Supersymmetric localization and AdS4 black holes

Alberto Zaffaroni

Milano-Bicocca

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Outline

- **Introduction**
- **Part I: Field Theory Perspective**
- **Part II: AdS Black Holes, localization and entropy**

Introduction

Lot of recent activity in the study of supersymmetric and superconformal theories in various dimensions in closely related contexts

- A deeply interconnected web of supersymmetric theories arising from branes, most often strongly coupled, related by various types of dualities.
- Progresses in the evaluation of exact quantum observables. Related to localization and the study of supersymmetry in curved space.
- Many results on indices and counting problems that can be also related to BH physics.

Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes [Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory

Introduction

No similar result for AdS black holes in $d \geq 4$ was known until very recently. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. People tried hard for AdS_5 black holes (states in N=4 SYM). Still an open problem.

For AdS_4 black holes this problem can be solved by using localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

Based on

F. Benini-AZ; arXiv 1504.03698 and 1605.06120

F. Benini-K.Hristov-AZ; arXiv 1511.04085 and 1608.07294

S. M. Hosseini-AZ; arXiv 1604.03122

S. M. Hosseini-A. Nedelin-AZ; arXiv 1611.09374

and also: Yoshida-Honda, arXiv 1504.04355; Closset-Cremonesi-Park, arXiv 1504.06308; Closset-Kim arXiv 1605.06531; Closset-Kim-Willet arXiv 1701.03171.

PART I : FIELD THEORY PERSPECTIVE

- Localization
- The topologically twisted index

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

$$\partial_t Z = \int \{Q, V\} e^{-S+t\{Q,V\}} = 0$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

Localization

Localization ideas apply to path integral of Euclidean supersymmetric theories

- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

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$$\int \prod_{i=1}^{N_1} du_i \prod_{j=1}^{N_2} dv_j \frac{\prod_{i<j} \sinh^2 \frac{u_i-u_j}{2} \sinh^2 \frac{v_i-v_j}{2}}{\prod_{i<j} \cosh^2 \frac{u_i-v_j}{2}} e^{\frac{ik}{4\pi}\left(\sum u_i^2 - \sum v_j^2\right)}$$

ABJM, 3d Chern-Simon theories, [Kapustin,Willet,Yakoov;Drukker,Marino,Putrov]

Localization

Carried out recently in many cases

- many papers on topological theories
- S^2, T^2
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$, Seifert manifolds
- S^4, $S^4/\mathbb{Z}_k$, $S^3 \times S^1$, ellipsoids
- S^5, $S^4 \times S^1$, Sasaki-Einstein manifolds

with addition of boundaries, codimension-2 operators, ...

Pestun 07; Kapustin,Willet,Yakoov; Kim; Jafferis; Hama,Hosomichi,Lee, too many to count them all $\cdots$

Localization

In all cases, it reduces to a finite-dimensional matrix model on gauge variables, possibly summed over different topological sectors

$$Z_M(y) = \sum_{\mathfrak{m}} \int_C dx\, Z_{\rm int}(x, y; \mathfrak{m})$$

with different integrands and integration contours.

When backgrounds for flavor symmetries are introduced, $Z_M(y)$ becomes an interesting and complicated function of y which can be used to test dualities

- Sphere partition function, Kapustin-Willet-Yakoov; $\cdots$
- Superconformal index, Spironov-Vartanov; Gadde,Rastelli,Razamat,Yan; $\cdots$
- Topologically twisted index, Benini,AZ; Closset-KIm; $\cdots$

The topological twist

Consider a very old idea: an $\mathcal{N}=2$ gauge theory on $S^2\times S^1$

$$ds^2 = R^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right)+\beta^2 dt^2$$

with a magnetic background for the R-symmetry:

$$A^R = -\frac{1}{2}\cos\theta\, d\varphi = -\frac{1}{2}\omega^{12}\,,$$

In particular A^R is equal to the spin connection so that

$$D_\mu\epsilon = \partial_\mu\epsilon+\frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon - iA_\mu^R\epsilon = 0 \qquad \Longrightarrow \qquad \epsilon = \mathrm{const}$$

This is just a topological twist. [Witten '88]

The partition function

The path integral on $S^2 \times S^1$ reduces by localization to a matrix model depending on few zero modes of the gauge multiplet $V = (A_\mu, \sigma, \lambda, \lambda^\dagger, D)$

