Black Holes in the $^{1}/_{D}$ expansion

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Who

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Keisuke Izumi

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Kentaro Tanabe

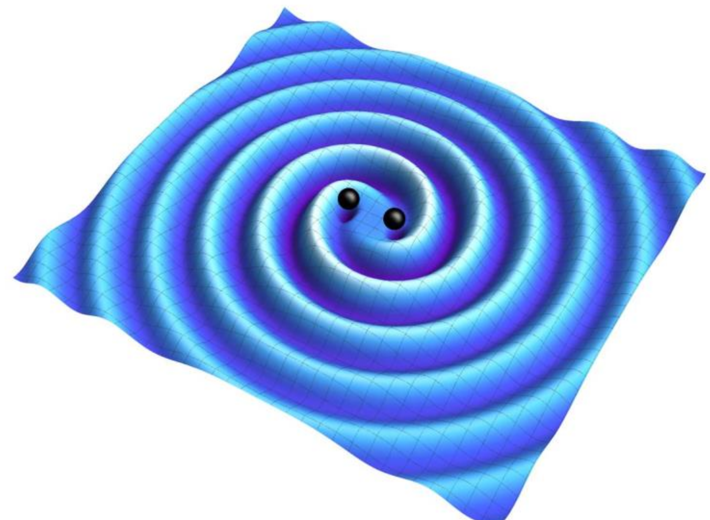
Takahiro Tanaka

Parallel line of work: Bhattacharya, Minwalla et al

Why

The Too Perfect Theory

$$R_{\mu\nu} = 0$$



No scale

No parameter

Fiendish complexity

D-diml General Relativity

Well-defined for all D

Many problems can be formulated keeping D arbitrary

$\rightarrow$ D = continuous parameter

$\rightarrow$ expand in 1/D

Large-D in General Relativity

$D^2 \sim$ # local degrees of freedom at a point

akin to Large N SU(N) gauge theory

$D \sim$ # connections between nearby points

= directions out of a point

akin to Mean Field Theory limit in Stat Mech

What's useful is

D^2 ~ # local degrees of freedom at a point

akin to Large N SU(N) gauge theory

D ~ # connections between nearby points

= directions out of a point

Exploit large gradients of gravitational potential $\frac{1}{r^{D-3}}$

D-diml General Relativity

Large D:

Keeps essential physics of D=4

∃ black holes

∃ gravitational waves

What

BH in D dimensions

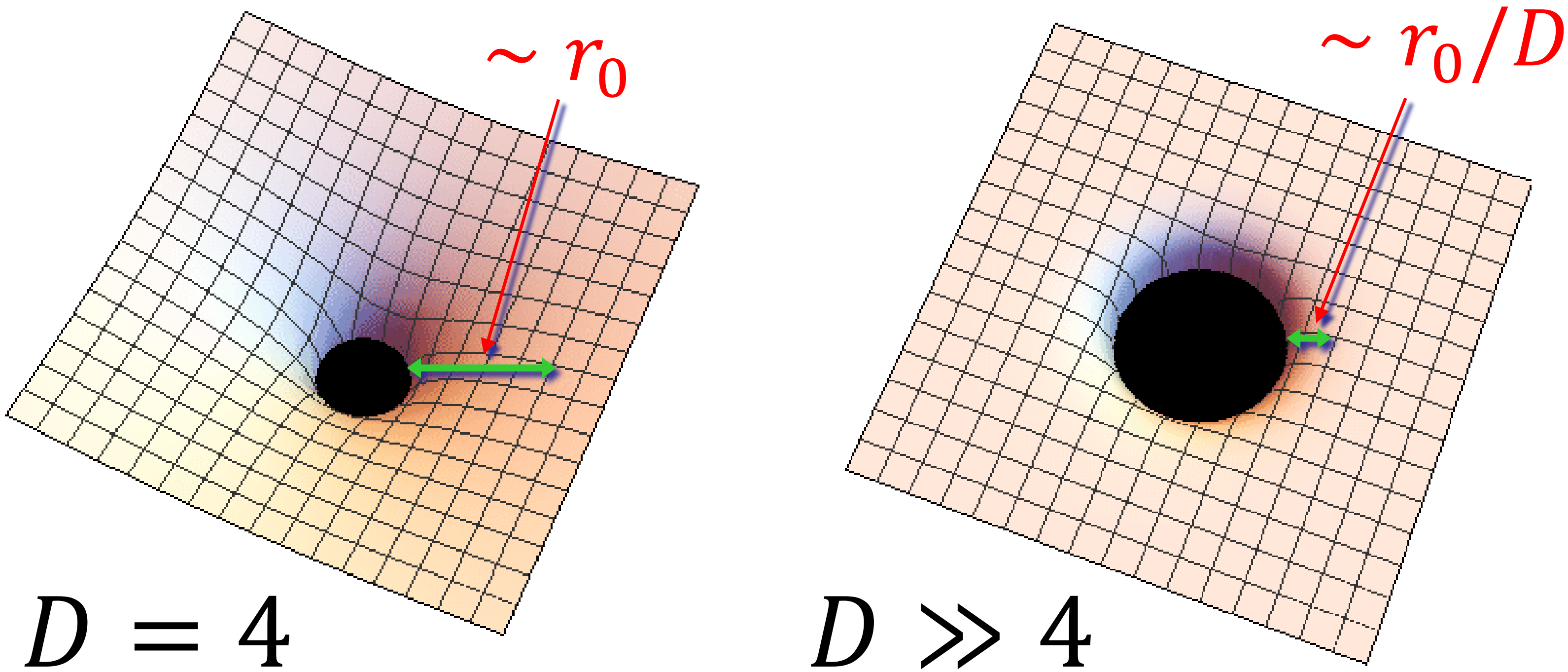
$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi|_{r_0} \sim \frac{D}{r_0} \gg \frac{1}{r_0}$$

