Non-Abelian Black Holes and Microstate Geometries

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Based on 1608.01330
and 1702.XXXX with Patrick Meessen and Tomás Ortín

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Motivation

Classical black holes: Event horizon, M, Q, J ...
To prevent a violation of the laws of thermodynamics black holes must have some entropy and temperature. Bekenstein, Hawking '70s

$$S \sim A_h \qquad\qquad T \sim \frac{1}{M},$$

But... this implies they radiate!

Semiclassical analysis: Hawking radiation, but violating unitarity!? What about microstates?

Quantum black holes: Must solve this problem and explain the origin of the degeneracy associated to the entropy.

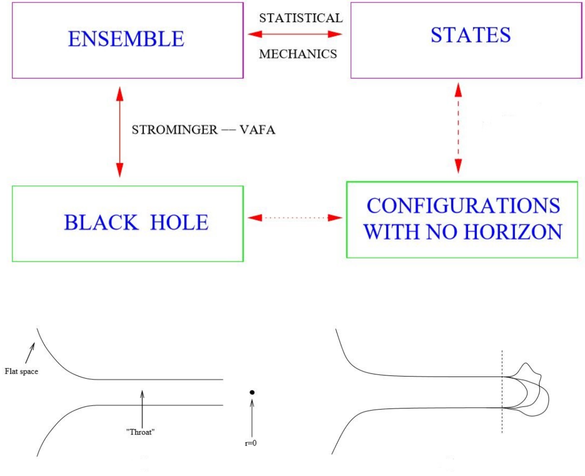
Motivation. Fuzzball proposal

In the context of string theory, the fuzzball proposal offers a possible solution: *A black hole is a quantum ensamble of string theory states interacting unitarily.* Lunin, Mathur ('02)

- No loss of information and explains microstates.
- Degeneracy of states is not found near the singularity, but horizon-sized.

Some of the microstates, when considered individually, might be captured as smooth horizonless supergravity solutions: **microstate geometries**.

Microstate geometries can be assigned a mass, charge and angular momentum that coincides with those of the black hole. Bena, Berglund, Gimon, Giusto, Levi, Lunin, Maldacena, Maoz, Mathur, Russo, Saxena, Skenderis, Srivastava, Turton, Warner, . . .



See Bena, Warner ('07)

Motivation. Non-Abelian black holes

Most of the work in microstate geometries on the (Abelian) 3-charge (Strominger-Vafa) black hole.

Non-Abelian gravitating objects remain quite unexplored. Hard to solve e.o.m. Mostly numerical results.

Do not forget: The world is non-Abelian. New fetures: hair, global monopoles, modify entropy but not the mass, ...

Realizable in String Theory. I will work on $\mathcal{N} = 1$, $d = 5$ Super-Einstein-Yang-Mills SU(2) supergravity. Heterotic Supergravity compactified on T^5 at a special point of the moduli space.

Motivation. Summary

Summary of results:

1. I will present a solution generating technique to construct multicenter non-Abelian gravitating solutions to any model of $\mathcal{N}=1$, $d=5$ SEYM. Briefly comment multicenter solutions. Meessen, Ortín, PFR ('15 & in progress)

2. I will show a general strategy to obtain families of non-Abelian microstate geometries. PFR ('16)

The theory $\mathcal{N} = 1$, $d = 5$ Super Einstein Yang Mills and its supersymmetric solutions.

$\mathcal{N}=1$, $d=5$ SEYM. The theory

The bosonic part of the action of $\mathcal{N}=1$, $d=5$ SEYM is

Günaydin, Sierra, Townsend '85

$$
\begin{aligned}
S \quad = \quad & \int d^5x\sqrt{g}\,\Big\{R+\tfrac{1}{2}g_{xy}\mathfrak{D}_\mu\phi^x\mathfrak{D}^\mu\phi^y-\tfrac{1}{4}a_{IJ}F^{I\,\mu\nu}F^J{}_{\mu\nu}+\tfrac{1}{12\sqrt{3}}C_{IJK}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}\Big[F^I{}_{\mu\nu}F^J{}_{\rho\sigma}A^K{}_\alpha \\
& -\tfrac{1}{2}gf_{LM}{}^I F^J{}_{\mu\nu}A^K{}_\rho A^L{}_\sigma A^M{}_\alpha+\tfrac{1}{10}g^2 f_{LM}{}^I f_{NP}{}^J A^K{}_\mu A^L{}_\nu A^M{}_\rho A^N{}_\sigma A^P{}_\alpha\Big]\Big\}.
\end{aligned}
$$

