STATISTICAL MECHANICS FOR COMPLEX SYSTEMS

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Lisboa, Outubro 2021

PILLARS OF CONTEMPORARY THEORETICAL PHYSICS

- Newtonian mechanics

- Maxwell electromagnetism

- Einstein special relativity
  (when the velocities approach that of light)

- Quantum mechanics
  (when masses are very small)

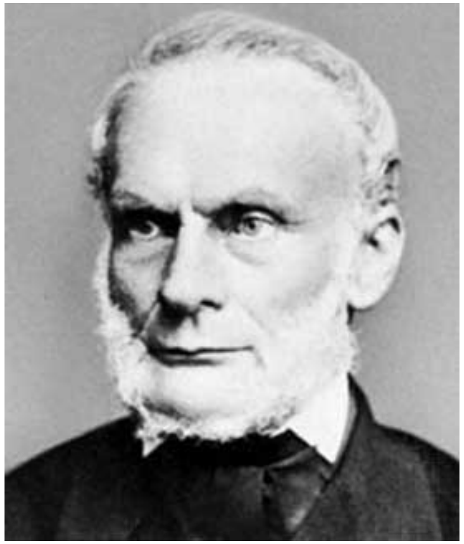
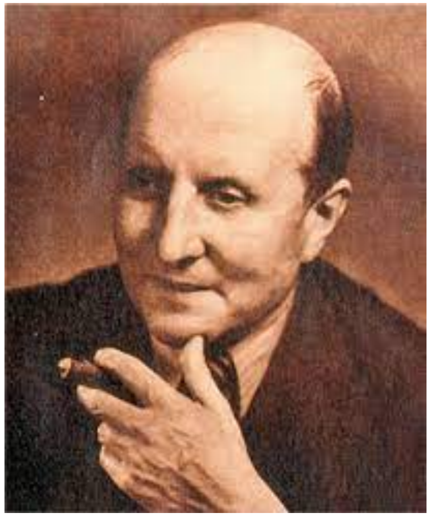
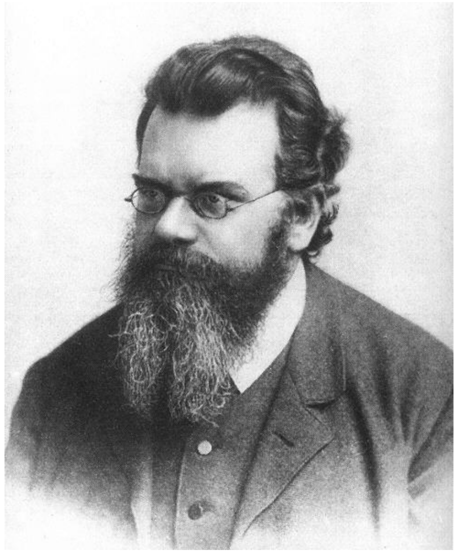
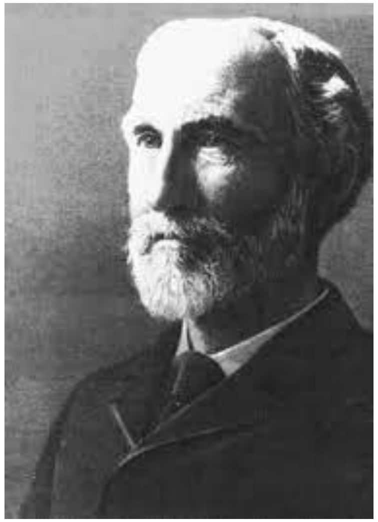
- Boltzmann-Gibbs statistical mechanics (→ Thermodynamics)

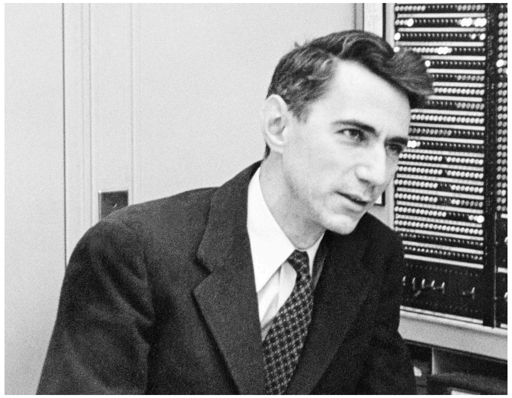
Previous four + Theory of probabilities

	QUANTUM MECHANICS TEXTBOOKS	BOLTZMANN-GIBBS STATISTICAL MECHANICS TEXTBOOKS
square well	YES	YES
harmonic oscillator	YES	YES
rigid rotator	YES	YES
spin 1/2 in magnetic field	YES	YES
nonionized Hydrogen atom	YES	NO! WHY?

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.



ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG **entropy** **(*q =1*)**	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* **(*q real*)**	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable (**unique** trace form; Enciso-Tempesta)

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van universal thermostat independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J. Stat. Phys. **52**, 479 (1988)

DEFINITIONS: $q-logarithm$: $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\; \ln_1 x = \ln x)$

$q-exponential$: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k\sum_{i=1}^{W} p_i \ln\frac{1}{p_i}$
entropy S_q $(q\in R)$	$k\ \ln_q W$	$k\sum_{i=1}^{W} p_i \ln_q\frac{1}{p_i}$

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since $$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\,\frac{S_q(A)}{k}\,\frac{S_q(B)}{k}$$

S_{BG} and $S_q^{Renyi}\ (\forall q)$ are additive, and $S_q\ (\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty\ ,\ i.e.,\ \ S(N) \propto N\ \ (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

$W \equiv$ total number of possibilities with nonzero probability,

assumed to be equally probable

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow \; S_{BG}(N) = k_B \ln W(N) \; \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow \; S_q(N) \; = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow \; S_{q=1-1/\rho}(N) \; \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\; 0 < \gamma < 1)$

$$\Rightarrow \; S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

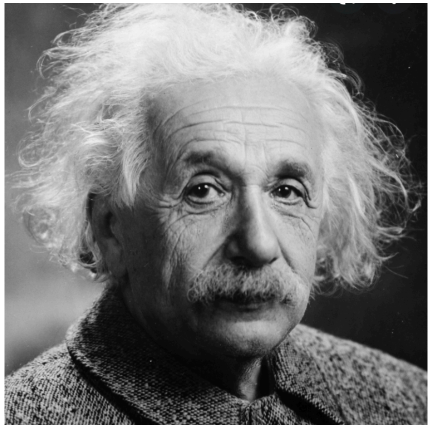
$$\Rightarrow \; S_{\delta=1/\gamma}(N) \; \propto N \qquad \text{OK!}$$

IMPORTANT: $\boxed{\mu^N >> \nu^{N^\gamma} >> N^\rho \quad \text{if} \;\; N >> 1}$

All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy (*Anna Karenina*, 1875-1877)

SYSTEMS $W(N)$ $\boxed{(equiprobable)}$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$e.g.,\ \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$e.g.,\ N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE $(q = 1 - 1/\rho)$	NONEXTENSIVE
$e.g.,\ \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\delta = 1/\gamma)$



A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown. Albert Einstein (1949)

COMPOSITION OF VELOCITIES OF INERTIAL SYSTEMS (d=1)

$$v_{13} = v_{12} + v_{23} \qquad \text{(Galileo)}$$

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \qquad \text{(Einstein)}$$

Newton mechanics:
It satisfies Galilean additivity **but** violates Lorentz invariance (hence mechanics can not be unified with Maxwell electromagnetism)

Einstein mechanics (Special relativity):
It satisfies Lorentz invariance (hence mechanics is unified with Maxwell electromagnetism) **but** violates Galilean additivity

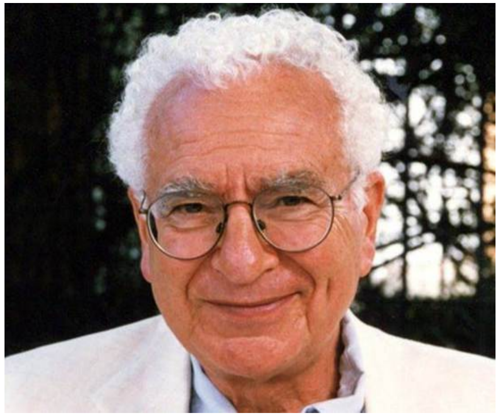
Question: which is physically more fundamental, the additive composition of velocities **or** the unification of mechanics and electromagnetism?

Milan j. math. 76 (2008), 307–328
1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg



M. Gell-Mann

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

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(Received 10 November 2009; accepted 4 January 2010; published online 3 March 2010)

See also:
H.J. Hilhorst, JSTAT P10023 (2010)
M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)
M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)
A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)
A. Plastino and M.C. Rocca, Physica A **392**, 3952 (2013)
S. Umarov and C. T., J Phys A **49**, 415204 (2016)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ *- scaled attractor* $\mathbb{F}(x)$ *when summing* $N \to \infty$ q*-independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) = Gaussian\ G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & if\ \lvert x \rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & if\ \lvert x \rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) = Levy\ distribution\ L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad if\ \lvert x \rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad if\ \lvert x \rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\ \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (intermediate\ regime) \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\ 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \quad (distant\ regime) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

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SCIENTIFIC REPORTS

OPEN

The standard map: From Boltzmann-Gibbs statistics to Tsallis statistics

Received: 10 December 2015

Accepted: 09 March 2016

Published: 23 March 2016

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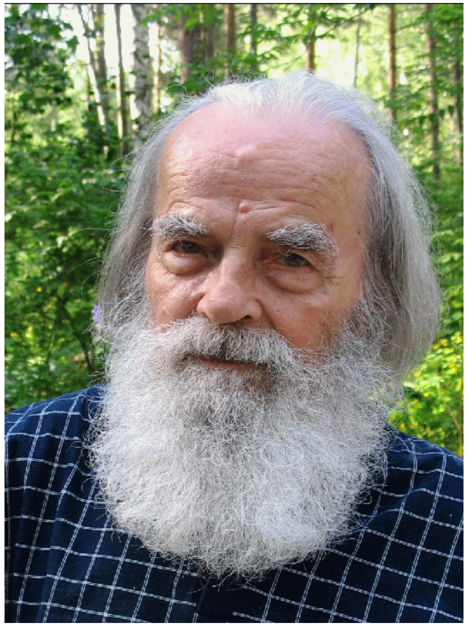
As well known, Boltzmann-Gibbs statistics is the correct way of thermostatistically approaching ergodic systems. On the other hand, nontrivial ergodicity breakdown and strong correlations typically drag the system into out-of-equilibrium states where Boltzmann-Gibbs statistics fails. For a wide class of such systems, it has been shown in recent years that the correct approach is to use Tsallis statistics instead. Here we show how the dynamics of the paradigmatic conservative (area-preserving) standard map exhibits, in an exceptionally clear manner, the crossing from one statistics to the other. Our results unambiguously illustrate the domains of validity of both Boltzmann-Gibbs and Tsallis statistical distributions. Since various important physical systems from particle confinement in magnetic traps to autoionization of molecular Rydberg states, through particle dynamics in accelerators and comet dynamics, can be reduced to the standard map, our results are expected to enlighten and enable an improved interpretation of diverse experimental and observational results.

