Interacting Giant Gravitons from Spin Matrix Theory

Troels Harmark, Niels Bohr Institute

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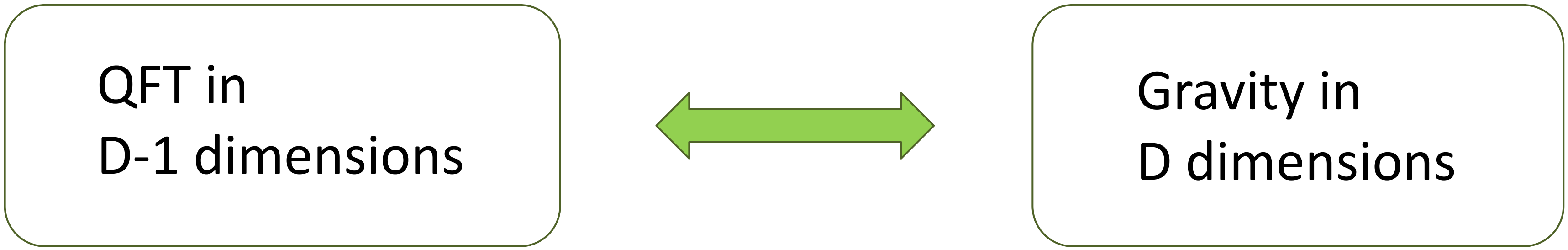
Talk mainly based on:

- "Interacting Giant Gravitons from Spin Matrix Theory" by TH, Phys. Rev. D94 (2016) no. 6, 066001 (ArXiv:1606.06296 [hep-th])
- "Spin Matrix Theory: A Quantum Mechanical Model of the AdS/CFT Correspondence" by TH and Orselli, JHEP 1411:134 (arXiv:1409.4417 [hep-th])

Question: How do space, time and gravity emerge from quantum theory?

Answer: Holographic duality



Can in principle solve many questions:

- What is space? Time? Gravity?
- Black holes?
- Confinement in QCD?

But to address these questions one needs quantitative framework

The AdS/CFT correspondence:

$\mathcal{N}$=4 SYM theory
with gauge group SU(N)

't Hooft coupling:

$$\lambda = g_{\rm YM}^2 N$$

$$g_s = \frac{\lambda}{N}$$

$$R^4 = \lambda l_s^4$$

Type IIB string theory
on AdS_5 x S^5

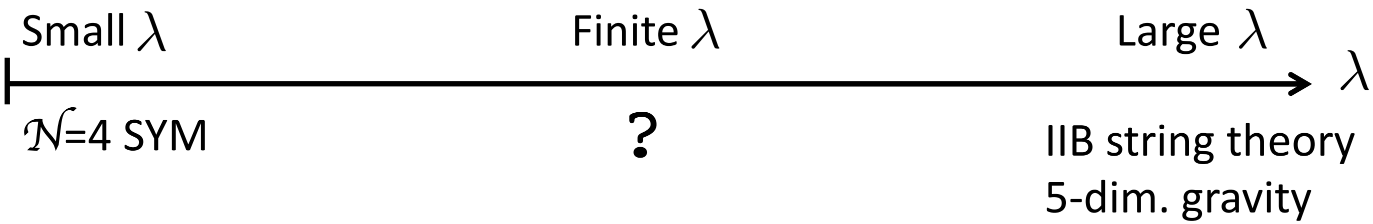
R: Radius of AdS_5 and S^5
g_s: string coupling
l_s: string length