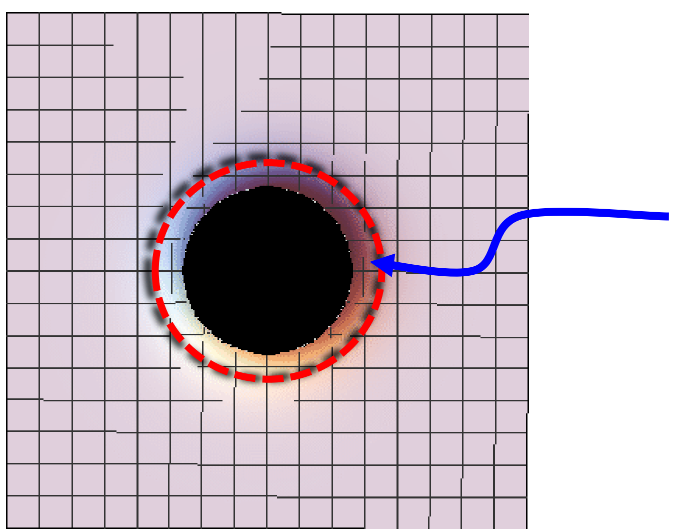
Large D introduces new, parametrically separated scales

$$r_0 \gg \frac{r_0}{D}$$


~ r_0
~ r_0/D
$D = 4$
$D \gg 4$

∃ well-defined, universal near-horizon geometry

Take $D \to \infty$ keeping finite $\left(\frac{r}{r_0}\right)^{D-3}$



$r - r_0 \sim \frac{r_0}{D}$

Goal

Understand & Reformulate
Black Hole dynamics
in 1/D expansion

Small fluctuations of black hole horizon

Quasinormal modes

Most QN modes have high frequencies

$$\omega \sim D/r_0$$

$$\frac{D}{r_0} \sim \text{surface gravity: natural frequency}$$

Featureless oscillations of a hole in space

A few slow, long-lived QN modes localized in near-horizon región

$$\omega \sim \frac{1}{r_0}$$

Decoupled from far-zone

They capture ***interesting horizon dynamics***

Hydro-like (sound & shear fluctuations)

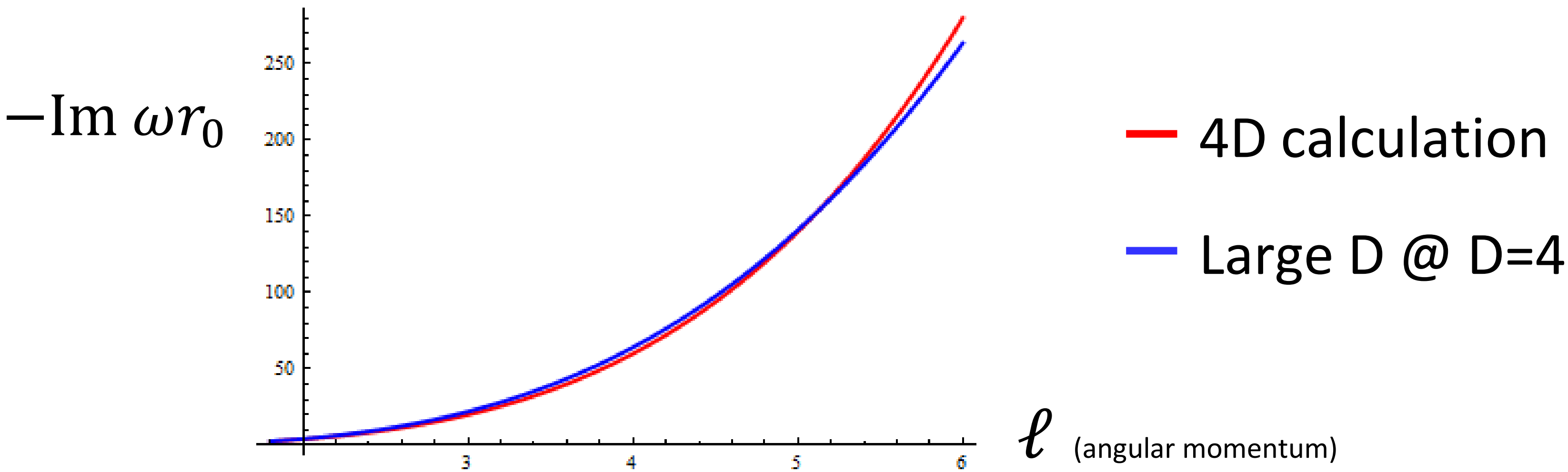
Have 4D counterparts

Large D can be a very good approximation

for moderate, even small D

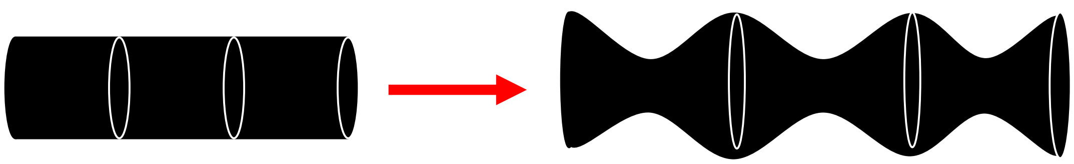
Quasinormal frequency of Schw bh in $D = 4$

(vector-type)



6% accuracy in $D = 4$

Gregory-Laflamme threshold mode of black string

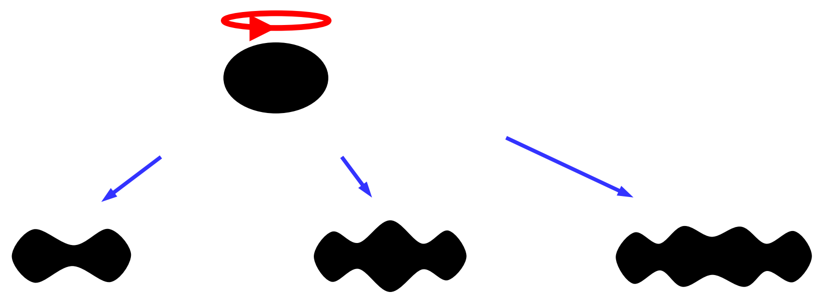


$$k_{GL}|_{\boldsymbol{D=6}} = 1.238 \quad \text{large-D analytical}$$

$$1.269 \quad \text{numerical}$$

2.4% accuracy

Ultraspinning bifurcations of Myers-Perry black holes



Numerical: $\frac{a}{r_+} = 1.77, \quad 2.27, \quad 2.72 \ldots$ (D=8)

