T-duality, α' corrections and BH entropy

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Work in progress with:

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T-duality reminder

If nothing depends on ψ, write the background as (Kaloper, Meissner 97):

$$\begin{aligned} \mathrm{d}\bar{s}^2 &= e^a_\mu e^b_\nu \eta_{ab}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2\sigma}(\mathrm{d}\psi + V)^2, \\ \bar{B} &= \mathcal{B} + \frac{1}{2} W \wedge V + W \wedge \mathrm{d}\psi \qquad\qquad e^{-2\bar{\phi}} = e^{-2\phi} e^{-\sigma} \end{aligned}$$

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The leading order Buscher rules are:

$$\begin{aligned} \sigma &\rightarrow -\sigma, \qquad \left(\sim \mathrm{R} \rightarrow \frac{1}{\mathrm{R}}\right), \\ V &\leftrightarrow W, \qquad (\sim \mathrm{n} \leftrightarrow \mathrm{m}). \end{aligned}$$

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The effective Lagrangian is manifestly invariant when ψ is cyclic:

$$
\begin{aligned}
L &= e^{-2\bar{\phi}}\sqrt{-g}\left(\bar{R} + 4(\bar{\nabla}\bar{\phi})^2 - \frac{\bar{H}^2}{12}\right) \\
&= e^{-2\phi}\sqrt{-g}\left(R + 4(\nabla\phi)^2 - \frac{H^2}{12} - (\nabla\sigma)^2 - \frac{e^{2\sigma} V^2_{\mu\nu}}{4} - \frac{e^{-2\sigma} W^2_{\mu\nu}}{4}\right)
\end{aligned}
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α' corrected theory (Marques, Nunez 2015)

Perturbative correction with two $O(\alpha')$ parameters:

$$L = e^{-2\bar{\phi}}\sqrt{-\bar{g}}\left(L^{(0)} + L^{(1)}\right),$$

$$L^{(0)} = \bar{R} - 4\bar{\nabla}_M\bar{\phi}\bar{\nabla}^M\bar{\phi} + 4\bar{\nabla}_M\bar{\nabla}^M\bar{\phi} - \frac{1}{12}\bar{H}^2$$

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\bar{\Omega} &= \mathrm{Tr}\Big(\bar{\omega}\wedge \mathrm{d}\bar{\omega} + \frac{2}{3}\bar{\omega}\wedge\bar{\omega}\wedge\bar{\omega}\Big).
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- Invariant under corrected T-duality.
- Invariant under (anomalous) Lorentz transformation.

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Leading order: YES (Horowitz, Welch 93). Arvanitakis, Blair 16 using Double Field Theory.

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3. Invariant exclusively for string theory $\mathbf{\Lambda}_-$, $\mathbf{\Lambda}_+$. Most interesting.

BH ansatz and corrected entropy

We consider very general non-degenerate BHs with coordinates $\{t, r, \theta_1, .., \theta_{D-3}, \psi\}$, $\bar{e}_M^A$ lower triangular and $W_r = 0$, the horizon given by $r = r_+$ and of bifurcate type ($\iff \kappa = const$) with:

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Schematically, the action is:

$$L = L^{(0)} + \boldsymbol{\Lambda}_+ \left(\bar{R}ie^2 + \bar{R}\bar{H}\bar{H} + \bar{H}^4 + (\bar{H}^2)^2 + \bar{\nabla}\bar{H}\bar{\nabla}\bar{H} \right) + \boldsymbol{\Lambda}_- \bar{H}\bar{\Omega}$$

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$\mathbf{\Lambda}_- H\Omega$ term forbids Wald formula; we used conical singularity method:

$$S_{BH} = (\alpha\partial_\alpha - 1)\mathcal{W}[\alpha]|_{\alpha=1}, \quad \mathcal{W}[\alpha] = \int_0^{2\pi\alpha} d^D x\, e^{-2\bar{\phi}}\sqrt{-\bar{g}}\, L_E[\bar{g}, \bar{B}, \bar{\phi}] \quad \alpha = \beta/\beta_H,$$

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The result can be written as:

$$S_{BH} = \frac{1}{4G_D}\int_{\bar{\Sigma}} d^{D-2}x\, e^{-2\bar{\phi}}\sqrt{\bar{g}_h}\left(1 + \left(\mathbf{\Lambda}_+ \frac{\delta \bar{L}^{(1)}_{\mathbf{\Lambda}_-=0}}{\delta \bar{R}_{MNRS}} + \mathbf{\Lambda}_- \bar{H}^{AMN}\bar{\omega}_A^{RS}\right)\bar{n}_{MN}\bar{n}_{RS}\right)$$

$\bar{n}_{MN}$ (trivial) transformation

$\bar{n}_{MN} = \xi_M \chi_N - \xi_N \chi_M$, ξ generator and χ orthogonal to Σ. We only need leading order.