Planar regime: N = ∞ ⟷ Tree-level string theory: g_s = 0

1/N corrections ⟷ Perturbative expansion in g_s

Finite-N effects ⟷ Non-perturbative string theory: D-branes, Black holes

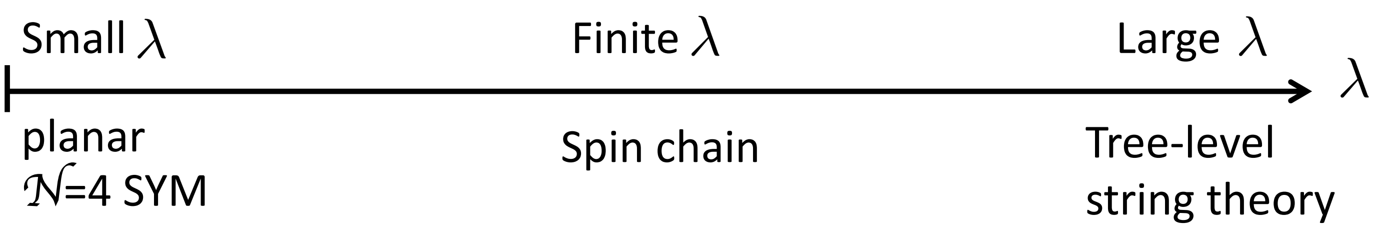
When are the two dual sides a valid description?



How can we make a quantitative connection between the two sides ?

We need a unifying framework to interpolate between weak and strong coupling

In planar regime ($N = \infty$ and $g_s = 0$) we have a unifying framework:



Can we find a unifying framework that generalizes the spin chain beyond this?

Can we find a unifying framework of AdS/CFT for finite, large N?

A finite N generalization of the spin chain?

$\mathcal{N}$=4 SYM simplifies near unitarity bounds / zero-temperature critical points:

Effective description by **Spin Matrix Theory**

TH & Orselli 2014

What is Spin Matrix theory? A well-defined quantum mechanical theory

Hilbert space built from harmonic oscillators:

$$(a_s^\dagger)^i{}_j$$

s: Index for representation of (super) Lie group (the "spin" group)

i,j: Matrix indices for adjoint representation of U(N)

Extra demand:
Only singlets of U(N)

$$\mathrm{Tr}(a_{s_1}^\dagger a_{s_2}^\dagger \cdots a_{s_k}^\dagger)\,\mathrm{Tr}(a_{s_{k+1}}^\dagger \cdots)\cdots\mathrm{Tr}(\cdots a_{s_L}^\dagger)|0\rangle$$

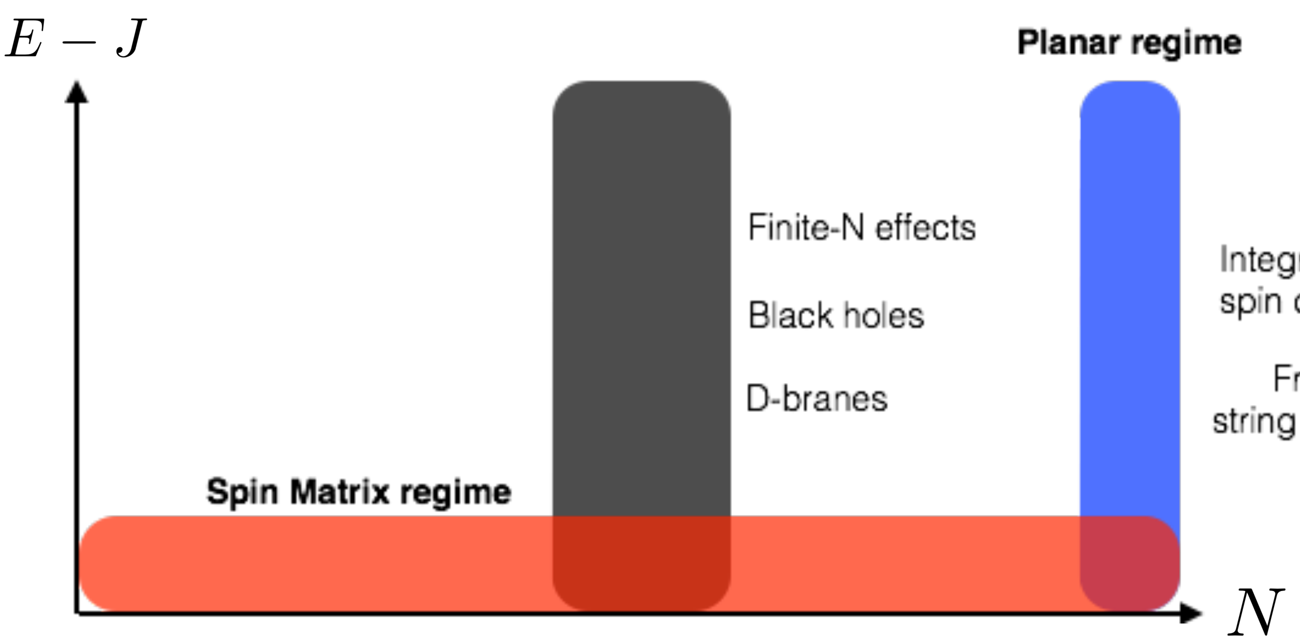
Interaction Hamiltonian: 1) Annihilates 2 excitations, creates 2 new ones.
2) Commutes with "spin" generators. 3) "spin" and "matrix" parts factorize.

