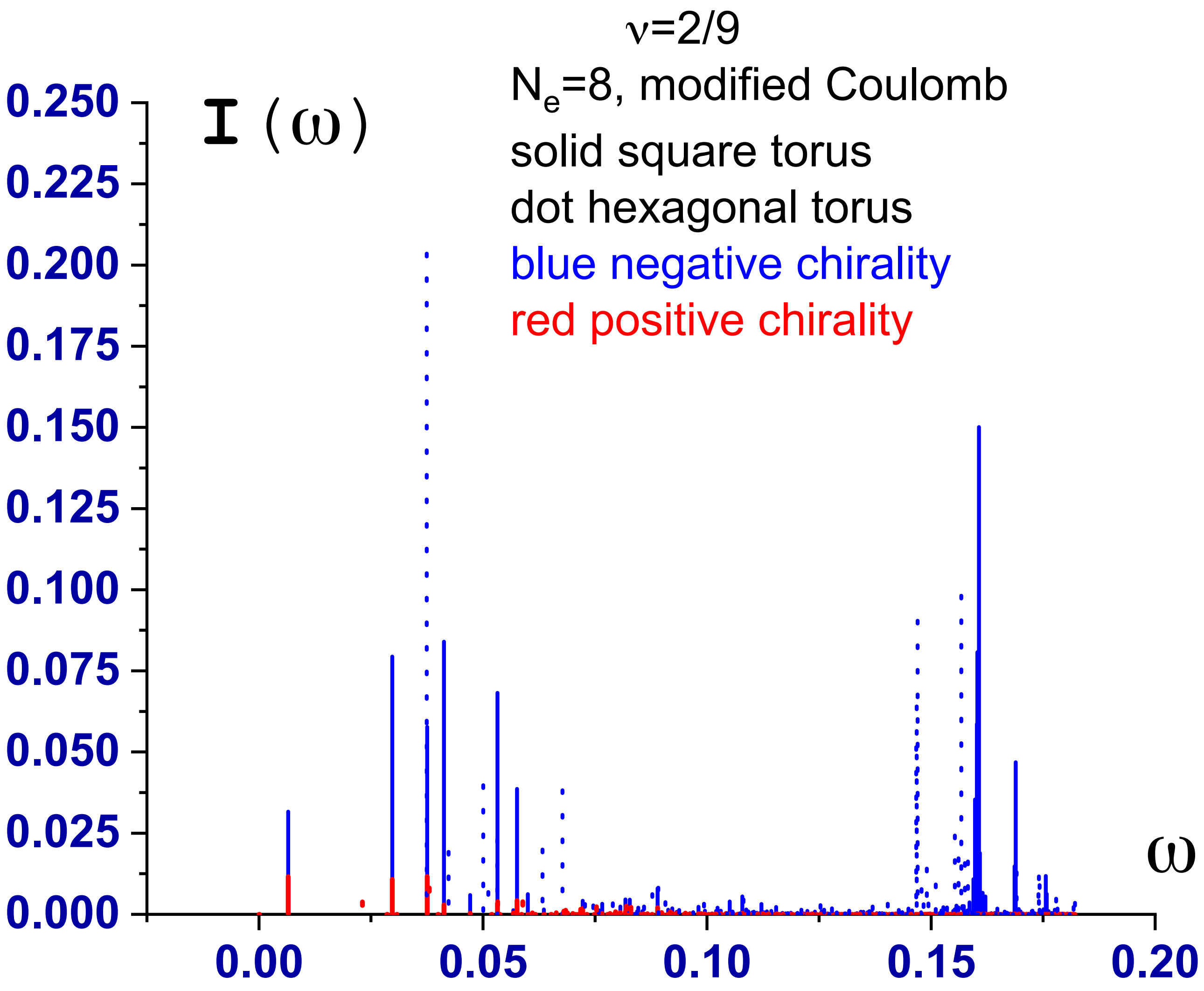
$$I\_(\omega) = \frac{1}{4}\omega^2\delta(\omega - \omega_H), \qquad I_+(\omega) = \frac{p-1}{8}\omega^2\delta(\omega - \omega_L)$$





