Thermodynamics of Supersymmetric AdS_5 Black Holes

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Thermodynamics of AdS_5 Black Holes

The **black hole entropy**:

$$S = \frac{A}{4G_5}$$

Thermodynamics: study dependence on mass M, charge Q, ...

Black holes in AdS_5 interesting: dual to $\mathcal{N} = 4$ SYM in $D = 4$.

Very well studied.

This talk: elucidate **supersymmetric** black holes in AdS_5.

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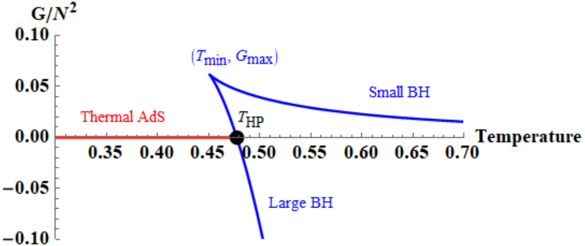
Nizar Ezroura, Billy Liu, and Yangwenxiao Zeng: arXiv:2108.11452

Review: Schwarzchild AdS_5 Black Hole



- **Small** (upper) black hole branch: nearly flat space.
- **Large** (lower) branch: **conformal fluid** in $\mathcal{N} = 4$ SYM.
- **Cusp**: minimal temperature, maximal free energy.
- **Hawking-Page transition**: "thermal AdS gas" with $G = 0$ (**red line**) dominates when black holes have $G > 0$.

Supersymmetric AdS_5 Black Holes

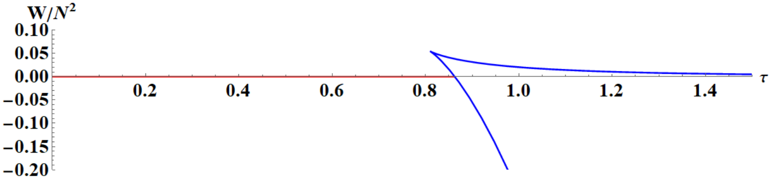
BPS black holes do not support conventional themodynamics:

- They are **extremal** $T = 0$.
- Their **mass is determined by conserved charges**:

$$M = \underbrace{\sum_{I=1}^{3} Q_I}_{\text{on } S^5} \quad + \quad \underbrace{\sum_{i=a}^{b} J_i}_{\text{in AdS}_5}$$

- "Thermodynamics": dependence on potentials (Φ, Ω) conjugate to charges (Q, J).

Phase Diagram of SUSY AdS_5 Black Holes



- Phase diagram **strikingly similar** to AdS-Schwarzchild.
- Free energy $G \equiv 0$ and temperature $T \equiv 0$.
- The W and the τ are **BPS analogues**.

Outline

More on **non-BPS** thermodynamics:

- Compare rotation and electric charge.
- The **approach** to the BPS limit.

BPS black holes:

- General BPS thermodynamics.
- The **constraint** on charges.
- The potential φ'.

The Grand Canonical Ensemble

Physically: free energy G a function of

- **temperature** T.
- **electric potentials** Φ_I (often equal $\Phi \equiv \Phi_I$, $I = 1, 2, 3$).
- **angular velocities** $\Omega_{a,b}$ (often equal $\Omega \equiv \Omega_a = \Omega_b$).

In practice:
Physical variables functions of **parameters** (a, b, q, m) and coordinate **position of horizon** r_+^2:

$$\Delta_r = \frac{(r_+^2 + a^2)(r_+^2 + b^2)(1 + r_+^2) + q^2 + 2abq}{r_+^2} - 2m = 0$$

Parametric Representations

Four **physical potentials** in terms of four parameters (a, b, q, r_+^2):

$$
\begin{aligned}
T &= \frac{r_+^4[1 + (2r_+^2 + a^2 + b^2)] - (ab + q)^2}{2\pi r_+[(r_+^2 + a^2)(r_+^2 + b^2) + abq]} \\
\Phi &= \frac{3qr_+^2}{(r_+^2 + a^2)(r_+^2 + b^2) + abq} \\
\Omega_a &= \frac{a(r_+^2 + b^2)(1 + r_+^2) + bq}{(r_+^2 + a^2)(r_+^2 + b^2) + abq} \\
\Omega_b &= \frac{b(r_+^2 + a^2)(1 + r_+^2) + aq}{(r_+^2 + a^2)(r_+^2 + b^2) + abq}
\end{aligned}
$$