with the indices $I, J\ldots$ splitted in the Abelian $(i, j, \ldots)$ and non-Abelian $(\alpha, \beta, \ldots)$ sectors,

$$
F^i{}_{\mu\nu}=2\partial_{[\mu}A^i{}_{\nu]}\,,\qquad F^\alpha{}_{\mu\nu}=2\partial_{[\mu}A^\alpha{}_{\nu]}+gf^\alpha{}_{\beta\gamma}A^\beta{}_\mu A^\gamma{}_\nu\,.
$$

The theory is completely characterized by the symmetric tensor

$$
C_{0xy}=\eta_{xy}=diag(+,-,-,\ldots)\,,
$$

$\mathcal{N}=1$, $d=5$ SEYM. Supersymmetric solutions

In adapted coordinates, Abelian fields: Gauntlett, Gutowski, Reall '00s

$$ds^2 = f^2 (dt+\omega)^2 - f^{-1} ds^2_{(4)}\,,$$

$$\phi^x = h_x/h_0\,, \qquad A^I = -\sqrt{3} h^I f (dt+\omega) + \hat{A}^I\,,$$

The solutions are completely determined by Bellorín, Ortín '07

1. A choice of 4-dimensional hyperKähler metric

$$ds^2_{(4)}\,.$$

2. A set of vector fields $\hat{A}^I$ with self dual field strengths

$$\star_4 \hat{F}^I = +\hat{F}^I\,. \qquad \text{Yang-Mills instanton}$$

3. A set of scalar functions (h_I/f) solving

$$\hat{\mathfrak{D}}^2 (h_I/f) = \tfrac{1}{6} C_{IJK} \hat{F}^J \cdot \hat{F}^K\,.$$

4. A one form ω satisfying

$$d\omega + \star_4 d\omega = \tfrac{\sqrt{3}}{2} (h_I/f)\, \hat{F}^I\,.$$

$\mathcal{N}=1$, $d=5$ SEYM. Solutions with spatial isometry

I will look for solutions with an additional spatial isometry. The fields can be decomposed in terms of seed functions,

$$
\begin{aligned}
ds^2_{(4)} &= H^{-1}(d\varphi+\chi)^2 + H dx^r dx^r\,, && \text{Gibbons, Hawking '78}\\
\hat{A}^I &= -2\sqrt{6}\left[-H^{-1}\Phi^I(d\varphi+\chi)+\breve{A}^I\right]\,, && \text{Kronheimer '85}\\
h_I/f &= L_I + 8C_{IJK}\Phi^J\Phi^K H^{-1}\,,\\
\omega &= \omega_5(d\varphi+\chi)+\breve{\omega}\,,
\end{aligned}
$$

Substituting on the e.o.m. we find that the solution is specified in terms of $2n+4$ three-dimensional functions satisfying

$$
\begin{aligned}
d\star_3 dM &= 0\,, & \star_3\breve{\mathfrak{D}}\Phi^I - \breve{F}^I &= 0\,,\\
\star_3 dH - d\chi &= 0\,, & \breve{\mathfrak{D}}^2 L_I - g^2 f_{IJ}{}^{L} f_{KL}{}^{M}\Phi^J\Phi^K L_M &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
\star_3 d\breve{\omega} &= \left\{H dM - M dH + 3\sqrt{2}(\Phi^I\breve{\mathfrak{D}}L_I - L_I\breve{\mathfrak{D}}\Phi^I)\right\}\,,\\
\omega_5 &= M + 16\sqrt{2}H^{-2}C_{IJK}\Phi^I\Phi^J\Phi^K + 3\sqrt{2}H^{-1}L_I\Phi^I\,,
\end{aligned}
$$

Multicenter solutions.

