Twists of supergravity theories via algebraic geometry

- w/ Brian Williams (any day...)
- w/ Walcher, Eager ⊛
- w/ Eager, Williams, Hahner

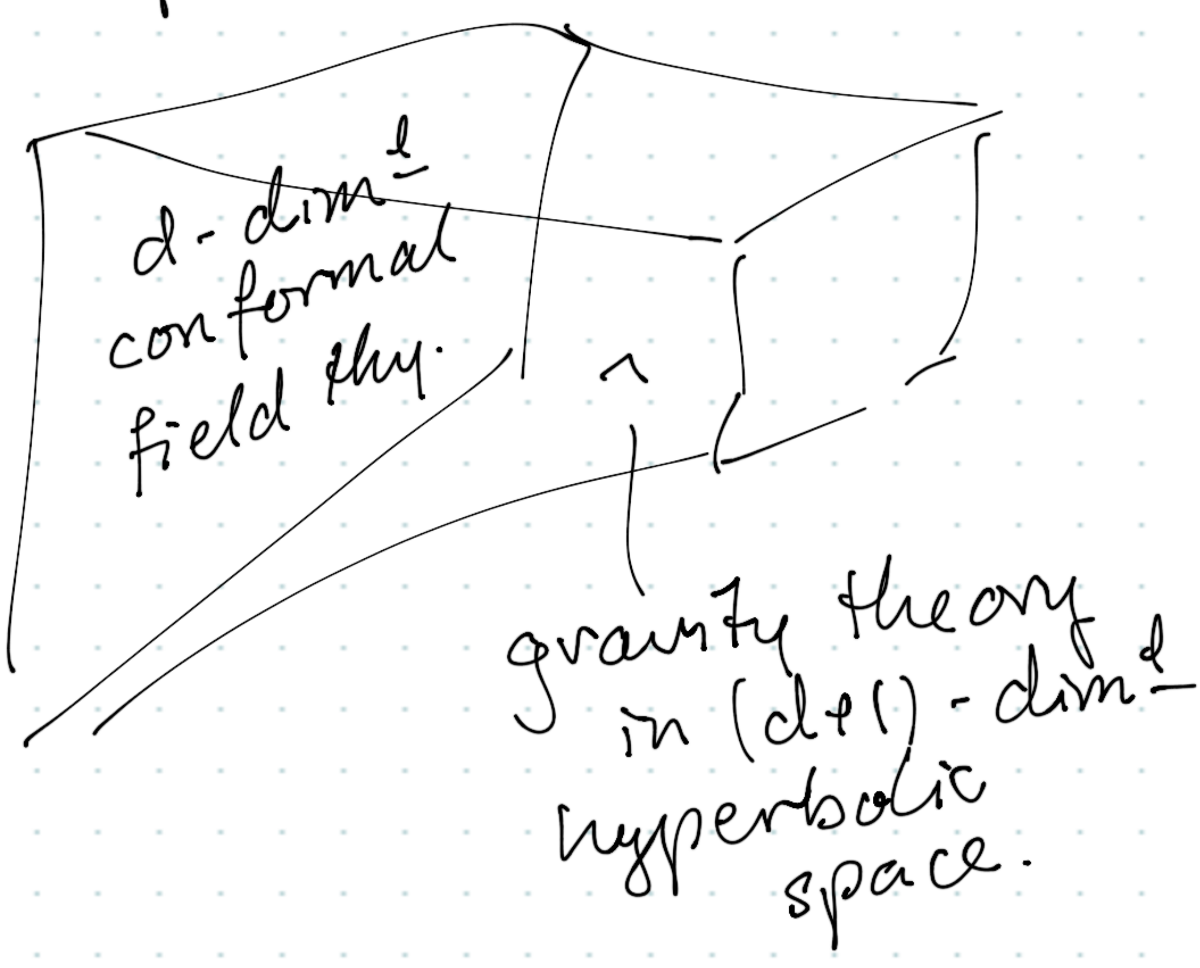
Twists of supersymmetric QFTs are an interesting source of new math and physics.

- TQFT, or generalizations (holomorphic field theories)
- "Dualities" $\rightsquigarrow$ surprising relations betw. inv't's.

(Mirror symmetry ...)

- Twists allow for exact computations / new structural insights.

A duality that remains to be explored: AdS/CFT.



Classic example:

$\mathcal{N}=4$ SYM theory (four dim'l) $\hookrightarrow$ type IIB supergravity on $H_5 \times S^5$

(AdS$_5$)

<u>Hindrance</u>: twisting IIB.

Costello/Li gave a conjectural formulation of the holo. twist of this duality.

Their description of IIB:

$X = CY5$,

"BCOV theory": $(PV^{\bullet,\bullet}(X)[[t]][2], \bar\partial + t\partial_\Omega)$

Motivation: topological B-model.

The holomorphic twist of IIB SUGRA is "minimal" BCOV theory, up to some interesting issues about degeneracy. (zero modes of some fields).

BCOV theory

$\cup|$

$$\left(\bigoplus_{i+j<5} t^i PV^{j,\bullet}(X)[2],\ \bar\partial + t\partial_\Omega \right) \quad *$$

"minimal" version

(IIB SUGRA $\subseteq$ IIB string theory)

Let's review briefly what some of these words really mean.

$V = \mathbb{R}^d$ = some affine space.

A "theory" can be described (classically) by:

- a chain complex of affine vector bundles (free) $\begin{array}{c} E^\bullet \\ \downarrow \\ V \end{array}$ *

<u>Note</u>: This could be a BRST or a BV theory.