- A magnetic flux on S^2, $\mathfrak{m} = \frac{1}{2\pi}\int_{S^2} F$ in the co-root lattice
- A Wilson line A_t along S^1
- The vacuum expectation value σ of the real scalar

The path integral reduces to an r-dimensional contour integral of a meromorphic form

$$\boxed{\frac{1}{|W|}\sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}}\oint_C Z_{\text{int}}(u,\mathfrak{m})} \qquad u = A_t + i\sigma$$

The partition function

- In each sector with gauge flux $\mathfrak{m}$ we have a meromorphic form

$$Z_{\text{int}}(u,\mathfrak{m}) = Z_{\text{class}} Z_{\text{1-loop}}$$

$$\boxed{Z^{\text{CS}}_{\text{class}} = x^{k\mathfrak{m}}} \qquad x = e^{iu}$$

$$\boxed{Z^{\text{chiral}}_{\text{1-loop}} = \prod_{\rho\in\mathfrak{R}} \left[\frac{x^{\rho/2}}{1-x^{\rho}}\right]^{\rho(\mathfrak{m})-q+1}} \qquad q = \text{R charge}$$

$$\boxed{Z^{\text{gauge}}_{\text{1-loop}} = \prod_{\alpha\in G} (1-x^{\alpha})\,(i\,du)^{r}}$$

- Supersymmetric localization selects a particular contour of integration C and picks some of the residues of the form $Z_{\text{int}}(u,\mathfrak{m})$.

[Jeffrey-Kirwan residue - similar to Benini,Eager,Hori,Tachikawa '13; Hori,Kim,Yi '14]

The partition function

We can introduce background fluxes $\mathfrak{p}$ and fugacities y for flavor symmetries

$$x^\rho \to x^\rho y^{\rho_f} , \qquad \rho(\mathfrak{m}) \to \rho(\mathfrak{m}) + \rho_f(\mathfrak{n}) ,$$

where ρ_f is the weight under the flavor group,

$$A^F = -\frac{\mathfrak{p}^F}{2} \cos\theta \, d\varphi = -\frac{\mathfrak{p}^F}{2} \omega^{12}$$

and

$$x = e^{iu} , \qquad y = e^{iu^F} , \qquad u = A_t + \sigma , \qquad u^F = A_t^F + \sigma^F$$

The path integral becomes a function of a set of magnetic charges $\mathfrak{p}$ and chemical potentials y.

A topologically twisted index

The path integral can be re-interpreted as a twisted index: a trace over the Hilbert space $\mathcal{H}$ of states on a sphere in the presence of a magnetic background for the R and the global symmetries,

$$\mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ_F A^F} e^{-\beta H}\right)$$

$Q^2 = H - \sigma^F J_F$

holomorphic in u^F

where J_F is the generator of the global symmetry.

A Simple Example: SQED

The theory has gauge group $U(1)$ and two chiral Q and $\tilde{Q}$

$$Z = \sum_{\mathfrak{m}\in\mathbb{Z}} \int \frac{dx}{2\pi i\, x} \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{1-xy} \right)^{\mathfrak{m}+\mathfrak{n}} \left(\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{1-x^{-1}y} \right)^{-\mathfrak{m}+\mathfrak{n}}$$

	$U(1)_g$	$U(1)_A$	$U(1)_R$
Q	1	1	1
$\tilde{Q}$	-1	1	1

Consistent with duality with three chirals with superpotential XYZ

$$Z = \left(\frac{y}{1-y^2} \right)^{2\mathfrak{n}-1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}} \right)^{-\mathfrak{n}+1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}} \right)^{-\mathfrak{n}+1}$$

Dualities and generalizations

Many generalizations

- We can add refinement for angular momentum on S^2.
- We can consider higher genus $S^2 \to \Sigma_g$ [also Closset-Kim '16]

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$$Z_{\text{int}}(u, \mathfrak{m}) \to Z_{\text{int}}(u, \mathfrak{m}) \det \left(\frac{\partial^2 \log Z_{\text{int}}}{\partial u \partial \mathfrak{m}} \right)^g$$

relation to Gauge/Bethe correspondence [Nekrasov-Shatashvili; Okuda-Yoshida; Gukov-Pei]

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We can go up and down in dimension and compute

- amplitudes in gauged linear sigma models for $(2,2)$ theories in 2d on S^2 [also Cremonesi-Closset-Park '15]
- an elliptically generalized twisted index for $\mathcal{N}=1$ theory on $S^2 \times T^2$ [also Closset-Shamir '13;Nishioka-Yaakov '14;Yoshida-Honda '15]

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The index adds to and complete the list of existing tools (superconformal indices, sphere partition functions) for testing dualities: Aharony; Giveon-Kutasov in 3d; Seiberg in 4d,$\cdots$.