Dias et al

Large D: $\frac{a}{r_+} = \sqrt{3}, \quad \sqrt{5}, \quad \sqrt{7}, \ldots$

Suzuki+Tanabe

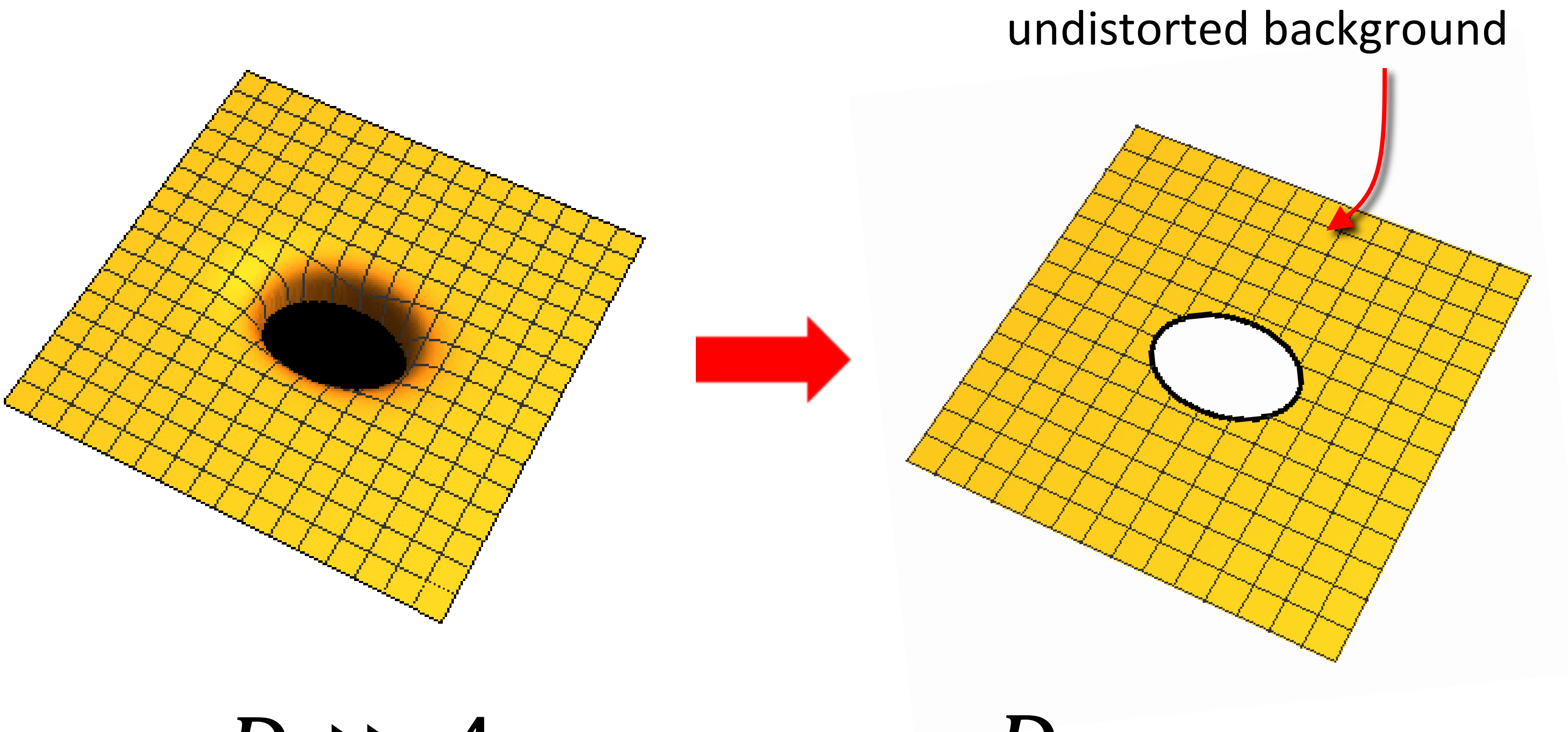
Error $\lesssim$ 2.7%

Non-linear fluctuations of black hole horizon

Effective Theory
of *decoupled* dynamics
of black holes

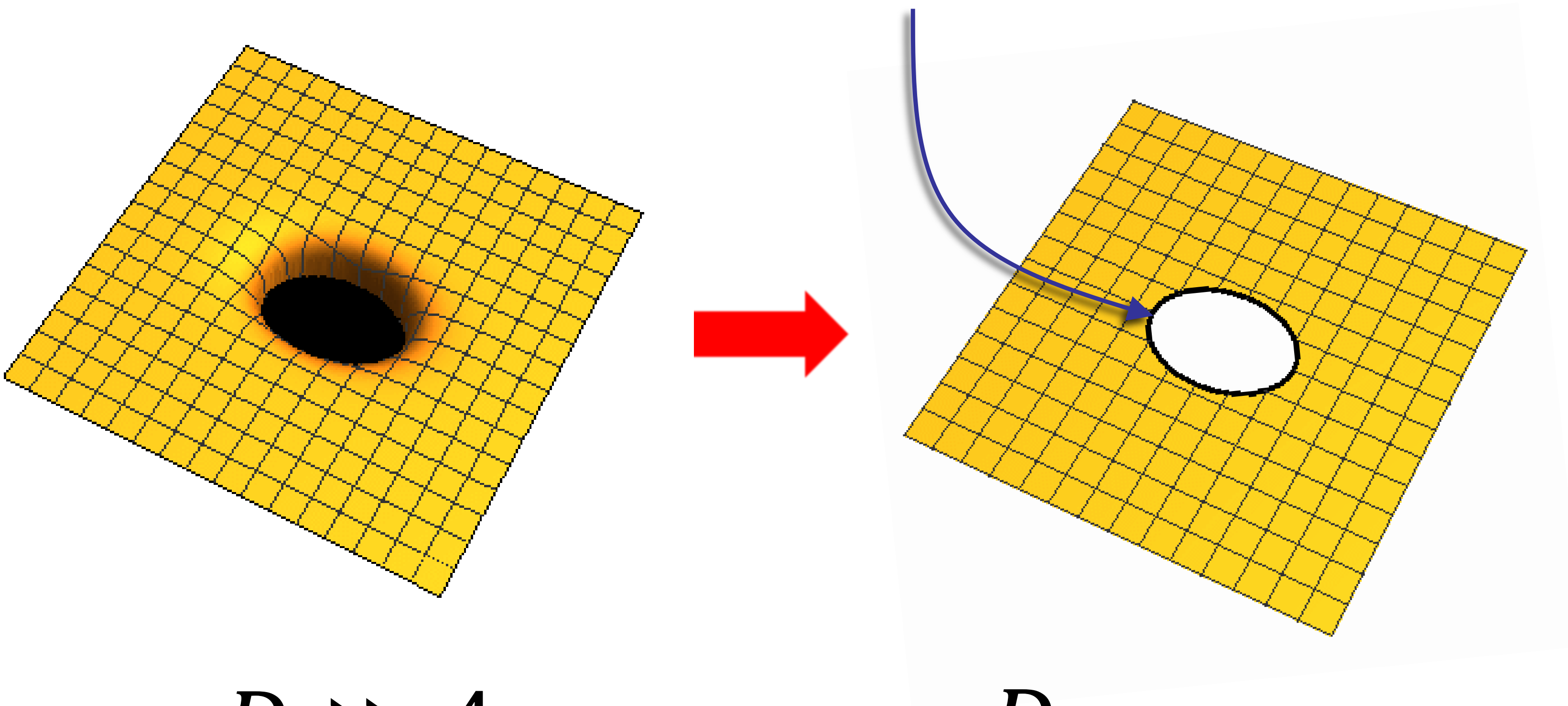
