The 3d Black String and its dual

Workshop on BHs, BPS & Quantum Information

IST, Lisbon, September 23 2021

Stéphane Detournay

Based on 1911.12359 with

Luis Apolo & Wei Song

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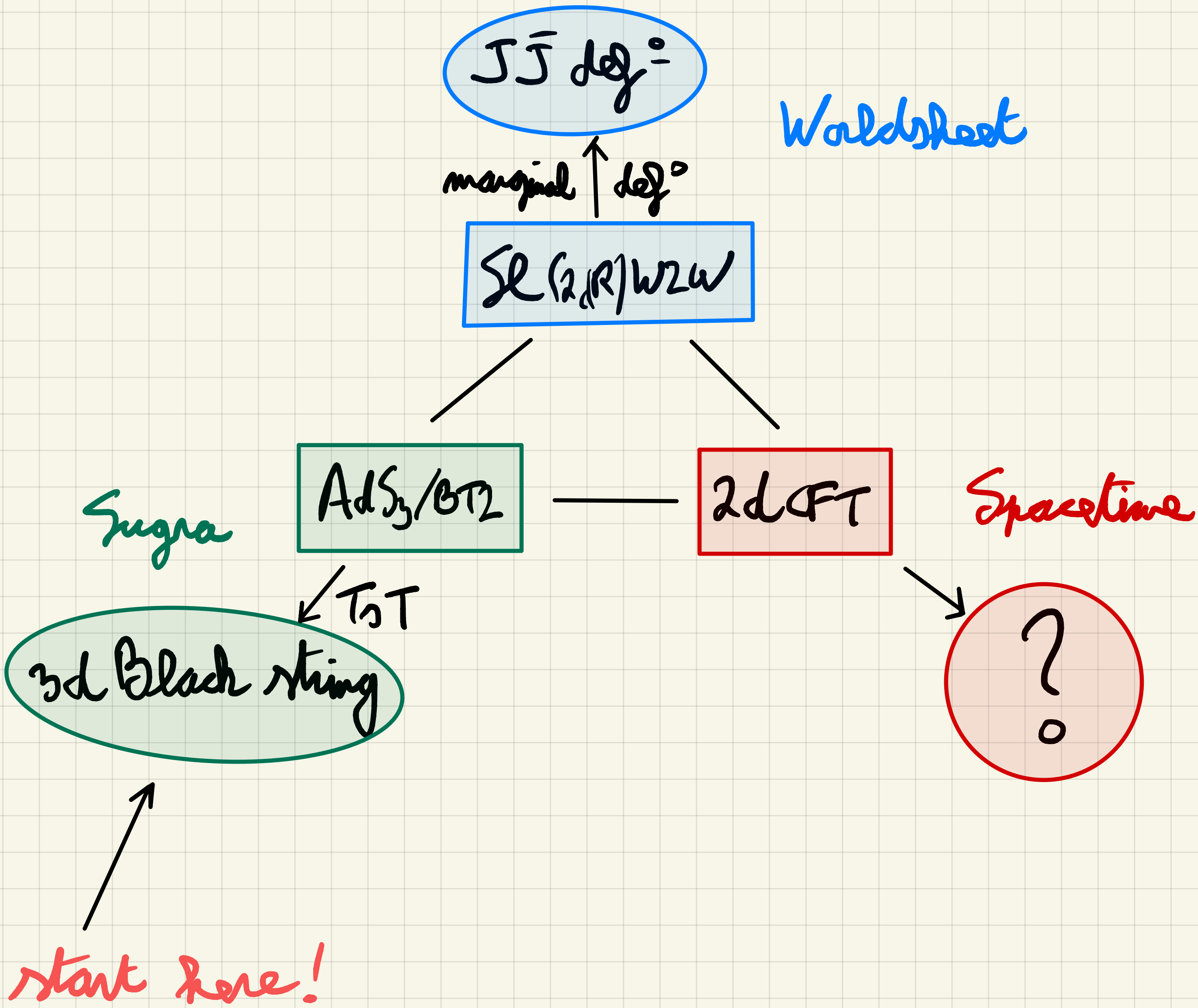

I. 3d Black string

"Exact Black String Solutions in Three dimensions"

[Horne, Horowitz] hep-th/9108001

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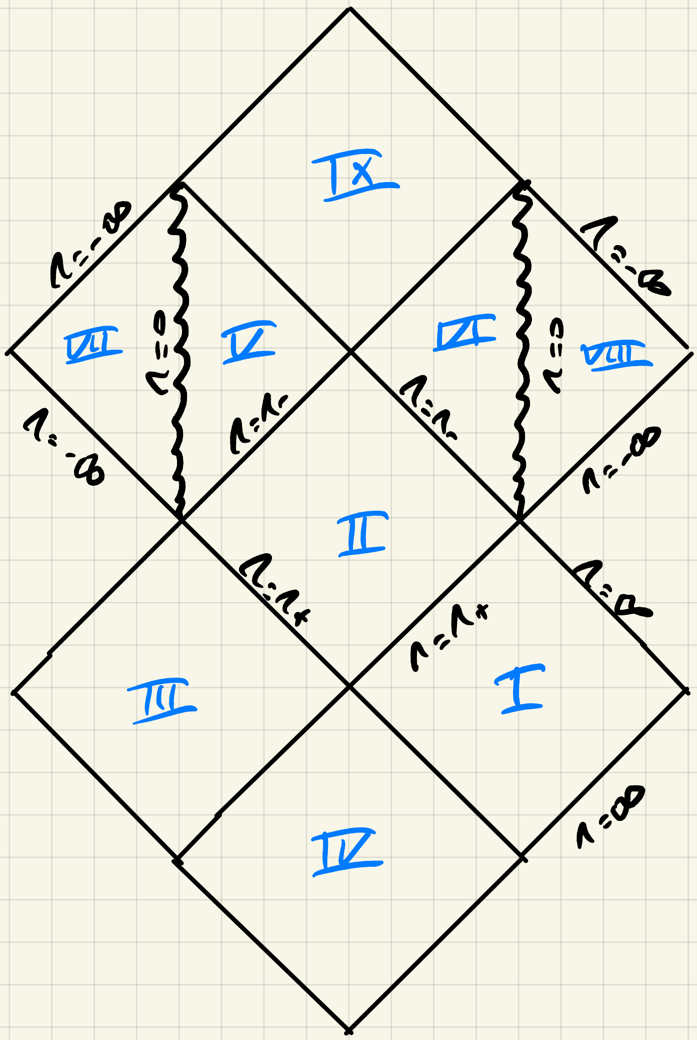
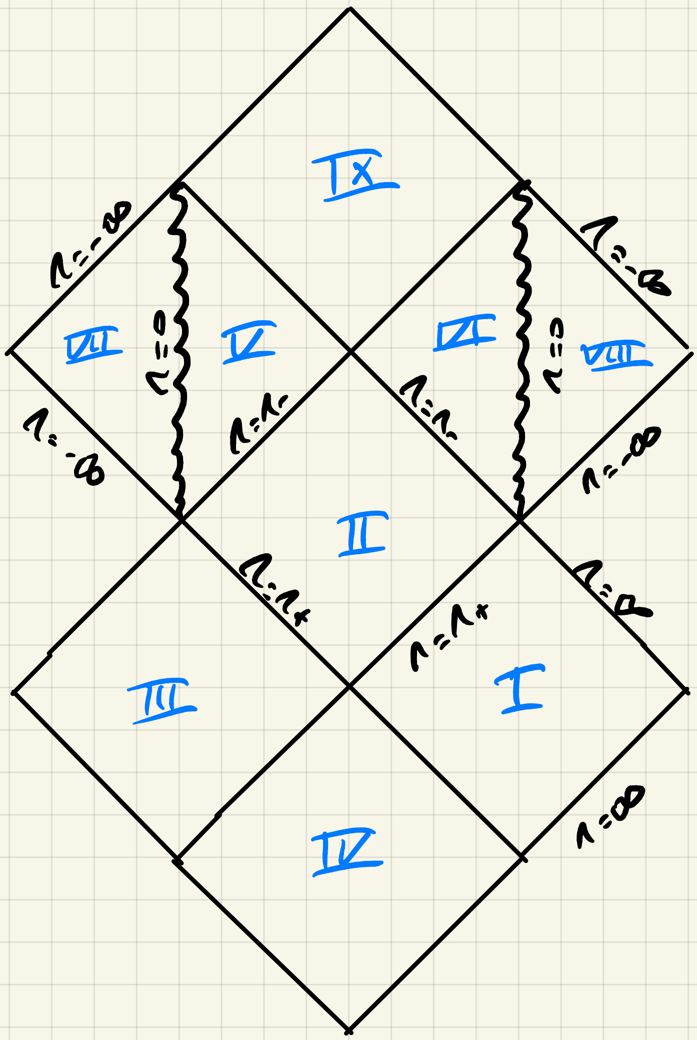
The authors introduced charged black strings as target space of a gauged WZW model:

