# Quantum Time crystals

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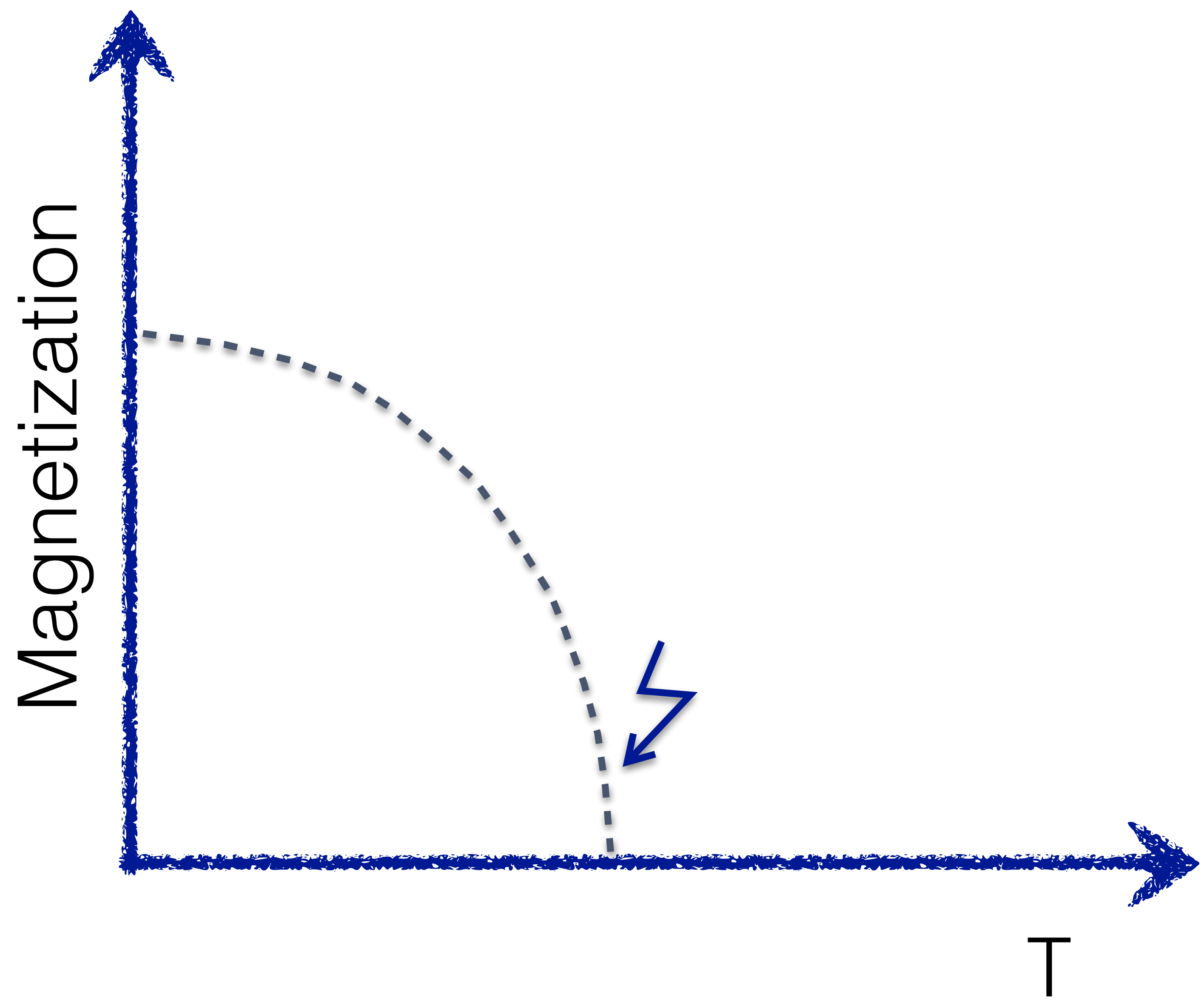
# Phase transitions

…to a superconducting state

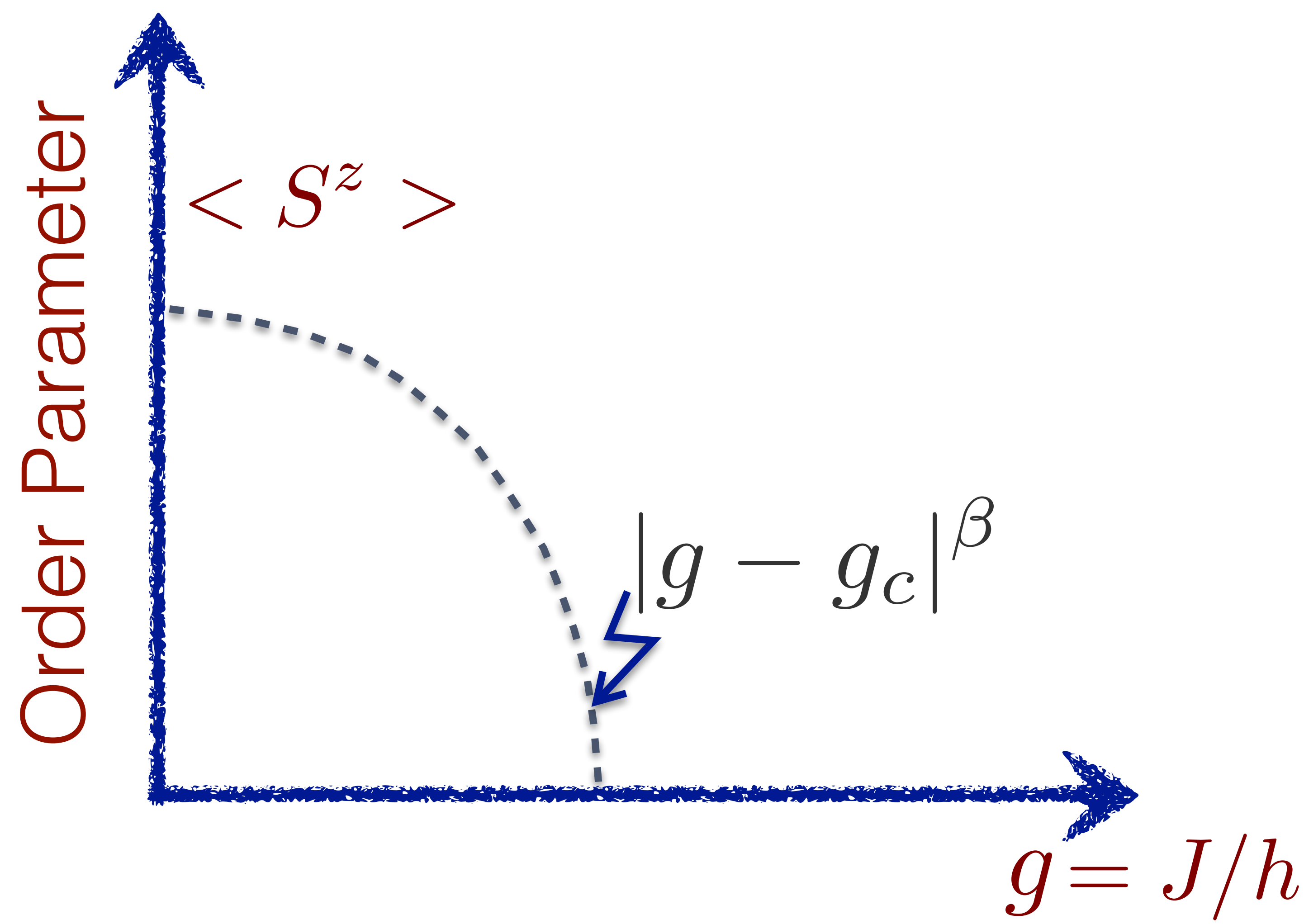
…to a ferromagnet

. . .



# Spontaneous symmetry breaking *[Example]*

$$H(h) = -\ J \sum_{<ij>}^{N} S_i^z\, S_j^z - 2\,h \sum_{i}^{N} S_i^x$$



# Can time-translational invariance be spontaneously broken?

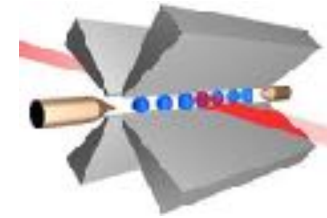
**Time-crystal**

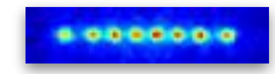
F. Wilczek, Physical Review Letters **109**, 160401 (2012)

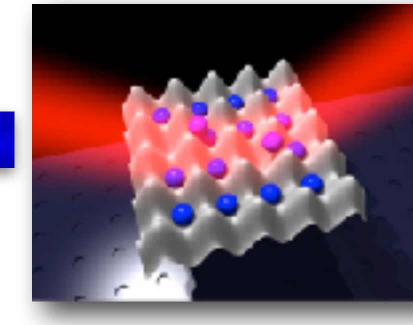
- Do laws of nature allow for the existence of a time-crystalline phase? ✓
- …if yes, how to define/characterise a time crystal? ✓
- …where to look for it? ✓
- How much do we know of its relations to other phenomena?
- Is it “useful”?