PART II : AdS_4 black holes

- Magnetically charged AdS_4 black holes
- Localization for ABJM

AdS$_4$ black holes

Consider BPS asymptotically AdS$_4$ static dyonic black holes

$$
\begin{aligned}
\mathrm{d}s^2 &= -e^{2U(r)}dt^2 + e^{-2U(r)}\left(dr^2 + V(r)^2 \mathrm{d}s^2_{S^2}\right) \\
X^i &= X^i(r)
\end{aligned}
$$

- vacua of $N = 2$ gauged supergravities arising from M theory on AdS$_4 \times S^7$
- electric and magnetic charges for $U(1)^4 \subset SO(8)$
- preserving supersymmetry via an R-symmetry twist

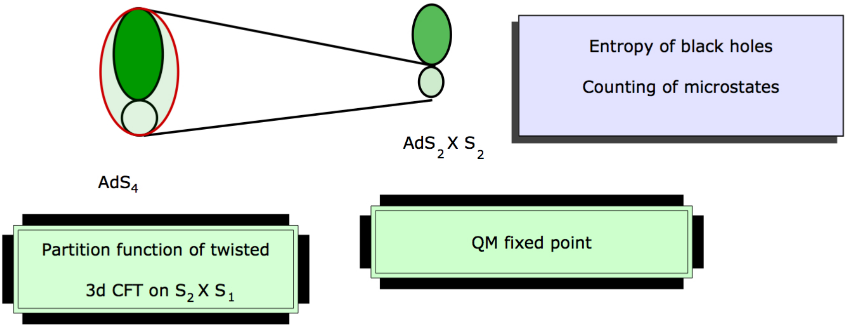
$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

[Cacciatori,Klemm; Gnecchi,Dall'agata; Hristov,Vandoren;Halmagyi;Katmadas]

AdS_4 black holes and holography

AdS black holes are dual to a topologically twisted CFT on $S^2 \times S^1$ with background magnetic fluxes for the global symmetries

$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A = A_{M_d} + O(1/r)$$



Dual Field Theory Perspective

The boundary is $S^2 \times \mathcal{R}$ or $S^2 \times S^1$ in the Euclidean, with a non vanishing background gauge field for the global symmetries on S^2

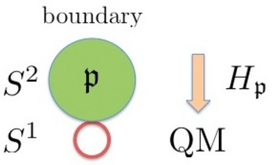
$$A^\Lambda = -\frac{p^\Lambda}{2} \cos\theta d\phi$$

- The magnetic charges p^Λ corresponds to a deformation of the boundary theory, which is topologically twisted, with a magnetic charge for the R-symmetry and for the global symmetries of the theory.
- The electric charges q_Λ gives sub-leading contributions at the boundary. They are a VEV in the boundary theory, meaning the the average electric charge of the CFT states is non zero.

Everything is easily generalized to $S^2 \to \Sigma_g$.

Dual Field Theory Perspective

It is then natural to evaluate the topologically twisted index with magnetic charges $\mathfrak{p}$ for the R-symmetry and for the global symmetries of the theory



$$Z^{\text{twisted}}_{S^2\times S^1}(\mathfrak{p},\Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{p}}}\right)$$

$$\Delta = A_t^F + i\sigma^F$$

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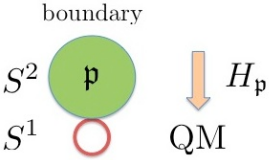

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$$\Delta = A^F_t + i\sigma^F$$

This is the Witten index of the QM obtained by reducing $S^2\times S^1 \to S^1$.