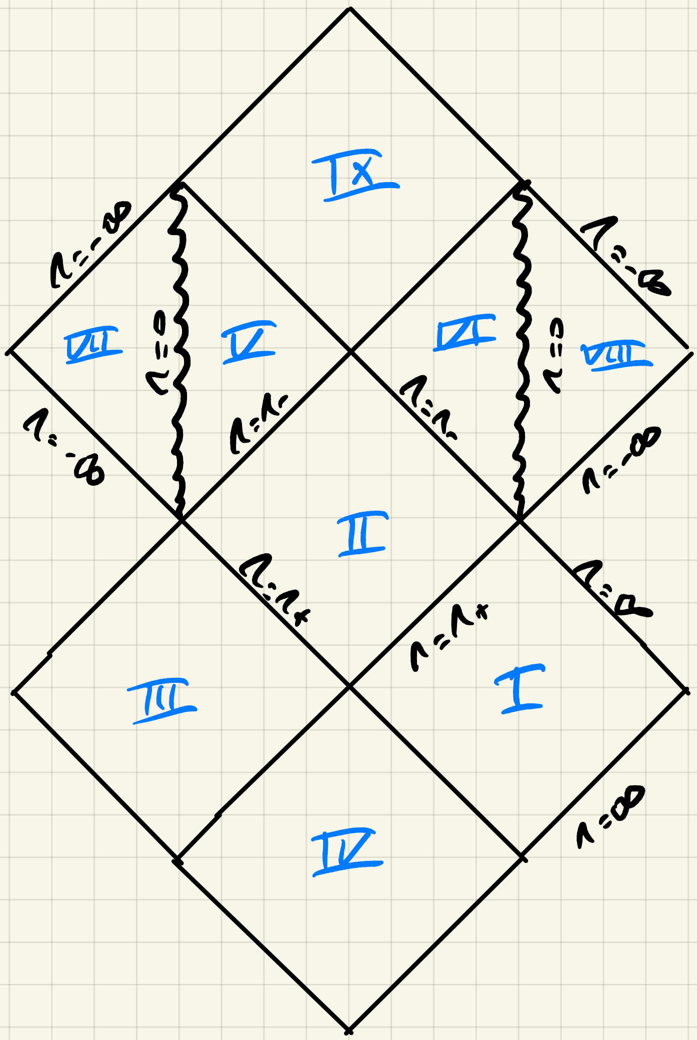
$$
\begin{cases}
ds^2 = -\left(1-\frac{\overbrace{M}^{\text{mass}}}{\rho}\right) dt^2 + \left(1-\frac{Q^2}{M\rho}\right) dx^2 + \left(1-\frac{M}{\rho}\right)^{-1}\left(1-\frac{\overbrace{Q}^{\text{charge}}{}^2}{M\rho}\right)^{-1} \underbrace{k}_{\text{level}} \frac{d\rho^2}{8\rho^2} \\
H_{rtx} = \frac{Q}{\rho^2} \\
\Phi = \ln \rho + \frac{1}{2} \ln \frac{k}{2}
\end{cases}
$$

Rather simple exact CFT producing a BG qualitatively similar to Reissner-Nordström in (2+1) dimensions.


IX
r = -∞
r = -∞
VII
r = 0
V
VI
r = 0
VIII
r = -∞
r = r₋
r = r₋
r = -∞
II
r = r₊
r = r₊
r = ∞
III
I
IV
r = ∞



- Outer and inner horizons $r_+ = M$, $r_- = \frac{Q^2}{M}$



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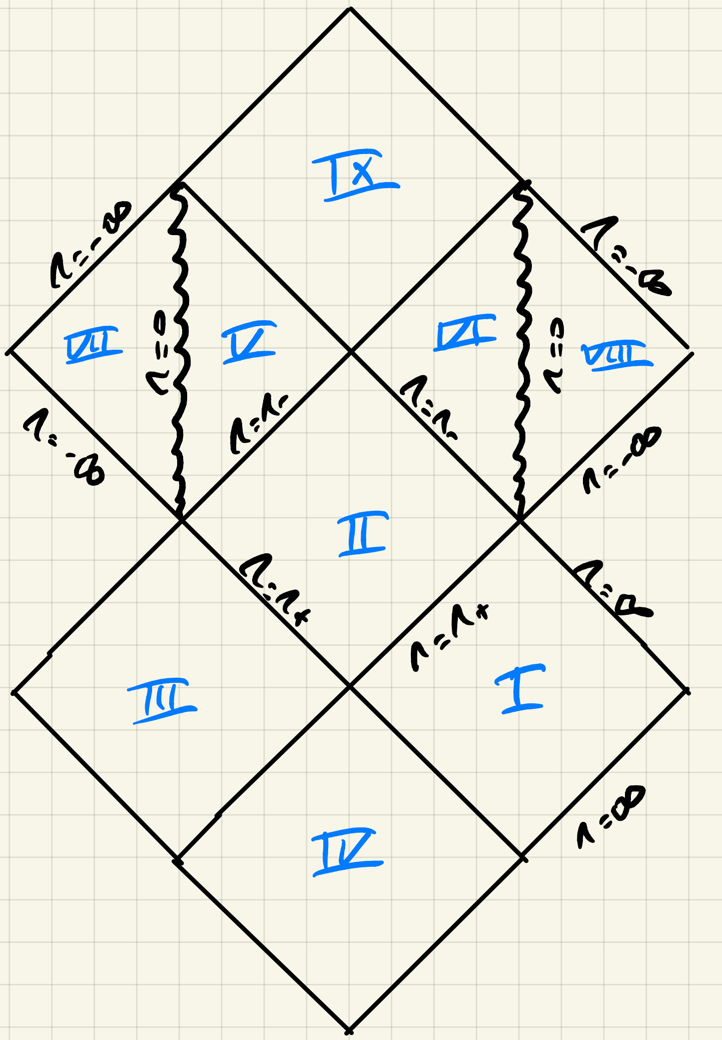
- Curvature singularity at $r=0$



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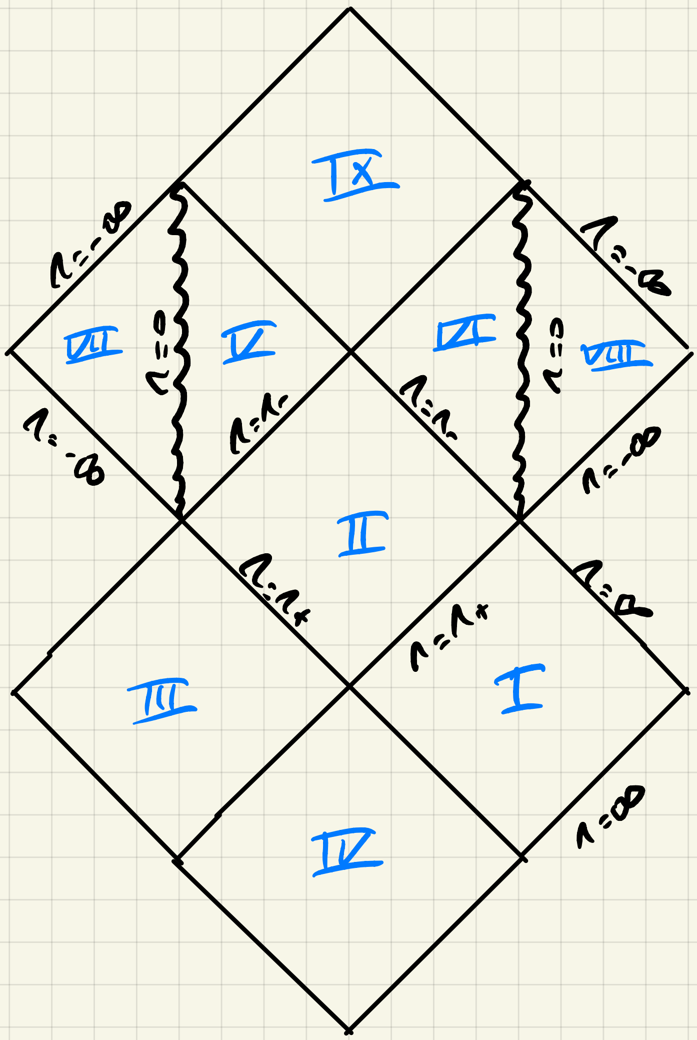
- $R \to 0$ as $\rho \to \infty$, "asymptotically flat"



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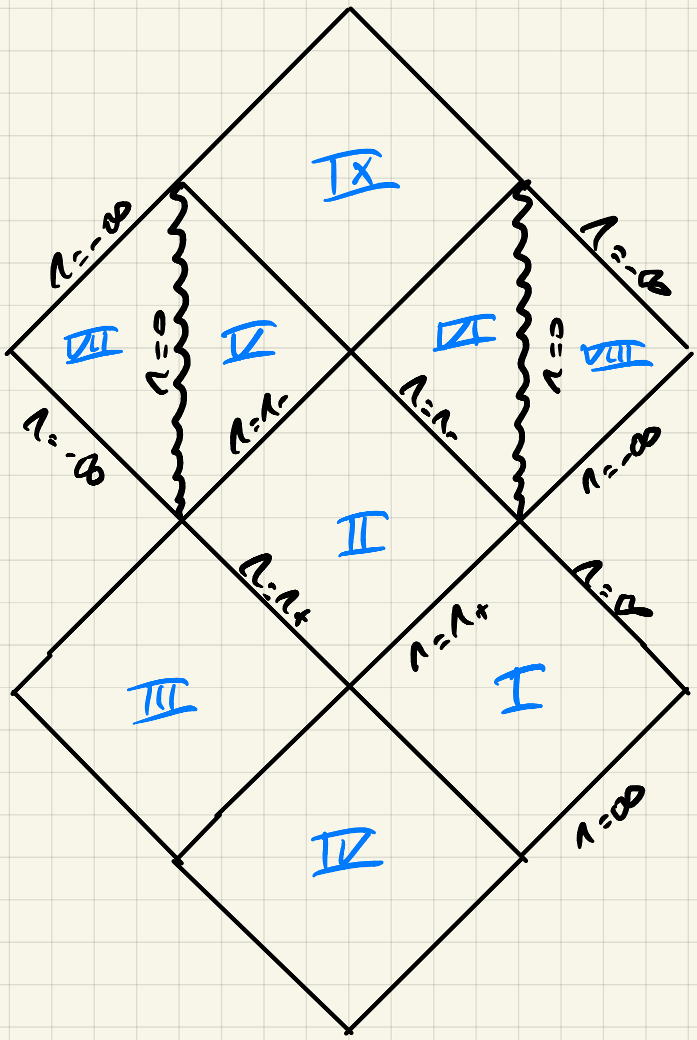
- Hawking T° $T_H = \frac{1}{\pi M}\sqrt{\frac{M^2-Q^2}{2\ell}}$

and

Bekenstein-Hawking entropy $S_{BH} = \frac{1}{4G\ell}\sqrt{M^2-Q^2}$



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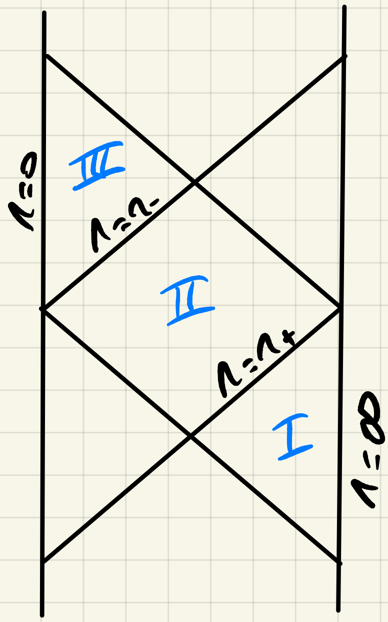
$= \log \rho$?