- State $\nu = 2/7$.
- High energy graviton has negative chirality
- Low energy graviton has positive chirality
- WZ shift sum rule: $\frac{\bar{s}}{4} = \frac{1}{8} = 0.125$

$\nu_+, n = 2$

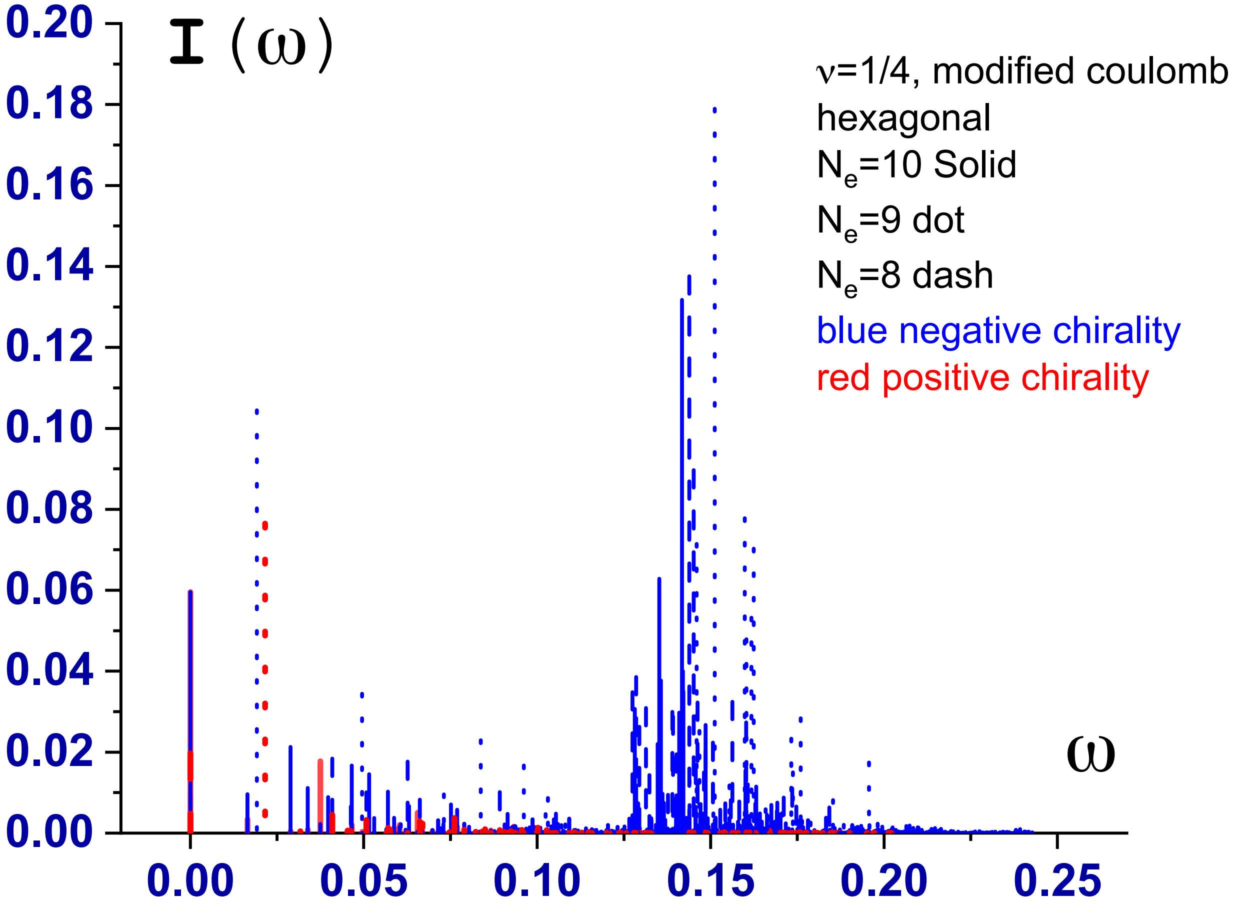


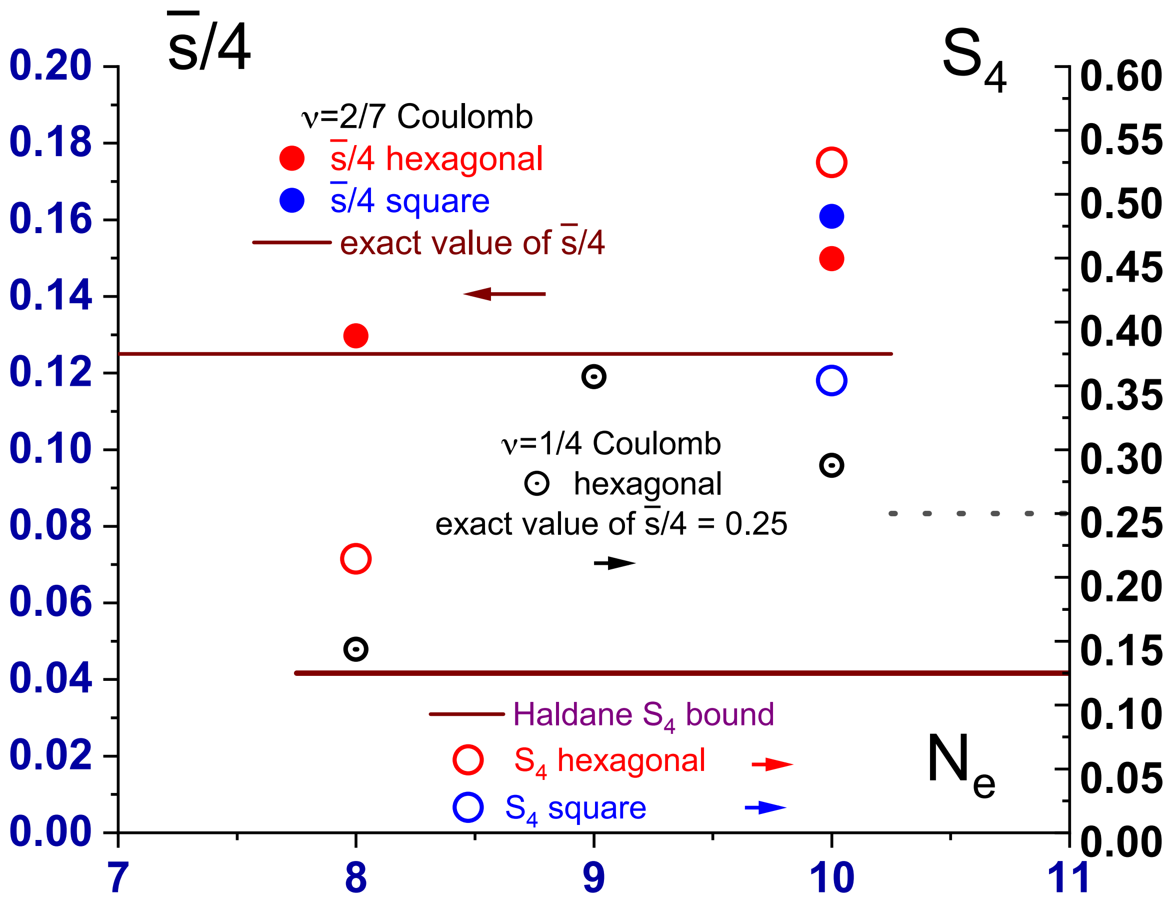
$$I\_(\omega) = \frac{p+1}{8}\omega^2\delta(\omega-\omega_L) + \omega^2\frac{1}{4}\delta(\omega-\omega_H),$$

$$I_+(\omega) = 0$$

- State $\nu = 2/9$.
- High energy graviton has negative chirality
- Low energy graviton has negative chirality
- The system size isn't big enough to has the meaningful numerical result of $\bar{s}$

Fermi liquid state $\nu = 1/4$





- $\dfrac{\bar{s}}{4} = \dfrac{1}{4} = 0.25$
- High energy graviton has negative chirality
- Low energy gravitons have both chiralities with equal weight.
- Graviton's spectrum of FL state $\nu = 1/2$ expected to be similar as the low energy graviton spectrum of $\nu = 1/4 \rightarrow$ two chiralities with equal weight (Particle-hole symmetry).