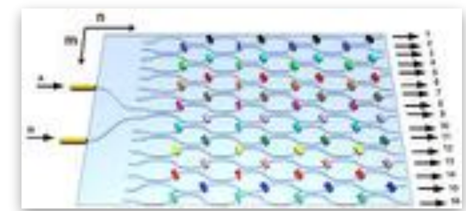
Bruno, Nozieres, Volovik, Sacha, ..

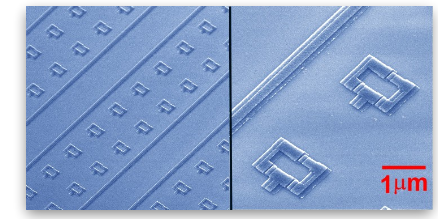
# Quantum Simulators





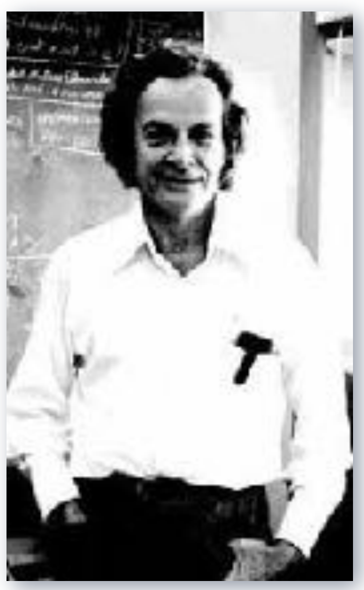







**Quantum simulator**: an experimental *controllable* system that reproduces the physics of a given model Hamiltonian

. . .



Questions **not accessible** via classical computation or questions that are not directly “tractable” in “nature” (e.g. thermalisation, defect formation, …)

Realise **new states of matter** —> “make possible phenomena real”

# Outline

- **Introduction time crystals**
  - Problem & definition (Wilczek 2013)
  - No-go theorem (Watanabe & Oshikawa 2015)
  - Floquet time-crystals (Else *et al* & Khemani *et al* 2013)
  - First exps on FTC (Choi *et al* & Zhang *et al* 2016)
- **Floquet time crystals in clock models**
  - Conditions for the existence
  - Direct transitions between different crystals
- **Floquet time crystals in clean systems**
  - Infinite range systems
- **Continuous (Boundary) time crystals**
  - Connections to many-body open quantum systems
- **Time crystals & Synchronization**

# In collaboration with

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A. Russomanno, F. Iemini, M. Dalmonte, R.F., Phys. Rev. B **95**, 214307 (2017)
F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte, and R. F., Phys. Rev. Lett. **121**, 035301 (2018)
O. Scarlatella, R. F., and M. Schirò, Phys. Rev. B **121**, 064511, (2019)
F. M. Surace, A. Russomanno, M. Dalmonte, A. Silva, R. F., and F. Iemini, Phys. Rev. B **99**, 104303 (2019)
R. Khasseh, R. F., S. Ruffo, A. Russomanno, Phys. Rev. Lett. **123**, 184301 (2019)
A. Russomanno, S. Notarnicola,F.M. Surace, R.F., M. Dalmonte, and Markus Heyl Phys. Rev. Research. **2,** 012003(R) (2020)

# TTSB - Definition:

$\phi(\vec{x}, t)$ local order parameter

$$\lim_{V \to \infty} \langle \phi(\vec{x}, t) \phi(\vec{x'}, t') \rangle \xrightarrow[|\vec{x} - \vec{x}'| \to \infty]{} f(t - t')$$

**No-go theorem:(*)** systems in the ground state or in thermal equilibrium cannot manifest any time-crystalline behaviour

Watanabe & Oshikawa 2015

(*) with sufficiently short-interactions

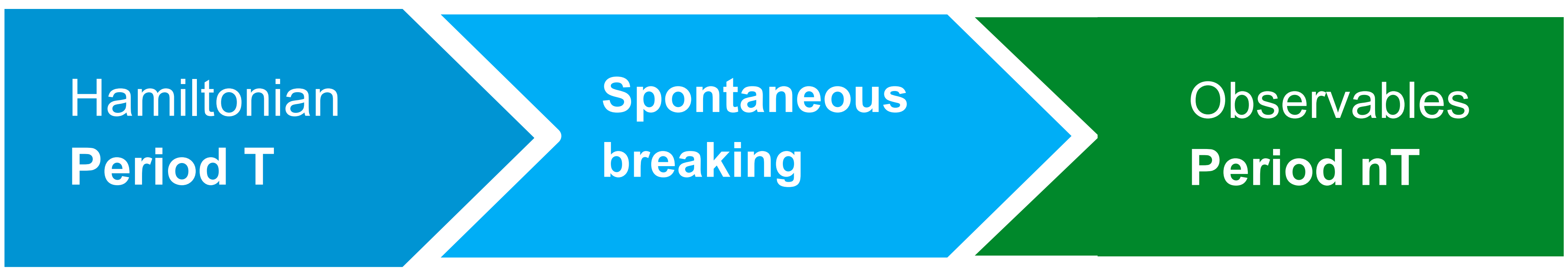
# Floquet time crystals

(TTSB in periodically driven systems)

*Theory*
D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
V. Khemani, A. Lazarides, R. Moessner, and S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

*Experiments*
J. Zhang *et al*, Nature **543**, 217 (2017)
S. Choi *et al*, Nature **543**, 221 (2017).

$$\mathcal{H}(t+T) = \mathcal{H}(t)$$




$$f(t) = \lim_{N\to\infty} \langle\psi|\hat{O}(t)|\psi\rangle$$

$$f(t+\tau_B) = f(t) \qquad \tau_B = nT$$

TTSB occurs if the eigenstates of the Floquet operator cannot be short-range correlated.

Every Floquet state violates cluster

$$U_f|\phi_\alpha\rangle = e^{-i\epsilon_\alpha T}|\phi_\alpha\rangle$$

Compare with the Ising ferromagnet

$$\frac{1}{\sqrt{2}}(|\uparrow\uparrow \dots \uparrow\uparrow\rangle \pm |\downarrow\downarrow \dots \downarrow\downarrow\rangle)$$

Short-range correlated states can only be formed by taking superpositions of Floquet eigenstates with different eigenvalues (not invariant under the discrete time translation by T

Rigidity: no fine-tuned Hamiltonian parameters.

Persistence: the non-trivial oscillation must persist for infinitely long time when first taking the thermodynamic limit

# Floquet time crystals

$T$ (one period)

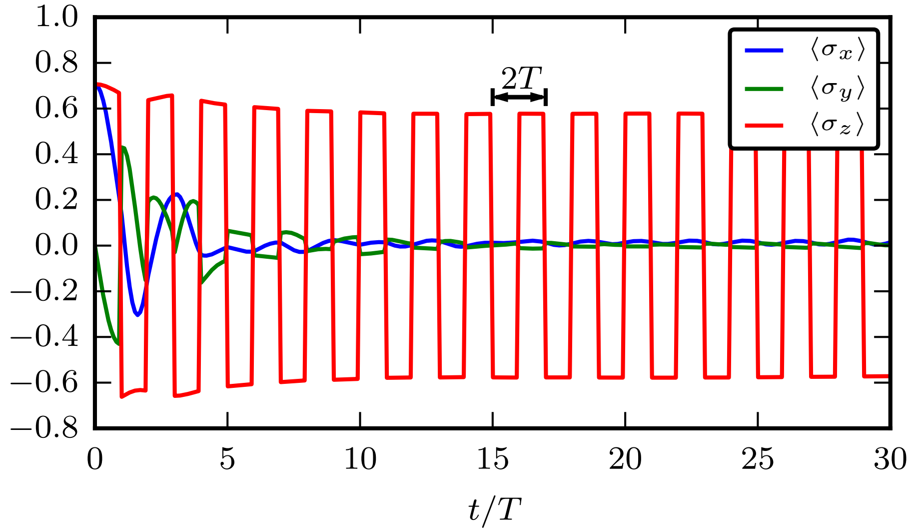
| Interactions + disorder | Flip pulse | Interactions + disorder | Flip pulse | Interactions + disorder |
|---|---|---|---|---|
| $H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$ | $\prod_i \sigma_i^x$ | $H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$ | $\prod_i \sigma_i^x$ | $H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$ |
| ↑ ↑ ↓ ↑ ↓ | | ↓ ↓ ↑ ↓ ↑ | | ↑ ↑ ↓ ↑ ↓ |

$$\hat{U} = \exp\left[-i\pi \sum_i^N \hat{\sigma}_i^x\right] \exp\left[-i\hat{H}(\hat{\sigma}_i^z)\right]$$