STANDARD MAP (Chirikov 1969)

$$p_{i+1} = p_i - K \sin x_i \quad (\mathrm{mod}\ 2\pi)$$

$$x_{i+1} = x_i + p_{i+1} \quad (\mathrm{mod}\ 2\pi)$$

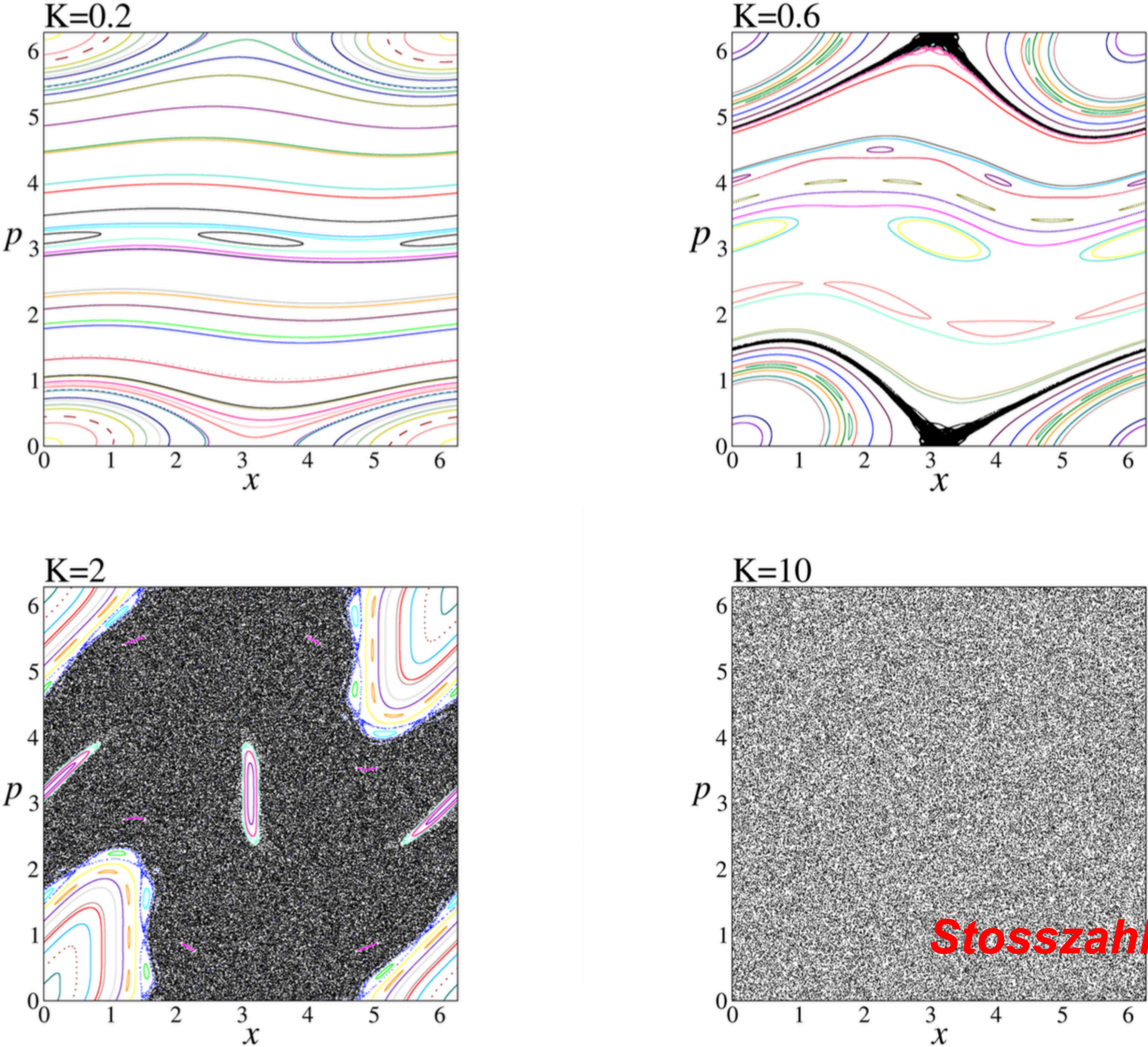
$$(i = 0, 1, 2, \ldots)$$



(1928-2008)

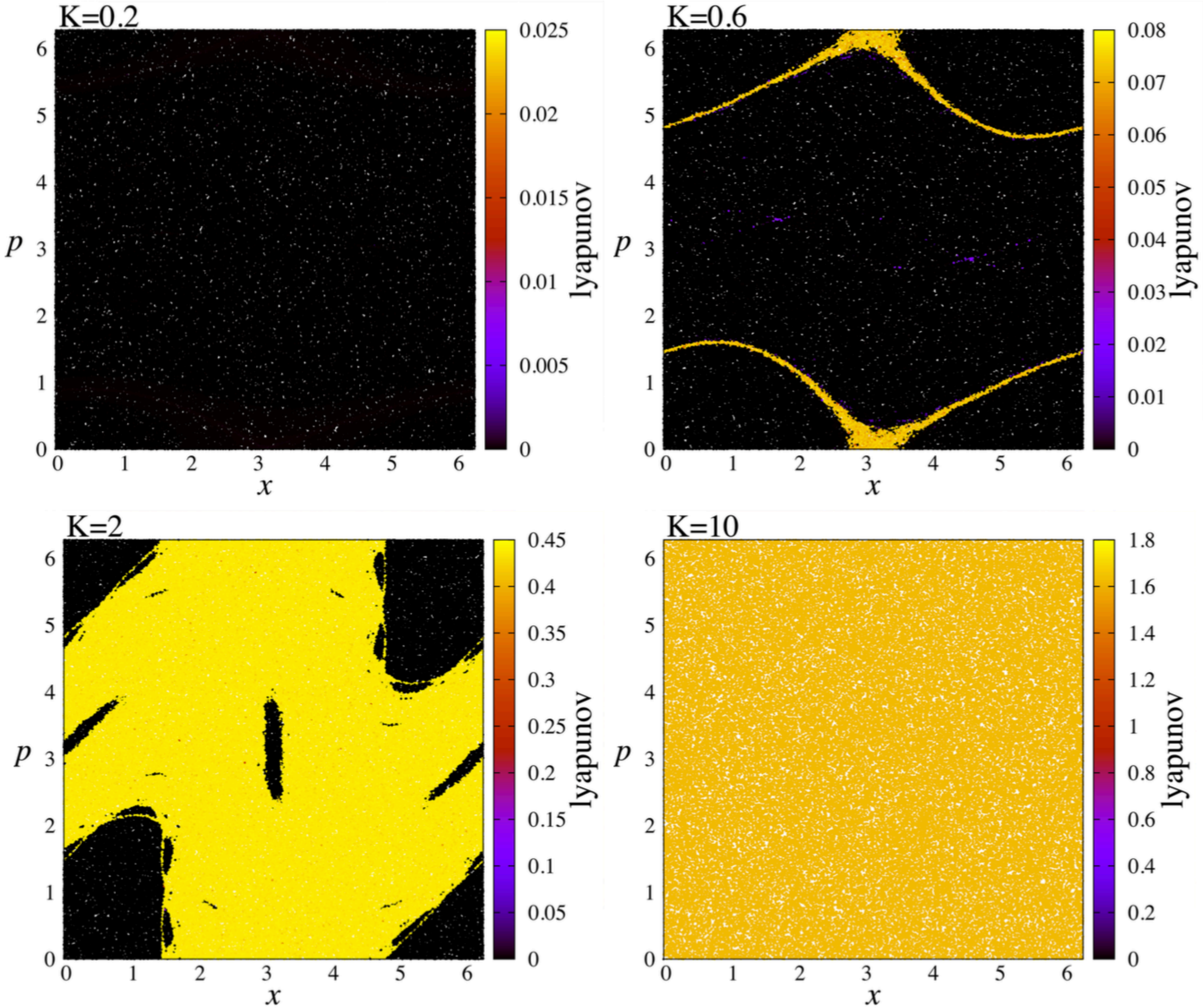
(area-preserving)

Particle confinement in magnetic traps,
particle dynamics in accelerators,
comet dynamics,
ionization of Rydberg atoms,
electron magneto-transport

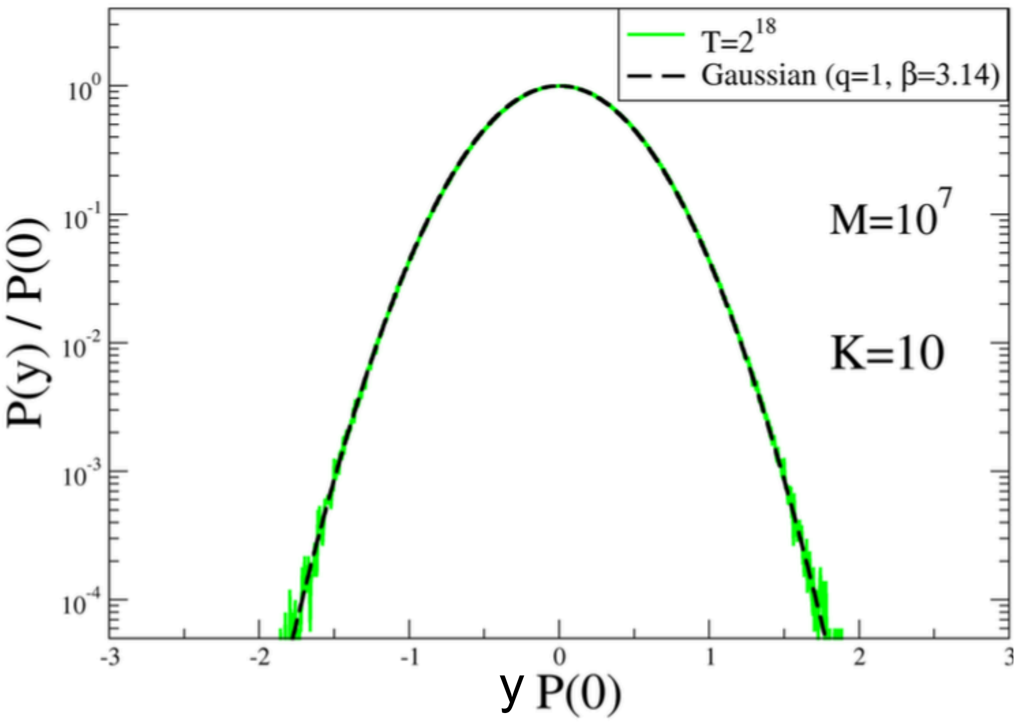


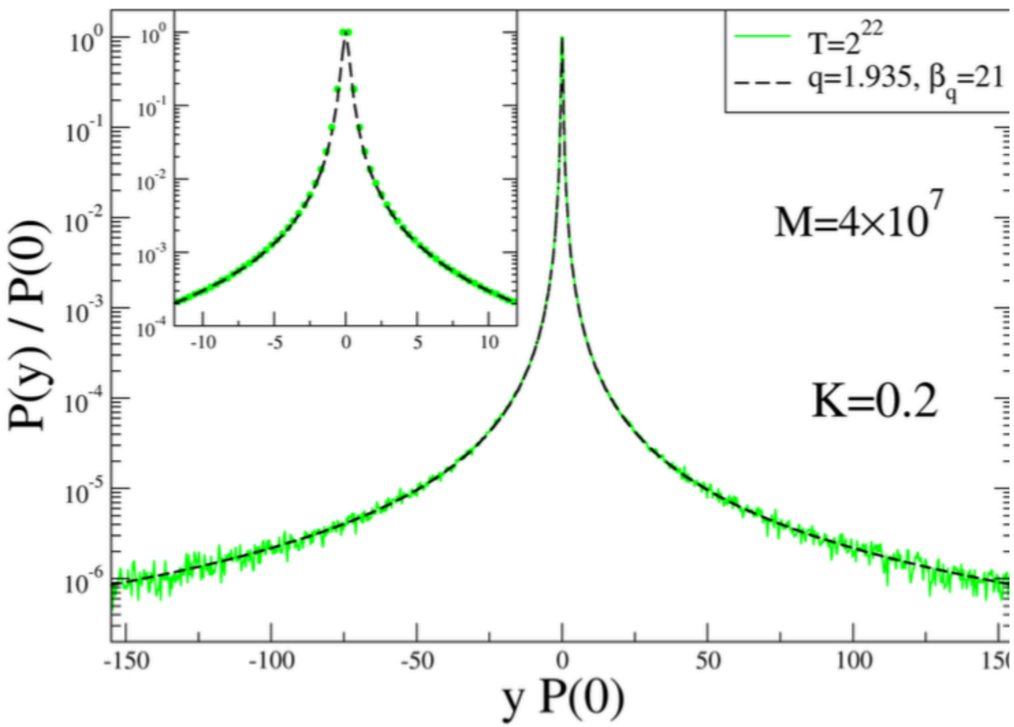
Stosszahlansatz

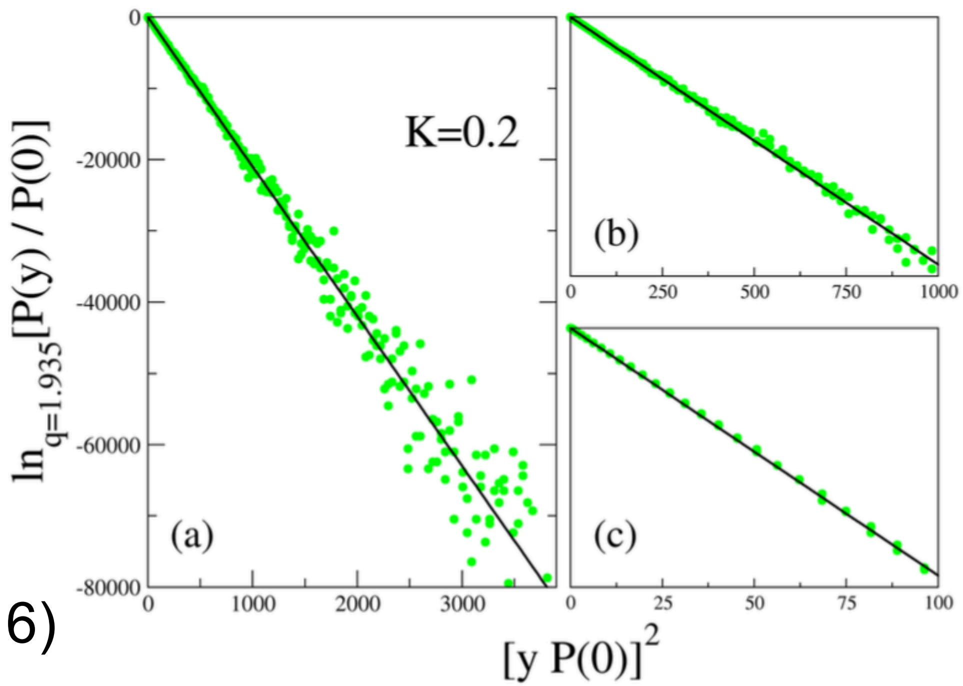
Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)



Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)







Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

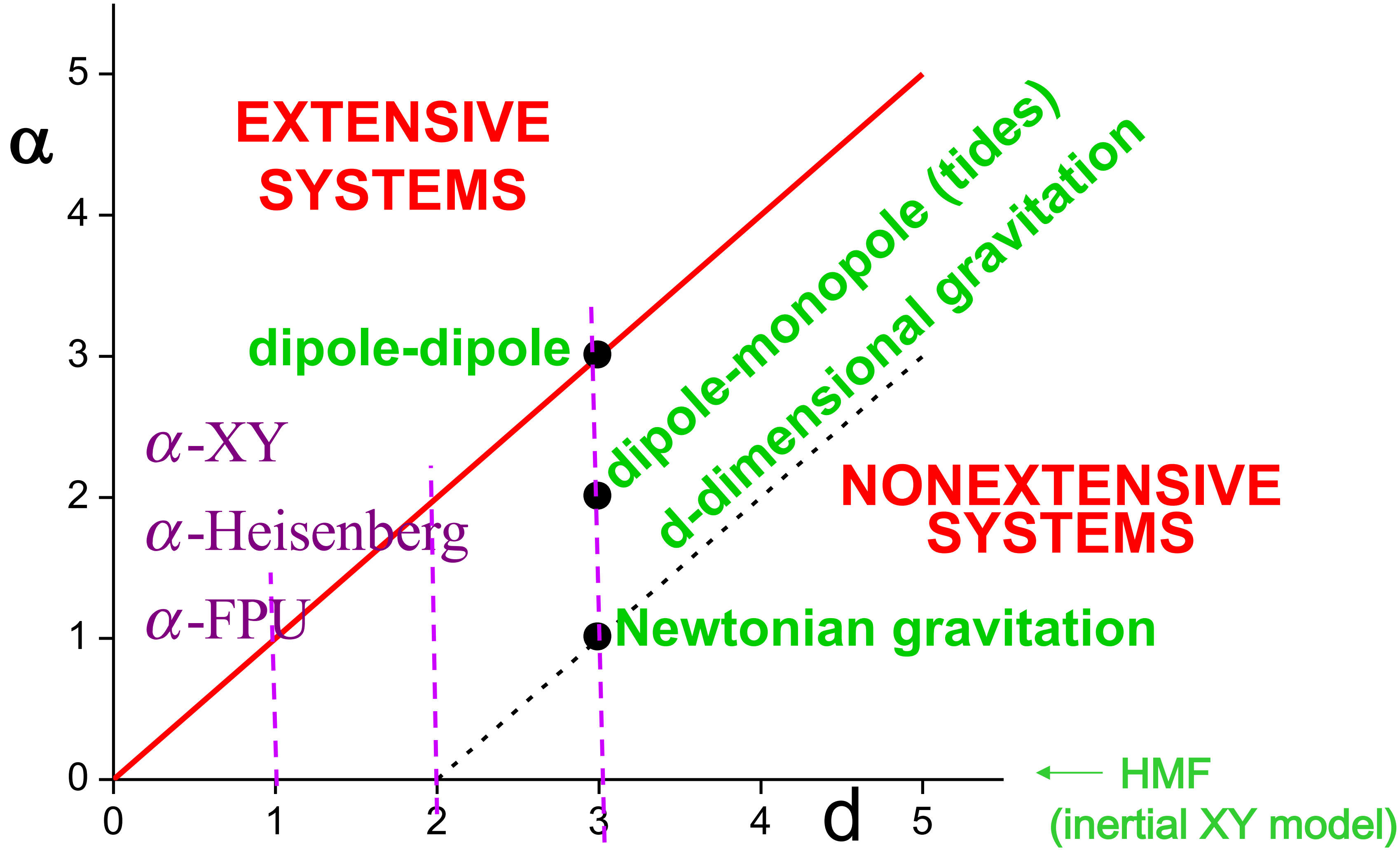
In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0,\ \ \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \qquad (\textit{short-ranged})$$

$$\textit{non-integrable if} \quad 0 \leq \alpha / d \leq 1 \qquad (\textit{long-ranged})$$



A Letters Journal Exploring the Frontiers of Physics

August 2018

EPL, **123** (2018) 30003 www.epljournal.org
doi: 10.1209/0295-5075/123/30003

Validity and failure of the Boltzmann weight

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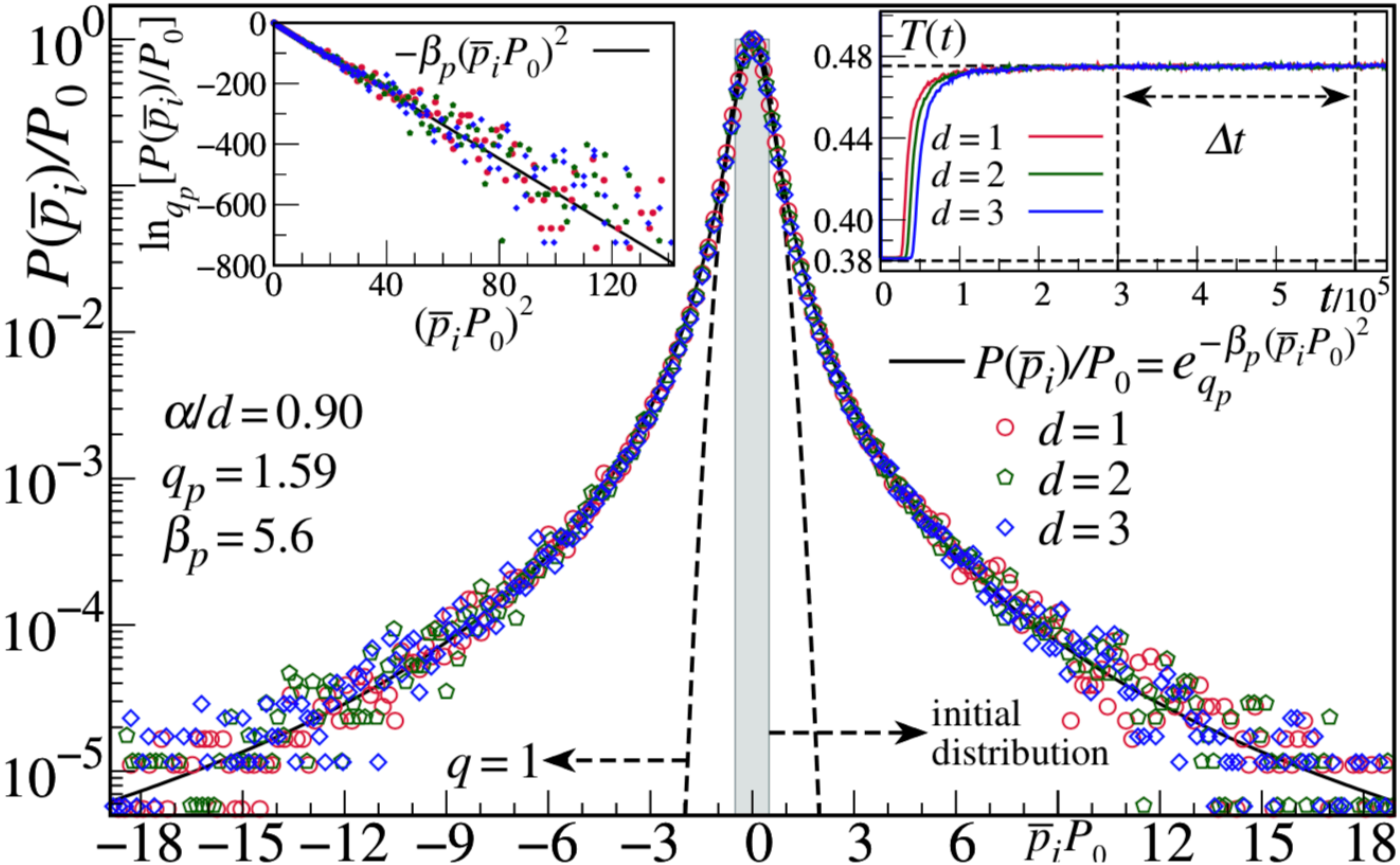
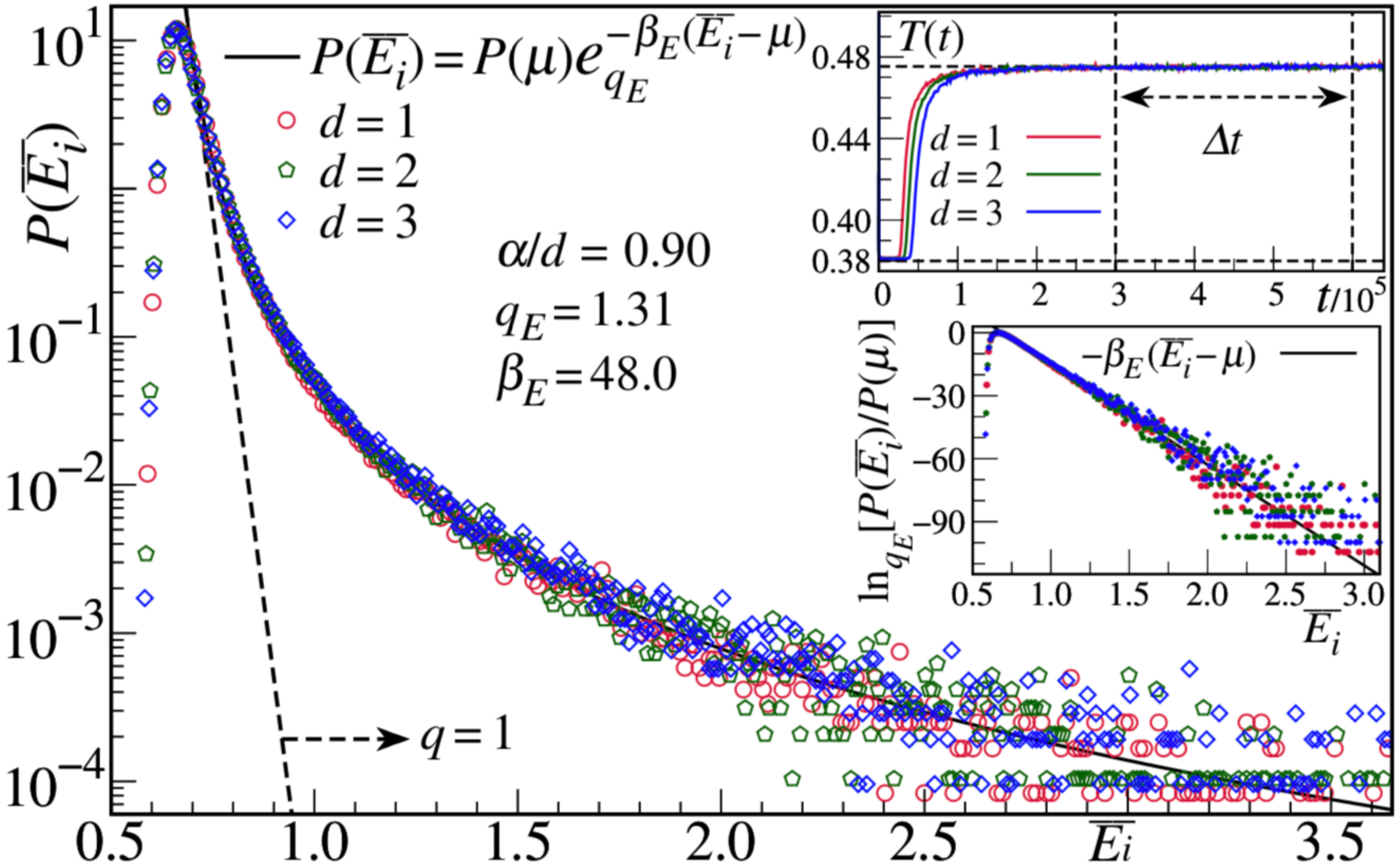
received 11 April 2018; accepted in final form 2 August 2018
published online 24 August 2018

d-DIMENSIONAL XY MODEL



L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)

d-DIMENSIONAL XY MODEL



L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)