How

Replace bh → Surface ('membrane')



$D \gg 4$ $D \to \infty$

What’s the dynamics of this membrane?

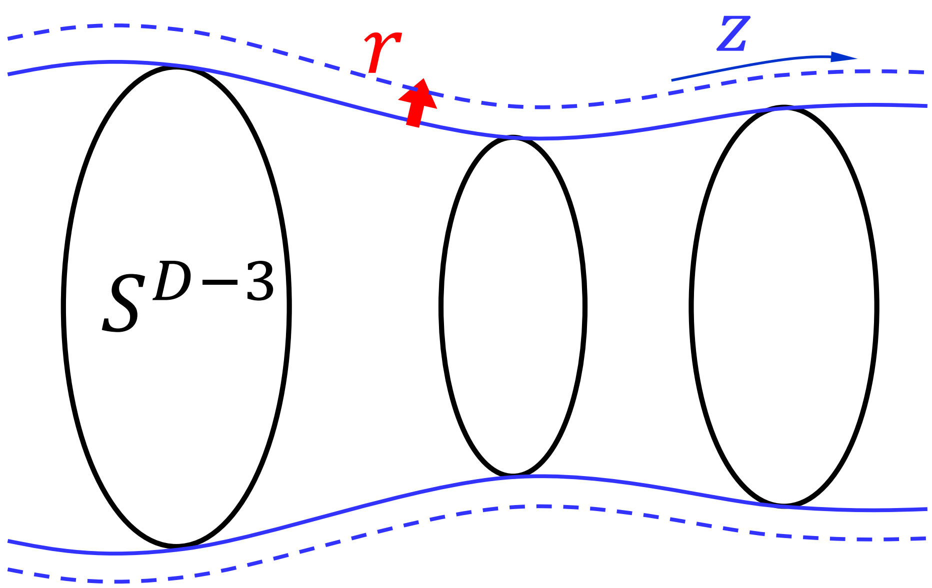


$D \gg 4$ $D \to \infty$

Gradient hierarchy

⊥ Horizon: $\partial_r \sim D$

∥ Horizon: $\partial_z \sim 1$ (or $\sim \sqrt{D}$)