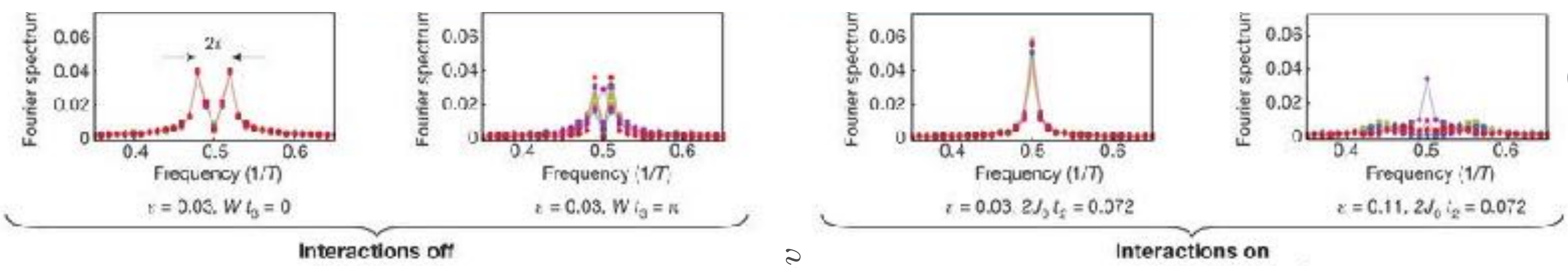
$$m_z = \frac{1}{N}\sum_i \langle \sigma_i^z(t) \rangle$$ will oscillate with a double period

The periodic drive, under generic conditions will heat the system up to infinite temperatures

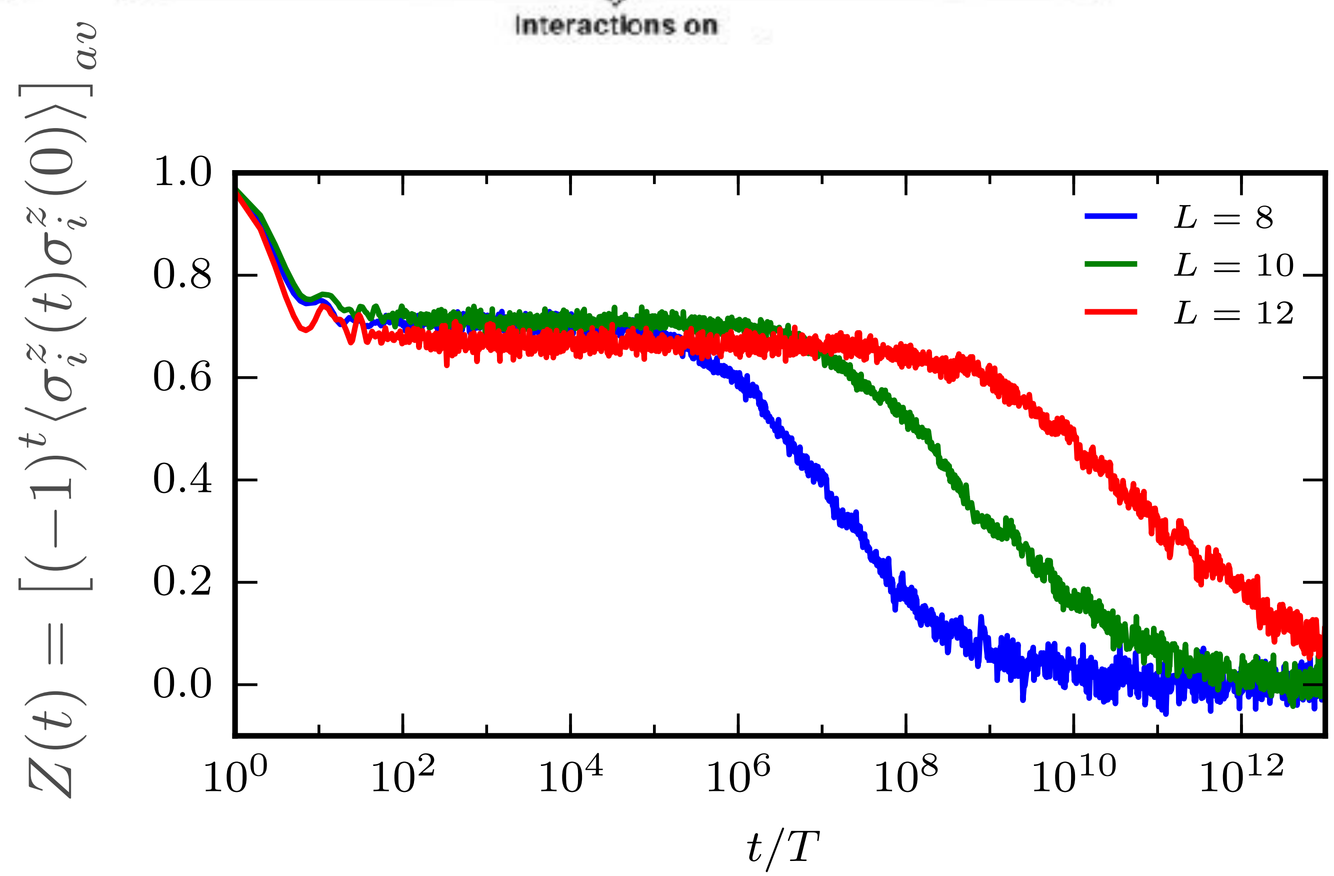
# Floquet time crystals




Disordered Ising model in an external field




Choi *et al* 2017




Else *et al* 2016

Is it possible to have a Floquet time-crystal in the absence of disorder?

Time-crystals crucially rely on ergodicity breaking dynamics

Examples: LMG model, $Z_2$ lattice gauge theory in 1D,..

A. Russomanno, F. Iemini, M. Dalmonte, R.F., Phys. Rev. B **95**, 214307 (2017)
FF. M. Surace, A. Russomanno, M. Dalmonte, A. Silva, R. F., and F. Iemini, Phys. Rev. B **99**, 104303 (2019)
A. Russomanno, S. Notarnicola,F.M. Surace, R.F.,  M. Dalmonte, and Markus Heyl Phys. Rev. Research. **2,** 012003(R) (2020)

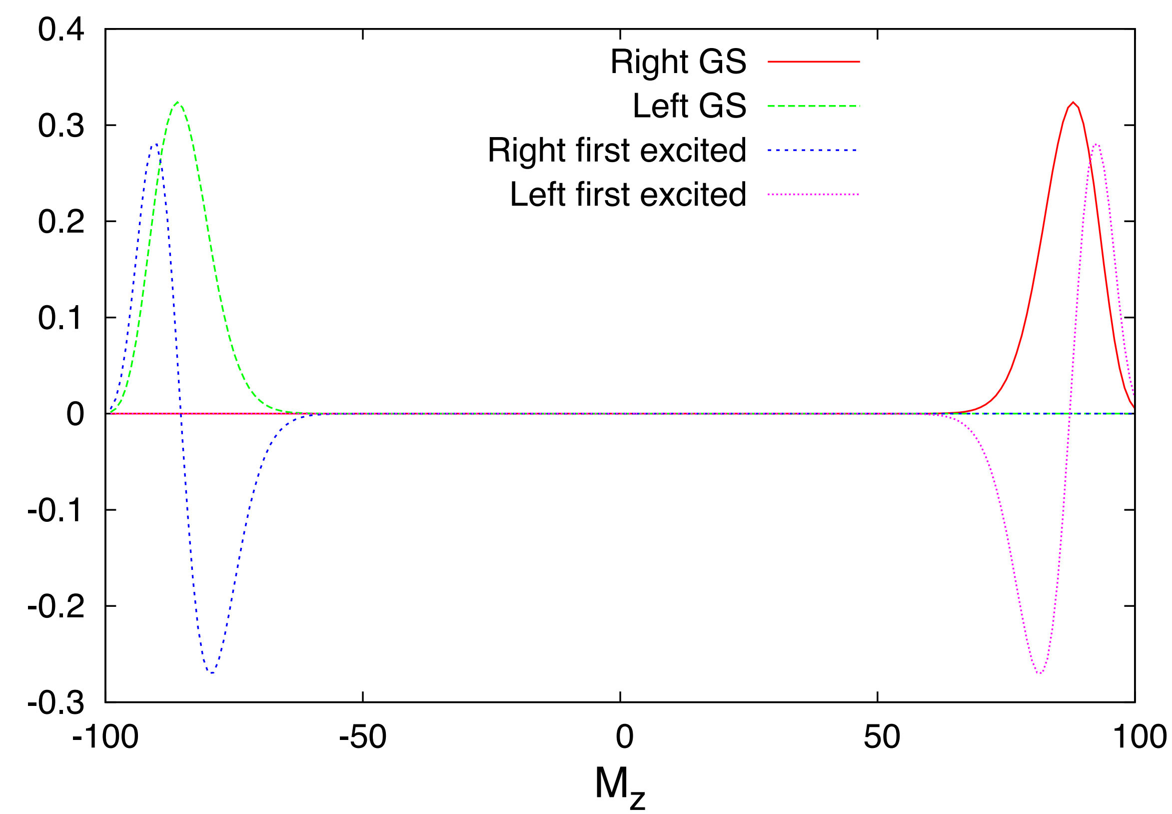
# Floquet time crystals in the LMG models

$$\mathcal{H}(h_0) = -\frac{2J}{N}\sum_{i,j}^{N}\hat{\sigma}_i^z\hat{\sigma}_j^z - 2h_0\sum_i^N\hat{\sigma}_i^x$$

When $h_0 < J$ there is $Z_2$ symmetry breaking that involves a finite fraction of all the spectrum.