Scale Free Network

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scientific reports

Connecting complex networks to nonadditive entropies

R. M. de Oliveira[1], Samuraí Brito[2]✉, L. R. da Silva[1,3] & Constantino Tsallis[3,4,5,6]

11, 1130 (2021)

RANDOM GEOMETRY ↔ *q* -THERMOSTATISTICS

All links are equally weighted:

$$P(w) = \delta(w-1)$$ → **degree distribution *P(k)***

D.J.B. Soares, C. T., A.M. Mariz and L.R. da Silva, EPL **70**, 70 (2005)
S. Thurner and C. T., EPL **72**, 197 (2005)
S. Brito, L.R. da Silva and C. T., Scientific Reports **6**, 27992 (2016)
T.C. Nunes, S. Brito, L.R. da Silva and C. T., J Stat Mech 093402 (2017)
S. Brito, T.C. Nunes, L.R. da Silva and C. T., PRE **99**, 012305 (2019)
N. Cinardi, A. Rapisarda and C. T., J Stat Mech 043404 (2020)

All links are randomly weighted:

$$P(w) = \frac{\eta}{w_0\,\Gamma\left(\frac{1}{\eta}\right)} e^{-(w/w_0)^\eta} \quad (w_0 > 0;\ \eta > 0)$$

R.M. Oliveira, S. Brito, L.R. da Silva and C.T., Scientific Reports **11**, 1130 (2021)

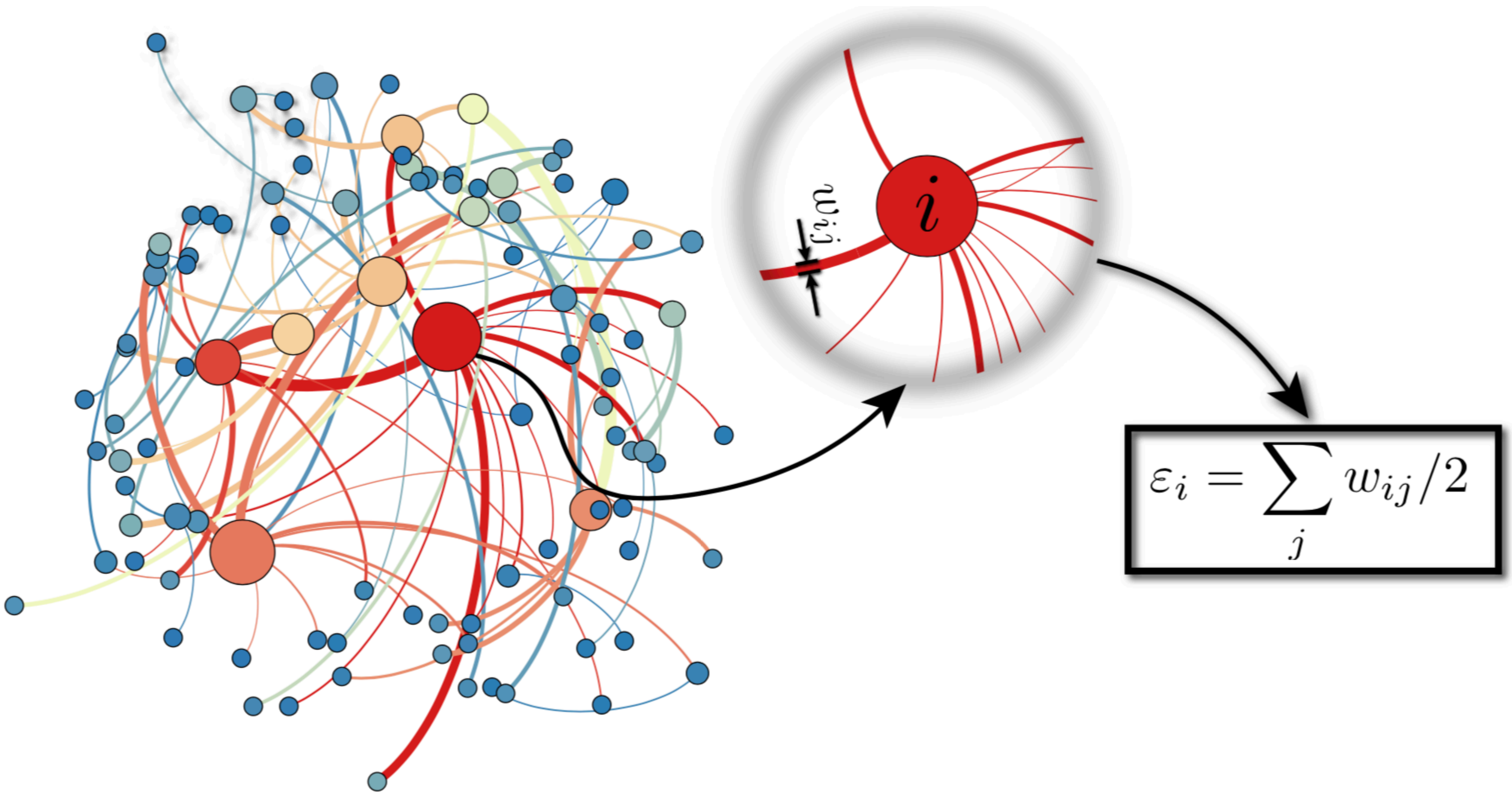
geographic localization of sites:

$$p(r) \propto 1 \big/ r^{d+\alpha_G} \quad (\alpha_G > 0,\ r \geq 1)$$

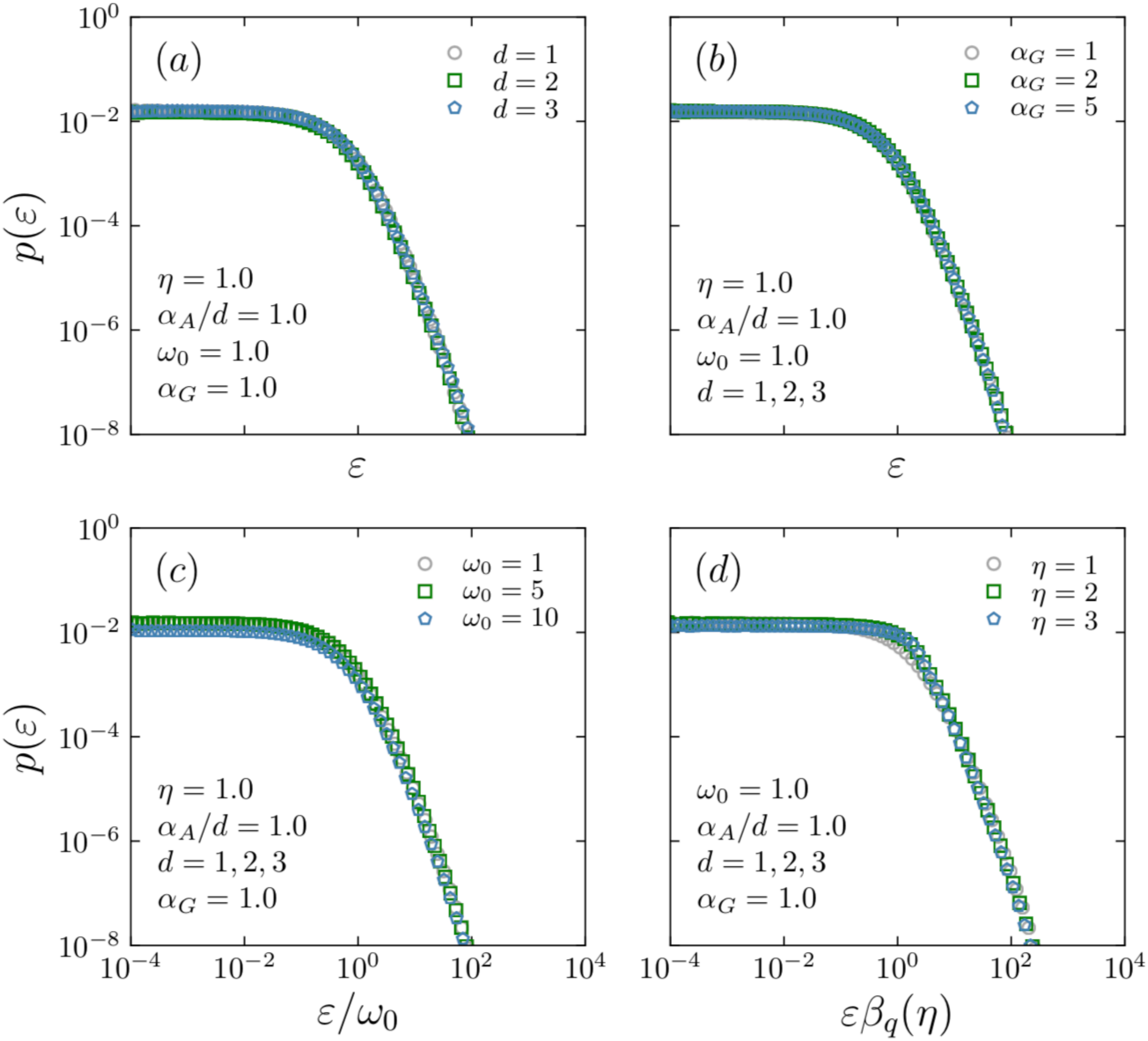
'energy' → $$\varepsilon_i \equiv \sum_{j=1}^{k_i} \frac{w_{ij}}{2} \quad (w_{ij} \geq 0)$$

preferential attachment:

$$\Pi_{ij} \propto \frac{\varepsilon_i}{d_{ij}^{\alpha_A}} \quad (\alpha_A \geq 0)$$