Seed functions in the Abelian sector are harmonic,

$$H = \sum \frac{q_a}{r_a}\,, \qquad M = m_0 + \sum \frac{m_a}{r_a}\,, \qquad \Phi^i = \sum \frac{k^i_a}{r_a}\,, \qquad L_i = l^i_0 + \sum \frac{l^i_a}{r_a}\,,$$

Multicenter dyon in the non-Abelian sector, PFR ('16)

$$\Phi^\alpha = \frac{1}{gP}\frac{\partial P}{\partial x^\alpha}\,, \quad L_\alpha = \frac{1}{gP}\frac{\partial R}{\partial x^\alpha}\,, \quad P = \lambda_0 + \sum \frac{\lambda_a}{r_a}\,, \quad R = \sum \frac{\sigma_a \lambda_a}{r_a}\,,$$

$$\lim_{r_a \to 0} \Phi^\alpha \sim \frac{k^\alpha_a}{r_a}\,, \qquad \lim_{r_a \to 0} L_\alpha \sim \frac{l^\alpha_a}{r_a}\,, \qquad \lim_{r \to \infty} \Phi^\alpha, L_\alpha \sim \frac{1}{r^2}\,.$$

Contribution at the center, but not asymptotically.

Define the local charges and angular momentum as

$$\tilde{Q}^I_a \equiv l^I_a q_a + 8 C_{IJK} k^J_a k^K_a\,, \qquad \tilde{J}_a \equiv m_a q_a^2 + 3\sqrt{2} q_a l^I_a k^I_a + 16\sqrt{2} C_{IJK} k^I_a k^J_a k^K_a\,,$$

Multicenter solutions.

There is a regular horizon at $r_a = 0$ when $C_{IJK}\tilde{Q}^I_a\tilde{Q}^J_a\tilde{Q}^K_a \neq 0$, whose topology is:

- $S^1 \times S^2$ if $q_a = 0 \to$ **black ring**.
- S^3 if $|q_a| = 1 \to$ **black hole**.
- $S^3/\mathcal{Z}_{|q_a|}$ if $|q_a| > 1 \to$ **black lens**.

In all cases, the area of the horizon is

$$A_h = \frac{8\pi^2}{|q_a|}\sqrt{27 C_{IJK}\tilde{Q}^I_a\tilde{Q}^J_a\tilde{Q}^K_a - \tilde{J}^2_a}$$

When all $\tilde{Q}^I_a = 0$ and $\tilde{J}_a = 0$ the center is just one smooth point in space $\to$ useful for **microstate geometries**.

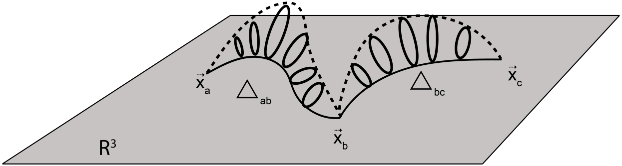
Microstate geometries

Key ingredient: ambipolar Gibbons-Hawking metric

$$ds^2_{(4)} = H^{-1}(d\varphi + \chi)^2 + H dx^r dx^r \,, \qquad H = \sum_a \frac{q_a}{r_a}\,, \qquad \sum_a q_a = 1\,.$$

All q_a are integers so at least one of them is negative.

This space has the form of a $U(1)$ fibration over a $\mathbb{R}^3$ base. The fiber collapses at the location of the centers and therefore any path in the $\mathbb{R}^3$ base connecting two centers defines a non-contractible 2-cycle.