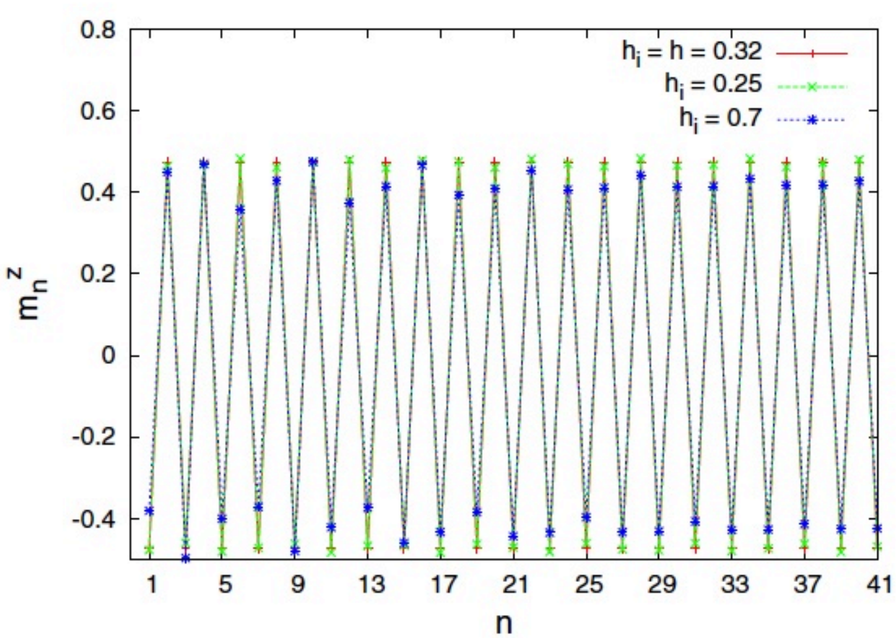
In the thermodynamic limit below the broken symmetry edge E* the corresponding energy eigenstates appear in degenerate doublets. Each member of the pair is localised in the basis of the eigenstates $M_z$.

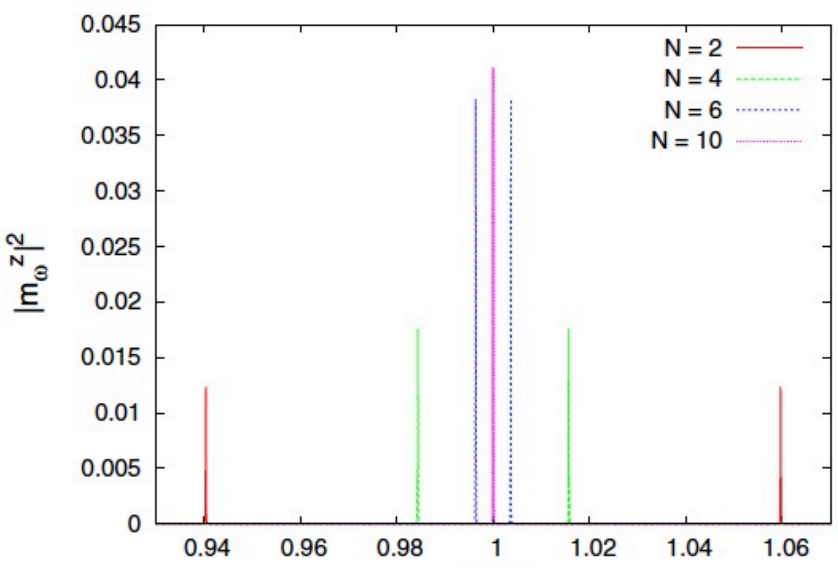
For finite sizes, the eigenstates are the even and odd superpositions of each doublet (with a spitting exponentially small in N).



# Floquet time-crystals in the LMG model

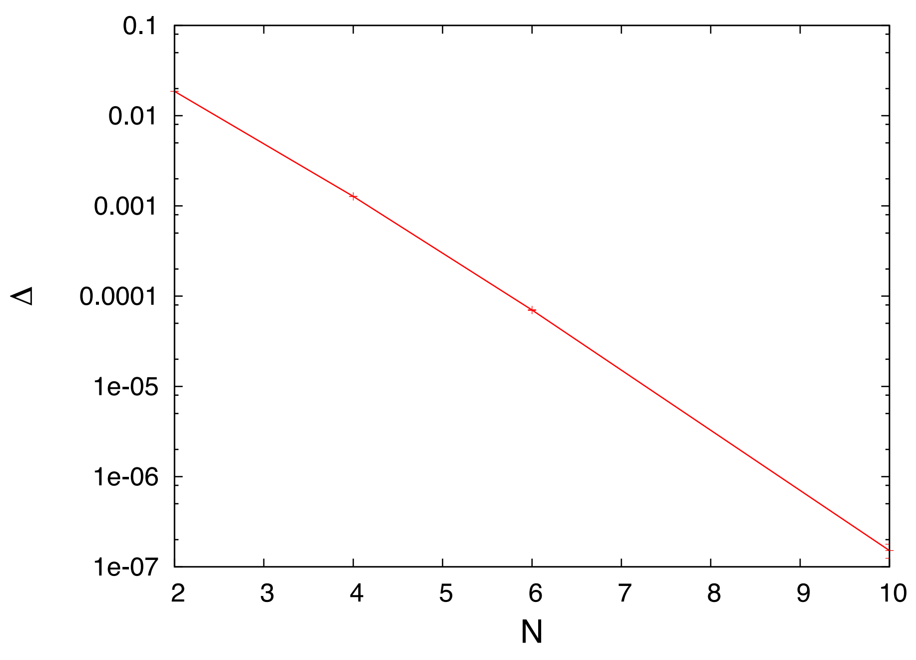
$$\widehat{U} = \widehat{U}_{\text{kick}} \exp\left[-i\widehat{H}(h)\tau\right] \text{ with } \widehat{U}_{\text{kick}} \equiv \exp\left[-i\phi\sum_{i}^{N}\widehat{s}_i^x\right],$$