Stationary solution

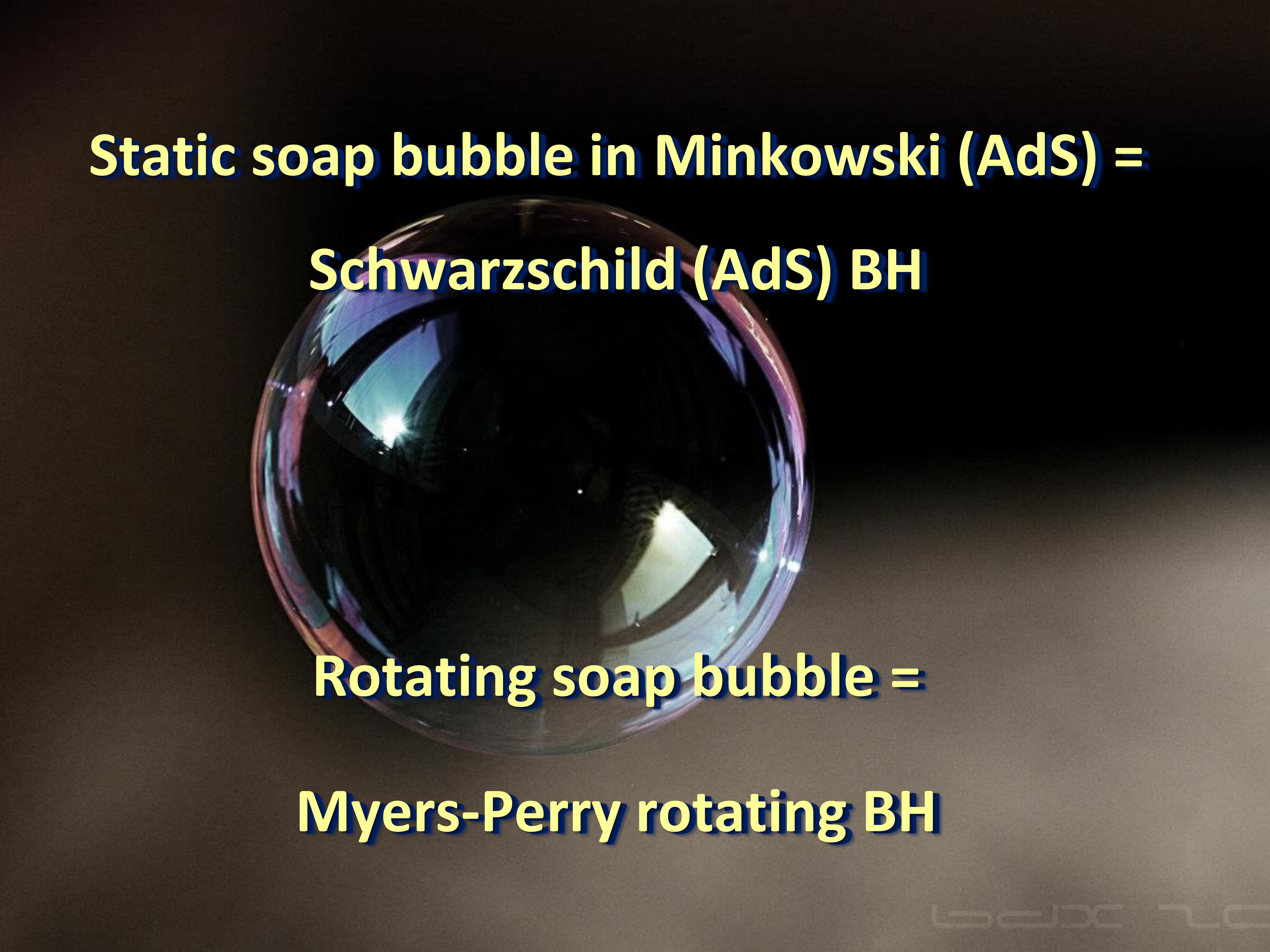
Soap-bubble equation (redshifted)

$$K = 2\gamma\kappa$$

$K =$ trace **extrinsic curvature** of membrane

$\gamma =$ **redshift** factor on membrane

$\kappa =$ **surface gravity**


Static soap bubble in Minkowski (AdS) =
Schwarzschild (AdS) BH
Rotating soap bubble =
Myers-Perry rotating BH

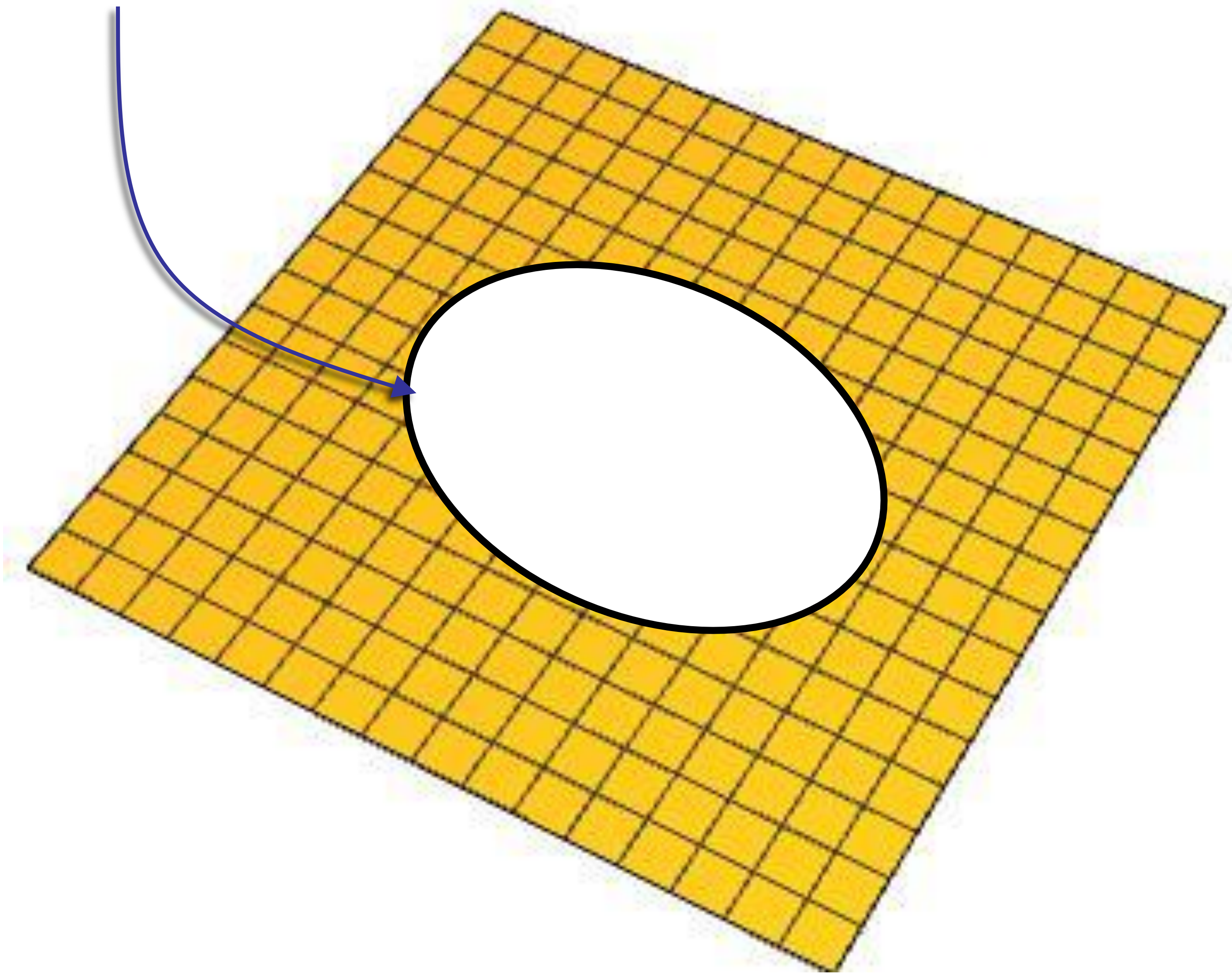
Time-dependent effective theory of black holes