- BTZ stealing the show '92 [Bañados, Teitelboim, Zanelli]



- AdS/CFT '97 [Maldacena]

- $N_{BS} \ll N_{BTZ}$

citations to HH = 202

citations to BTZ = 2842

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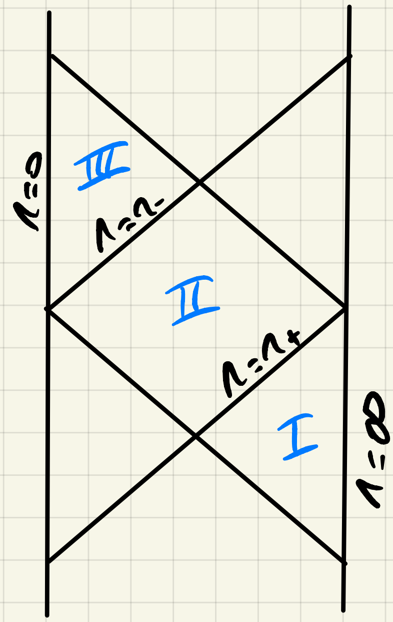

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II. AdS$_3$/CFT$_2$

[Bañados, Henneaux, Teitelboim, Zanelli] '92-'93

II.1 BTZ Black Holes & 3d gravity

II. AdS_3/CFT_2

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$\Lambda = -\frac{1}{\ell^2}$

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2}dt^2 + \frac{\ell^2 r^2\, dr^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\left(d\phi^2 + \frac{r_+ r_-}{r^2}dt^2\right)^2$$

used later

$$= \ell^2\left[\frac{dr^2}{4(r^2-4T_u^2T_v^2)} + r\,du\,dv + T_u^2\,du^2 + T_v^2\,dv^2\right]$$

$$u, v = \phi \pm \frac{t}{\ell}$$

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$$+\, r^2\left(d\phi^2 + \frac{r_+ r_-}{r^2}\,dt^2\right)^2$$

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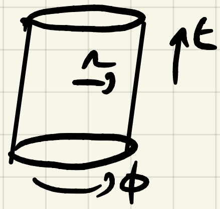

Asymptotically AdS$_3$

Belong to a phase space endowed with a 2d conformal symmetry generated by

$(x^\pm = \frac{t}{\ell} \pm \phi)$ $\qquad l_n^\pm = e^{inx^\pm}(\partial_\pm - i n r \partial_r)$

Corresponding charges $Q_{l_n^\pm} \equiv L_n^\pm$ satisfy a Virasoro algebra with

$$\boxed{c = \frac{3\ell}{2G}}$$

[Brown-Henneaux] '86

Suggests that quantum gravity in AdS_3 would be a CFT_2

Observation that the Bekenstein - Hawking entropy of BTZ is [Strominger] '97

Cardy formula

$$S_{BH} = \frac{\text{Area}}{4G} = \frac{2\pi r_+}{4G} = 2\pi\sqrt{\frac{c E_L}{6}} + 2\pi\sqrt{\frac{c E_R}{6}}$$

$$\begin{cases} E_L = L_0^+ = \frac{1}{2}(\ell M + J) \\ E_R = L_0^- = \frac{1}{2}(\ell M - J) \end{cases}$$

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Cardy regime is $E_{L,R} \gg c$, with c fixed.

The above result seems however to be valid for $c \gg$ ($\ell_{AdS} \gg \ell_p = G$) but $E_{L,R} \sim c$

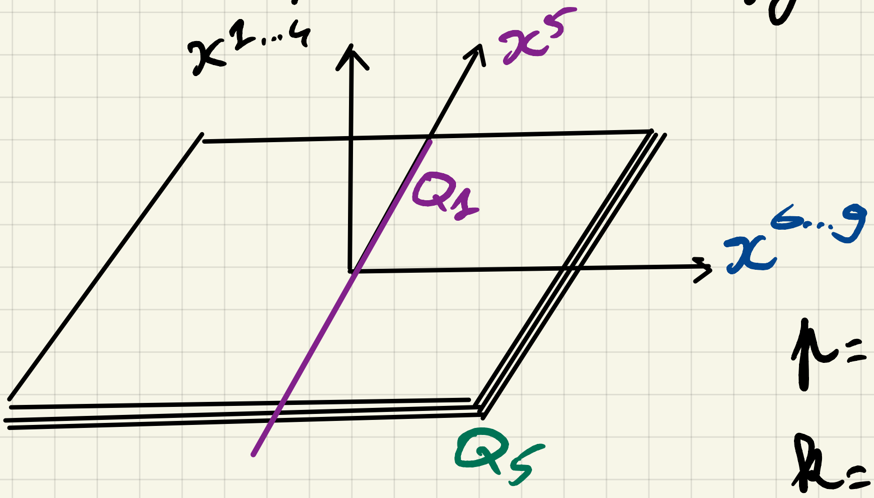
This suggests that the CFT has to be quite special

see [Hartman, Keller, Stoica]

II.2 Stringy AdS_3/CFT_2

[Maldacena '97; David, Mandal, Wadia '02]

Within string theory, one can take a bottom-up approach to identify the dual CFT_2.



Type IIB on T^4 with

$p = Q_1 = \#$ D1/F1

$k = Q_5 = \#$ D5/NS5

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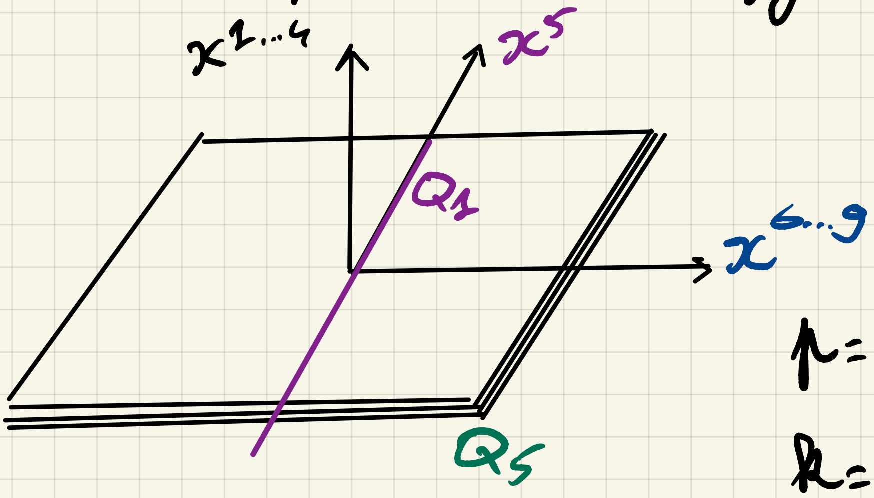

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Taking the decoupling limit of this system:

$$\text{IIB ST on } AdS_3 \times S^3 \times T^4 \xleftrightarrow{\text{dual}} (\text{deformation of}) \ \mathrm{Sym}^N(X) \ \text{CFT}$$

w/ RR / NSNS fluxes

with $c = 6 Q_1 Q_5$

$$\mathrm{Sym}^N(X) \equiv X^N / S_N$$

$$\begin{cases} N = Q_1 Q_5 \ / \ Q_1 \\ c_X = 6 \ / \ 6 Q_5 \end{cases}$$

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[Maldacena, Strominger '98; David, Mandal, Wadia '02]

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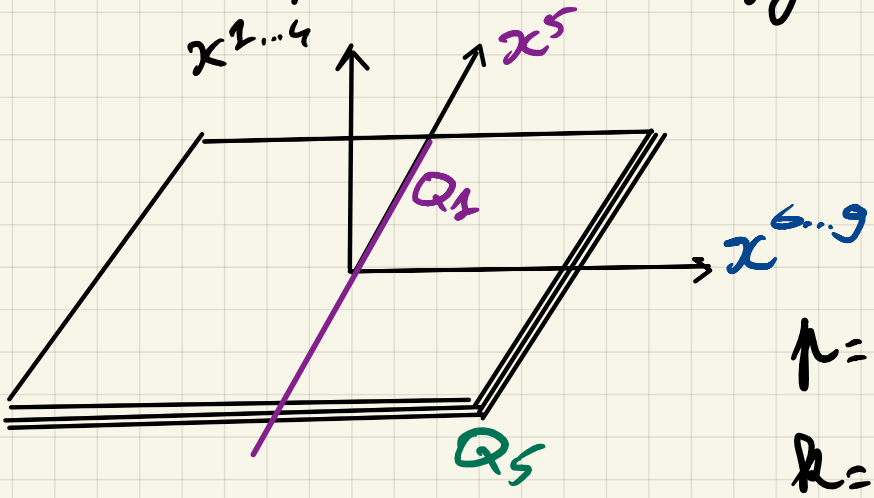

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Taking the decoupling limit of this system:

IIB ST on $AdS_3 \times S^3 \times T^4$ w/ RR/NSNS fluxes $\xleftrightarrow{\text{dual}}$ (deformation of) $\mathrm{Sym}^N(X)$ CFT with $c = 6Q_1Q_5$

$$\mathrm{Sym}^N(X) \equiv X^N/S_N$$

$$\begin{cases} N = Q_1Q_5 \,/\, Q_1 \\ c_X = 6 \,/\, 6Q_5 \end{cases}$$

This is the counterpart of

IIB ST on $AdS_5 \times S^5$ $\xleftrightarrow{\text{dual}}$ $SU(N)$ $\mathcal{N}=4$ SYM

derived from a stack of D3 branes

but is weaker in the former case

Identifying the precise CFT dual is still an open question

see [Eberhardt, Gaberdiel, Gopakumar '12 '19 ;
'19 Chakraborty, Giveon, Kutasov ; ...
'00 Argurio, Giveon, Shomer ; ...]