- magnetic charges $\mathfrak{p}$ are not vanishing at the boundary and appear in the Hamiltonian
- electric charges $\mathfrak{q}$ can be introduced using chemical potentials Δ

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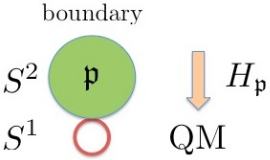


$$Z^{\text{twisted}}_{S^2\times S^1}(\mathfrak{p},\Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{p}}}\right)$$

$$\Delta = A^F_t + i\sigma^F$$

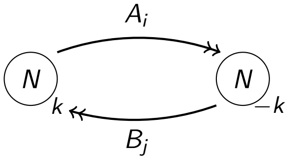
The BH entropy is related to a Legendre Transform of the index [Benini-Hristov-AZ]

$$S_{BH}(\mathfrak{q},\mathfrak{n}) \equiv \mathbb{R}\mathrm{e}\,\mathcal{I}(\Delta) = \mathbb{R}\mathrm{e}(\log Z(\mathfrak{p},\Delta) - i\Delta\mathfrak{q})\,, \qquad \frac{d\mathcal{I}}{d\Delta} = 0$$

[similar to Sen's formalism, OSV, etc]

The dual field theory

The dual field theory to $\mathrm{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

with R and global symmetries

$$U(1)^4 \subset SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

.

The dual field theory

It is useful to introduce a basis of four R -symmetries R_a, $a = 1, 2, 3, 4$

	R_1	R_2	R_3	R_4
A_1	2	0	0	0
A_2	0	2	0	0
B_1	0	0	2	0
B_2	0	0	0	2

A basis for the three flavor symmetries is given by $J_a = \frac{1}{2}(R_a - R_4)$. Magnetic fluxes $\mathfrak{n}_a$ and complex fugacity y_a for the symmetries can be introduced. They satisfy

$$\sum_{a=1}^{4} \mathfrak{p}_a = 2\,, \qquad \text{supersymmetry}$$

$$\prod_{a=1}^{4} y_a = 1\,, \qquad \text{invariance } of\ W$$

ABJM twisted index

The ABJM twisted index is

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\tilde{\mathfrak{m}}\in\mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\tilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right) \times$$

$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_1}}{1-\frac{x_i}{\tilde{x}_j} y_1}\right)^{\mathfrak{m}_i-\tilde{\mathfrak{m}}_j-\mathfrak{p}_1+1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_2}}{1-\frac{x_i}{\tilde{x}_j} y_2}\right)^{\mathfrak{m}_i-\tilde{\mathfrak{m}}_j-\mathfrak{p}_2+1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_3}}{1-\frac{\tilde{x}_j}{x_i} y_3}\right)^{\tilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_3+1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_4}}{1-\frac{\tilde{x}_j}{x_i} y_4}\right)^{\tilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_4+1}$$

$$\prod_i y_i = 1\,, \qquad \sum \mathfrak{p}_i = 2$$

where $\mathfrak{m}, \tilde{\mathfrak{m}}$ are the gauge magnetic fluxes, $y_i = e^{i\Delta_i}$ are fugacities and $\mathfrak{n}_i$ the magnetic fluxes for the three independent $U(1)$ global symmetries

ABJM twisted index

We need to evaluate it in the large N limit. Strategy:

- Re-sum geometric series in $\mathfrak{m}, \widetilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^{N}(e^{iB_i} - 1) \prod_{j=1}^{N}(e^{i\widetilde{B}_j} - 1)}$$

- Step 1: find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N

- Step 2: evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

[Benini-Hristov-AZ]

[extended to other models Hosseini-AZ; Hosseini-Mekareeya]

The large N limit

Step 1: solve the large N Limit of the algebraic equations $e^{iB_i} = e^{i\tilde{B}_i} = 1$ giving the positions of poles

$$1 = x_i^k \prod_{j=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}$$

- We call them *Bethe Ansatz Equations* because the same expressions can be reintepreted in the 2d integrability approach [Nekrasov-Shatashvili; Okuda-Yoshida; Gukov-Pei]
- They can be derived by a BA potential $\mathcal{V}_{BA}$

$$e^{iB_i} = e^{i\tilde{B}_i} = 1 \qquad \Longrightarrow \qquad \frac{d\mathcal{V}_{BA}}{dx_i} = \frac{\mathcal{V}_{BA}}{d\tilde{x}_i} = 0$$