It consists of two elements:

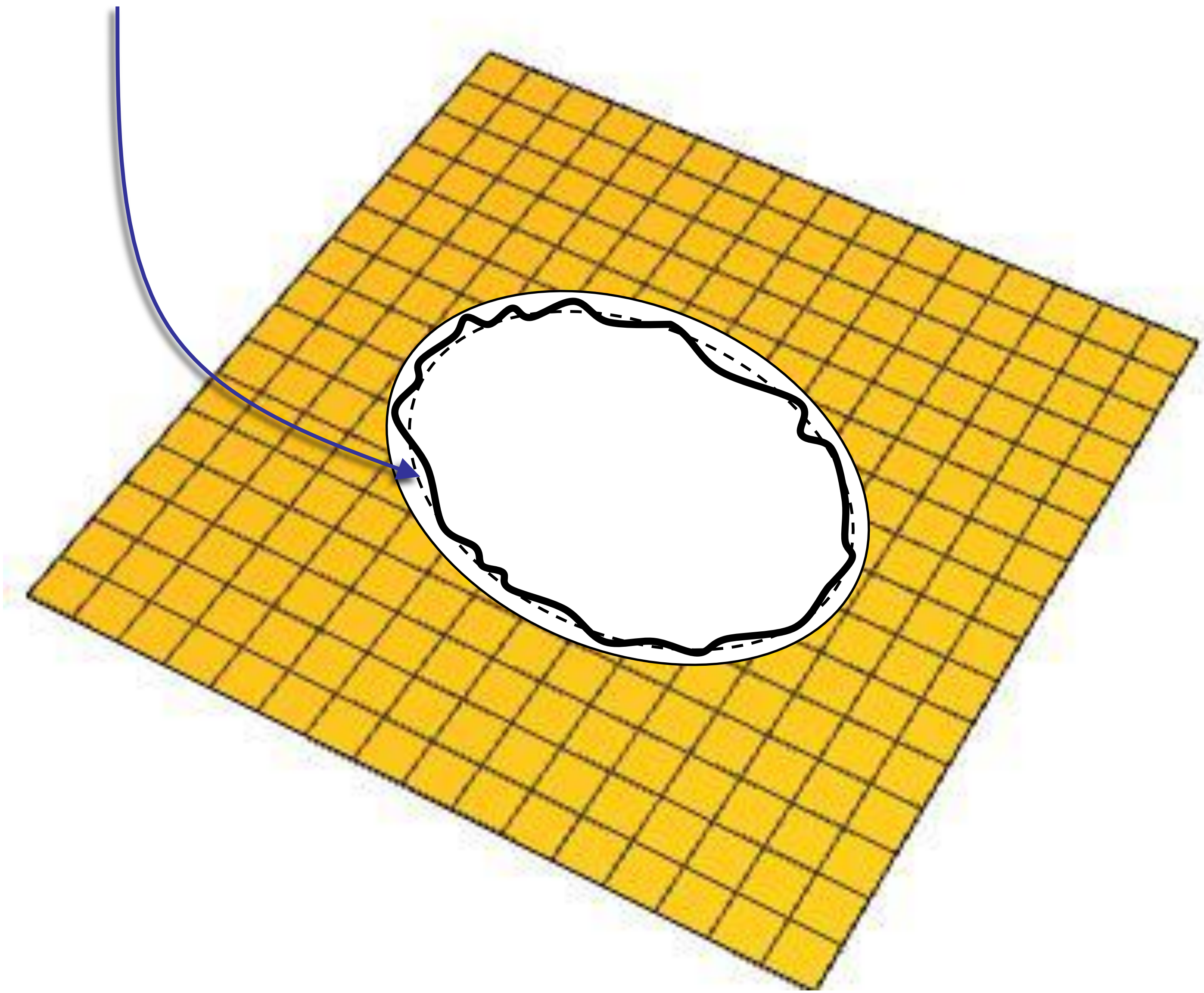
a stationary embedding in the background

a hydrodynamic-like theory of fluctuations of the embedding, with amplitude $\sim 1/D$

Stationary shape (soap-bubble)