Microstate geometries. Absence of CTCs

There is one last differential equation to solve,

$$\star_3 d\breve{\omega} = \left\{ HdM - MdH + 3\sqrt{2}(\Phi^I \breve{\mathfrak{D}} L_I - L_I \breve{\mathfrak{D}} \Phi^I) \right\} ,$$

Its integrability equation gives a set of conditions that fix the distances between the centers, the **bubbling equations**, that prevent the appearance of Dirac-Misner strings:

$$\sum_{b\neq a} \frac{q_a q_b}{r_{ab}} \left[C_{ijk} \Pi^i_{ab} \Pi^j_{ab} \Pi^k_{ab} - \frac{1}{2g^2} \Pi^0_{ab} \mathbb{T}_{ab} \right] = \frac{3}{8} l^i_0 \left(\sum_b q_a k^i_b - k^i_a \right) ,$$

$$\Pi^i_{ab} \equiv \left(\frac{k^i_b}{q_b} - \frac{k^i_a}{q_a} \right) , \qquad \mathbb{T}_{ab} \equiv g^2 \left(\frac{1}{q_a^2} + \frac{1}{q_b^2} \right) .$$

After those are solved, $\breve{\omega}$ can be found by integration.

$$\breve{\omega} = \breve{\omega}^A + \breve{\omega}^{nA} , \qquad \breve{\omega}^{nA} = \frac{3\sqrt{2}\varepsilon_{rst}}{g^2 P^2} \frac{\partial Q}{\partial x^s} \frac{\partial P}{\partial x^t} dx^r .$$

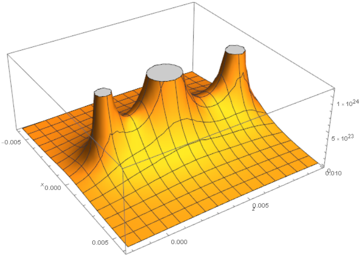
Microstate geometries. Absence of CTCs

Now the field configuration is completely specified. We have ensured smoothness locally at the centers. The global absence of singularities and CTCs is a tricky problem and usually requires numerical analysis. The metric can be written as

$$ds^2 = f^2 dt^2 + 2f^2 dt\omega - \frac{\mathcal{I}}{f^{-2}H^2}\left(d\varphi + \chi - \frac{\omega_5 H^2}{\mathcal{I}}\breve{\omega}\right)^2 - f^{-1}H\left(d\vec{x}\cdot d\vec{x} - \frac{\breve{\omega}^2}{\mathcal{I}}\right)$$

It is enough to impose the global condition

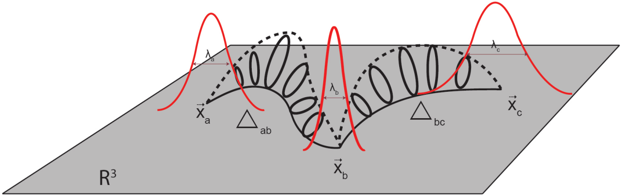
$$\mathcal{I} \equiv f^{-3}H - \omega_5^2 H^2 \geq 0\,.$$



The parameters λ_a of the multicenter dyon are related to the size of the multicenter instanton on the hyperKähler base.

For the single center colored monopole, this instanton is the BPST and this relation is transparent.

For multicenter instantons, the local expansion of the vector fields around the centers suggests this interpretation can be extrapolated.



Final remarks

Multicenter *purely* non-Abelian black holes can be placed anywhere. They are not subjected to a *Denef* constraint, contrary to Abelian configurations.

The colored parameters λ_a do not affect the mass, angular momentum or charge of the microstate. However they influence the fields and allow for the construction of classically infinite microstate geometries for a given black hole or black ring.

This new type of solutions need further study. The presence of a set of free continuous parameters on the microstate geometries is simply shocking. The construction of explicit solutions should provide valuable information for the microscopic interpretation.

The End

$\mathcal{N} = 1$, $d = 5$ SEYM. Matter content

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Matter content,

- Supergravity multiplet, $\{g_{\mu\nu}, \psi^i_\mu, A^0_\mu\}$
- n vector multiplets, $\{A^x_\mu, \lambda^{xi}, \phi^x\}$, with $x = 1, ..., n$

All fermions are minimal spinors (8 real components).