For $N \to \infty$: Spin Matrix Theory reduces to a nearest neighbor spin chain

For a given unitarity bound: $E \geq J$

The planar regime: $N \to \infty$ with $E - J$ fixed

The Spin Matrix regime: $E - J \to 0$ with N fixed



Spin Matrix regime includes SUSY states with E = J and finite N

Spin Matrix Theory from $\mathcal{N}$=4 SYM near unitarity bound:

For a given unitarity bound: $E \geq J$

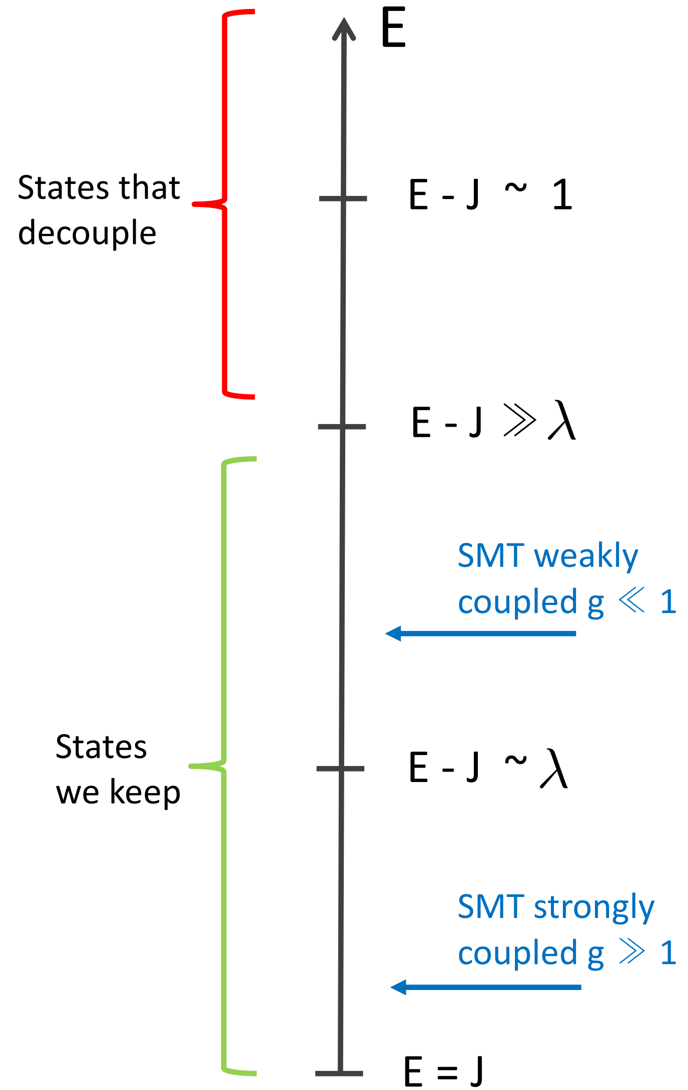
SMT limit:

$$H = J + \lim_{\lambda \to 0} \frac{g}{\lambda}(E - J)$$

g: Coupling constant of Spin Matrix theory

E: Energy of states in $\mathcal{N}$=4 SYM on R x S^3
(in units of inverse radius of S^3)
= Scaling dim. of operator of $\mathcal{N}$=4 SYM on R^4

N is fixed in limit



Case 1: $E \geq J_1$

Angular momenta on S^3 : S_1 , S_2

R-charges: J_1 , J_2 , J_3

Berenstein's toy-model for AdS/CFT

Berenstein 2004

$$H = \mathrm{Tr}(a^\dagger a)$$

Singlet condition: $\Phi^i{}_j |\phi\rangle = 0$ with $\Phi^i{}_j = \sum_{k=1}^{N} \left[(a^\dagger)^i{}_k \, a^k{}_j - (a^\dagger)^k{}_j \, a^i{}_k \right]$