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see [Eberhardt, Gaberdiel, Gopakumar '18 '19; '19 Dei, Kutasov; ... '20 Argurio, Giveon, Shomer; ...]

The NS5-F1 setup is very interesting because string theory on AdS_3 w/ only NSNS fluxes is exactly solvable: it is described by a $SL(2,\mathbb{R})$ WZW at level $k = Q_5$.

$$\left(k = Q_5 = \frac{\ell_{AdS}^2}{\ell_s^2} = \frac{R^2}{\alpha'} \right)$$

So, finite $k \Rightarrow$ solving classical string theory to all orders in α'

(we omit the $S^3 \times T^4$ factors, but they are still there)

II.3 Strings on AdS_3

[Maldacena, Ooguri + Son] 100

The $SL(2,\mathbb{R})$ WZW model describes string theory on $AdS_3 \sim SL(2,\mathbb{R})$ group manifold supported by an NSNS B-field.

The proof of the consistency of the model and the determination of the spectrum was achieved by [MO + S], also building on work from the end of the 80s onwards.

Sl(2,R) WZW model

$$S_{WZW} = k \int d^2x \, \mathrm{Tr}\left(g^{-1}\partial g \, g^{-1}\bar{\partial} g\right) + k\,\Gamma_{WZ}$$

Basic field: $\quad g(z,\bar{z}) \in Sl(2,\mathbb{R}) \equiv G$

$\hookrightarrow$ WS coordinates

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↪ WS coordinates

Symmetries: Left/Right affine symmetry $(g \to \Omega(z)\, g\, \bar{\Omega}(\bar{z}))$ generated by (anti-) holomorphic currents (Lie algebra valued)

$$\boxed{J(z) = k\, g^{-1}\partial g} \equiv J^a T_a \qquad \boxed{\bar{J}(\bar{z}) = k\, \bar{\partial} g\, g^{-1}} \equiv \bar{J}^a T_a$$

Modes $J^a_n, \bar{J}^a_n \longrightarrow$ affine current algebra $\hat{g}$

Zero modes J^3_0 and $\bar{J}^3_0$ generate shifts in t and ϕ, so

$$\boxed{E = J^3_0 + \bar{J}^3_0} \quad \text{and} \quad \boxed{J = J^3_0 - \bar{J}^3_0}$$

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Sugawara construction

$$\begin{cases} T = \dfrac{1}{k}\, J^a J^a \\ \bar{T} = \dfrac{1}{k}\, \bar{J}^a \bar{J}^a \end{cases} \qquad \begin{cases} L_n \sim \sum_m :J^a_m J^a_{n-m}: \\ \bar{L}_n \sim \sum_m :\bar{J}^a_m \bar{J}^a_{n-m}: \end{cases}$$

conformal invariance

<u>Spectral flow symmetry</u>

$$\begin{cases} J^3_n \longrightarrow \tilde{J}^3_n = J^3_n + \frac{k}{2} w \,\delta_{n,0} \\ J^\pm_n \longrightarrow \tilde{J}^\pm_n = J^\pm_{n\mp w} \end{cases} \qquad w \in \mathbb{Z}$$

is an automorphism of $\hat{g}$.

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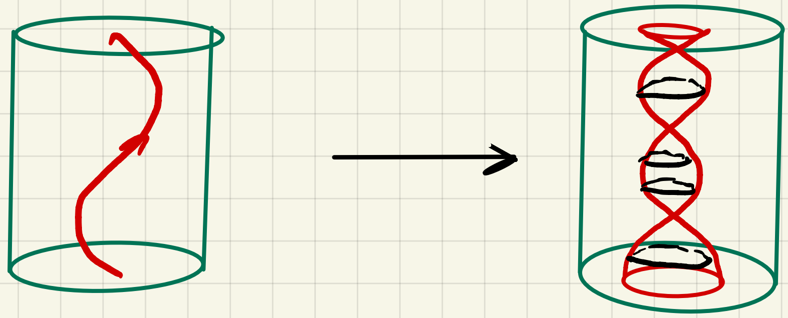
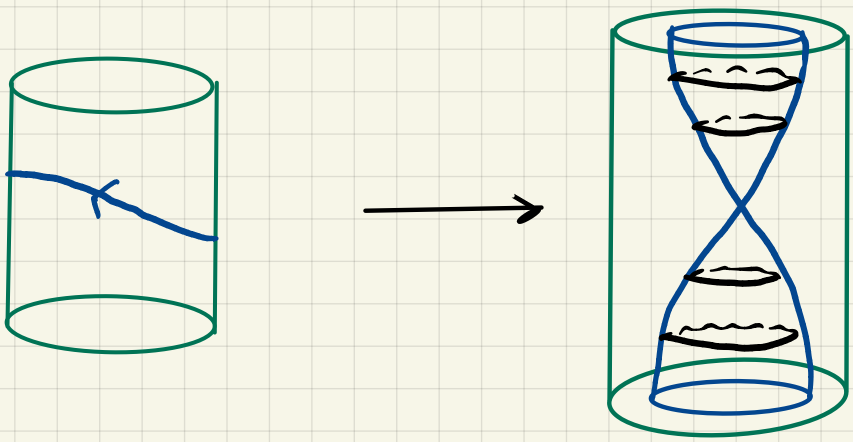
At the classical level, it generates string-like solutions from geodesics:



short strings

(bound states)



long strings

(scattering states)

At the quantum level, it generates new, inequivalent, representations of the affine algebra obtained by acting with $\tilde{J}^a_n$ and $\bar{\tilde{J}}^a_n$ on $SL(2,\mathbb{R})$ representations $|\psi\rangle$ of the zero modes:

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$$|\psi\rangle \begin{cases} \nearrow |j\, m\rangle \text{ Principal discrete } \mathcal{D}^{\pm}_{j>1/2} \\ \searrow |j\, \alpha; m\rangle \text{ Principal continuous } \mathcal{C}^{\alpha}_{j=\frac{1}{2}+is} \end{cases}$$

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$|\psi\rangle \rightarrow |j\,\alpha; m\rangle$ Principal continuous $\mathcal{C}^{\alpha}_{j=\frac{1}{2}+is}$

The corresponding affine representations are

$\hat{\mathcal{D}}^{\pm,w}_j$	$\hat{\mathcal{C}}^{\alpha,w}_{j=\frac{1}{2}+is}$
short strings	long strings
discrete energy spectrum	continuous energy spectrum

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The corresponding affine representations are

$\hat{\mathcal{D}}^{\pm,w}_{j}$ — short strings — discrete energy spectrum

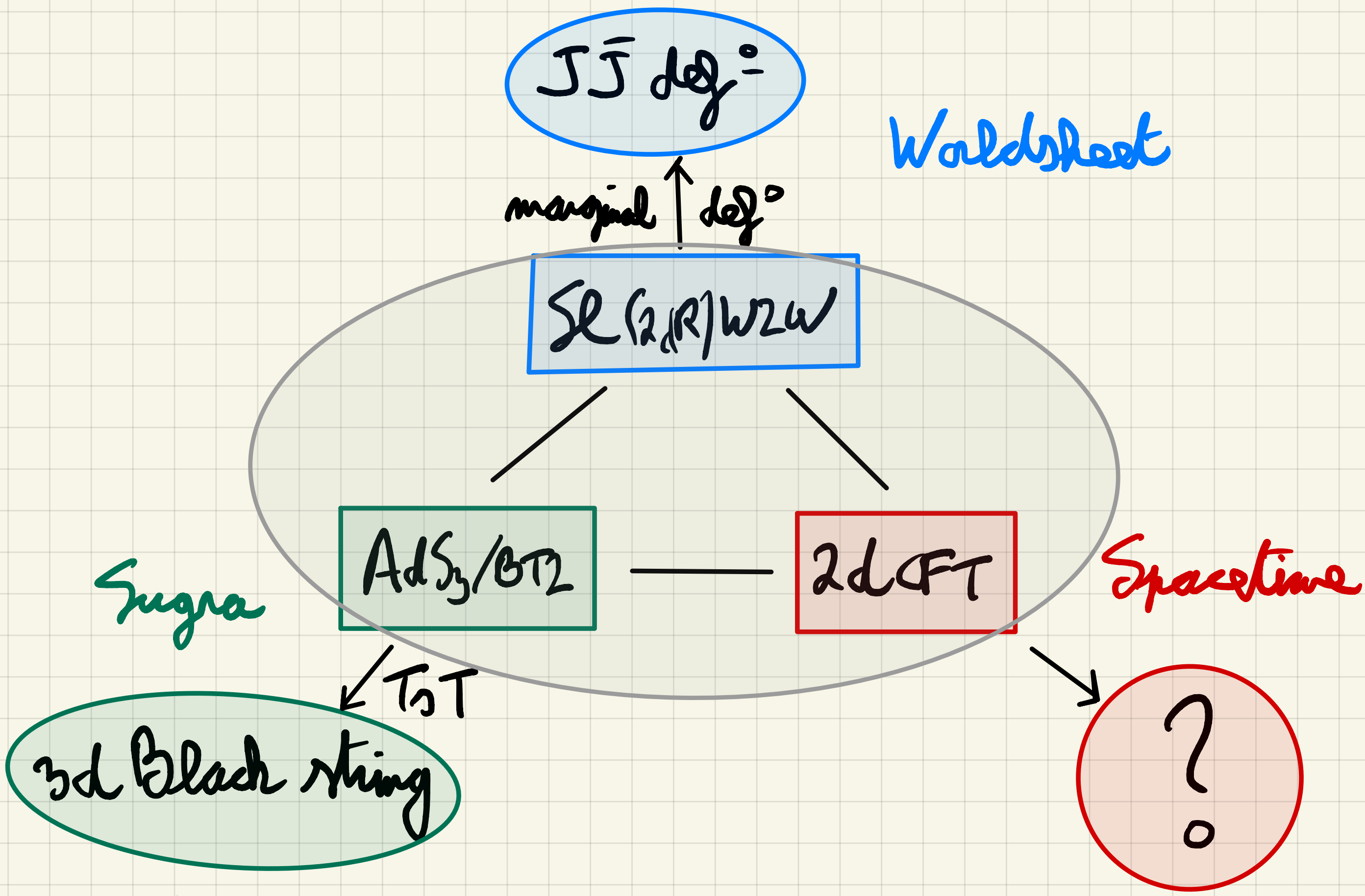
$\hat{\mathcal{C}}^{\alpha,w}_{j=\frac{1}{2}+is}$ — long strings — continuous energy spectrum