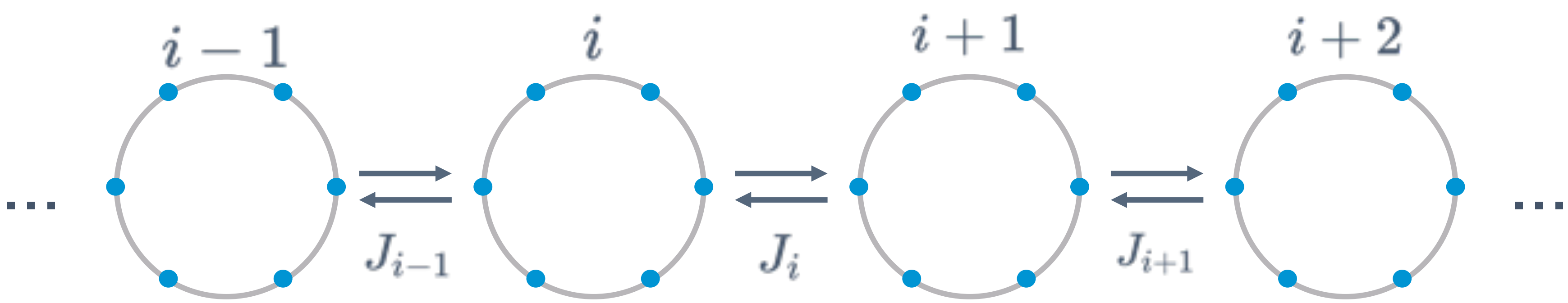

The gap closes in the thermodynamic limit

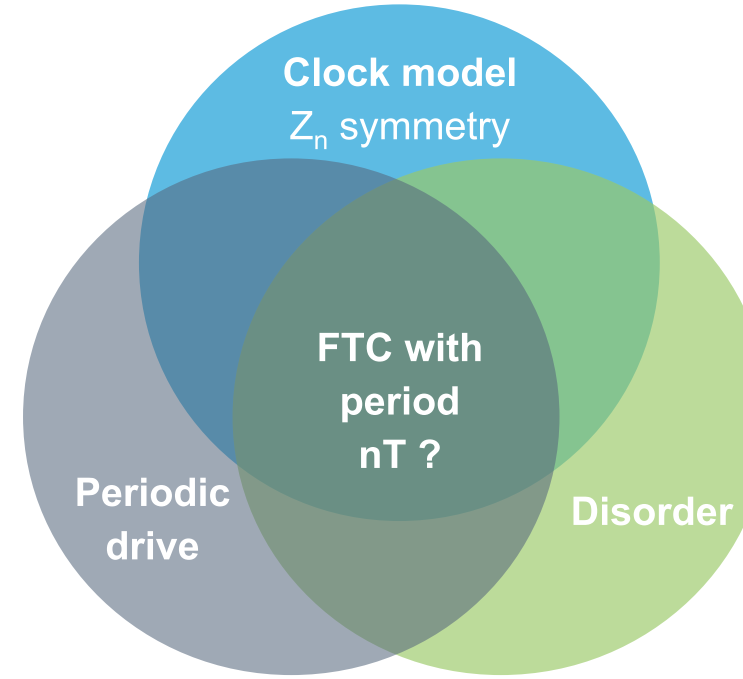
$$m_z = \frac{1}{N}\sum_i \langle \sigma_i^z(t) \rangle$$




will oscillate with a double period

# Floquet time crystals in clock models

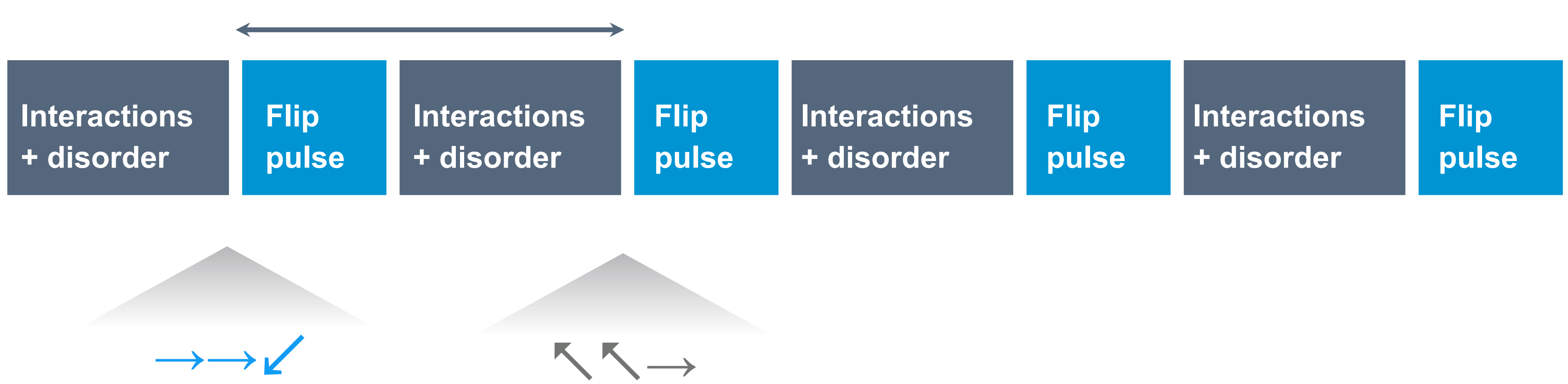



$$\tau = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \sigma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \omega^{n-1} \end{pmatrix}$$

$$X = \prod_i \tau_i$$

$$H = \sum_i J_i (e^{i\varphi} \sigma_i^\dagger \sigma_{i+1} + h.c.) + \sum_i h_i^z (e^{i\varphi_z} \sigma_i + h.c.) + \sum_i h_i^x (e^{i\varphi_x} \tau_i + h.c.)$$




$$\omega = e^{2\pi i/n}$$

$$\sigma\tau = \omega\tau\sigma$$

$$\sigma^n = 1$$

$$\tau^n = 1$$

# Floquet time crystals in clock models

$$h_z = h_x = 0$$

**FLOQUET STATES**
*p=0,1,2*

(|↖ ↖ →⟩ + $\omega^{-p}$|↙ ↙ ↖⟩ +$\omega^{-2p}$|→ → ↙⟩ )/√3

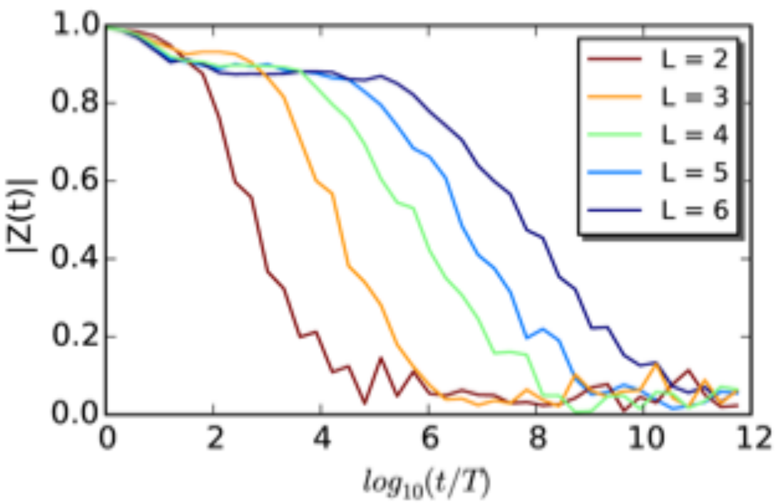
**Disorder** → **Local spectral gap** → **Stability**

If $\varphi = 0 \bmod \pi/3$

➜ the spectrum has degeneracies

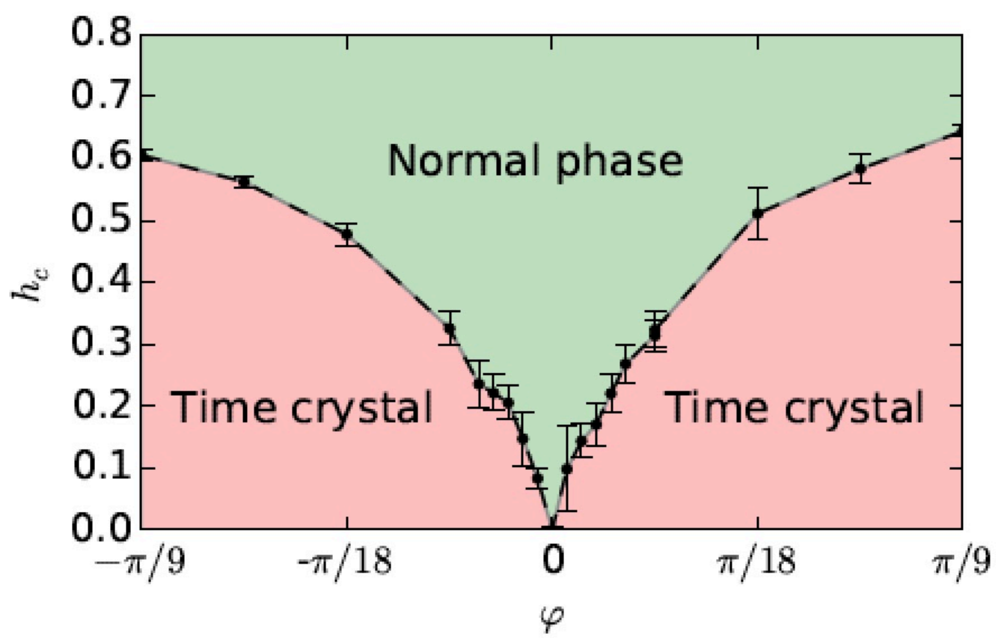
**Non-chiral clock model**

**CHIRAL MODEL**



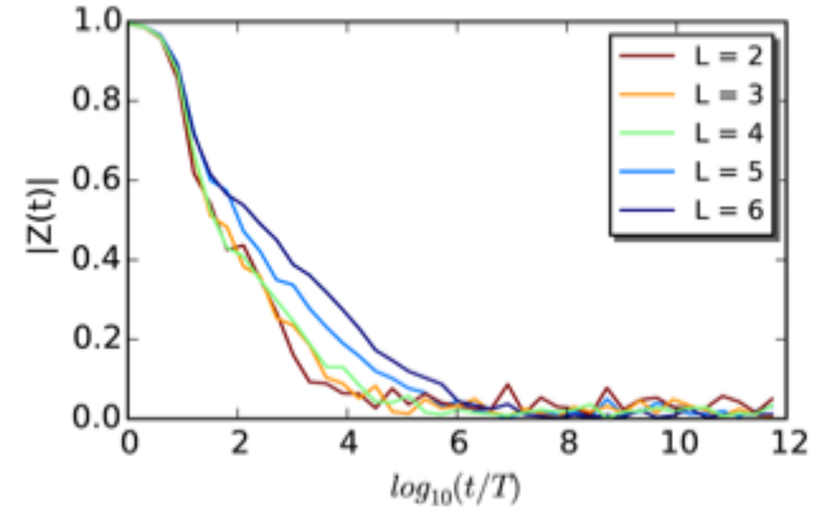

Decay time $t^* \sim O(e^{cL})$

➜ **Robust**




$h_z = 0.3$

**NON-CHIRAL MODEL**




Similar analysis of the spectral gap gives conditions for the existence of time-crystals in $Z_n$ models

For $Z_n$ possibility to observe transitions between different crystalline phases

# Time-crystals
# &
# dissipative many-body open systems

F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M.Dalmonte, and R. F., Phys. Rev. Lett. **121**, 035301 (2018)

# Time-crystals & dissipative many-body open systems

## Also investigated in …

In an ensemble of atoms collectively coupled to a leaky cavity mode.