i
w_{ij}
$\varepsilon_i = \sum_j w_{ij}/2$

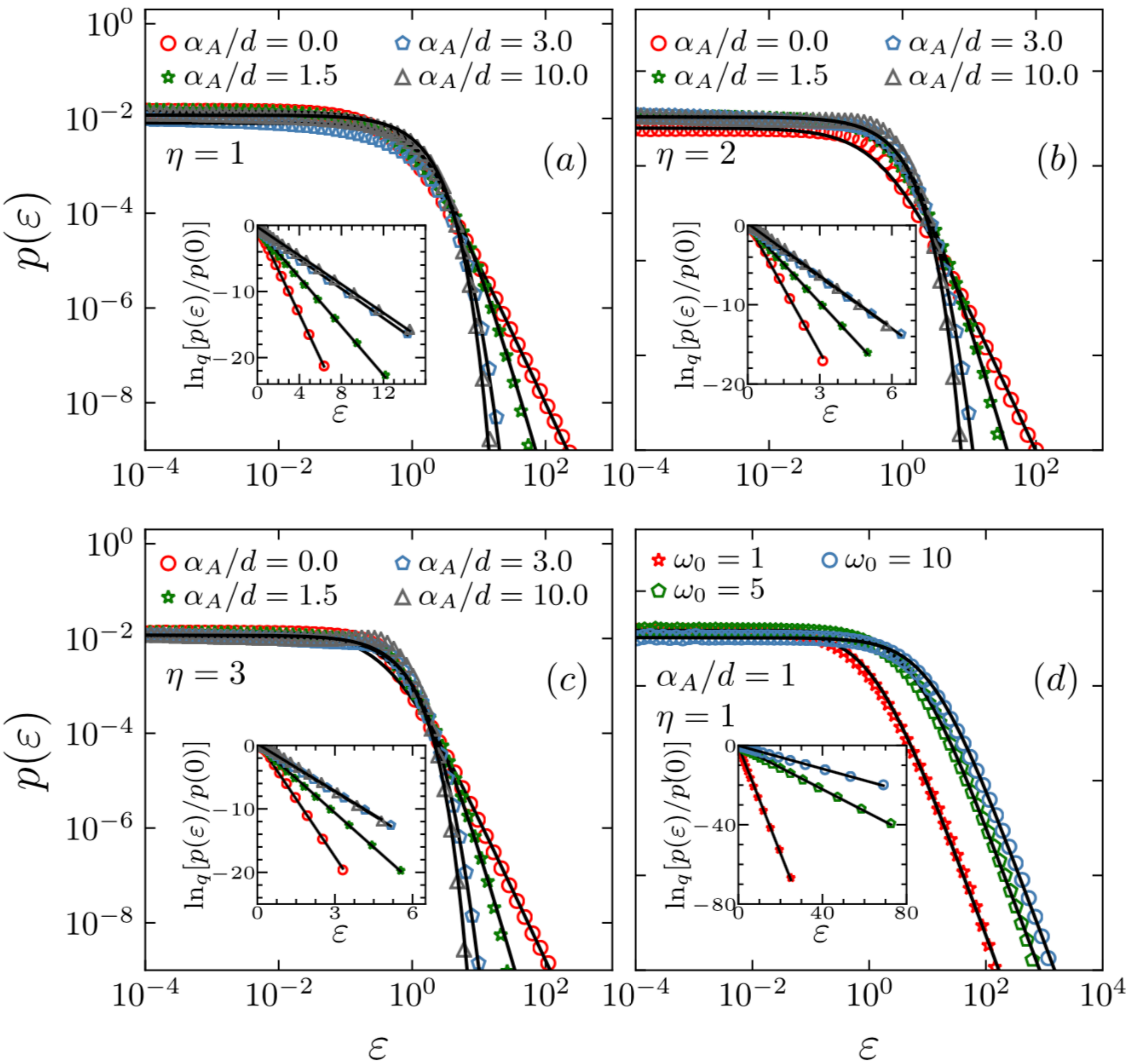


(a)
d = 1
d = 2
d = 3
η = 1.0
α_A/d = 1.0
ω_0 = 1.0
α_G = 1.0
(b)
α_G = 1
α_G = 2
α_G = 5
η = 1.0
α_A/d = 1.0
ω_0 = 1.0
d = 1, 2, 3
(c)
ω_0 = 1
ω_0 = 5
ω_0 = 10
η = 1.0
α_A/d = 1.0
d = 1, 2, 3
α_G = 1.0
(d)
η = 1
η = 2
η = 3
ω_0 = 1.0
α_A/d = 1.0
d = 1, 2, 3
α_G = 1.0
p(ε)
ε
ε/ω_0
εβ_q(η)
10^0
10^-2
10^-4
10^-6
10^-8
10^2
10^4

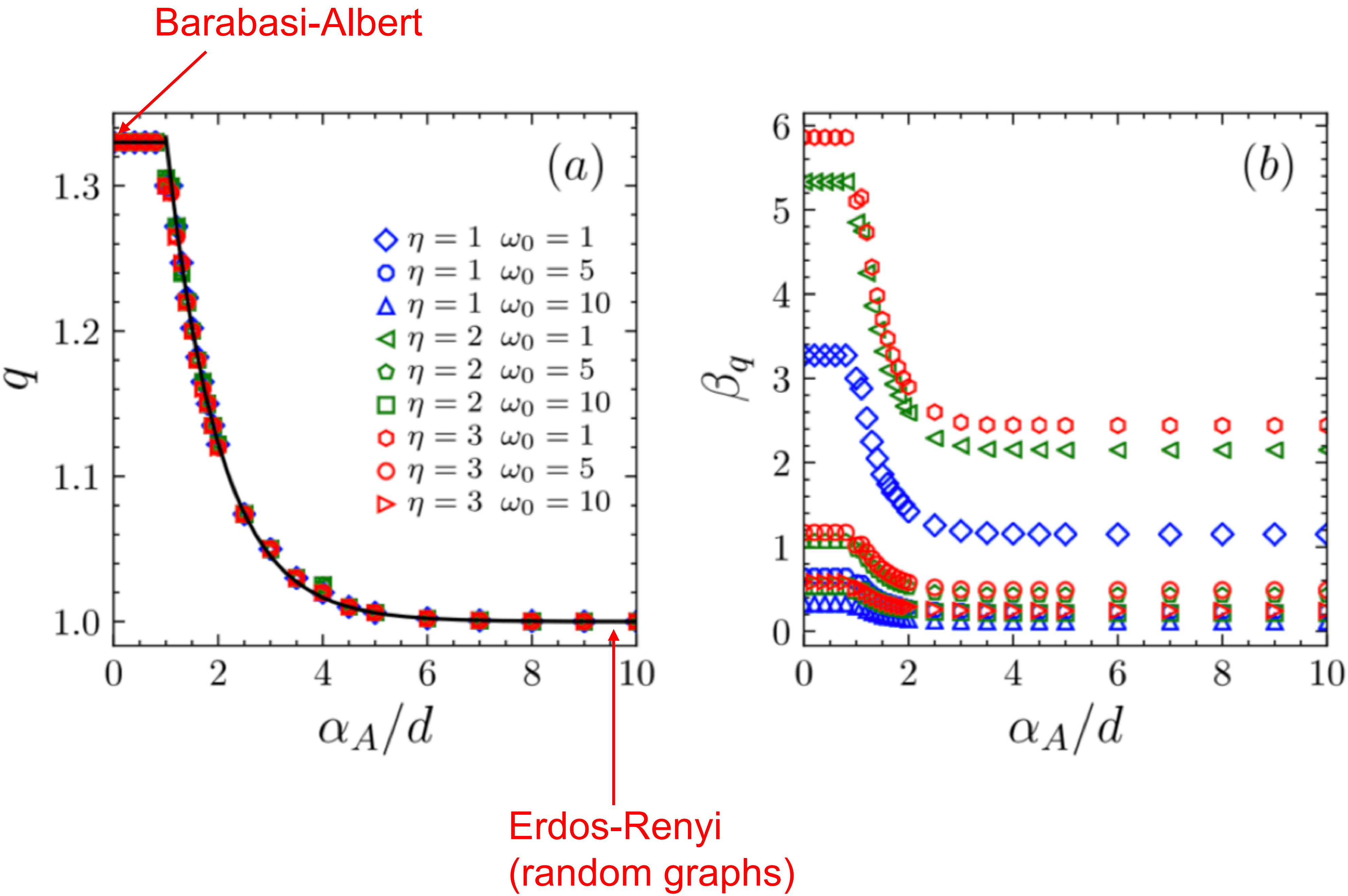
$$p_q(\varepsilon) = \frac{e_q^{-\beta_q \varepsilon}}{Z_q}$$

where $e_q^z \equiv \left[1+(1-q)\,z\right]_+^{\frac{1}{1-q}} \quad \left(e_1^z = e^z\right)$

and $\ln_q z \equiv \dfrac{z^{1-q}-1}{1-q} \quad \left(\ln_1 z = \ln z\right)$



$\alpha_A/d = 0.0$
$\alpha_A/d = 1.5$
$\alpha_A/d = 3.0$
$\alpha_A/d = 10.0$
$\eta = 1$
(a)
$\eta = 2$
(b)
$\eta = 3$
(c)
$\omega_0 = 1$
$\omega_0 = 5$
$\omega_0 = 10$
$\alpha_A/d = 1$
$\eta = 1$
(d)
$p(\varepsilon)$
ε
$\ln_q[p(\varepsilon)/p(0)]$



Barabasi-Albert
Erdos-Renyi
(random graphs)
(a)
(b)
q
β_q
α_A/d
$\eta = 1 \quad \omega_0 = 1$
$\eta = 1 \quad \omega_0 = 5$
$\eta = 1 \quad \omega_0 = 10$
$\eta = 2 \quad \omega_0 = 1$
$\eta = 2 \quad \omega_0 = 5$
$\eta = 2 \quad \omega_0 = 10$
$\eta = 3 \quad \omega_0 = 1$
$\eta = 3 \quad \omega_0 = 5$
$\eta = 3 \quad \omega_0 = 10$

Limoges - France

Journal of
Applied
Crystallography
ISSN 0021-8898

Strain-profile determination in ion-implanted single crystals using generalized simulated annealing

Alexandre Boulle[a]* and Aurélien Debelle[b]

Received 29 March 2010
Accepted 29 July 2010

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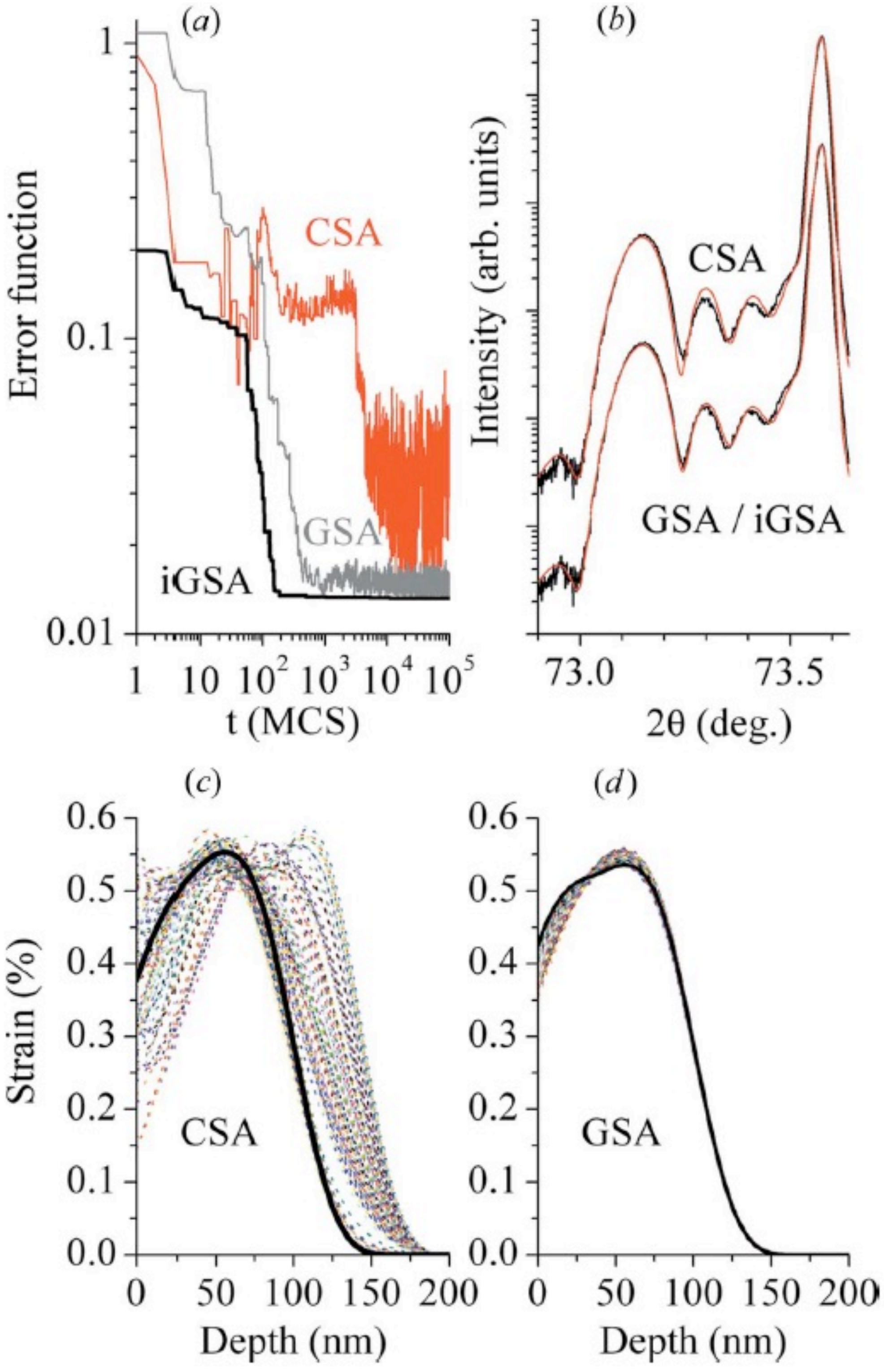
A novel least-squares fitting procedure is presented that allows the retrieval of strain profiles in ion-implanted single crystals using high-resolution X-ray diffraction. The model is based on the dynamical theory of diffraction, including a B-spline-based description of the lattice strain. The fitting procedure relies on the generalized simulated annealing algorithm which, contrarily to most common least-squares fitting-based methods, allows the global minimum of the error function (the difference between the experimental and the calculated curves) to be found extremely quickly. It is shown that convergence can be achieved in a few hundred Monte Carlo steps, *i.e.* a few seconds. The method is model-independent and allows determination of the strain profile even without any 'guess' regarding its shape. This procedure is applied to the determination of strain profiles in Cs-implanted yttria-stabilized zirconia (YSZ). The strain and damage profiles of YSZ single crystals implanted at different ion fluences are analyzed and discussed.