Spectrum: N bosons in a harmonic oscillator potential

At high energies: N decoupled harmonic oscillators

Hilbert space: ½ BPS states in $\mathcal{N}$=4 SYM

String side: Giant gravitons, LLM geometry

Case 2: $E \geq J_1 + J_2$

Angular momenta on S^3: S_1, S_2

R-charges: J_1, J_2, J_3

SU(2) Spin Matrix Theory

$$H = \mathrm{Tr}(a_1^\dagger a_1 + a_2^\dagger a_2) - \frac{g}{8\pi^2 N} \mathrm{Tr}([a_1^\dagger, a_2^\dagger][a_1, a_2])$$

Singlet condition:

$$\Phi^i{}_j |\phi\rangle = 0 \quad \text{with} \quad \Phi^i{}_j = \sum_{s=1}^{2} \sum_{k=1}^{N} \left[(a_s^\dagger)^i{}_k (a^s)^k{}_j - (a_s^\dagger)^k{}_j (a^s)^i{}_k \right]$$

Two tractable regimes:

The “planar regime”: N large and $H \ll N$

Described by the spin ½ ferromagnetic Heisenberg spin chain

The “matrix regime”: $H \gg N^2$

Described by classical matrix model

Planar regime:

For N large and $H \ll N$ the single traces are approximately independent

Single-trace of length J ⟷ Spin chain of length J $(J = J_1 + J_2)$

SU(2) SMT becomes a nearest neighbor spin chain:
The spin ½ ferromagnetic Heisenberg spin chain

Minahan & Zarembo 2002

Strong coupling limit $g \gg 1$:

Strong coupling limit zooms into low energy spectrum of spin chain for $J \gg 1$

Lowest excitations = magnons

In classical limit (many magnons): Described by Landau-Lifshitz sigma-model

Kruczenski 2003

$$I = \frac{J}{4\pi} \int dt \int_0^{2\pi} d\sigma \left[\sin\theta \dot{\phi} - \frac{\theta'^2 + \sin^2 \phi'^2}{4} \right]$$

Amazingly, one gets the same action from the string theory side, but seemingly in a different regime:

Gauge theory/SMT side: $g_s N \ll 1$ and $J \gg 1$

String theory side: $g_s N \gg 1$ and $J^2 \gg g_s N$

$\lambda = 4\pi g_s N$

→ The famous "one-loop match" in early post-BMN days

A coincidence? No, it is not! TH, Orselli & Kristjansson 2008

We can take the SMT limit also on the string theory side

We can take the SMT limit also on the string theory side TH, Orselli & Kristjansson 2008

$$H = J + \lim_{g_s \to 0} \frac{g}{4\pi g_s N}(E - J)$$

Consider the planar regime: We should take limit of the string sigma-model on AdS_5 x S^5 background

- Naively: We enter the quantum string regime is string tension goes like $\sqrt{g_s N}$

  However, in the actual limit, the sigma-model action remains large for large J and one gets a different effective string tension

- What about corrections to sigma-model? It is protected by 32 SUSY

- What about other modes? → They become infinitely heavy and decouple

- Zero-mode fluctuation contribution? → Absent due to SUSY of unitarity bound

A match between strongly coupled SU(2) SMT and string theory!

Can one do this in the Matrix regime as well?

Matrix regime:

For H ≫ N^2 the SU(2) SMT becomes approximately classical

We can find classical limit using coherent states:

$$|\lambda\rangle = \mathcal{N}_\lambda \exp\left(\sum_s \mathrm{Tr}(\lambda_s a_s^\dagger)\right)|0\rangle \quad \text{with} \quad \langle\lambda|\lambda\rangle = 1$$

$$\lambda_s = \frac{1}{\sqrt{2}}(X_s + iP_s)$$ with X_s and P_s Hermitian N x N matrices

Singlet condition $0 = \langle\lambda|\Phi^i{}_j|\lambda\rangle$ becomes a Gauss constraint $\sum_{s=1}^{2}[X_s, P_s] = 0$