Free energy:

$$
\frac{G}{N^2} = \frac{-2r_+^{10} + [1 - 2a^2 - 2b^2]r_+^8 + [-(a^2 + b^2)^2 + 2(a^2 + b^2 - a^2b^2) - abq]r_+^6 + [-q^2 + a^4 + b^4 - abq(a^2 + b^2) + 4a^2b^2}{4(1 - a^2)(1 - b^2)r_+^2((r_+^2 + a^2)(r_+^2 + b^2) + abq)}
$$

$$
\frac{-2a^2b^2(a^2 + b^2)]r_+^4 + [q^2(a^2 + b^2) + 2abq(a^2 + b^2)2a^2b^2(a^2 + b^2) - a^3b^3(1 + ab)]r_+^2 + ab(q + ab)(q^2 + a^2b^2 + 3abq)}{4(1 - a^2)(1 - b^2)r_+^2((r_+^2 + a^2)(r_+^2 + b^2) + abq)}
$$

Strategy: study nonlinear dependences using **parametric plots**.

Critical Values of Potentials

The BPS mass interpreted in terms of potentials:

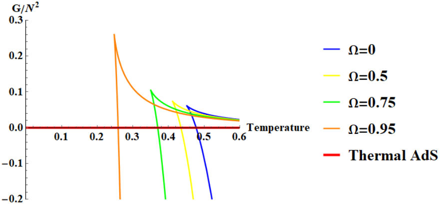
$$M_{\rm BPS} = \sum_{I=1}^{3} Q_I + \sum_{i=a,b} J_i = \Phi^* Q + \Omega_a^* J_a + \Omega_b^* J_b$$

Critical potentials:
$\Phi^* = 3$, $\Omega_a^* = 1$, $\Omega_b^* = 1$ essential also for non-BPS black holes.

Note: angular velocities $\Omega_{a,b} \leq 1 \quad \Rightarrow \quad \Omega_{a,b}^*$ **maximal**.

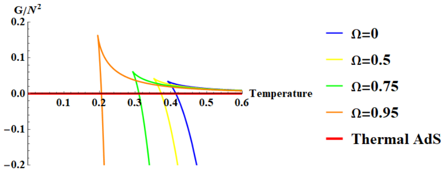
Plan: study Ω, then Φ, then **approach to BPS**.

Angular Velocity



- Maximal free energy (at cusp) is higher.
- Hawking-Page phase transition reached at lower temperature.
- Interpretation: rotation **destabilizes** black hole.

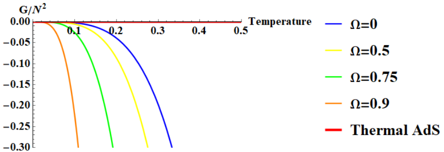
A Small Electric Potential $\Phi < \Phi_*$



Same angular velocities, but with electric potential $\Phi = 1.5$.

- Maximal free energy (at cusp) is lowered.
- HP phase transition reached at even lower temperature.
- Electric potential **stabilizes** black hole.

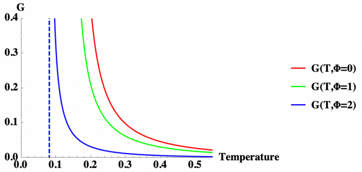
The Critical Electric Potential $\Phi = \Phi_*$



Same angular velocities, **critical** electric potential $\Phi = \Phi_* = 3$.

- Free energy never positive: **no small black hole branch**.
- Electric potential **stabilizes** black hole.
- The BPS case $\Omega = \Omega^* = 1$:
  not a curve: $G = T = 0$ identically.

Critical Angular Velocity $\Omega = \Omega_*$



Small electric potential, **critical** angular velocity $\Omega = \Omega_* = 1$.

- Free energy always positive: **no large black hole branch**.
- For each Φ:
  a **minimal temperature** where free energy diverges.
- Minimal temperature $\to 0$ as $\Phi \to (\Phi_*)_-$ but "dangerous" approach to BPS.

Status

- Thermodynamics of **non-BPS** black holes in AdS_5.
- Rotation Ω has critical value $\Omega_* = 1$ and tends to **destabilize**.
- Electric potential Φ has critical value $\Phi_* = 3$ and **stabilizes**.

We now turn to **thermodynamics of BPS black holes** in AdS_5.

BPS Potentials

BPS black holes all have:

- Potentials $T = 0$, $\Phi = \Phi^*$, $\Omega = \Omega^*$ **identically**.

The **BPS potentials**

$$\Phi' = \frac{\Phi - \Phi^*}{T} \ , \quad \Omega' = \frac{\Omega - \Omega^*}{T}$$

(Basic) interpretation of **"prime"**:
derivative with respect to temperature $\partial_T \big|_{T=0}$.