Probe gravitons in the bulk

DXN, Son Phys. Rev. Research **3**, 023040

Raman scattering experiment

Energy momentum conservation:

- Momentum of incident and scattered lights $\mathbf{k}_I, \mathbf{k}_S$.
- Frequency of incident and scattered lights ω_I, ω_S
- Magneto-roton's momentum and energy (dispersion)

$$\mathbf{q} = \mathbf{k}_I - \mathbf{k}_S, \quad \omega = \omega_I - \omega_S$$

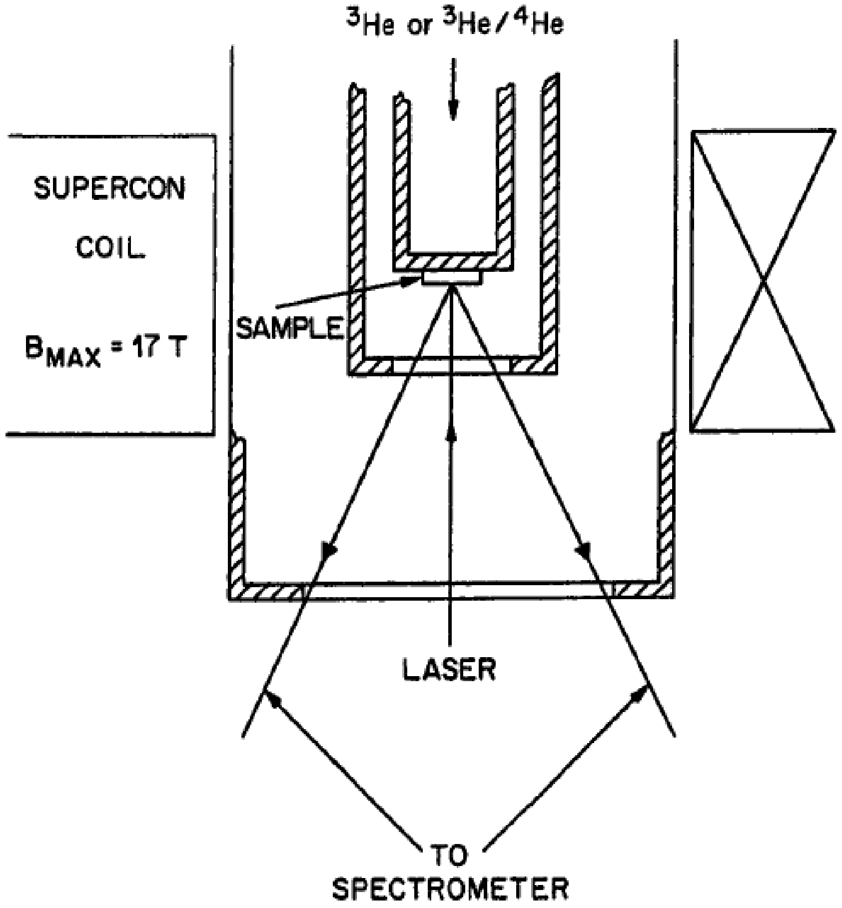


Figure 8.3. Schematic representation of the low-temperature inelastic light-scattering experiment. The magnetic field is perpendicular to the two-dimensional electron layer. In this configuration the in-plane component of the light-scattering wavevector is $k < 10^4\,\mathrm{cm}^{-1}$.