The theory is now unitary (no-ghost theorem for physical states $L_0 = 1$) and consistent (e.g. no upper bound on mass of string states)

This result is important in that it contains the nature and spectrum of the CFT dual to AdS_3 string theory.

One proposal [Eberhardt, Gaberdiel, Gopakumar '18 '19] for the dual to superstring theory on $AdS_3 \times S^3 \times T^4$ at level k:

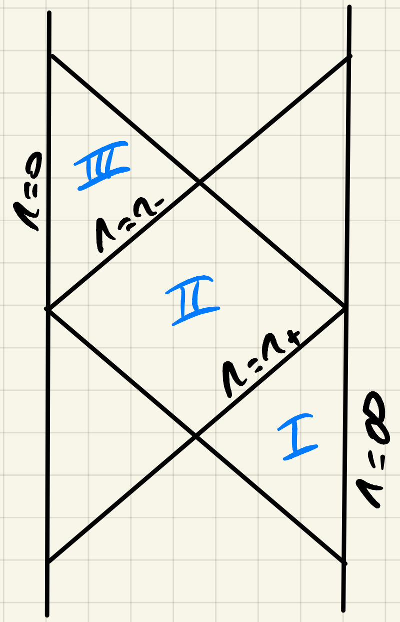
$$\mathrm{Sym}^N\left([\mathcal{N}=4 \text{ Liouville w/ } c = 6(k-1)] \oplus T^4 \right)$$



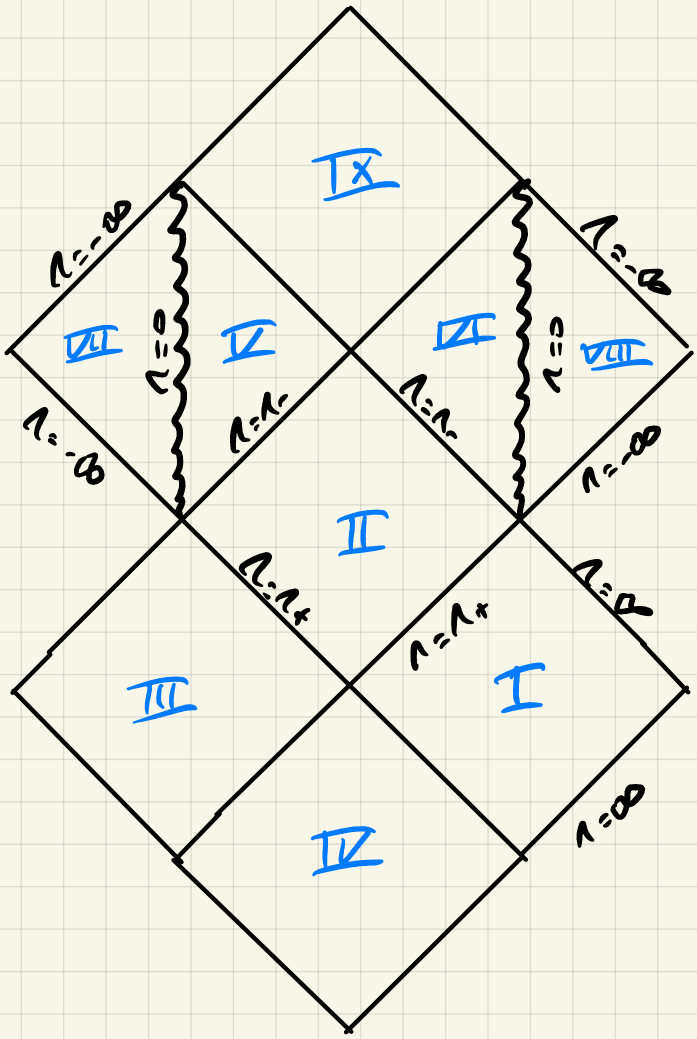
$J\bar{J}$ def$^{\underline{o}}$
Worldsheet
marginal
def^{o}
$SL(2,\mathbb{R})$ WZW
AdS_3/BTZ
2d CFT
Sugra
Spacetime
$T\bar{T}$
3d Black string
?

III. Away from AdS_3

From



to



III. Marginal deformations of WZW models

The $SL(2,\mathbb{R})$ WZW describes string propagation on AdS_3 through an exact worldsheet description.

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How to describe more general backgrounds?

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How to describe more general backgrounds?

It has been discussed that WZW models admit integrable marginal deformations, thus allowing to reach new exact string backgrounds:

$$\boxed{S_{\lambda,WZW} = S_{WZW} + \lambda_{WZW} \int d^2z \, \underbrace{\mathcal{O}(z,\bar{z})}_{(1,1)\text{ operator}}}$$

For instance, $\mathcal{O} = \overset{(1,0)}{J^1} \, \overset{(0,1)}{\bar{J}^1}$ is exactly marginal

[Chaudhuri, Schwartz; Hassan, Sen; Giveon, Kiritsis; Forste, Roggenkamp, ...]

Black string from $J_1\bar{J}_1$ marginal deformation

see also [Hora-Horowitz; Israël, Kounnas, Petropoulos, Orlando, Spindel, SD, ...] ~ '05

* Start with W2W with group element

$$g = e^{2T_u u T^1} e^{f(r, T_u, T_v) T^3} e^{2T_v v T^1} \in SL(2,\mathbb{R})$$

This corresponds to string propagation on the BTZ metric we saw earlier:

$$\begin{cases} \dfrac{ds^2}{\ell^2} = \dfrac{dr^2}{4(r^2 - T_u^2 T_v^2)} + r\, du\, dv + T_u^2 du^2 + T_v^2 dv^2 \\ B = \ldots \\ e^{2\Phi} = \text{cst}. \end{cases}$$

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generate translations in u and v, "i.e." Noether currents

* Turn on marginal operator $\mathcal{O} = J^1 \bar{J}^1$

The deformed action satisfies

$$\boxed{\frac{\partial S_{\lambda,\mathrm{W2W}}}{\partial \lambda} \sim \int d^2 z\, J^1 \bar{J}^1}$$

* The backgrounds after deformation are

$$\begin{cases} \dfrac{ds^2}{\ell^2} = \dfrac{dr^2}{4(r^2 - 4T_{uu}^2 T_{vv}^2)} + \dfrac{r\,du\,dv + T_{uu}^2\,du^2 + T_{vv}^2\,dv^2}{1 + 2\lambda r + 4\lambda^2 T_{uu}^2 T_{vv}^2} \\ B = \ldots \\ e^{2\Phi} = \dfrac{k}{p}\, \dfrac{(1 - 4\lambda^2 T_{uu}^2 T_{vv}^2)}{1 + 2\lambda r + 4\lambda^2 T_{uu}^2 T_{vv}^2}\, e^{-2\Phi_0} \end{cases}$$

After a change of radial coordinate

$$\rho = \left[\frac{(M+Q)^2 - J^2}{4M}\right] r + \frac{M^2 + Q^2 - J^2}{2M}$$

the metric becomes, for $\lambda = \frac{1}{2}$,

$$\frac{ds^2}{\ell^2} = \frac{d\rho^2}{4\left[\rho^2 - \left(\frac{M^2+Q^2}{M}\right)\rho + Q^2\right]} - \left(1 - \frac{M}{\rho}\right) dt^2$$

$$+ \left(1 - \frac{Q^2 - J^2}{M\rho}\right) d\varphi^2 + \frac{2J}{\rho}\, dt\, d\varphi$$

For $J=0$, this exactly the HH black string:

(pick on p3)

$$ds^2 = -\Big(1 - \underbrace{\frac{M}{\rho}}_{\text{mass}}\Big) dt^2 + \Big(1 - \frac{Q^2}{M\rho}\Big) dx^2 +$$

$$\Big(1 - \frac{M}{\rho}\Big)^{-1} \Big(1 - \overbrace{\frac{Q^2}{M\rho}}^{\text{charge}}\Big)^{-1} k\, \frac{d\rho^2}{8\rho^2}$$

* The deformed background can be obtained from the undeformed one by a

TsT transformation

T duality along u

shift along $v \to v - \frac{2\lambda}{R} u$

T duality along u once more

(up to change of coords and axial currents ~ Noether currents)

	Undeformed		Deformed
Sugra	AdS_3/BTZ	TsT	Black string
Worldsheet	$SL(2,R)$ WZW	Marginal def.	$S_{WZW} + \lambda \int J^1 \bar{J}^1$
Spacetime	CFT_2	?	

asymptotics changed

→ irrelevant deformation?