Hamiltonian: $H(X_s, P_s) = \langle\lambda|H|\lambda\rangle$

$$H = \frac{1}{2}\sum_{s=1}^{2}\mathrm{Tr}(P_s^2 + X_s^2) - \frac{g}{32\pi^2 N}\mathrm{Tr}\Big([X_1, X_2]^2 + [P_1, P_2]^2 + [X_1, P_1]^2 + [X_2, P_2]^2 + [X_1, P_2]^2 + [X_2, P_1]^2\Big)$$

TH & Orselli 2014

Classical matrix model

Strong coupling limit g $\gg$ 1:

For g = ∞:

All four matrices mutually commute → They become diagonal

$$H = \frac{1}{2}\sum_{i=1}^{N}\left((P_{s,ii})^2 + (X_{s,ii})^2\right)$$

2N decoupled harmonic oscillators → Match with giant gravitons

For g large:

Correction from the commutator terms: The harmonic oscillators interact

We would like to match the strong coupling limit of the classical matrix model, including the interaction term, to giant gravitons in type IIB string theory on AdS_5 x S^5

Our proposal for a match:

Classical matrix model from SU(2) SMT for $g \gg 1$

⇕

k weakly interacting AdS giant gravitons
at sufficiently large energy and with $k \ll N$

Why AdS giant gravitons?
Classical matrix model limit = high energy limit
→ Representations with #columns $\gg$ #rows dominate
→ AdS giant gravitons

Why $k \ll N$:
D-branes in probe limit, no backreaction

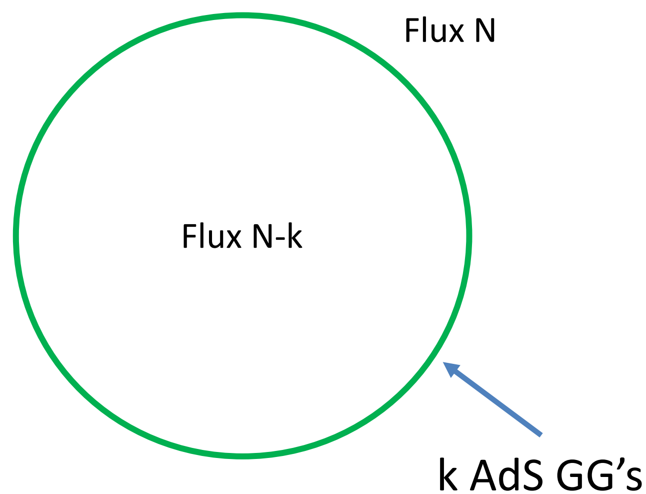
Our proposal for a match:

Classical matrix model from SU(2) SMT for $g \gg 1$

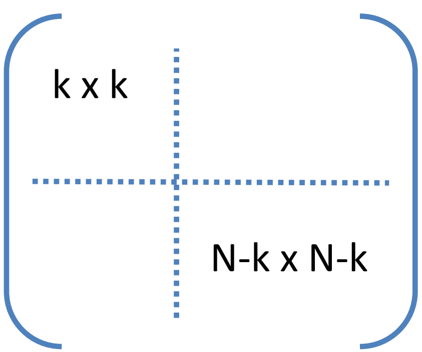
⇕

k weakly interacting AdS giant gravitons
at sufficiently large energy and with $k \ll N$

Put k spherical D3-branes: Breaks U(N) → U(k) x U(N-k)
U(k): The k AdS giant gravitons
U(N-k): The $AdS_5 \times S^5$ inside with N-k units of flux



In SU(2) SMT: U(N) → U(k) x U(N-k)
N x N matrix:



We only turn on k x k part of the matrices

N-k x N-k part zero: Dual to $AdS_5 \times S^5$ inside with N-k units of flux

Our proposal for a match:

Classical matrix model with k x k matrices from SU(2) SMT (with U(k) symmetry) for $g \gg 1$

⇕

k weakly interacting AdS giant gravitons at sufficiently large energy and with $k \ll N$

For k *non-interacting* AdS giant gravitons: $U(k) \rightarrow U(1)^k$
Use abelian DBI action