Corrolary: convention so $\Omega' < 0$ (because $\Omega < 1$ for $T > 0$).

BPS Potentials: Statistical Mechanics

Partition function **adapted to the BPS limit** $M = M_*$:

$$Z = \mathrm{Tr}\ \exp\Big(-\beta\Big[(M - M^*) - (\Omega - \Omega^*)J - (\Phi - \Phi^*)Q\Big]\Big)$$

$$\underset{T\to 0}{=}\ \mathrm{Tr}\ \exp\Big(\Omega' J + \Phi' Q\Big)$$

At linear order: the BPS surface itself, **nothing more**.

(Sufficient) assumption: there is **gap**.

Projective Thermodynamics

BPS black holes have free energy $G = 0$ **identically**.

At **linear order** G gives: $M = M_*$ and **nothing more**.

Conclusion: **BPS thermodynamics** is **projective**:

$$W(\Phi', \Omega') = \frac{G}{T} = -S - \Phi' Q - \Omega' J$$

The **first law of BPS black hole thermodynamics**:

$$dW = -Q d\Phi' - J d\Omega'$$

Black hole entropy follows from W by a Legendre transform.

BPS Temperature

"The" temperature $T = 0$.

"Small" values of Φ', $\Omega'_{a,b}$ favors **highly excited states**
$\Rightarrow$ a **notion of temperature**.

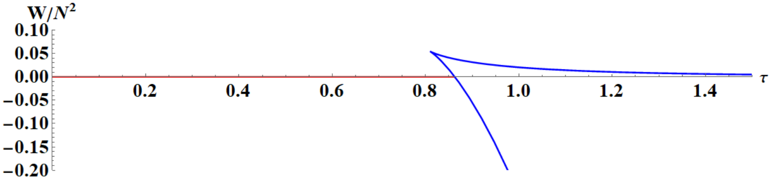
Some features of physical temperature:

- Large for highly excited states $\Rightarrow$ large free energy.
- It is **positive**.

Definition of **BPS temperature**:

$$\tau = -\frac{1}{\Omega'_a + \Omega'_b}$$

Phase Diagram of SUSY AdS_5 Black Holes Revisited



- The "free energy" is the BPS free energy W.
- The "temperature" is the BPS temperature τ.
- Phase diagram **strikingly similar** to AdS-Schwarzchild.

Comparison with AdS-Schwarzchild

SUSY AdS_5 and AdS-Schwarzchild have **similar** phase diagrams.

They are **not identical**:

- High temperature behavior on the **small branch**:

$$G \sim T^{-3} \quad , \quad W \sim \tau^{-3}$$

- High temperature behavior on the **large branch**:

$$G \sim -T^{3} \quad , \quad W \sim -\tau^{2}$$

Interpretation:
Large BPS black holes behave like **gas in two spatial dimensions**.

Parametric Representation

The BPS free energy depends on $(\Phi', \Omega'_a, \Omega'_b)$:

$$\frac{W}{N^2} = -\frac{(a+b)(2-a-b)}{12(1-a)(1-b)}\Phi' + \frac{(a+b)^2(1+a)}{4(1-a)^2(1-b)}\Omega'_a + \frac{(a+b)^2(1+b)}{4(1-a)(1-b)^2}\Omega'_b$$

The parameters a, b are solutions to the **constraints**

$$-\frac{\sqrt{a+b+ab}}{1-a}\Omega'_a - \frac{\sqrt{a+b+ab}}{1-b}\Omega'_b - \frac{\sqrt{a+b+ab}}{3(a+b)}\Phi' = \pi \ ,$$

$$\frac{(a+b)(1+a)}{1-a}\Omega'_a - \frac{(a+b)(1+b)}{1-b}\Omega'_b + \frac{a-b}{3}\Phi' = 0 \ .$$

Strategy: study BPS thermodynamics using **parametric plots**.