(a)
Error function
CSA
GSA
iGSA
t (MCS)
(b)
Intensity (arb. units)
CSA
GSA / iGSA
2θ (deg.)
(c)
Strain (%)
CSA
Depth (nm)
(d)
GSA
Depth (nm)

Genova, 2018

Eur. Phys. J. Special Topics **229**, 841–849 (2020)

https://doi.org/10.1140/epjst/e2020-800232-7

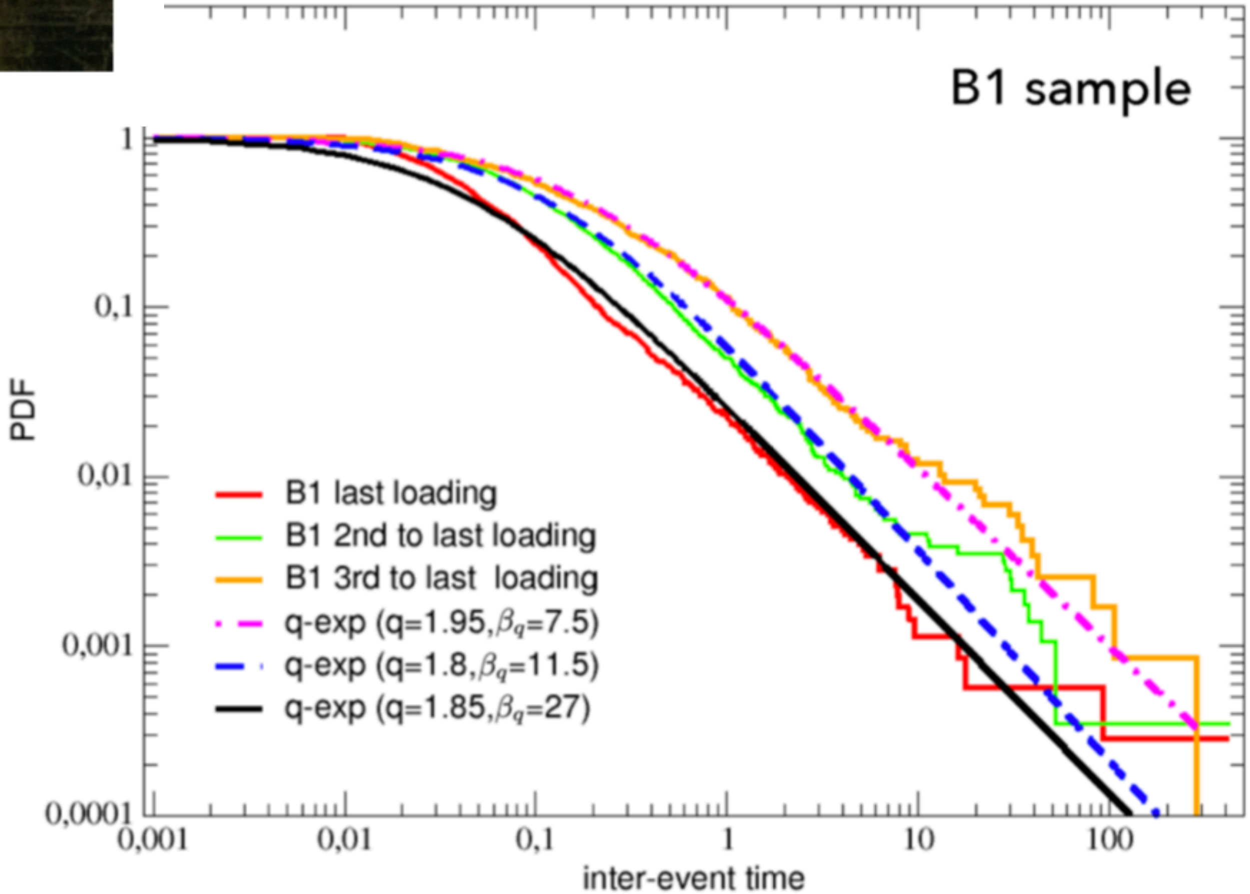
THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

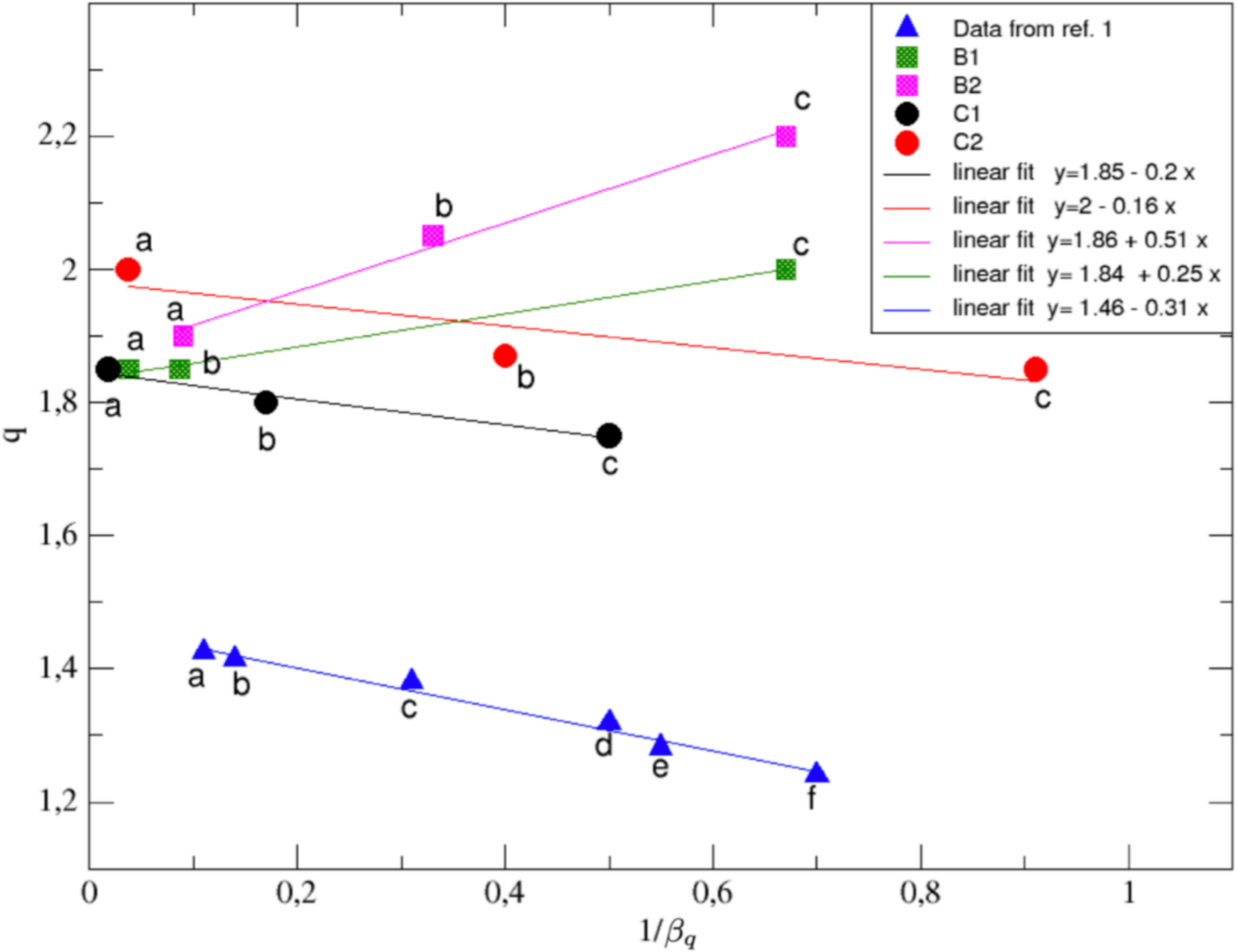
Regular Article

Acoustic emissions in compression of building materials: q-statistics enables the anticipation of the breakdown point

Annalisa Greco[1], Constantino Tsallis[2,3,4], Andrea Rapisarda[4,5,6], Alessandro Pluchino[5,6,a], Gabriele Fichera[1], and Loredana Contrafatto[1]

$$\frac{P(t)}{P(0)} = e_q^{-\beta_q t}$$





Data from ref. 1
B1
B2
C1
C2
linear fit y=1.85 - 0.2 x
linear fit y=2 - 0.16 x
linear fit y=1.86 + 0.51 x
linear fit y= 1.84 + 0.25 x
linear fit y= 1.46 - 0.31 x
q
1/βq
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
2
2,2
a
b
c
d
e
f

Computers and Mathematics with Applications 60 (2010) 2426–2432

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

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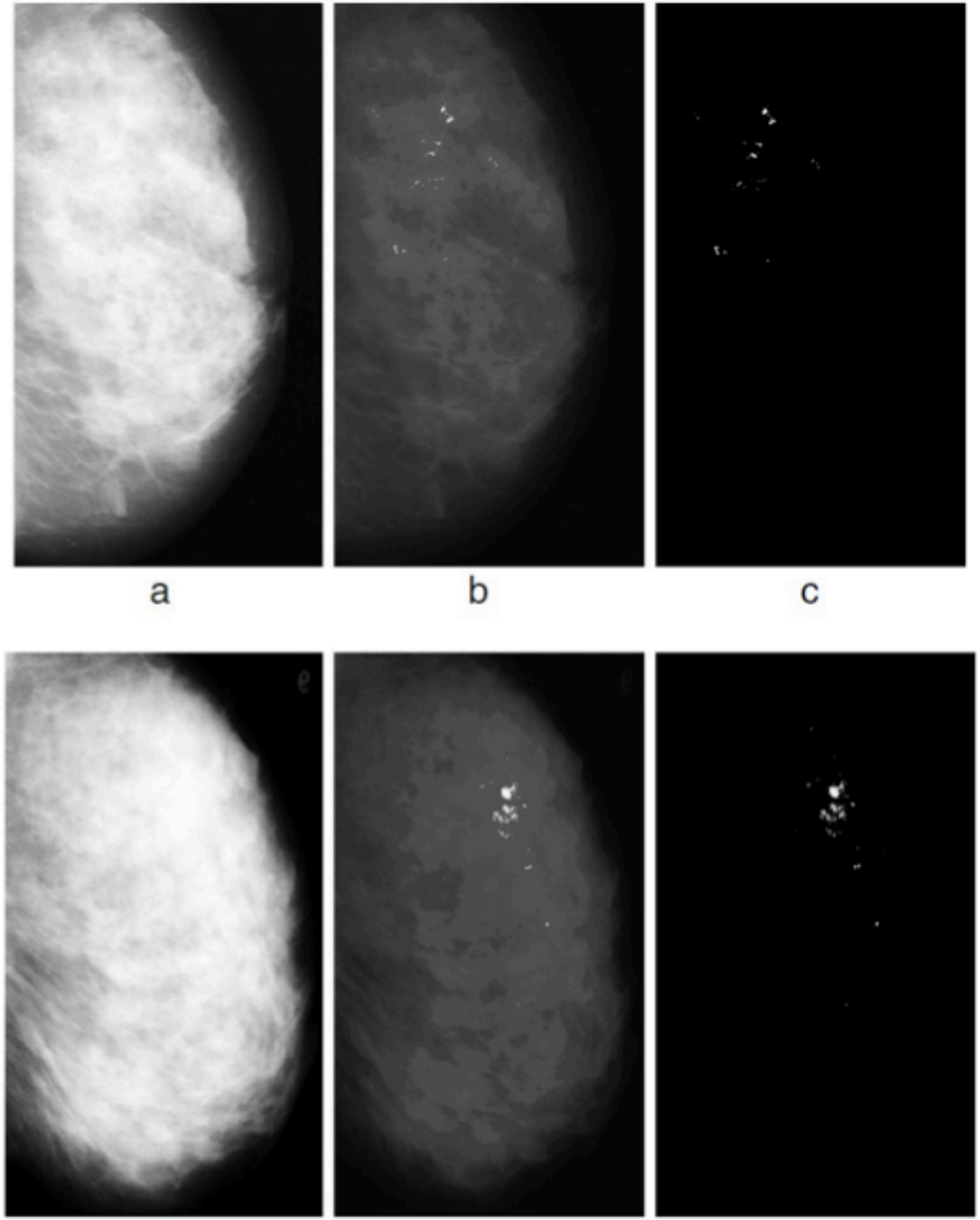
ARTICLE INFO