→ 2k harmonic oscillators at high energies

Mandal & Suryanarayana 2006

Matches the $g = \infty$ of the classical matrix model of SU(2) SMT

TH & Orselli 2014

Our proposal for a match:

Classical matrix model with k x k matrices from SU(2) SMT (with U(k) symmetry) for $g \gg 1$

$\Updownarrow$

k weakly interacting AdS giant gravitons at sufficiently large energy and with $k \ll N$

For k interacting AdS giant gravitions: Use non-abelian DBI action

Myers 1999
Taylor & van Raamsdonk 1999

We should take the SU(2) SMT limit of the non-abelian DBI action for the k interacting giant gravitons

Does this match matrix regime of SU(2) SMT for $g \gg 1$,
including the interaction terms in the matrix model?

Problem: Non-abelian DBI action is impossibly complicated
Even more so in a non-flat background ($AdS_5 \times S^5$)

Non-abelian DBI action up to terms of order F^6:

Myers 1999
Taylor & van Raamsdonk 1999

$$\mathcal{L} = \mathrm{STr}\left(-\sqrt{-\det(g_{ab} + F_{aI}F_{bJ}(g^{IJ} + F^{IK}g_{KL}F^{LJ}) + 2\pi l_s^2 F_{ab})\det(\delta^I_J + F^{IK}g_{KJ})} + C_{0123}\right)$$

$$F_{aI} = g_{IJ}(\partial_a x^I + i[A_a, x^I])\ , \quad F_{ab} = \partial_a A_b - \partial_b A_a + [A_a, A_b]\ , \quad F^{IJ} = \frac{i}{2\pi l_s^2}[x^I, x^J]$$

STr(…) means one symmetrizes over the field strengths and coordinate matrices (from background metric etc.) before taking the trace

Fortunately:
1) Match is in the weak interacting limit → Only need F^2 terms
2) It is for large energies → Corresponds to large AdS radius for D3-branes
3) Velocities of D3-branes are small (consequence of large radius)

→ Effectively one should take matrix model limit,
so only quadratic terms in the action!

Taking then the SU(2) SMT limit (and after quite some work) one gets:

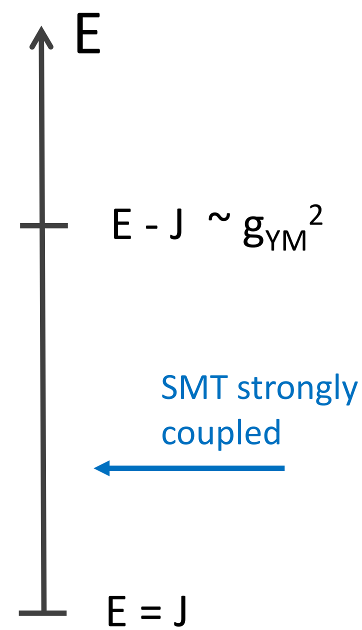
$$H = \frac{1}{2}\sum_{s=1}^{2}\mathrm{Tr}(P_s^2 + X_s^2) - \frac{4\pi g_s}{32\pi^2}\,\mathrm{Tr}\Big([X_1, X_2]^2 + [P_1, P_2]^2 + [X_1, P_1]^2$$
$$+[X_2, P_2]^2 + [X_1, P_2]^2 + [X_2, P_1]^2\Big) \quad \text{with} \quad \sum_{s=1}^{2}[X_s, P_s] = 0$$

This is the same classical matrix model as computed from SU(2) Spin Matrix theory (using $g_{YM}^2 = 4\pi g_s$)