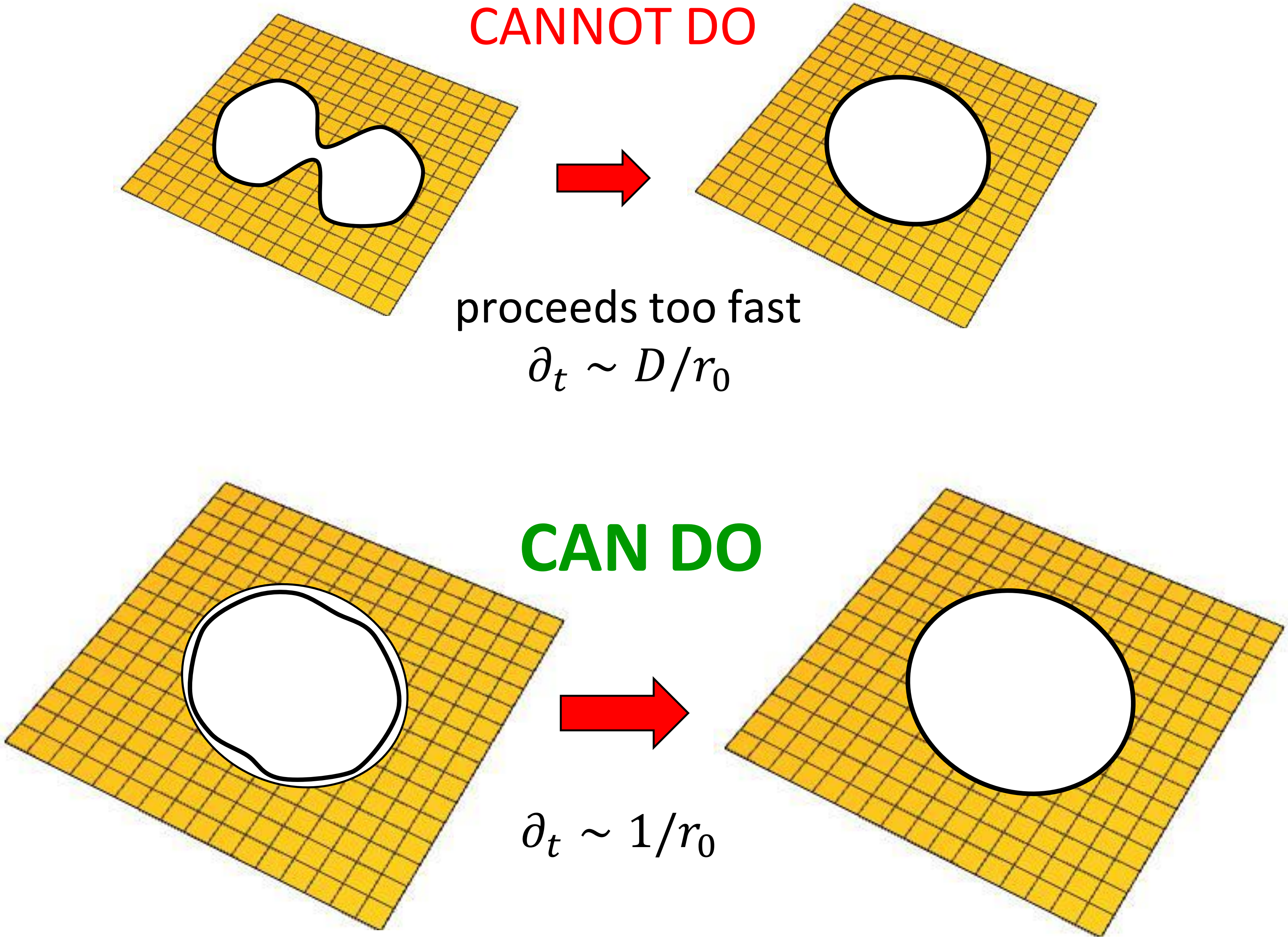
Non-linear fluctuations of amplitude 1/D



Dynamic fluctuations decouple from far-zone

only as long as they occur

within the near-zone


CANNOT DO
proceeds too fast
$\partial_t \sim D/r_0$
CAN DO
$\partial_t \sim 1/r_0$