Properties:

$\bar{e}^2_M \xi^M|_\Sigma = \bar{e}^2_t|_\Sigma = 0$, $\bar{B}_{MN}\xi^M(\partial_\psi)^N|_\Sigma = \bar{B}_{t\psi}|_\Sigma = W_t|_\Sigma = 0$. Vierbein before $\to$ after duality (4d example):

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$$\implies \quad \partial_t, \partial_r \perp \partial_{\theta_i}, \partial_\psi \implies \bar{n} = \bar{n}_{tr}\, \mathrm{d}t \wedge \mathrm{d}r + O(\alpha') = \bar{n}_{tr} E^t_0 E^r_1\; e^0 \wedge e^1 + O(\alpha') = \hat{\bar{n}},$$

Using more rigorous near horizon expansion with regular coordinates U, V, and regularity of $e^{a>1}_t$ or simply $\hat{\nabla}_M \xi_N = \kappa \bar{n}_{MN}$ yields the same result at the bifurcation surface B:

$$\boxed{\bar{n}_{01} = \hat{\bar{n}}_{01} = 1 \qquad , \text{ others } 0.}$$

$e^{-2\bar{\phi}}\sqrt{\bar{g}_h}$ and $\left(\Lambda_+ \frac{\delta L^{(1)}}{\delta \bar{R}_{MNRS}} + \Lambda_- \bar{H}^{AMN}\bar{\omega}_A^{RS}\right)\bar{n}_{MN}\bar{n}_{RS}$ invariance

Area law invariant by itself! (every factor is non trivially invariant under corrected rules):

$$e^{-2\bar{\phi}}\sqrt{\bar{g}_h} \quad = \quad e^{-2\phi}\sqrt{g_h} = e^{-2\phi} e^2_{\theta^1} \ldots e^{D-2}_{\theta^{D-3}} = e^{-2\widehat{\phi}}\widehat{e}^2_{\theta^1}\ldots\widehat{e}^{D-2}_{\theta^{D-3}} = e^{-2\widehat{\bar{\phi}}}\sqrt{\widehat{\bar{g}}_h}.$$

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Corrections invariant under $\sigma \to -\sigma$, $V \leftrightarrow W$:

$$\left(\Lambda_+ \frac{\delta L^{(1)}}{\delta \bar{R}_{MNRS}} + \Lambda_- \bar{H}^{AMN}\bar{\omega}_A^{RS}\right)\bar{n}^{MN}\bar{n}^{RS} = \Lambda_+\left(4\bar{R}^{01}{}_{01} - 3\bar{H}^{A01}\bar{H}_{A01}\right) - \Lambda_-\, 4\bar{\omega}^{A01}\bar{H}_{A01} =$$

$$= \Lambda_+\left(4R^{01}{}_{01} - 3H^{a01}H_{a01} - 3(e^{2\sigma}V^{01}V_{01} + e^{-2\sigma}W^{01}W_{01})\right) + \Lambda_-\left(-4\omega^{a01}H_{a01} + 2V^{01}W_{01}\right).$$

Full S_{BH} invariance

$$\hat{s}_{BH} = \frac{\hat{S}_{BH}}{\widehat{\Delta\psi}} = \frac{1}{4G_D}\int_{\hat{\Sigma}} d^{D-3}x\, e^{-2\hat{\bar{\phi}}}\sqrt{\hat{\bar{g}}_h}\left(1+\left(\boldsymbol{\Lambda}_+\frac{\widehat{\delta L^{(1)}}}{\delta \bar{R}_{0101}}+\boldsymbol{\Lambda}_- 4\hat{\bar{H}}^{A01}\hat{\bar{\omega}}_A^{01}\right)\hat{\bar{n}}_{01}\hat{\bar{n}}_{01}\right)$$

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Invariance for $\boldsymbol{\Lambda}_- = 0$ (bosonic) and $\boldsymbol{\Lambda}_+ = \boldsymbol{\Lambda}_-$ (heterotic) separately implies invariance of integrand for all $\boldsymbol{\Lambda}_-, \boldsymbol{\Lambda}_+$ (non-stringy).

T-DUALITY IS ALSO A SOLUTION GENERATING TECHNIQUE!

We solved BTZ BH and generated its T-dual for $\boldsymbol{\Lambda}_+ = 0$. We can now compute explicitly the entropy:

$$S_{BH} \quad = \quad \frac{\pi r_+}{2G_3} + \frac{4\pi\boldsymbol{\Lambda}_- \, h \, r_-}{l \, G_3} = \widehat{S}_{BH}.$$

Example: BTZ and its corrected dual

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$$S^{(1)}_{BH} = \quad \frac{4\,\pi \mathbf{\Lambda}_-}{G_3} \, e^{-2\bar{\phi}} \sqrt{\bar{g}_{\psi\psi}} \qquad \bar{H}^{201} \qquad \bar{\omega}_{2\,01}$$

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\widehat{S}^{(1)}_{BH} = & \frac{4\,\pi \mathbf{\Lambda}_-}{G_3} \, r_+ & \frac{-2r_-}{l r_+} & \frac{h}{2}
\end{array}$$

There is a exchange of terms, as expected from $V^{01}\, W_{01}$ invariance.

Conclusions, open questions and future directions

S_{BH} is invariant at $O(\alpha')$ even for non-stringy cases. (for very general BHs)

- Invariance for bosonic and heterotic implies it for any $\boldsymbol{\Lambda}_-, \boldsymbol{\Lambda}_+$.

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- Use as generating technique for α' corrected theories. Possibly extension to Non-Abelian T-duality.

THANK YOU!