- In previous theoretical model (Platzman, He 1994), the scattered light intensity I_S measures the dynamic structure factor $S = \langle \rho\rho \rangle$

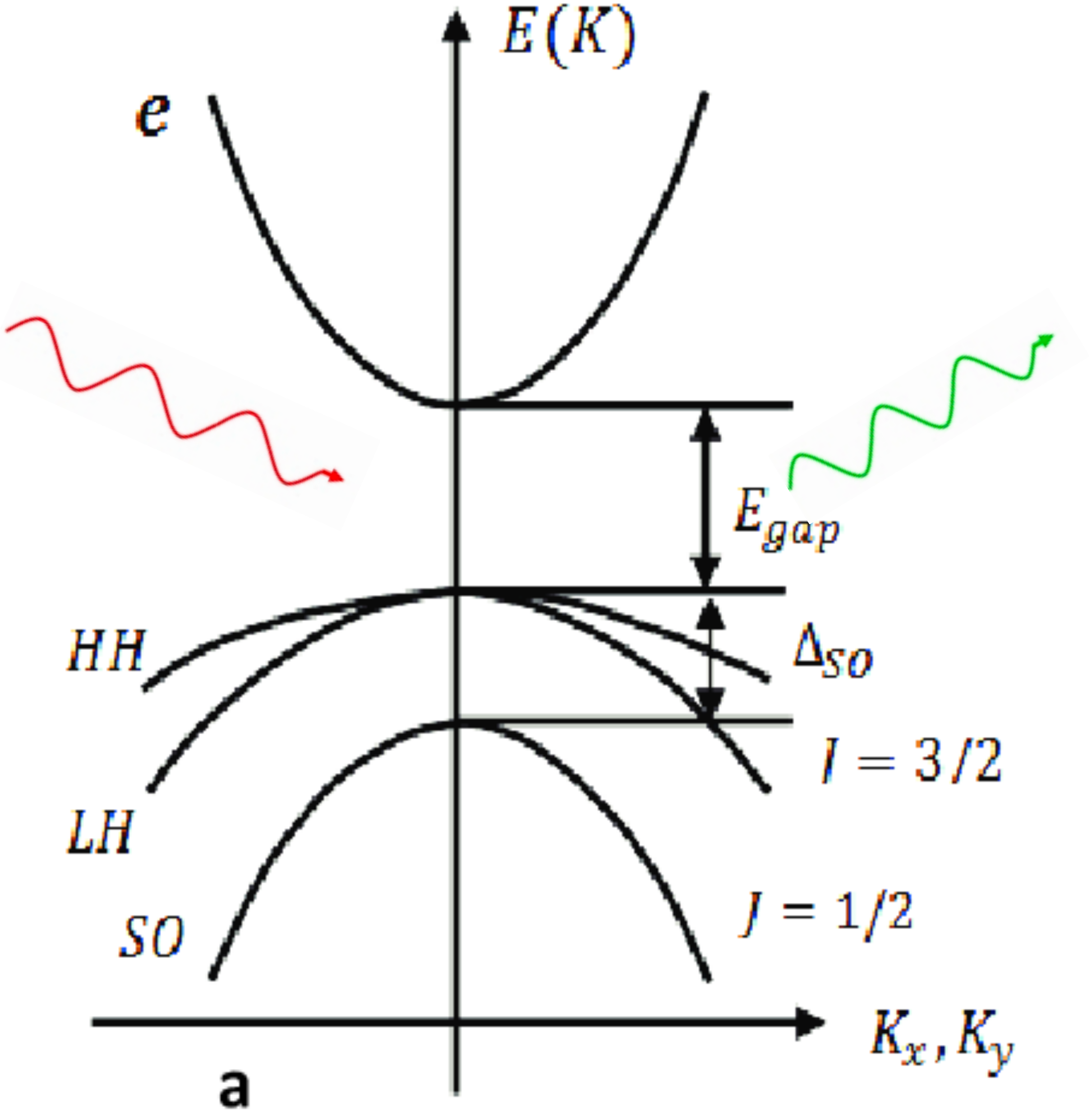
$$I_S(\omega, q) \sim S(\omega, q) \sim q^4 \qquad \text{(GMP 1986)}$$