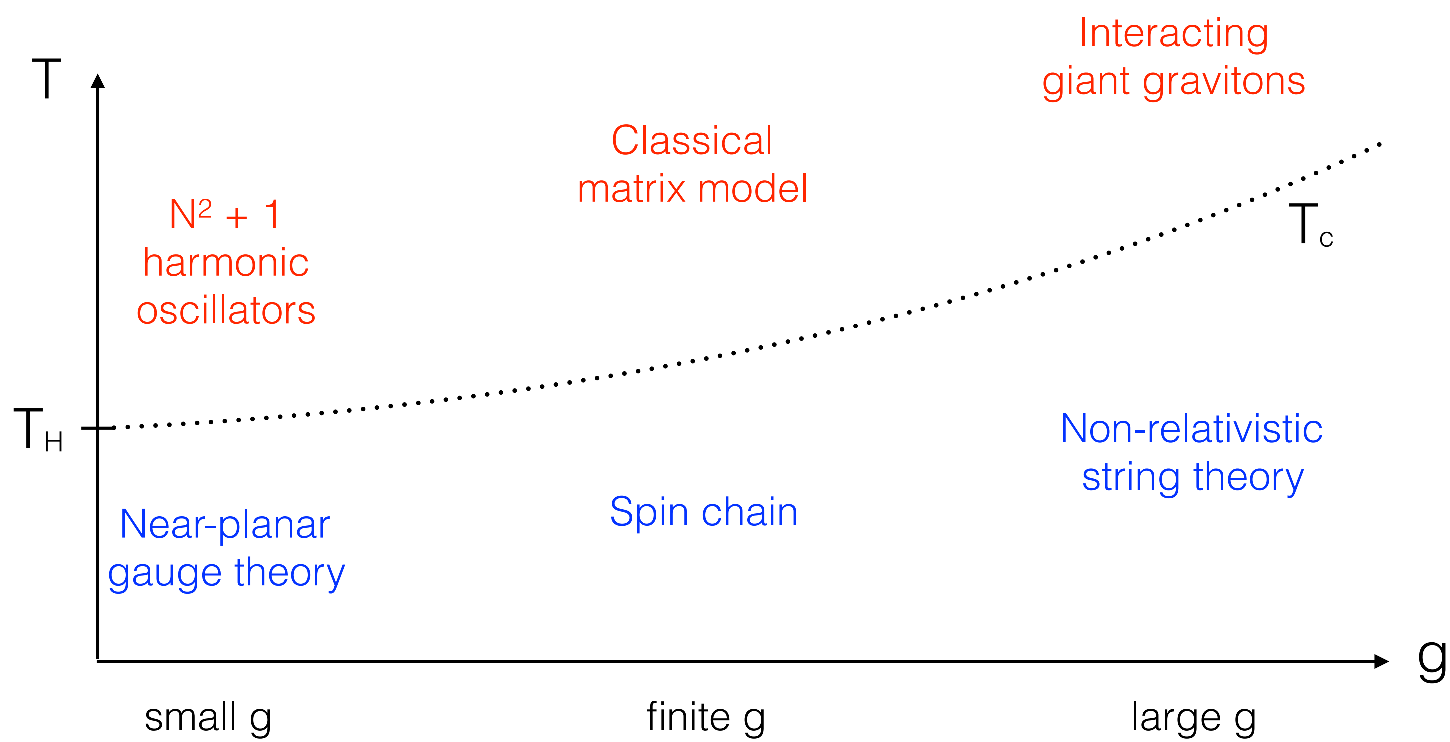
An exact match between gauge and string theory close to the unitarity bound:

$$E - J \ll g_{ym}^2$$

where SU(2) SMT is strongly coupled



We have matched SU(2) SMT for $g \gg 1$ both in the planar regime and in the matrix regime!



Just like in the planar case:
Non-abelian DBI action and gauge theory side valid in two different regimes
However, we expect one can make similar arguments as in the planar case
→ The match is not a coincidence

Other Spin Matrix theories from 𝒩=4 SYM?

Unitarity bound	Spin group G_s	Cartan diagram for algebra	Representation R_s
$E \geq J_1 + J_2$	$SU(2)$	○	$[1]$
$E \geq J_1 + J_2 + J_3$	$SU(2\|3)$	○—⊗—○—○	$[0,0,0,1]$
$E \geq S_1 + J_1 + J_2$	$SU(1,1\|2)$	⊗—○—⊗	$[0,1,0]$
$E \geq S_1 + S_2 + J_1 + J_2 + J_3$	$SU(1,2\|3)$	○—⊗—○—○—⊗	$[0,0,0,1,0]$

What are the analogues of the classical matrix model for the SU(1,1|2) SMT and SU(1,2|3) SMT?
The free spectra suggest 2D and 3D field theories?
In case it would be field theories with very interesting symmetry groups

Can one make a similar match with AdS giant gravitons for these SMT's?