- additional data:

BV: a (-1)-shifted symplectic structure (*)

BRST: an action functional of degree zero

- a local L_∞ algebra structure on $E^\circ[-1]$

(BV: interactions + gauge sym.
BRST: gauge symmetries

You should just think of a "classical theory", for now, as

$$\begin{array}{c} E^\circ \\ \downarrow \\ V \end{array}.$$

What is supersymmetry?

$$\mathfrak{aff}(V) = \mathfrak{so}(V) \ltimes \underline{V}$$

$$\begin{array}{c} \mathfrak{aff}(V) \\ \downarrow \\ \mathfrak{p} \end{array}$$

$\mathfrak{p}$ = super Lie algebra

$$\mathfrak{p} = \mathfrak{aff}(V) \oplus \mathfrak{aut}(R) \oplus \Pi(\underline{S} \otimes \underline{R})$$

$\mathfrak{aut}(R) = \mathfrak{sp}(R),\ \mathfrak{so}(R),\ \mathfrak{gl}(R)$

depending on dimension of V.

A supersymmetric theory is a (homotopy) $\mathfrak{p}$-module structure on $E^\bullet$ compatible w/ affine structure.

What's a twist?

Pick $Q \in \Pi(S \otimes R)$ such that $\{Q, Q\} = 0$

$$E^\bullet_Q = \left(E^\bullet[u,u^{-1}],\ \underset{(1,+)}{d_E} + \underset{(1,-)}{u}\cdot \underset{(0,-)}{Q}\right).$$

Could have just considered the family over $\underline{Y} = \{Q \mid [Q,Q]=0\}$

$$\left(\underline{E}^\bullet \otimes \mathcal{O}_{\underline{Y}},\ d_E + \langle \underline{\lambda}, Q\rangle\right)$$

λ = a coordinate on Y.

$$\left(Y = \operatorname{Spec} \underset{\lambda \in \Pi(S\otimes R)[+1]^\vee}{\mathbb{C}[\lambda]} \big/ \langle \lambda\cdot\gamma\cdot\lambda\rangle\right)$$

γ = structure constants of susy. algebra

In fact, the pure spinor formalism uses exactly this trick to build E° itself!

$$\hat{V} = V \oplus \Pi(S \otimes R)$$

is a supermanifold w/ 2 $\mathfrak{p}$-actions ~ on the left and on the right. ($\underset{\text{left}}{Q}$'s and $\underset{\text{right}}{D}$'s)

$\Rightarrow$ free superfield

$$A^\bullet = \underline{C^\infty(\hat{V})} \otimes \underline{\mathcal{O}_Y}, \quad d_A = \langle \lambda, D \rangle$$

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has lots of cool properties:

- Left $\mathfrak{p}$-action is both unbroken and strict.

$\Rightarrow (A^\circ, d_A)$ is a multiplet
(or supersymmetric thy.)

- It's freely resolved over $\hat{V}$, not just V.

- Has a multiplicative structure.

Can construct every multiplet in the physics literature by using this technique — (replace $\mathcal{O}_Y$ by any $\mathcal{O}_Y$-module.)

In particular, it makes it (astonishingly) easy to compute twists.

Rk. $$C^\infty(V) \otimes \overbrace{\mathbb{C}[\theta]}^{\Pi(S\otimes R)} \otimes \overbrace{\mathbb{C}[\lambda]/\langle \lambda^2 \rangle}^{\mathcal{O}_Y}$$

$$d_A = \underbrace{\lambda^a \left(\frac{\partial}{\partial \theta^a}\right.}_{} - \left.\underbrace{\theta^b \gamma^\mu_{ab} \frac{\partial}{\partial x^\mu}}_{\text{BRST/BV differential}} \right)$$

Computes Koszul homology of Y! $\cong$ Component fields.

Fact: Upon twisting, I just consider Y_Q.

$$\mathfrak{p} \rightsquigarrow (\mathfrak{p}, [Q, -])$$

$$H^*(\mathfrak{p}, \mathrm{ad}_Q) = \mathfrak{p}_Q$$

Y = square-zero elements in $\mathfrak{p}_Q$.

I have to replace $C^\infty(V)$ by holomorphic functions on V.

In the example of IIB,

$$\mathfrak{p}_Q^{(-1)} = \underset{\lambda_1}{\Lambda^1 T^*} \oplus \underset{\lambda_3}{\Lambda^3 T^*}$$

$$\mathcal{O}(Y_Q) = \mathbb{C}[\lambda_1, \lambda_3] \big/ \lambda_1 \wedge \lambda_3 = 0.$$

Koszul homology:

$$\Omega^0_{hol} \xrightarrow{\partial} \Omega^1_{hol} \xrightarrow{\partial} \Omega^2_{hol}$$

$$\Omega^0_{hol} \xrightarrow{} \Omega^1_{hol}$$

Ω^0_{hol} $\qquad$ PV^0_{hol}

$$PV^1_{hol} \xrightarrow{\partial} PV^0_{hol}$$

$\downarrow \partial$

$$\Omega^1 \to \Omega^2 \cdots \to \Omega^5$$

$$\Omega^2 \to \cdots \to \Omega^5$$

$$\Omega^3 \to \cdots \Omega^5$$

$\cong PV^{\leq 2}, PV^{\leq 3}, PV^{\leq 4}.$

$PV^{i,j}[t](z)$ is a

Poisson-BV theory

we see something "presymplectic"