In dissipative spin arrays

In a driven Bose-Hubbard dimer

…

Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018)
K. Tucker, B. Zhu, R. J. Lewis-Swan, J. Marino, F. Jimenez, J. G. Restrepo, and A. M. Rey, New J. Phys. **20**, 123003 (2018)
N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, F. Nori, Phys. Rev. A **98**, 063815 (2018)
T. L. Heugel, M. Oscity, A. Eichler, O. Zilberberg, and R. Chitra, Phys. Rev. Lett. **123**, 124301 (2019)
B. Zhu, J. Marino, N. Y. Yao, M. D. Lukin, and E. Demler, New J. Phys. **21**, 073028 (2019)
C. Lledó, T. Mavrogordatos, and M. Szymańska, Phys. Rev. B **100**, 054303 (2019)
A. Lazarides, S. Roy, F. Piazza, and R. Moessner, arXiv:1904.04820 (2019)
A. Riera-Campeny, M. Moreno-Cardoner, and A. Sanpera, arXiv:1909.11339 (2019)
K. Seibold, R. Rota, and V. Savona, arXiv:1910.03499 (2019)
…

# (Boundary) Time crystals in dissipative systems

- **Is the coupling to a bath always detrimental?**

"Many-body" limit cycles as time-crystals in open systems …

… they can be interpreted as "boundary" phases

The no-go theorem does not apply since the steady state is non-equilibrium …

… an idealised model vs possible exp realisations

These limit cycles can be understood as a macroscopic synchronised dynamics characterised by a time-dependent order parameter

# A toy model

$$\hat{H}_{\rm b} = \omega_0 \sum_j \hat{\sigma}_j^x$$

The steady state diagram of the model has two distinct phases

J. Hannukainen and J. Larson Phys. Rev. A **98**, 042113 (2018)

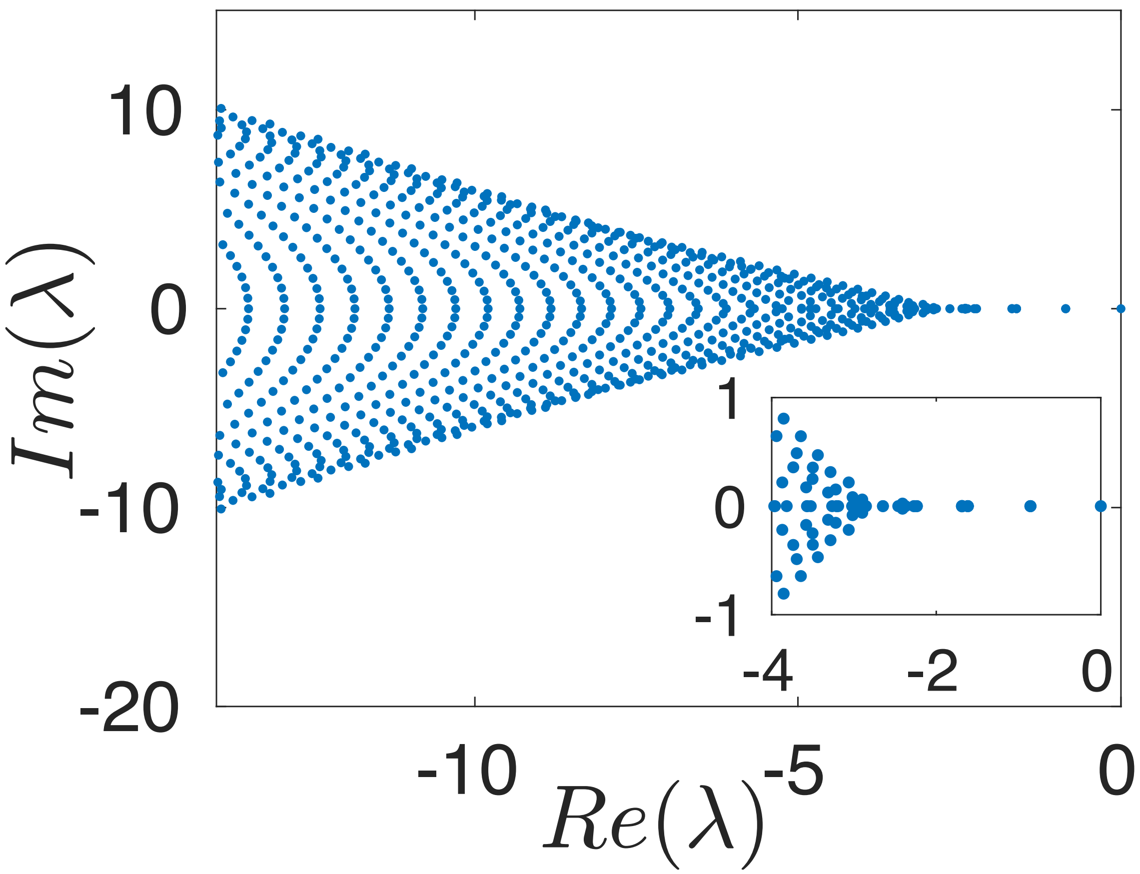
$$\hat{S}^\alpha = \sum_j \hat{\sigma}_j^\alpha$$

| $\omega_0/\kappa < 1$ | $\omega_0/\kappa > 1$ |
|---|---|
| $\langle \hat{S}^z \rangle \neq 0$ | $\langle \hat{S}^z \rangle = 0$ |

$$\frac{d}{dt}\hat{\rho}_{\rm b} = i\omega_0[\hat{\rho}_{\rm b}, \hat{S}^x] + \frac{\kappa}{S}\left(\hat{S}_-\hat{\rho}_{\rm b}\hat{S}_+ - \frac{1}{2}\{\hat{S}_+\hat{S}_-, \hat{\rho}_{\rm b}\}\right)$$