	Undeformed		Deformed
Sugra	AdS_3/BTZ	TsT	Black string
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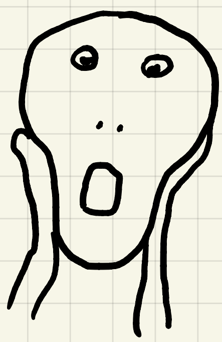
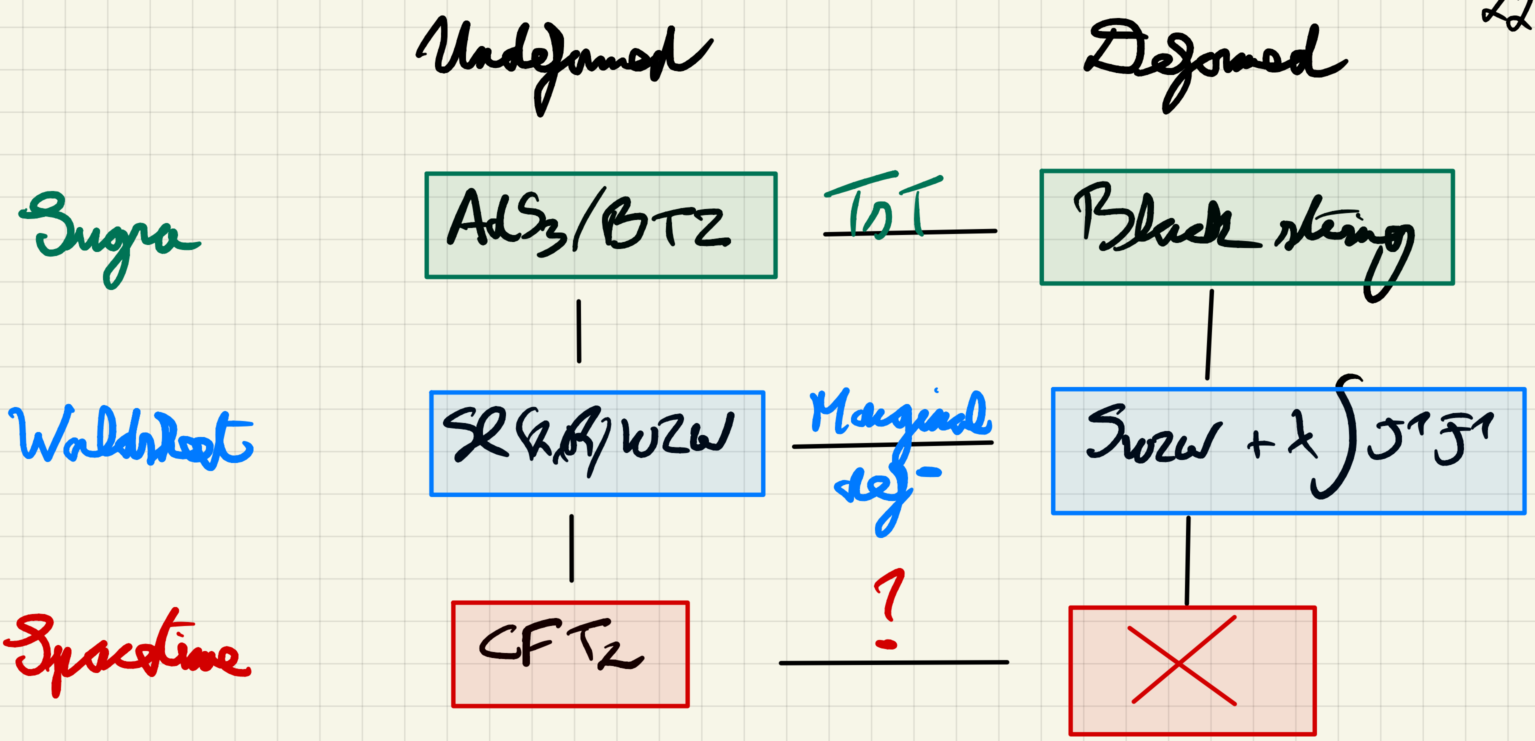
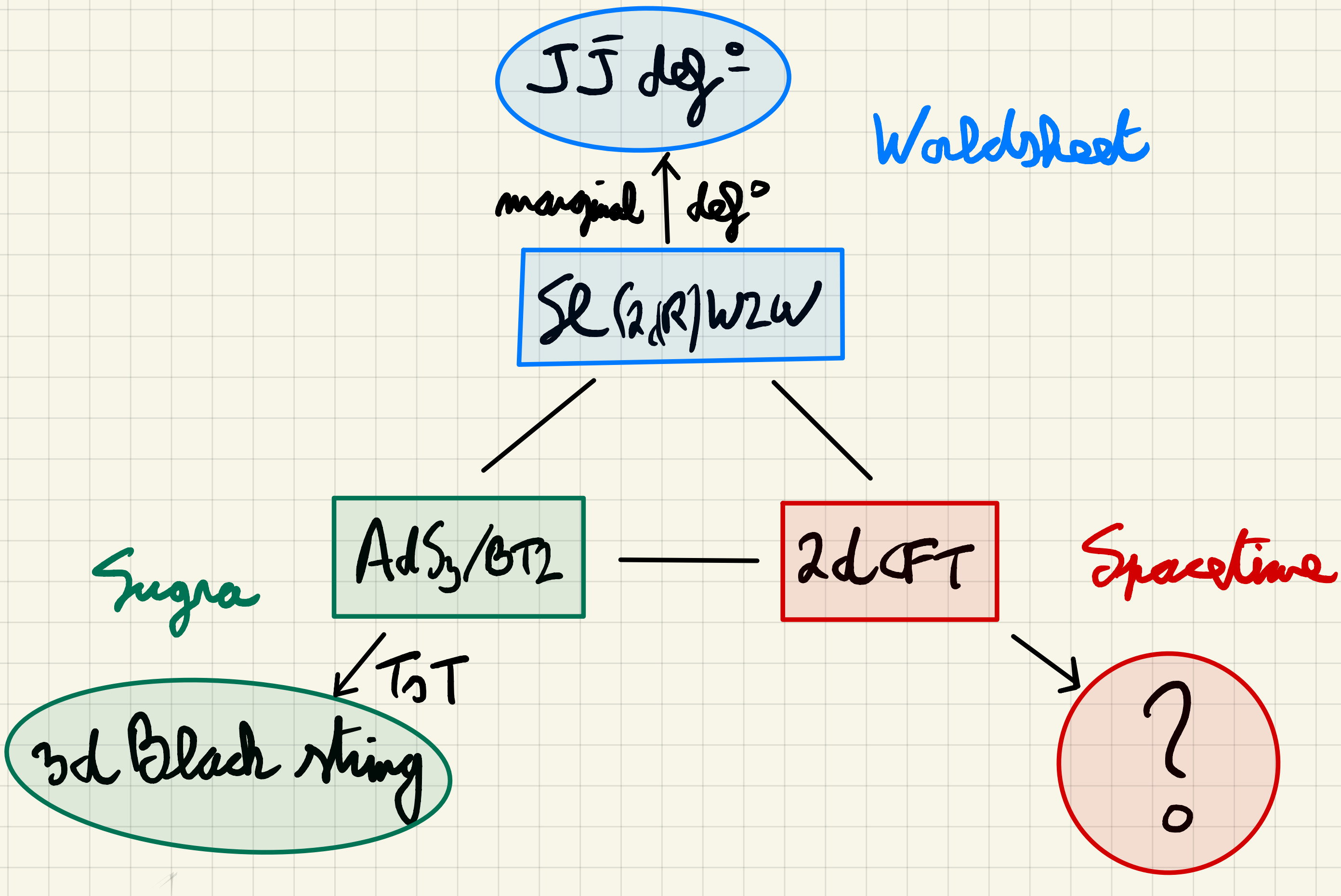
Spacetime	CFT_2	?	$\times$

asymptotics changed

→ irrelevant deformation?




Undeformed
Deformed
Sugra
AdS_3/BTZ
$T\bar{T}$
Black string
Worldsheet
$SL(2,\mathbb{R})$ WZW
Marginal def$^{\underline{n}}$
$S_{WZW} + \lambda \int J^1 \bar{J}^1$
Spacetime
CFT_2
?


$J\bar{J}$ def$^{\underline{n}}$:
Worldsheet
marginal def$^{\underline{n}}$
$SL(2,\mathbb{R})$ WZW
Sugra
AdS_3/BTZ
2d CFT
Spacetime
$T\bar{T}$
3d Black string
?

IV. $T\bar{T}$ deformations [Smirnov, Zamolodchikov '16; Cavaglià, Negro, Szécsényi, Tateo]

Irrelevant def. of 2d QFTs triggered by

$$T\bar{T} \equiv -\pi^2 \det(T_{\mu\nu})$$

The corresponding deformed action then satisfies

$$\frac{\partial S_{QFT}}{\partial \mu} = \int d^2x \left(T_{zz} T_{\bar{z}\bar{z}} - T_{z\bar{z}} T_{\bar{z}z} \right)$$

$$\sim \int J_{(1)} \wedge J_{(\bar{1})}$$

$$\begin{cases} J_{(1)} = T_{zz}\, dz + T_{z\bar{z}}\, d\bar{z} \\ J_{(\bar{1})} = T_{\bar{z}z}\, dz + T_{\bar{z}\bar{z}}\, d\bar{z} \end{cases}$$

Many physically interesting quantities can be computed exactly and explicitly in terms of the data from the undeformed theory.