→ Can't explain the peak at zero momentum $q \sim 0$

Resonant Raman scattering

- $E_\gamma \approx E_{\text{gap}}$
- $\gamma_I \rightarrow$ particle-hole pair
- $\rightarrow \gamma_S$+excitation

$$\omega_I, \omega_S \gg \omega$$



Luttinger model of GaAs

Effective coupling

- After performing second-order perturbation theory (integrating out hole-bands): coupling of EM field to electron in the conductance band

- The effective coupling for circular polarised Raman scattering

$$\mathscr{L}_{\gamma\psi} \sim E^z E^z (T_{zz} + 0.16\, T_{\bar{z}\bar{z}}) + \text{h.c.} \qquad (z = x + iy)$$

  - predominantly spin-conserving, not changing by 4 (dues to slightly broken rotational symmetry of the Luttinger model)

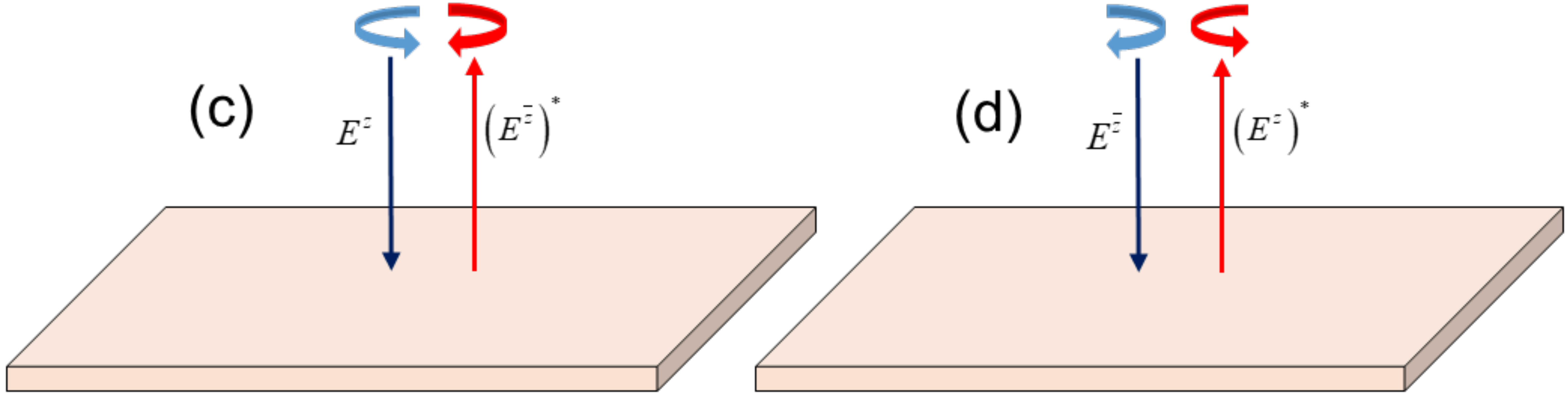
- The scattered light intensity measures the stress tensor correlation function

$$I_S(q) \sim \langle TT \rangle \sim q^0$$

Explains the peak in Raman scattering near $q = 0$

Determining the spin structure of the magnetoroton

- Use circular polarized Raman scattering with $\mathbf{q} = 0$



$I_S(\omega,0)$ measures $I\_(\omega)$

Large peak if magnero-roton has spin 2

$I_S(\omega,0)$ measures $I_+(\omega)$

Large peak if magnero-roton has spin -2

Prediction: dominance by one sign of photon spin flip for chiral sates

Conclusions:

- We introduce massive graviton excitation(s) in FQH which are mangneto-roton(s). (And higher spin fields)
- We proposed the microscopic description of the spin-2 modes.
- **Gravitons' chirality** and **gravitational spectral sum rules** are verified numerically.
- Massive gravitons in FQH can be probed in experiments.
- Dirac composite fermion model is an example of duality (electrons ↔ CFs) in field theory.
    - Lowest Landau level → higher rank symmetry

Du, Mehta, DXN, Son, **arXiv:2103.09826**