The large N limit

Step 1: the Bethe Ansatz equations can be solved with the ansatz

$$u_i = i\sqrt{N} t_i + v_i \,, \qquad \log \tilde{u}_i = i\sqrt{N} t_i + \tilde{v}_i \qquad (x_i = e^{iu_i}\,, \tilde{x}_i = e^{i\tilde{u}_i})$$

which has the property of selecting contributions from $i \sim j$ and makes the problem local.

$$\rho(t) = \frac{1}{N}\frac{di}{dt}\,, \qquad \delta v(t) = v_i - \tilde{v}_i$$

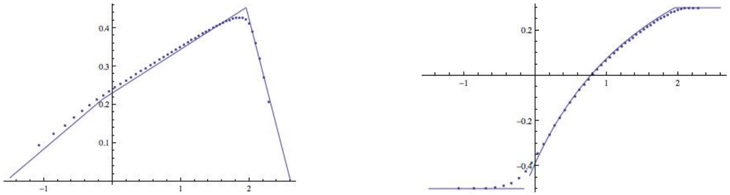
$$\frac{\mathcal{V}_{BA}}{iN^{\frac{3}{2}}} = \int dt \left[t\,\rho(t)\,\delta v(t) + \rho(t)^2 \left(\sum_{a=3,4} g_+\big(\delta v(t) + \Delta_a\big) - \sum_{a=1,2} g_-\big(\delta v(t) - \Delta_a\big) \right) \right]$$

where $g_\pm(u) = \frac{u^3}{6} \mp \frac{\pi}{2} u^2 + \frac{\pi^2}{3} u$.

The large N limit

Step 1: the equations can be then explicitly solved

$$u_i = i\sqrt{N}t_i + v_i\,, \qquad \log \tilde{u}_i = i\sqrt{N}t_i + \tilde{v}_i$$



$\rho(t)$ $\qquad$ $\delta v(t)$

and

$$\mathcal{V}_{BA} \sim N^{3/2}\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$$

The large N limit

Step 2: plug into the partition function. It is crucial to keep into account exponentially small corrections in tail regions where $y_i x_i/\tilde{x}_i = 1$

$$\log Z = N^{3/2}(\text{finite}) + \underbrace{\sum_{i=1}^{N} \log(1 - y_i x_i/\tilde{x}_i)}_{O(N)} \qquad\qquad y_i x_i/\tilde{x}_i = 1 + e^{-N^{1/2} Y_i}$$

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One can by-pass it by using a general simple formula [Hosseini-AZ; arXiv 1604.03122]

$$\log Z = -\sum_a \mathfrak{p}_a \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_a}$$

The final result

The Legendre transform of the index is obtained from $\mathcal{V}_{BA} \sim \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$:

$$\mathcal{I}(\Delta) = \frac{1}{3} N^{3/2} \sum_a \left(-\mathfrak{p}_a \frac{d\mathcal{V}_{BA}}{d\Delta_a} - i\Delta_a \mathfrak{q}_a \right) \qquad y_a = e^{i\Delta_a}$$

$$\log Z$$

This function can be extremized with respect to the Δ_a and

$$\boxed{\mathcal{I}|_{crit} = \mathrm{BH\,Entropy}(\mathfrak{p}_a, \mathfrak{q}_a)}$$

$$\boxed{\Delta_a|_{crit} \sim X^a(r_h)}$$

[Benini-Hristov-AZ]

AdS_4 black holes

- Notice that the explicit expression for the entropy of the $AdS_4 \times S^7$ black hole is quite complicated. In the case of purely magnetical black holes with just

$$\mathfrak{p}^1 = \mathfrak{p}^2 = \mathfrak{p}^3$$

is given by

$$S = \sqrt{-1 + 6\mathfrak{p}^1 - 6(\mathfrak{p}^1)^2 + (-1 + 2\mathfrak{p}^1)^{3/2}\sqrt{-1 + 6\mathfrak{p}^1}}$$

The attractor mechanism

The BPS equations at the horizon imply that the gauge supergravity quantity

$$\mathcal{R} = \left(F_\Lambda \mathfrak{p}^\Lambda - X^\Lambda \mathfrak{q}_\Lambda\right) , \qquad F_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}$$

with $(\mathfrak{q}, \mathfrak{n})$ electric and magnetic charges, is extremized with respect to the scalar fields at the horizon and its critical value gives the entropy.