$T\bar{T}$ deformations

TT deformations

Finite volume spectrum

$(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$\begin{cases} E(\mu) = \dfrac{R}{2\mu}\left[\sqrt{1 + \dfrac{4\mu E}{R} + \dfrac{4\mu^2 J^2}{R^2}} - 1\right] \\ J(\mu) = J \end{cases}$$

In terms of $E_{L/R} = \dfrac{E \pm J}{2}$:

$$\boxed{E_{L/R} = E_{L/R}(\mu) + \frac{2\mu}{R} E_L(\mu) E_R(\mu)}$$

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Asymptotic growth of states

(deformed CFT_2)

['18 Datta, Jiang, Aharony, Giveon, Kutasov]

$$\boxed{S_{T\bar{T}} = 2\pi\left[\sqrt{\frac{c}{6} R E_L(\mu)\left(1 + \frac{2\mu}{R} E_R(\mu)\right)} + \sqrt{\frac{c}{6} R E_R(\mu)\left(1 + \frac{2\mu}{R} E_L(\mu)\right)}\right]}$$

of undeformed CFT

V. Dual to the 3d Black String

Proposal :

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Proposal:

see related proposals
(Giveon, Itzhaki, Kutasov

$$\boxed{\begin{array}{c}\text{String theory on a } T\bar{T} \text{ transf}^{\underline{o}} \text{ of} \\ AdS_3 \times M \text{ w/ NSNS fluxes} \\ \xleftrightarrow[\text{to}]{\text{dual}} \\ \text{Single trace } T\bar{T} \text{ deformation} \\ \text{of a } CFT_2\end{array}}$$

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see related proposals (Giveon, Itzhaki, Kutasov)

Assume the original CFT_2 is of the form

$$\mathrm{Sym}^{p}(X_{6k})$$

with
$$\begin{cases} c_X = 6k \ , \ X = \text{seed CFT} \\ p = Q_1 = \# F1 \\ k = Q_5 = \# NS5 \end{cases}$$

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Proposal:

see related proposals (Giveon, Itzhaki, Kutasov)

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with $\begin{cases} c_X = 6k \;,\; X = \text{seed CFT} \\ p = Q_1 = \#F1 \\ k = Q_5 = \#NS5 \end{cases}$

The single-trace version of $T\bar{T}$ deformation is

$$\frac{\partial S}{\partial \mu} = \sum_{i=1}^{p} \int J^i_{(1)} \wedge J^i_{(2)}$$

$$J^i_{(1)} = \underbrace{T^i_{xx}}_{\text{stress tensor in the } i^{\text{th}} \text{ copy}} dx + T^i_{x\bar{x}}\, d\bar{x}$$

Evidences :

1) Thermodynamics

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The total entropy of the single trace dof's of $Sym^N(X)$ is given by

$$\frac{S_{T\bar{T}}(\epsilon_L,\epsilon_R)}{2\pi} = \sqrt{\frac{c}{6}R\epsilon_L\left(1+\frac{2\mu}{R\lambda}\epsilon_R\right)} + \sqrt{\frac{c}{6}R\epsilon_L\left(1+\frac{2\mu}{R\lambda}\epsilon_L\right)}$$

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Let's compare to the Bekenstein-Hawking entropy of the Black string.

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Let's compare to the Bekenstein-Hawking entropy of the Black string.

Its conserved charges can be computed:

$$\begin{cases} Q_L = Q_{T_{ra}} = \frac{c}{6}\frac{(1+2\lambda T_{v}^2)T_{u}^2}{1-4\lambda^2 T_u^2 T_v^2} \\ Q_R = Q_{T_{vr}} = \dots \\ Q_e = p = Q_1 \;\; , \;\; Q_m = \mathcal{R} = Q_5 \\ \quad\;\; \#F1 \qquad\qquad\qquad\;\; \#NS5 \end{cases}$$

$= \frac{l_{AdS}^2}{l_s^2} =$ level

measured by dof's

$$\frac{S_{BH}}{2\pi} = \sqrt{Q_L(Q_eQ_m+2\lambda Q_R)} + \sqrt{Q_R(Q_eQ_m+2\lambda Q_L)}$$

with $RE_L = Q_L$, $RE_R = Q_R$

$c = 6Q_eQ_m$, $\lambda = \frac{\mu k}{R^2}$

Evidences :

1) Thermodynamics

The total entropy of the single trace def$^{=}$ of $\mathrm{Sym}^N(X)$ is given by

$$\frac{S_{TT}(E_L,E_R)}{2\pi} = \sqrt{\frac{c}{6} R E_L \left(1+\frac{2\mu}{Rp} E_R\right)} + \sqrt{\frac{c}{6} R E_L \left(1+\frac{2\mu}{Rp} E_L\right)}$$

Let's compare to the Bekenstein-Hawking entropy of the black string.

Its conserved charges can be computed:

$$\begin{cases} Q_L = Q_{T_u} = \dfrac{c}{6}\, \dfrac{(1+2\lambda T_v^2)T_u^2}{1-4\lambda^2 T_u^2 T_v^2} \\ Q_R = Q_{T_v} = \dots \\ Q_e = p = \underset{\#F1}{Q_1}\ , \quad Q_m = k = \underset{\#NS5}{Q_5} \end{cases}$$

$k = \dfrac{l_{AdS}^2}{l_s^2} =$ level

measured by def$^{=}$

$$\frac{S_{BH}}{2\pi} = \sqrt{Q_L(Q_e Q_m + 2\lambda Q_R)} + \sqrt{Q_R(Q_e Q_m + 2\lambda Q_L)}$$

with $R E_L = Q_L$, $R E_R = Q_R$

$c = 6 Q_e Q_m$, $\lambda = \dfrac{\mu k}{R^2}$

$\longrightarrow$ $\boxed{S_{TT} = S_{BH}!}$

2) String spectrum

It is possible to determine the string spectrum on the $T\bar{T}$ transformed background (G, B, Φ) in terms of that on the undeformed one $(\tilde{G}, \tilde{B}, \tilde{\Phi})$

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Feature of $T\bar{T}$: the EOM and Virasoro constraints of the deformed theory can be obtained from those of the undeformed theory by a non-local change of coords

[Alday, Arutyunov, Frolov] '05

2) String spectrum

It is possible to determine the string spectrum on the TsT transformed background (G, B, Φ) in terms of that on the undeformed one $(\tilde{G}, \tilde{B}, \tilde{\Phi})$

Feature of TsT : the EOM and Virasoro constraints of the deformed theory can be obtained from those of the undeformed theory by a non-local change of coords

[Alday, Arutyunov, Frolov] '05

One TsT : $\begin{cases} \text{T-duality along } u \\ \text{shift } v \to v - \frac{2\lambda}{R} u \\ \text{T-duality along } u \end{cases}$

Non-local change of coords : $\begin{cases} \partial \hat{v} = \partial v - \frac{2\lambda}{R} J^1 \\ \bar{\partial} \hat{u} = \bar{\partial} u - \frac{2\lambda}{R} \bar{J}^1 \end{cases}$ currents associated with u and v translations

Then : $(\hat{G}, \hat{B}, \hat{\Phi}) = (\tilde{G}, \tilde{B}, \tilde{\Phi})$ but w/ non-local BCs for $\hat{u}$ and $\hat{v}$

Non-local BCs on $\hat u$ and $\hat v$:

$$\hat u(\varphi+2\pi) = \hat u(\varphi) + 2\pi\gamma_{(u)}$$

$$\hat v(\varphi+2\pi) = \hat v(\varphi) + 2\pi\gamma_{(v)}$$

$$\gamma_{(u)} = \underbrace{w}_{\text{winding}} + \frac{\lambda}{2\pi}\underbrace{\oint \bar J^1 d\bar z}_{J_0^1 = E_R} \quad , \quad \gamma_{(v)} = \dots$$

These twisted BCs can be implemented by

$$\hat u = \tilde u + \gamma_{(u)} z \quad , \quad \hat v = \tilde v - \gamma_{(v)} \bar z$$

coord of the deformed theory

coordinates of the undeformed theory

This induces a spectral flow of the currents

$$\begin{cases} \hat J_1 = \tilde J_1 + \#\, \gamma_{(u)} \\ \hat{\bar J}_1 = \tilde{\bar J}_1 + \#\, \gamma_{(v)} \end{cases}$$

l = AdS radius

and of the Virasoro modes:

$$\begin{cases} \hat L_0(\lambda) = \tilde L_0 + l\, E_L(\lambda)\, w - \frac{2\lambda}{l} p^2 E_L(\lambda) E_R(\lambda) \\ \hat{\bar L}_0(\lambda) = \dots \end{cases}$$

Imposing the Virasoro constraints before and after deformation $\hat{C}_0(\lambda) = 1 = \hat{C}_0(0)$ one can extract

$$E_{L/R}(\lambda) = E_{L/R} + \frac{2\lambda\ell}{w R} E_L(\lambda) E_R(\lambda)$$

compare to $$\boxed{E_{L/R} = E_{L/R}(\mu) + \frac{2\mu}{R} E_L(\mu) E_R(\mu)}$$

$$\ell = R w, \quad \lambda = \frac{\mu k}{R^2}$$

TT spectrum!

<u>Then</u>

Imposing the Virasoro constraints before and after deformation $C_0(\lambda) = 1 = C_0(0)$ one can extract

$$E_{L/R}(\lambda) = E_{L/R} + \frac{2\lambda\ell}{wk} E_L(\lambda) E_R(\lambda)$$

compare to $$\boxed{E_{L/R} = E_{L/R}(\mu) + \frac{2\mu}{R} E_L(\mu) E_R(\mu)}$$

$\ell = R w$, $\lambda = \frac{\mu k}{R^2}$ $T\bar{T}$ spectrum!