Counting Parameters

The **three** black hole charges are functions of **two** parameters:

$$Q = \frac{1}{2}N^2 \frac{a+b}{(1-a)(1-b)} \ ,$$

$$J_a = \frac{1}{2}N^2 \frac{(a+b)(2a+b+ab)}{(1-a)^2(1-b)} \ ,$$

$$J_b = \frac{1}{2}N^2 \frac{(a+b)(a+2b+ab)}{(1-a)(1-b)^2} \ .$$

All BPS black holes satisfy a **constraint**:

$$\left(Q^{*3} + \frac{1}{2}N^2 J_a^* J_b^*\right) - \left(3Q^* + \frac{1}{2}N^2\right)\left(3Q^{*2} - \frac{1}{2}N^2(J_a^* + J_b^*)\right) = 0$$

Supersymmetry Breaking: Two Directions

Corollary: **two** distinct deformations **break supersymmetry**

- Recall: $\underbrace{T=0}_{\textbf{extremality}} \quad \Leftrightarrow \quad \underbrace{M = M_{\rm ext}}_{\rm lowest\ mass\ (given\ conserved\ charges)}$.
- **Standard** SUSY breaking: **mass exceeds** $M_{\rm ext}$.
  Description: **raise the temperature** $\Rightarrow \quad T > 0$.
- **Novel** SUSY breaking: violate **constraint** by **adjusting conserved charges** while preserving $T=0$ (so $M = M_{\rm ext}$).
- The potential analogous to the temperature T:

$$\varphi = (\Phi - \Phi^*) - (\Omega_a - \Omega_a^*) - (\Omega_b - \Omega_b^*)$$

Excitations above the BPS Limit

- Both SUSY breaking parameters **increase black hole mass** above the BPS bound:

$$M = M_{\rm BPS} + \frac{1}{2}\left(\frac{C_T}{T}\right)\ \left[T^2 + \left(\frac{\varphi}{2\pi}\right)^2\right]$$

- C_T is the black hole **heat capacity** (proportional to T so C_T/T is a constant).

- The excitations above the BPS limit are described by the **Schwarzian** theory.

- Two kinds of excitations have **same coefficient**. This is due to (spontaneously broken) $\mathcal{N} = 2$ supersymmetry.

The BPS Limiting Procedure

- Interpretation of φ:
  phase of supercharge relative to "the" preserved SUSY.
- The BPS black hole has $T = 0$ and $\varphi = 0$ but **any ratio**

$$\varphi' = \frac{\varphi}{T}$$

  (The BPS relation $M = M_*$ preserved for **any supercharge**).
- The dependence of the BPS free energy on φ' is interesting.

Dependence on φ' is Physical

The first law of BPS black hole thermodynamics:

$$dW = -Qd\Phi' - Jd\Omega'$$

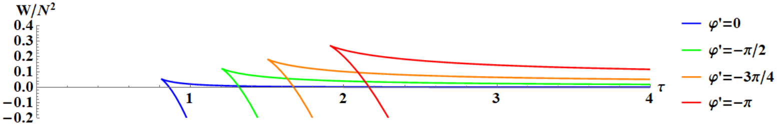
The charges Q, J are **distinct physical parameters**:

The BPS free energy W depends on Φ' and Ω' **independently**.

The potential $\varphi' = \Phi' - \Omega'$ is **not fixed**.

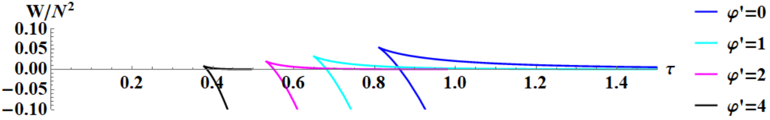
It determines "mixture" of Q, J (for given $M_* = 3Q + 2J$).

Dependence of BPS Free Energy on $\varphi' \leq 0$



- High temperature BPS free energy W **non-vanishing** on the small black hole branch.
- Larger φ' moves the cusp to smaller free energy and smaller temperatures.
- Larger φ' (smaller $|\varphi'|$) is **stabilizing**.

Dependence of Free Energy on $\varphi' \geq 0$



- Small black holes are subject to a **maximal BPS temperature**.
- The maximal temperature **decreases** for larger φ'.
- In the limit $\varphi' \to \infty$ there are **only large black holes**.

Interpretation of φ': Microscopics

The **supersymmetric index**

$$Z = \text{Tr}\,(-)^F \exp\left(\Omega' J + \Phi' Q\right)$$

counts states annihilated by **a given supercharge** $\mathcal{Q}$.
(the grading $(-)^F$ is determined by $\mathcal{Q}$).

The potential φ' determines "which" supercharge $\mathcal{Q}$.

Exactly one mixture of Q, J is realized as a black hole:
the **extremum**.

Summary

Main results:

- A formalism for **thermodynamics of BPS black holes**. ("Projective" thermodynamics).
- BPS black holes in AdS_5 are **strikingly similar** to AdS-Schwarzchild.
- The potential φ' plays a **central role** in BPS thermodynamics.