We consider only the bosonic content of the theory.

The full theory is formally invariant under a $SO(n+1)$ rotation that mixes the matter vectors and the graviphoton, so it is convenient to define the $SO(n+1)$ vectors

- $A^I = (A^0, A^1, ..., A^n)$
- $h^I = h^I(\phi^1, ..., \phi^n)$

$\mathcal{N} = 1$, $d = 5$ SEYM. Ungauged theory

Let us begin with the ungauged theory, Günaydin & Sierra & Townsend '84

$$S = \int d^5x\sqrt{g}\{R + g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y - \frac{1}{4}a_{IJ}F^I_{\mu\nu}F^{J\,\mu\nu} + \frac{1}{12\sqrt{3}}C_{IJK}\frac{\epsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\alpha\}$$

The scalars ϕ^x parametrize a real special manifold with σ-model metric $g_{xy}(\phi)$. The vector of functions $h^I(\phi)$ can be seen as coordinates in a $(n+1)$-dimensional Riemannian space with metric a_{IJ}. The real spacial manifold is a codimension-1 hypersurface defined by the constrain $C_{IJK}h^Ih^Jh^K = 1$.

The constant symmetric tensor C_{IJK} completely characterizes the theory and the real special geometry of the scalar manifold.

$$h_I = C_{IJK}h^Jh^K\,, \qquad a_{IJ} = -2C_{IJK}h^K + 3h_Ih_J\,, \qquad g_{xy} = 3a_{IJ}\partial_x h^I\partial_y h^J\,.$$

$\mathcal{N} = 1$, $d = 5$ SEYM. Isometries

The real special structure is invariant under reparametrizations generated by Killing vectors of the σ-model metric g_{xy}

$$\delta\phi^x = c^I k_I^x\,, \quad [k_I, k_J] = -f^K{}_{IJ} k_K\,, \quad \delta h^I = f^I{}_{JK} c^J h^K\,,$$

In our case they satisfy the $SU(2)$ algebra. The vectors k_I and the structure constants $f^I{}_{JK}$ will be non-vanishing only for a subset of all possible values of the indexes.

We want to promote this global symmetry to be local using the gauge fields already present in the theory

$$\begin{aligned} c^I &\rightarrow -g\Lambda^I(x)\,, \\ \delta_\Lambda \phi^x &= -g\Lambda^I k_I^x\,, \\ \delta_\Lambda h^I &= -g f_{JK}{}^I \Lambda^J h^K\,, \\ \delta_\Lambda A^I_\mu &= \partial_\mu \Lambda^I + g f_{JK}{}^I A^J_\mu \Lambda^K\,. \end{aligned}$$

$\mathcal{N}=1$, $d=5$ SEYM. Characterization of solutions

We first construct all possible spinor bilinears and its algebra

$$\hat{f} = i\bar{\epsilon}_i\epsilon^i\,, \qquad V^a = i\bar{\epsilon}_i\gamma^a\epsilon^i\,, \qquad \ldots$$
$$V^aV_a = f^2 \qquad \ldots$$

From the KSE evaluated on vanishing fermions

$$\delta_\epsilon\psi^i_\mu = \nabla_\mu\epsilon^i - \tfrac{1}{8\sqrt{3}}h_I F^{I\,\alpha\beta}\left(\gamma_{\mu\alpha\beta} - 4g_{\mu\alpha}\gamma_\beta\right)\epsilon^i = 0,$$

$$\delta_\epsilon\lambda^{i\,x} = \tfrac{1}{2}\left(\not{\mathfrak{D}}\phi^x - \tfrac{1}{2}h^x_I\ \not{F}^I\right)\epsilon^i = 0,$$

we obtain several tensor equations

$$d\hat{f} = \frac{1}{\sqrt{3}}h_I i_V F^I\,, \qquad \nabla_{(\mu}V_{\nu)} = 0\,, \qquad \ldots$$

V is a Killing vector of the space-time metric.