Thus

$$\begin{tikzcd}
\text{Spectrum of strings on } AdS_3\ (E, J) \arrow[r, leftrightarrow, "\text{should}", "\text{match}"'] \arrow[d, leftrightarrow, "TsT"] & \text{Spectrum of dual CFT } (E, J) \arrow[d, leftrightarrow, "T\bar{T}!"] \\
\text{Spectrum on } TsT\ (E(\lambda), J(\lambda)) \arrow[r, leftrightarrow, "\text{should}", "\text{match}"'] & \text{Spectrum of dual to } TsT\ (E(\lambda), J(\lambda))
\end{tikzcd}$$

Imposing the Virasoro constraints before and after deformation $C_0(\lambda) = 1 = C_0(0)$ one can extract

$$E_{L/R}(\lambda) = E_{L/R} + \frac{2\lambda\ell}{\omega k} E_L(\lambda)E_R(\lambda)$$

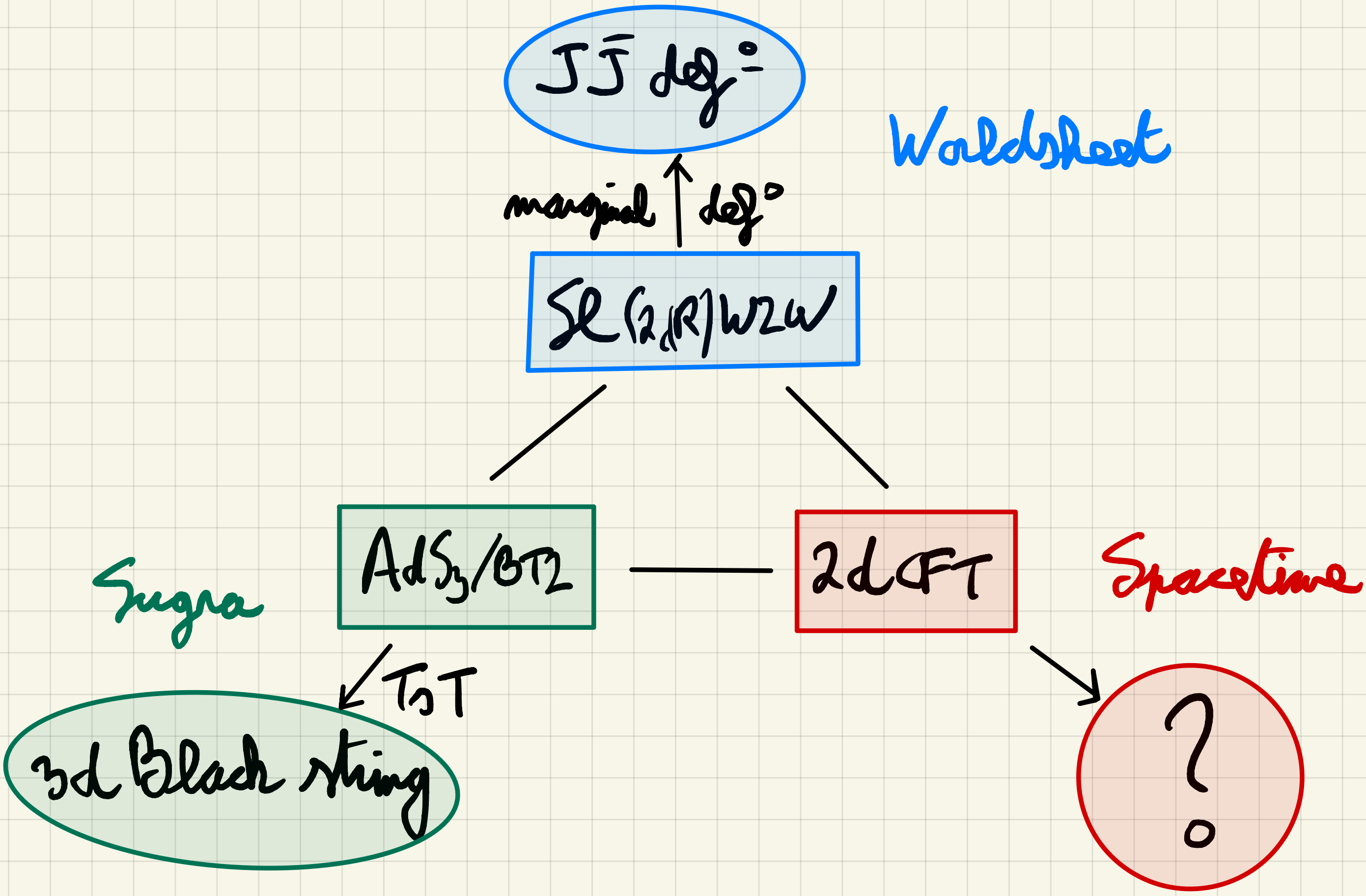
compare to
$$\boxed{E_{L/R} = E_{L/R}(\mu) + \frac{2\mu}{R} E_L(\mu)E_R(\mu)}$$

$\ell = R\omega$, $\lambda = \frac{\mu k}{R^2}$ $\quad$ $T\bar{T}$ spectrum!

Thus

$$\begin{tikzcd}
\text{Spectrum of strings on } AdS_3\ (E,J) \arrow[rr, leftrightarrow, "\text{should}", "\text{match}"'] \arrow[d, leftrightarrow, "T_s T"] & & \text{Spectrum of dual CFT } (E,J) \arrow[d, leftrightarrow, "T\bar{T}!"] \\
\text{Spectrum on } T_s T\ (E(\lambda), J(\lambda)) \arrow[rr, leftrightarrow, "\text{should}", "\text{match}"'] & & \text{Spectrum of dual to } T_s T\ (E(\lambda), J(\lambda))
\end{tikzcd}$$

We have showed that the relation between $(E(\lambda), J(\lambda))$ and (E, J) is precisely the one relating a CFT and its $T\bar{T}$ deformation (even if we don't know the details of the L.R.S.)


$J\bar{J}$ def:
Worldsheet
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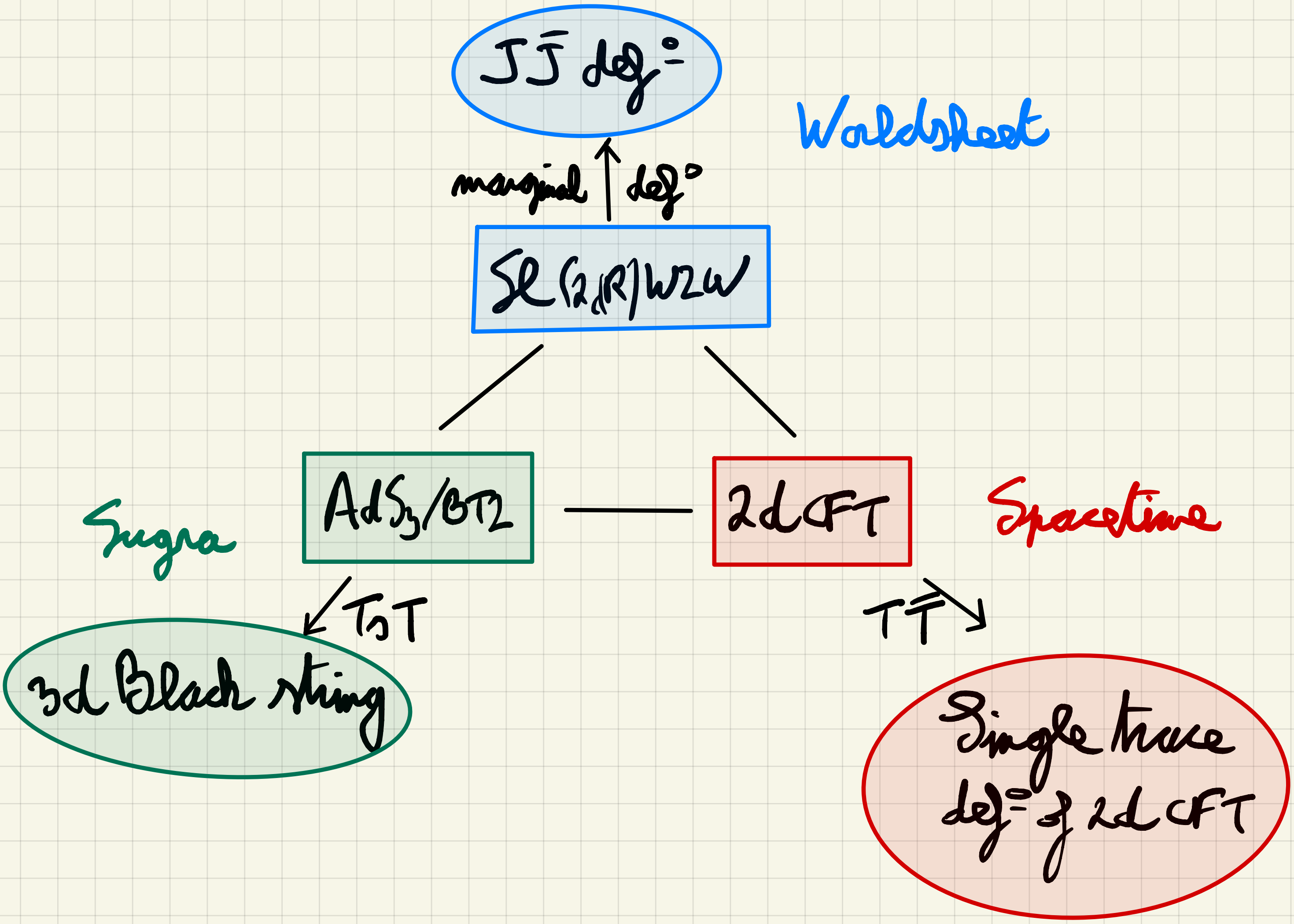
JJ̄ def^n
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Single trace def^n of 2d CFT

Thank you!

More :

- Other observables (EE, QNM) ?
- Gravitational Phase space ? (TsT of Brown-Henneaux)
- Extremal BS
- Study deformation of explicit SPO ? (Eberhardt, Gaberdiel)