Article history:
Received 18 August 2009
Received in revised form 12 August 2010
Accepted 12 August 2010

Keywords:
Tsallis entropy
Type II fuzzy index
Shannon entropy
Mammograms
Microcalcification

ABSTRACT

This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter '*q*', which depends on the non-extensiveness of a mammogram. In previous studies, '*q*' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of '*q*'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.



a b c

d e f

Efficient classification of COVID-19 CT scans by using q-transform model for feature extraction

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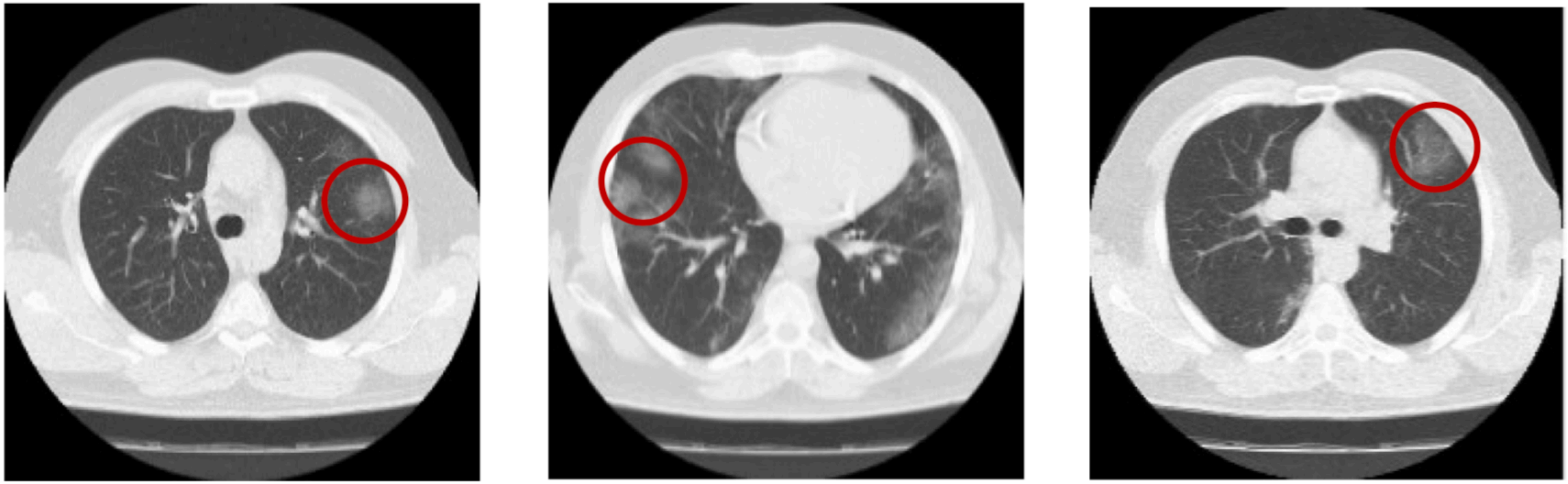
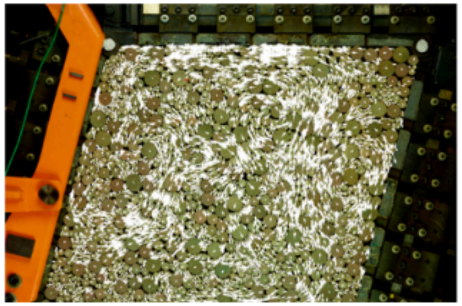
PeerJ Comput. Sci. 7:e553 DOI 10.7717/peerj-cs.553

(15 June 2021)



Original scans of infected lungs (fibrosis)

scans after q-enhanced processing (q=0.5)



EDITORS' SUGGESTION

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

The velocity distribution of sheared granular media shows unexpected similarities with turbulent fluid flows.

Gaël Combe, Vincent Richefeu, Marta Stasiak, and Allbens P.F. Atman

Phys. Rev. Lett. **115**, 238301 (2015)

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

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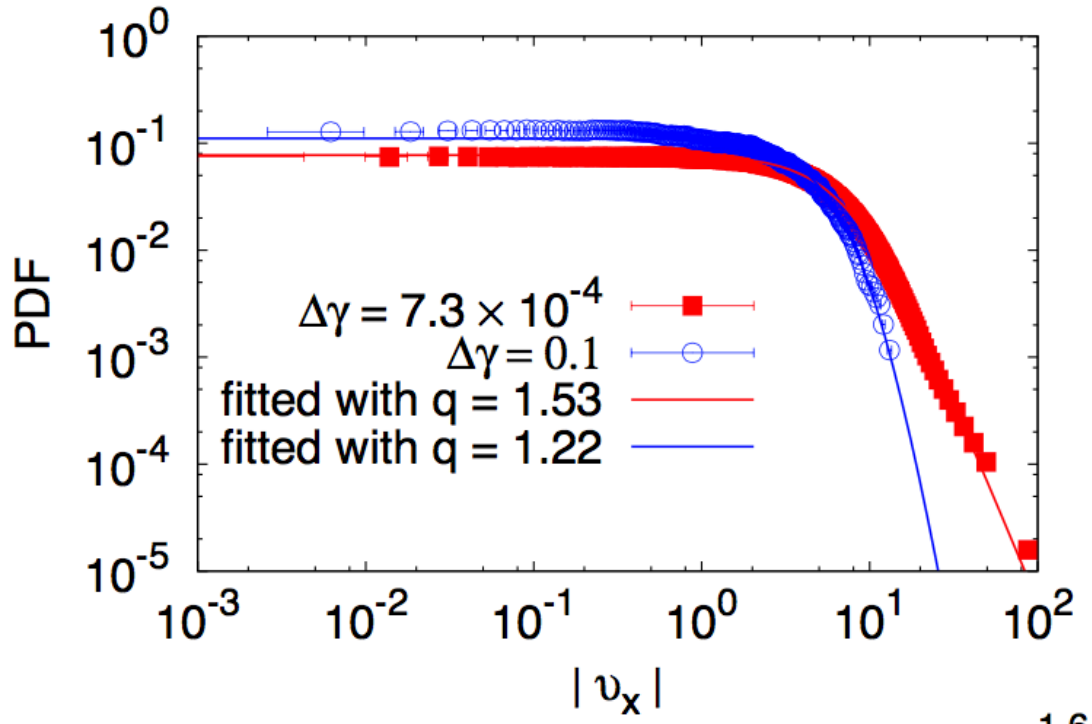
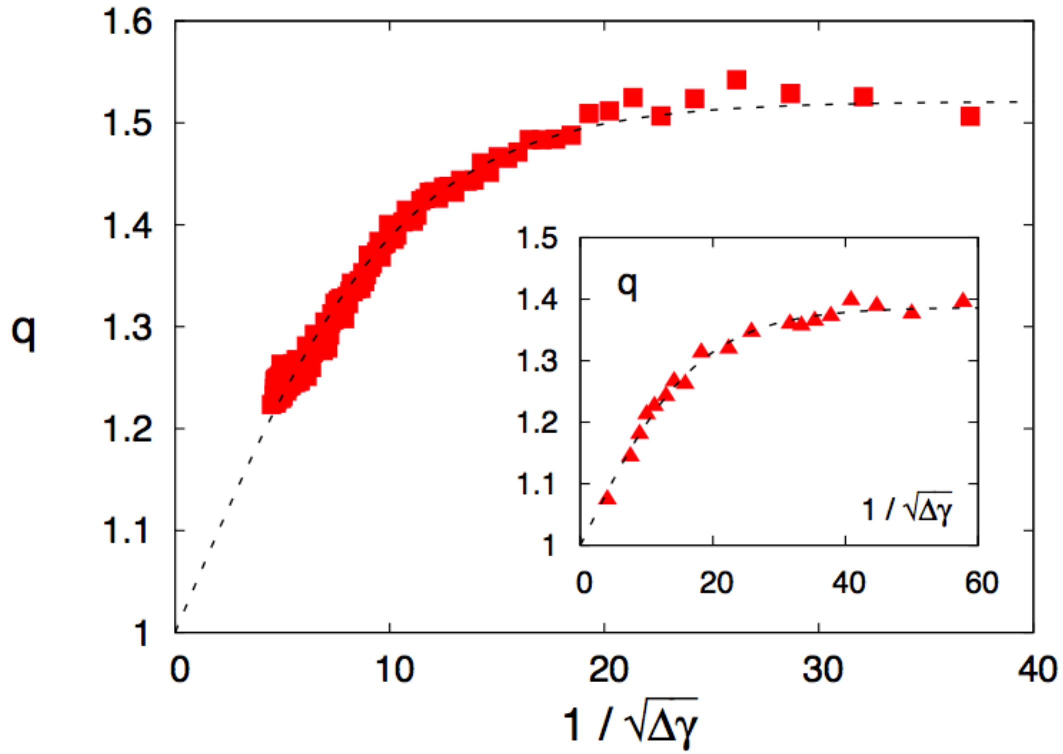
(Received 28 July 2015; published 1 December 2015)





Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

$$\langle x^2 \rangle \propto t^{\alpha}$$

Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

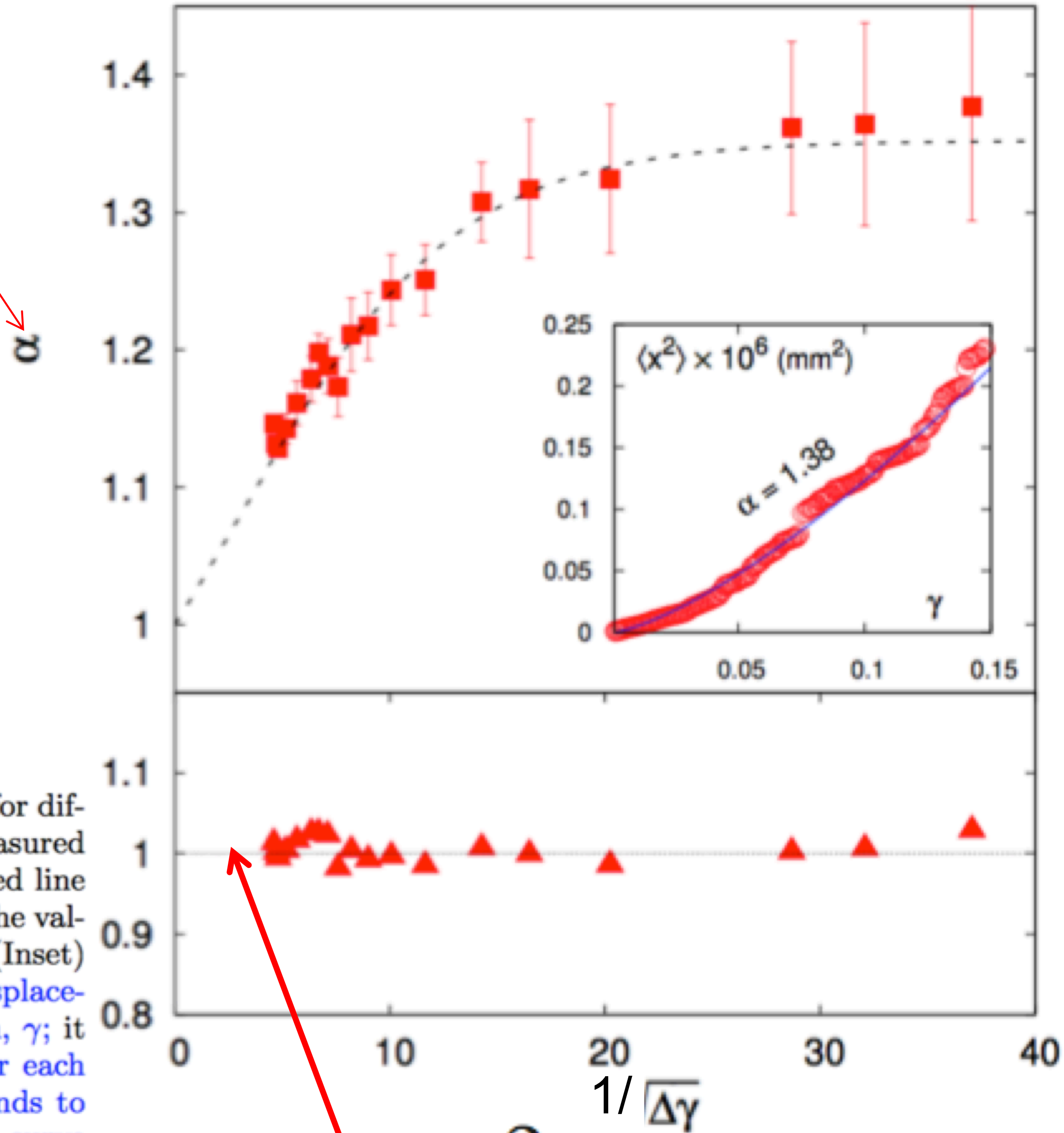


FIG. 4. Verification of the Tsallis-Bukman scaling law for different regimes of diffusion. (top) Evolution of the measured diffusion exponent α as a function of $1/\sqrt{\Delta\gamma}$ the dashed line is a direct application of the scaling law from the fit of the values shown in Fig. 3, $\alpha(1/\sqrt{\Delta\gamma}) = 2/[3 - q(1/\sqrt{\Delta\gamma})]$. (Inset) a typical diffusion curve showing the mean square displacement fluctuations, $\langle x^2 \rangle$, in function of the shear strain, γ; it allows the assessment of the diffusion exponent, α, for each strain window tested. In the case shown, it corresponds to the smallest strain window, the rightmost point in the curve at the main panel. Note that for a constant strain rate, γ is proportional to time. (Bottom) Measure of the deviation of the data relative to the scaling law prediction, as a function of $1/\sqrt{\Delta\gamma}$, showing an agreement on the order of $\pm 2\%$.