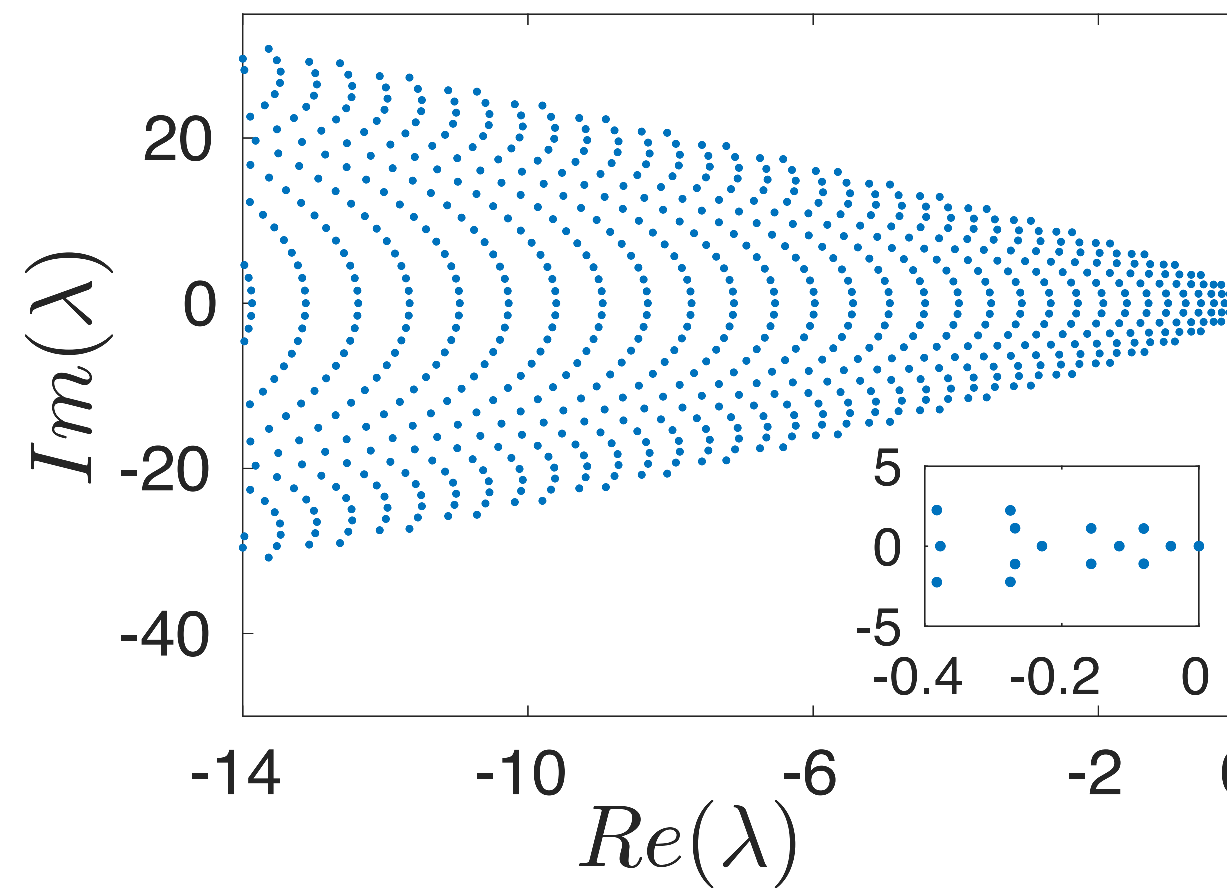
# A toy model

$$\omega_0/\kappa > 1$$

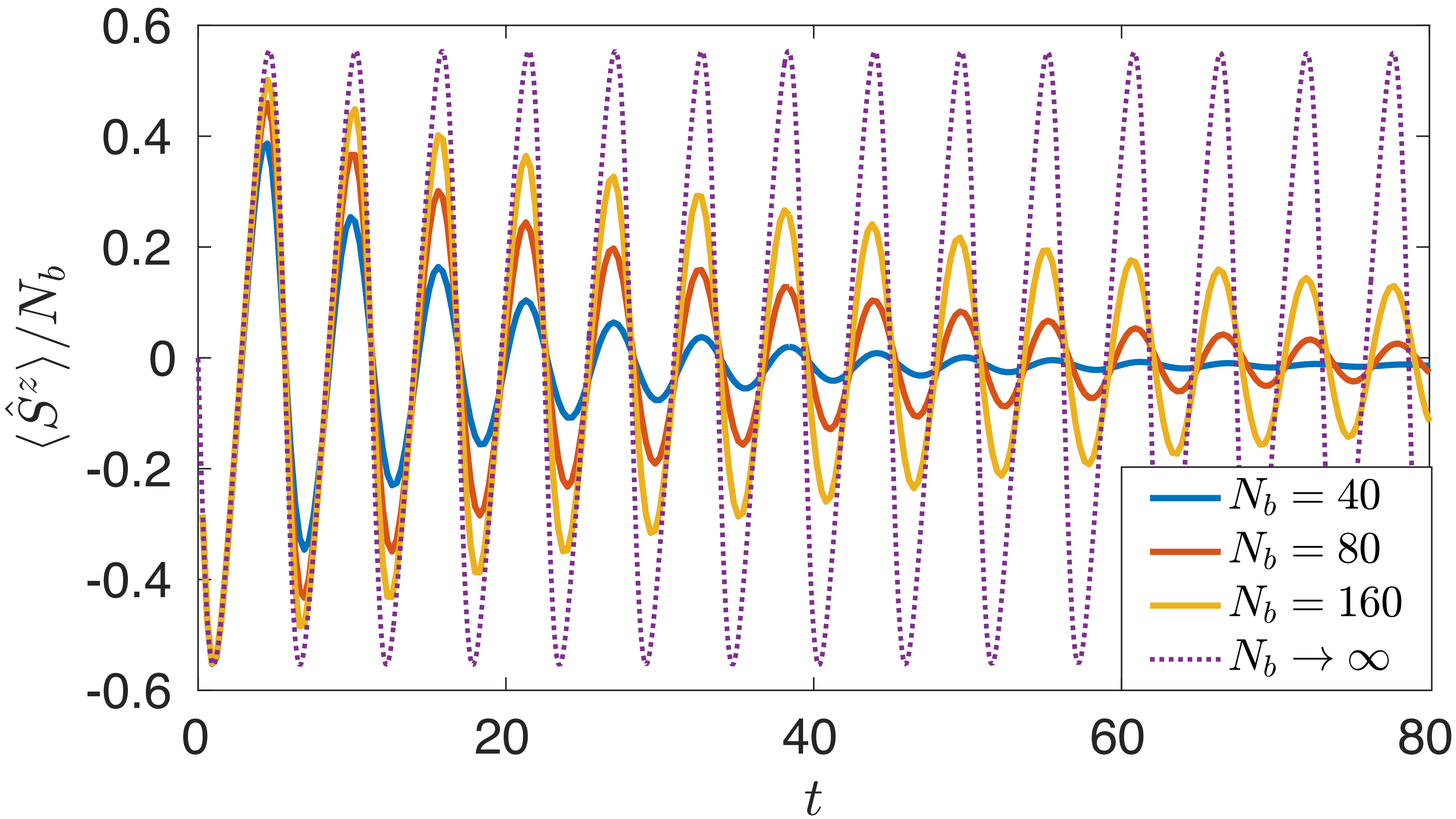



The spectrum is gapped and the low-lying eigenvalues of the Liouvillian have purely real values

$$\omega_0/\kappa < 1$$

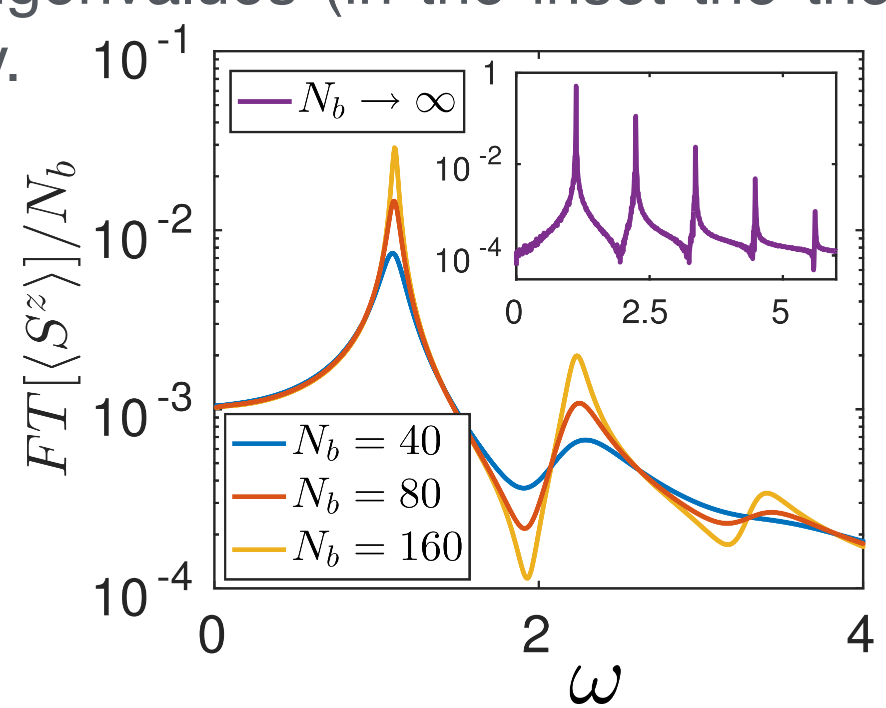


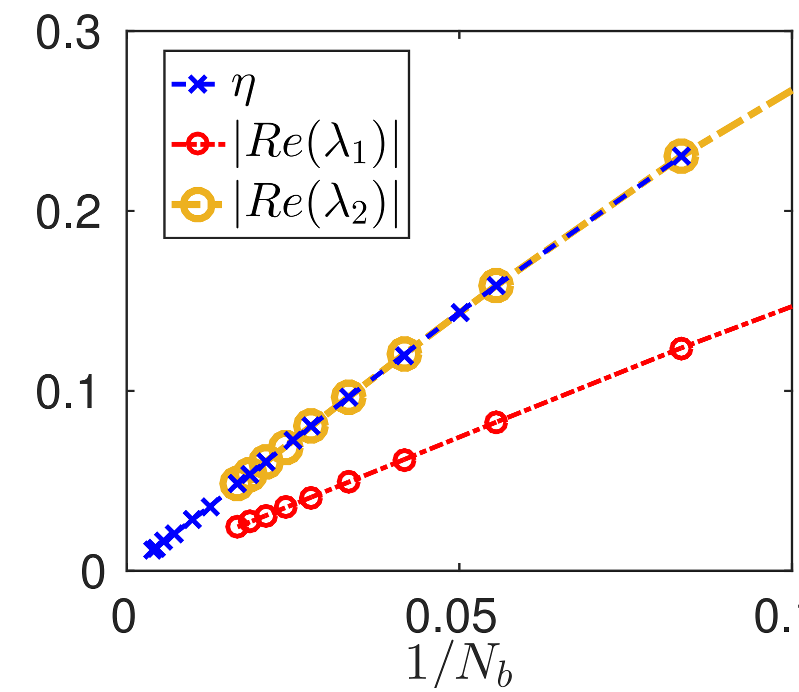

The spectrum becomes gapless and the low-lying excited eigenvalues have a non zero imaginary part




The peaks in the Fourier transform are associated to the band separations in the imaginary part of the Lindblad eigenvalues (in the inset the thermodynamic limit where the oscillations persist indefinitely.