Under $X^\Lambda \rightarrow \Delta^\Lambda$

$$\mathcal{F} = i\sqrt{X^0 X^1 X^2 X^3} \sim \mathcal{V}_{BA}(\Delta) = \sqrt{\Delta_1 \Delta_2 \Delta_3 \Delta_4}$$

$$\mathcal{R} = \sum \sum_\Lambda \left(\mathfrak{p}^\Lambda \frac{d\mathcal{F}}{dX^\Lambda} - \mathfrak{q}_\Lambda X^\Lambda\right) \sim \sum_a \left(-\mathfrak{p}_a \frac{d\mathcal{V}}{d\Delta_a} - i\Delta_a \mathfrak{q}_a\right) = \mathcal{I}(\Delta)$$

The previous discussion can be extended to higher genus, again with perfect agreement [Benini-Hristov-AZ].

Part III : Comments and discussions

A. Statistical ensemble

Δ_a can be seen as chemical potential in a macro-canonical ensemble defined by the supersymmetric index

$$Z = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_a J_a} e^{-\beta H}$$

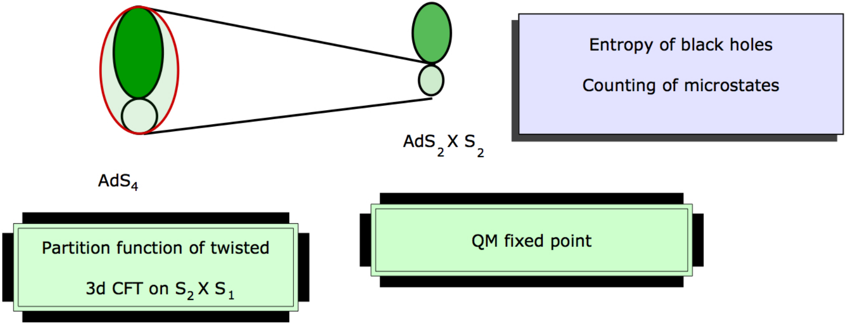
so that the extremization can be rephrased as the statement that the black hole has average electric charge

$$\frac{\partial}{\partial \Delta} log Z \sim < J >$$

- Similarities with Sen's entropy formalism based on AdS_2.
- Similarly to asymptotically flat BH, $(-1)^F$ does not cause cancellations at large N. What's about finite N?

B. R-symmetry extremization

Recall the cartoon



B. R-symmetry extremization

The extremization reflects exactly what's going on in the bulk. Consider no electric charge, for simplicity. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^{\Lambda} F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\boxed{\Delta_i\big|_{crit} \sim X^i(r_h)}$$

B. R-symmetry extremization

The twisted index depends on Δ_i because we are computing the trace

$$Z(\Delta) = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^{R(\Delta)}$$

where $R(\Delta) = F + \Delta_i J_i$ is a possible R-symmetry of the system.

For zero electric charges, the entropy is obtained by extremizing $\log Z(\Delta)$.

Some QFT extremization is at work?

B. R-symmetry extremization

The extremum $\log Z(\hat{\Delta})$ is the entropy.

- symmetry enhancement at the horizon AdS_2:

$$\mathrm{QM}_1 \to \mathrm{CFT}_1$$

- $R(\hat{\Delta})$ is the exact R-symmetry at the superconformal point
- all the BH ground states have $R(\hat{\Delta}) = 0$ because of superconformal invariance (AdS_2)

$$Z(\hat{\Delta}) = \mathrm{Tr}_{\mathcal{H}}(-1)^{R(\hat{\Delta})} = \sum 1 = e^{\mathrm{entropy}}$$

  and the extremum is obtained when all states have the same phase $(-1)^R$
- Z is the natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions
- similar results can be found for black strings in AdS_5 [Hosseini,Nedelin,AZ]

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions
- similar results can be found for black strings in AdS_5 [Hosseini,Nedelin,AZ]

But don't forget that we also gave a general formula for the topologically twisted path integral of 2d (2,2), 3d $\mathcal{N} = 2$ and 4d $\mathcal{N} = 1$ theories.

- Efficient quantum field theory tools for testing dualities.
- With many field theory questions/generalizations

Thank you for the attention !