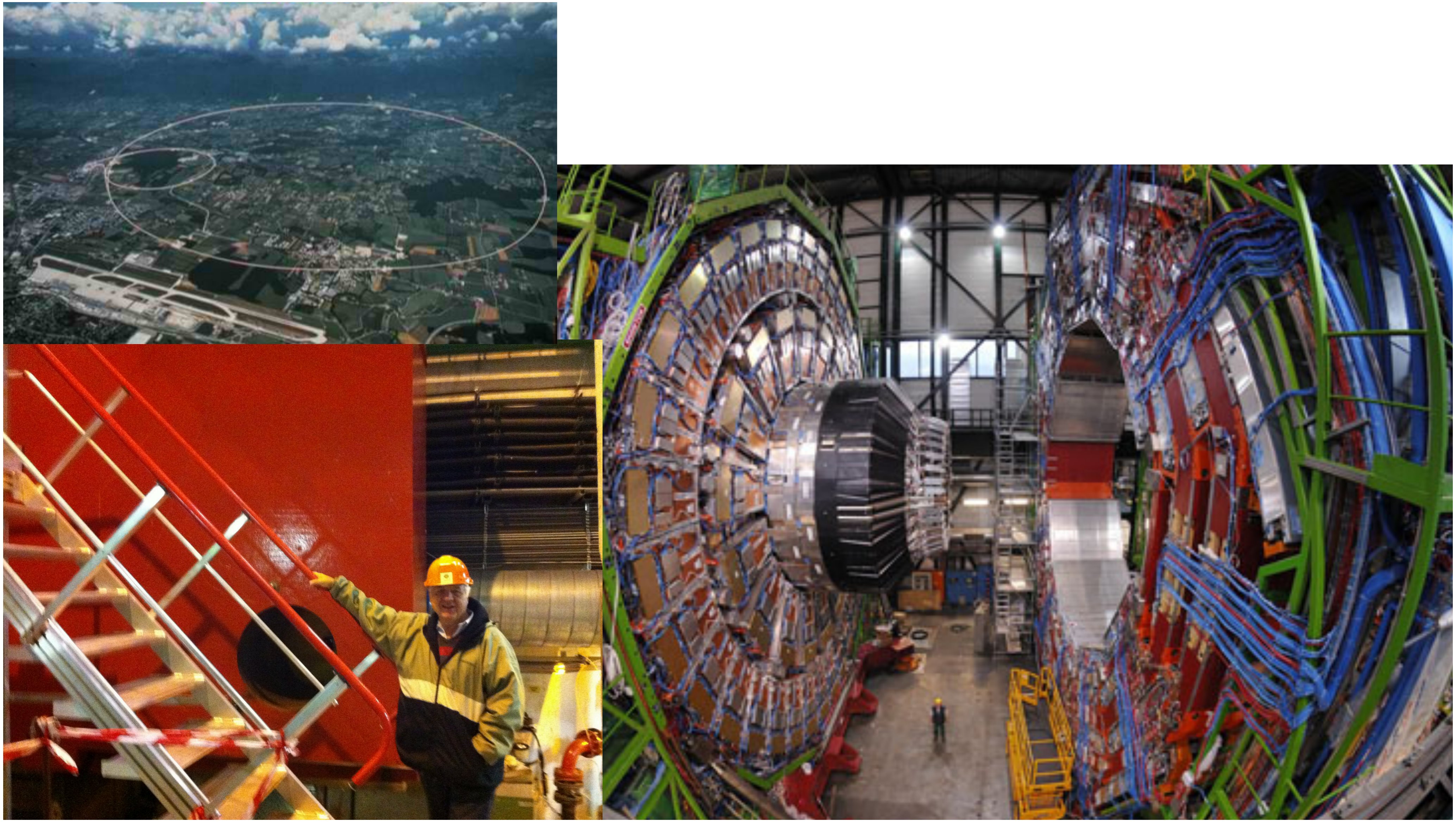
$$\alpha = \frac{2}{3 - q}$$

CT and DJ Bukman, PRE **54** (1996) R2197

LHC (Large Hadron Collider)

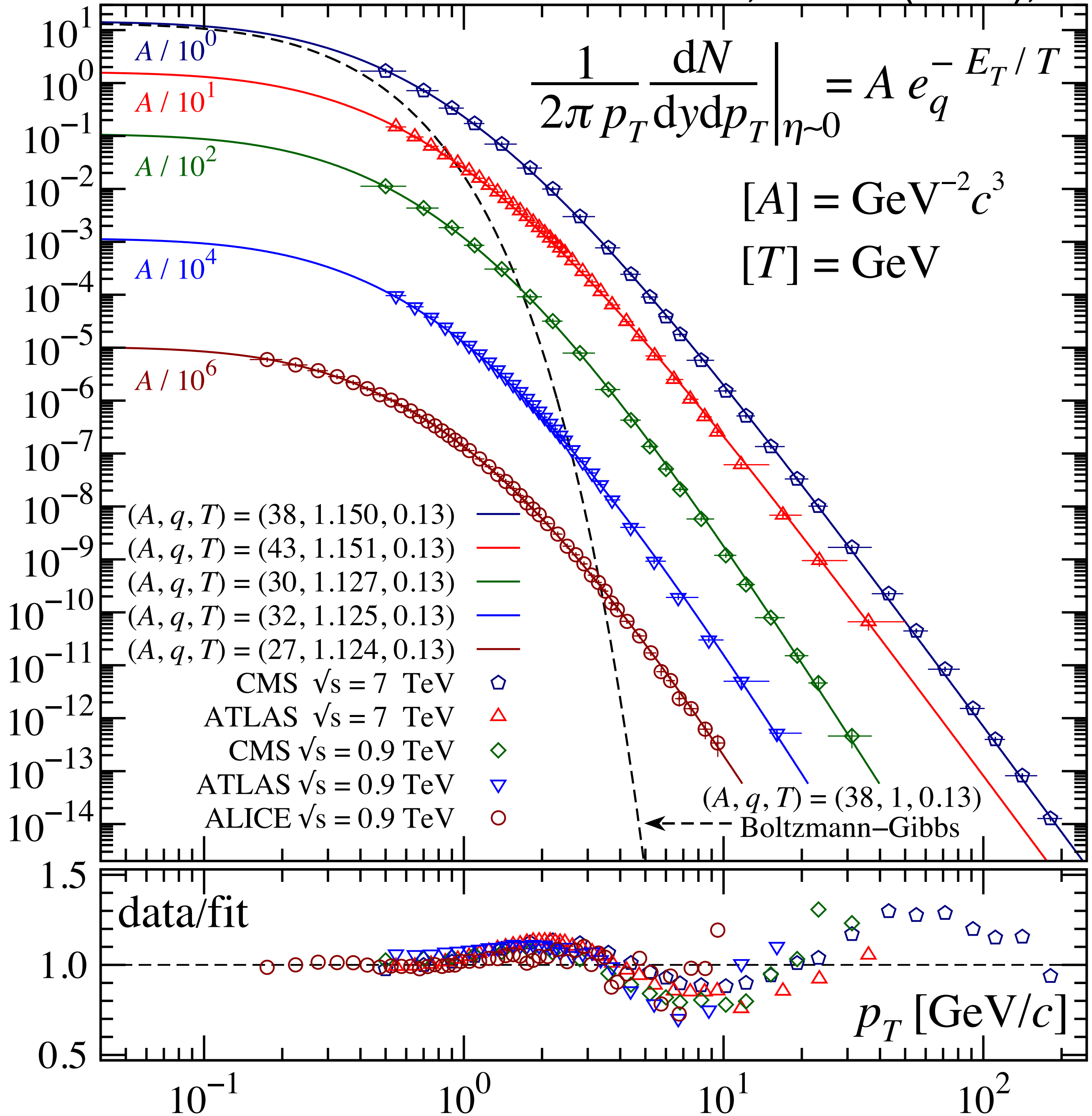
CMS, ALICE, ATLAS and LHCb detectors

~ 4000 scientists/engineers from ~ 200 institutions of ~ 50 countries



SIMPLE APPROACH: TWO-DIMENSIONAL SINGLE RELATIVISTIC FREE PARTICLE

C.Y. Wong, G. Wilk, L.J.L. Cirto and C. T.,
EPJ Web of Conferences **90**, 04002 (2015), and PRD **91**, 114027 (2015)

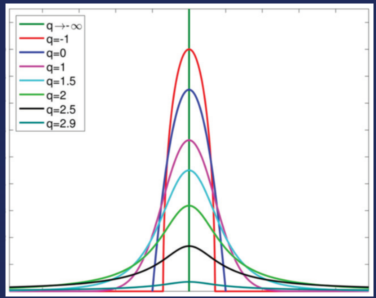


$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$

	BOLTZMANN-GIBBS STATISTICAL MECHANICS	NONEXTENSIVE STATISTICAL MECHANICS
Entropy	$S_{BG} = -k\sum_i p_i \ln p_i$ $= k\sum_i p_i \ln \frac{1}{p_i}$ (additive)	$S_q = k\frac{1-\sum_i p_i^q}{q-1}$ $= k\sum_i p_i \ln_q \frac{1}{p_i}$ (nonadditive)
Distribution of velocities v	$p(v) \propto e^{-mv^2/2kT}$ (Maxwellian) Central Limit Theorem	$p(v) \propto e_q^{-mv^2/2kT_q}$ (q-Gaussian) q-Central Limit Theorem
Distribution of energies E_i	$p_i \propto e^{-E_i/kT}$ (BG weight) Large Deviation Theory	$p_i \propto e_q^{-E_i/kT_q}$ (q-weight) q-Large Deviation Theory

$$\ln_q z \equiv \frac{z^{1-q}-1}{1-q} \quad (\ln_1 z = \ln z) \quad \text{and} \quad e_q^z \equiv \left[1+(1-q)z\right]^{\frac{1}{1-q}} \quad (e_1^z = e^z)$$

The book is devoted to the mathematical foundations of nonextensive statistical mechanics. This is the first book containing the systematic presentation of the mathematical theory and concepts related to nonextensive statistical mechanics, a current generalization of Boltzmann-Gibbs statistical mechanics introduced in 1988 by one of the authors and based on a nonadditive entropic functional extending the usual Boltzmann-Gibbs-von Neumann-Shannon entropy. Main mathematical tools like the q-exponential function, q-Gaussian distribution, q-Fourier transform, q-central limit theorems, and other related objects are discussed rigorously with detailed mathematical rational. The book also contains recent results obtained in this direction and challenging open problems. Each chapter is accompanied with additional useful notes including the history of development and related bibliographies for further reading.



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ISBN 978-981-124-515-2

Mathematical Foundations of Nonextensive Statistical Mechanics

Umarov
Tsallis

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Sabir Umarov
Constantino Tsallis

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THAN*q*