Black holes from SMT?

Previously:

Match between strongly coupled SMT and string theory for $J \sim N^0$
→ Strings

This talk:

Match between strongly coupled SMT and string theory for $J \sim N$
→ D-branes

Can we find a match for $J \sim N^2$ → Geometry

Emerging black hole in SU(1,2|3) Spin Matrix theory

How is geometry emerging in SMT?

Going beyond one-loop?

We were able to match the one-loop contribution in $g_{YM}^2 = 4\pi g_s$

→ Next step: Consider two-loops, higher loops

One can easily add the two-loop dilation operator as a perturbation of SU(2) Spin Matrix theory

String side: Consider F^4 terms in non-abelian DBI

→ Integrability of the classical matrix model? Or not?

Can one develop a similar program as for the planar regime?

Thank you!

Comparison to previous work:

We are able to match non-supersymmetric dynamics of D-branes on $AdS_5 \times S^5$ to finite-N regime of $\mathcal{N}=4$ SYM near unitarity bound

Previously:

- SUSY giant gravitons

Kinney, Maldacena, Minwalla & Raju 2005
Biswas, Gaitto, Lahiri & Minwalla 2006
Mandal & Suryanarayana 2006

- Dispersion relations for open strings stretching between giant gravitons

Balasubramanian et al 2002
Berenstein, Correa and Vazquez 2006
Carlson, de Mello Koch and Lin 2011
Berenstein and Dzienkowski 2013
de Mello Koch, Taharidimbisoa and Mathwin 2015

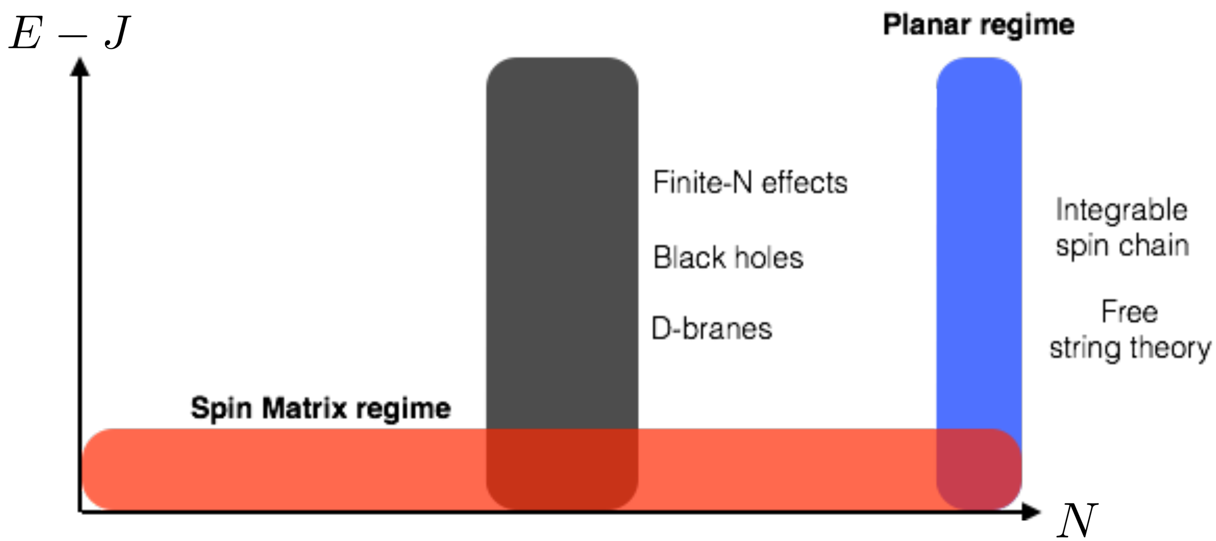
Giant gravitons not dynamical:
They only provide boundary conditions for the open string

One takes the $N \to \infty$ limit

Hence: Not in the finite-N regime

Why Spin Matrix Theory?

1. To match gauge theory and string theory beyond the planar regime



2.
- Emergence of non-lorentzian gravity and geometry from Spin Matrix theory?
- Is Spin Matrix theory a unified framework for a simpler type of holographic duality?