The decay rate of the oscillations scales as $N_b^{-1}$



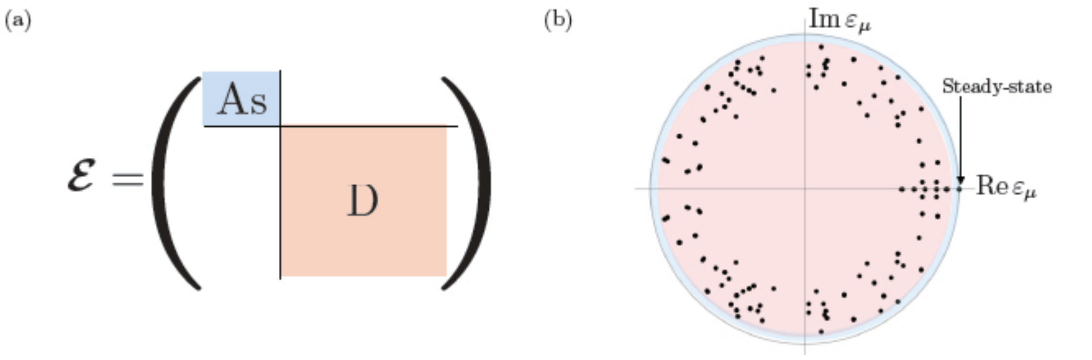



# Lindblad dynamics

$$\hat{\mathcal{L}}[\cdot] = \sum_{\alpha} \left\{ \hat{\ell}_\alpha \cdot \hat{\ell}_\alpha^\dagger - \frac{1}{2}\{\hat{\ell}_\alpha^\dagger \hat{\ell}_\alpha, \cdot\} \right\}$$

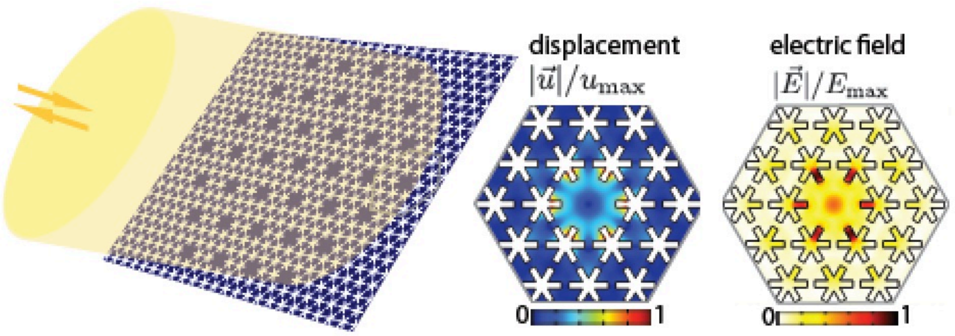
- Closing of the Liouvillian gap making the non-equilibrium steady state subspace degenerate in the thermodynamic limit
- Oscillating coherences appearing in the degenerate subspace
- Liouvillian gap above the degenerate subspace, in order to stabilise the time-crystal against perturbations

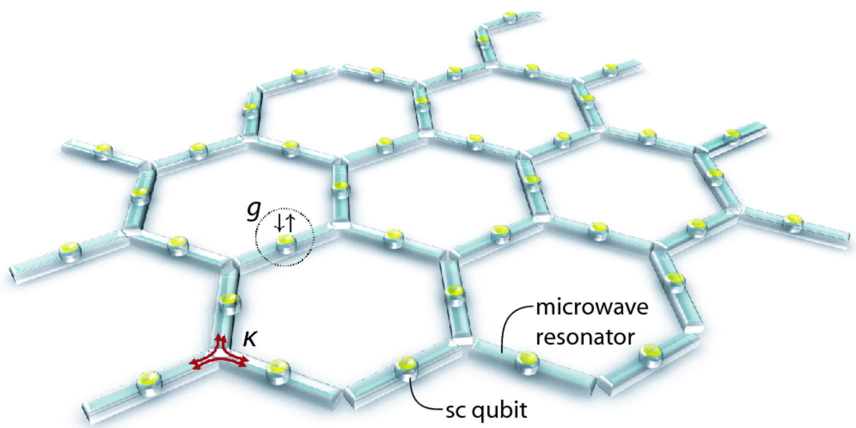
General discussion on the possibility of a dissipative time-crystal in A. Riera-Campeny, *et al.*, arXiv:1909.11339 (2019)

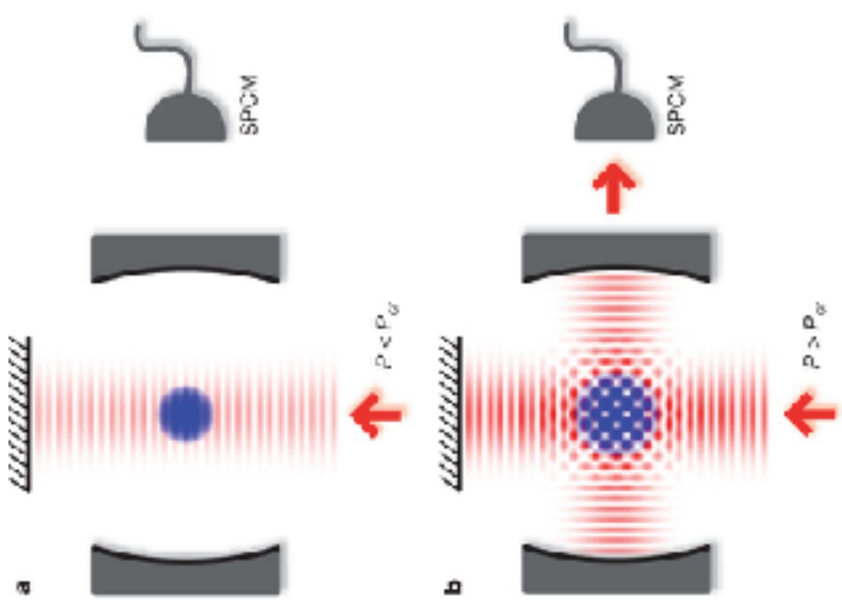



**Open questions**: conserved quantities, short- vs long-range systems, connection to decoherence-free subspaces, connection to dissipative state preparation, …

# Quantum synchronisation in many-body open systems




Identical features have been already seen in model systems of interacting Rydberg atoms, opto-mechanical arrays, coupled cavity arrays, atoms in cavities, interacting spin-systems …

These phases were all found however in a mean-field approximation, it is not clear to which extent they will survive when fluctuations are included.

Lee, Haffner, and Cross (2011)
M. Ludwig and F. Marquardt,(2013)
Jin, et al (2013)
Schiro', et al (2016)
Chan, Lee, and Gopalakrishnan (2015)
….

# Finite-frequency criticality

Lattice bosons (Bose-Hubbard like Hamiltonian) in the presence of drive and dissipation

$$\psi \longrightarrow \langle a_i \rangle$$

**Question**: related to whether finite frequency modes can become critical, giving rise to genuine time-domain instabilities of the quantum dynamics and to an associated breaking of time-translational invariance,

$$\chi_R = \frac{1}{-2J\sum_\alpha \cos q_\alpha - G^R_{loc}(\omega)}$$

$$G^R_{loc}(\omega) = \int dt e^{i\omega t}\langle [a(t), a(t')]\rangle$$

Exact single-site retarded local Green's function in presence of interaction, drive and dissipation (J=0)

O. Scarlatella, R. F., and M. Schirò, Phys. Rev. B **121**, 064511, (2019)

# Conclusions

- Quantum time crystals manifest in a variety of non-equilibrium conditions with ergodicity-breaking dynamics
- Many-Body limit cycles in open systems can be understood as dissipative (boundary) time-crystals
- Connections to synchronisation, finite-frequency criticality, …
- Possible experimental realisations