Effective equations for black brane

effective fields for fluctuating horizon

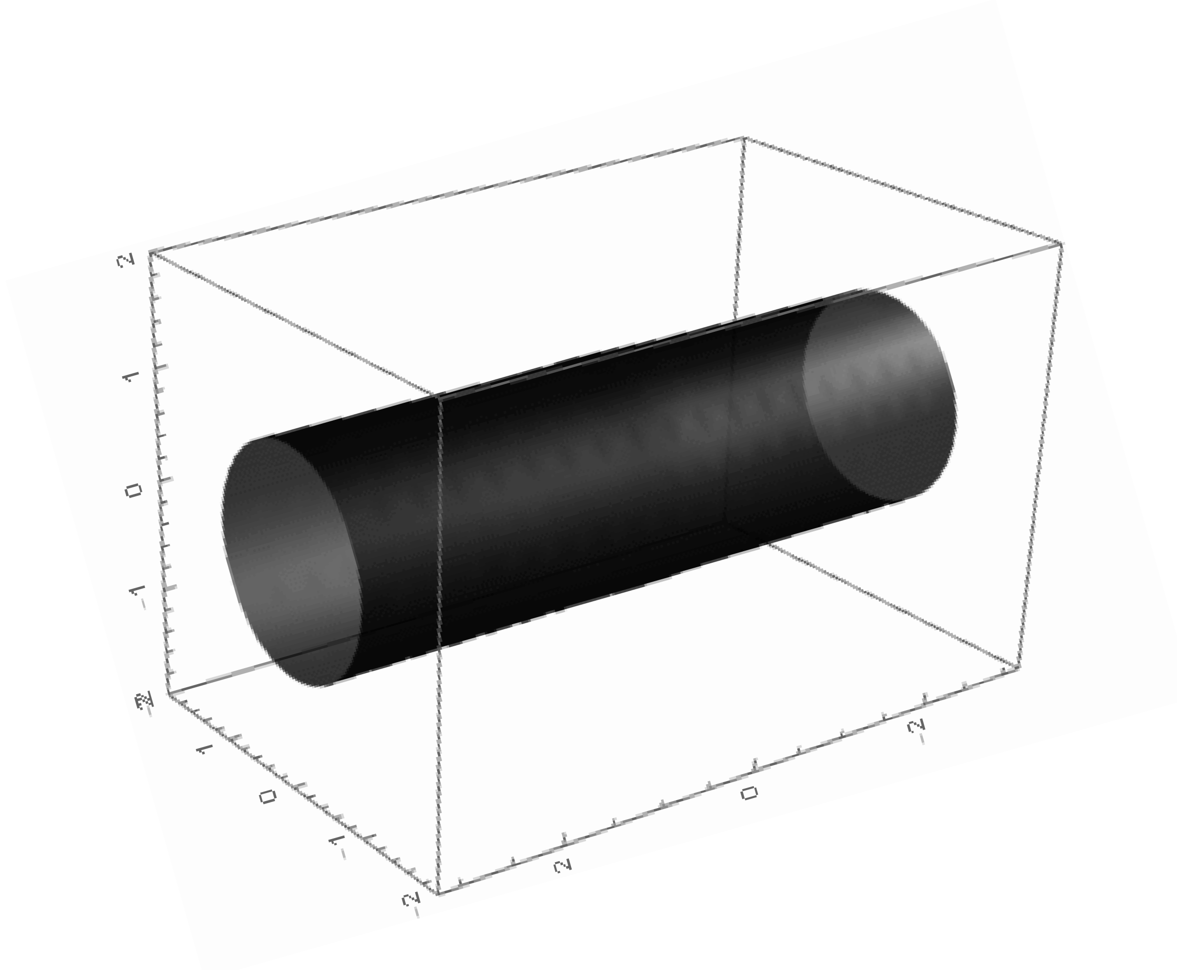
$$\rho(t, z^i)\,,\ v_i(t, z^j)$$

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

$$\partial_t(\rho v_i) + \partial^j \left(\pm\rho\, \delta_{ij} + \rho v_i v_j - 2\,\rho\, \partial_{(i} v_{j)} - \rho\, \partial^2_{ij} \ln\rho\right) = 0$$

Hydrodynamic-like, but truncate exactly

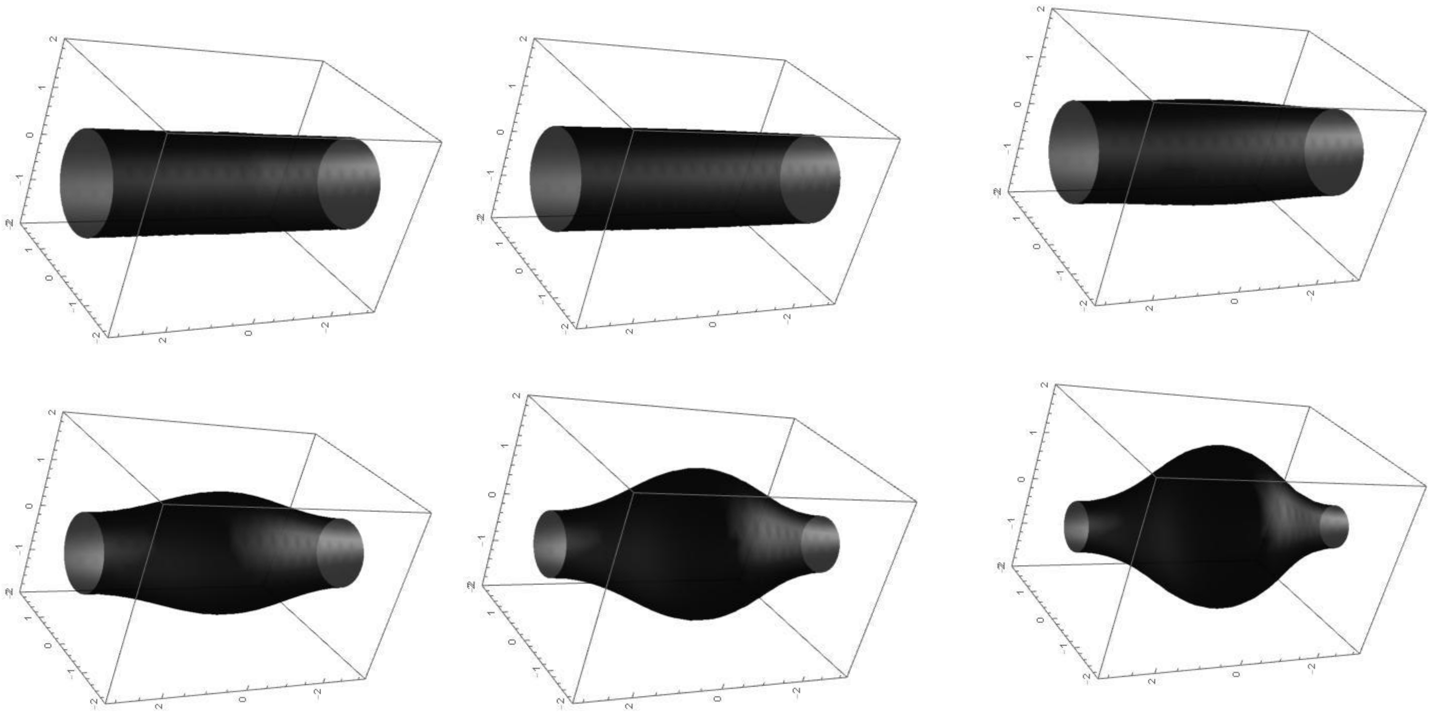
Can study phenomena at finite wavelengths

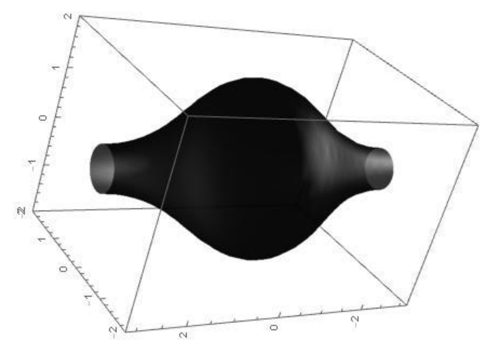


$D \to \infty$ endpoint:

stable non-uniform black string

Horowitz+Maeda





$D \to \infty$ endpoint:

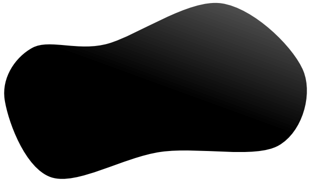
stable non-uniform black string

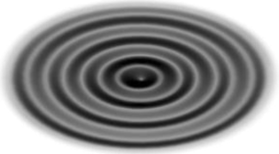
Horowitz+Maeda

So

The main thing we've learned so far

Large D is very efficient for describing and solving **horizon deformations and fluctuations**





Thanks