Evidence for the Holographic dual of N = 3 AdS_4xM_6 Solution in Massive II A

Yi Pang

Max Planck Institute for Gravitational Physics

Albert Einstein Institute, Potsdam, Germany

Iberian Strings, 1.16-19, 2017

1511.08223 1508.05376 wih Junchen Rong

- This work is aimed at looking for the supergravity dual of the simplest Chern-Simons matter CFT, inspired by[Jafferis Talk @ Texas AM '15, Guarino, Jafferis, Varela' 15].
- In $D = 3$, we know that the $\mathcal{N} = 6$ ABJM model possesses a IIA dual description. It consists of two gauge groups and 4 bifundamental adjoint chirals. There are also deformations or quiver generalizations ABJM whose dual supergravity backgrounds are known [Ceresole, Dall'Agata, D'Auria, Ferrara, Fré, Gualtieri, Lozano, Martelli, Petrini, Sparks, Tomasiello, Zaffaroni...]
- We also know that there are many Chern-Simons matter CFTs with a single gauge group [Schwarz '04, Gaiotto and Yin'07]. Do they have supergravity duals?

- Chern-Simons matter theory with a single gauge group can have maximal $\mathcal{N} = 3$ SUSY.

- Most of the Chern-Simons matter CFTs with a single gauge group cannot have supergravity duals, because in the spectrum of BPS single trace operators, there are higher spin operators and their number grows exponential with energy. [Minwalla, P. Narayan, T. Sharma, V. Umesh and X. Yin '11]

- However, a particular $\mathcal{N} = 2$ Chern-Simons matter theory with a single gauge group admits supergravity description in the large N limit [Guarino, Jafferis, Varela' 15].

- This $\mathcal{N}=2$ Chern-Simons matter theory consists of a single SU(N) Chern-Simons vector, 3 SU(N) adjoint chirals with cubic superpotential $W = \mathrm{Tr}(X, [Y, Z])$ invariant under $SU(3)_f$.

- This theory does not have a Lagrangian description. It can be defined via RG flow from maximal SYM deformed by a CS term which breaks the supersymmetry from $\mathcal{N}=8$ to $\mathcal{N}=2$.

- In the large N limit, it is dual to the $\mathcal{N}=2$ SU(3)xU(1) invariant warped $AdS_4 \times M_6$ solution in massive IIA, where M_6 is topologically S^6. This solution is found by lifting the $\mathcal{N}=2$ SU(3)xU(1) invariant critical point in dyonic ISO(7) gauged maximal supergravity [Guarino, Jafferis, Varela' 15].

Can we find a CFT even simpler than the $\mathcal{N} = 2$ CFT which admits a supergravity dual?

- There is a potential candidate. Deforming the $\mathcal{N} = 2$ CFT by adding $\mathcal{N} = 2$ mass term to the chiral multiplet Z flows to an $\mathcal{N} = 3$ SU(N) Chern-Simons coupled to two chiral multiplets with quartic superpotential $W = \mathrm{Tr}([X, Y][X, Y])$ invariant under SO(3) flavor symmetry [Gaiotto and Yin'07].
- It admits a Lagrangian description.
- There are no higher spin operators present in the spectrum of BPS single trace operators. The number of BPS single trace operators grows with energy in a roughly Kaluza-Klein fashion [Minwalla, P. Narayan, T. Sharma, V. Umesh and X. Yin '11].

- It was known that dyonic ISO(7) gauged maximal SUGRA admits an $\mathcal{N} = 3$ SO(3)×SO(3) invariant critical point with negative cosmological constant [Gallerati, Samtleben and Trigiante'14].

- It can be lift to a warped $AdS_4 \times M_6$ solution in massive IIA using the uplift formulas found by [Guarino, Varela' 15]. Its free energy is lower than that of the $\mathcal{N} = 2$ solution. There may exist an $\mathcal{N} = 2$ domain wall interpolating between the $\mathcal{N} = 2$ and $\mathcal{N} = 3$ solutions (yet to be constructed).

- Could this solution be the dual of the $\mathcal{N} = 3$ SCFT Chern-Simons matter theory?

- To apply the uplift formulae, we need to construct the metric $\mathcal{M}_{\underline{MN}}$ (square of the 56-bein $\mathcal{V}_{\underline{M}}{}^{N}$) on the scalar coset at the critical point.

- The $\mathcal{N}=3$ critical point is captured by the SO(3)$_D\times$SO(3)$_R$ invariant sector of the dyonic ISO(7) gauged maximal SUGRA[Pang, Pope, Rong'15].

- See [Dall'Agata, Inverso and Marrani'14, Guarino and Varela'15] for details about dyonic ISO(7) gauged maximal SUGRA. SO(7) is gauged electrically, while $\mathbb{R}^7$ is gauged dyonically

$$D = d - gA^{IJ}t_{[I}{}^{K}\delta_{J]K} + (g\delta_{IJ}A^I - m\tilde{A}_J)t_8{}^J.$$

- ISO(7)$\equiv$ SO(7)$\ltimes\mathbb{R}^7$

$$\begin{aligned} \mathrm{SO}(7) \supset \mathrm{SO}(4)\times\mathrm{SO}(3)_V &\supset \mathrm{SO}(3)_R\times\mathrm{SO}(3)_L\times\mathrm{SO}(3)_V \\ &\supset \mathrm{SO}(3)_R\times\left[\mathrm{SO}(3)_L\times\mathrm{SO}(3)_V\right]_D \end{aligned}$$

- $\mathrm{SO}(3)_R \times \mathrm{SO}(3)_D$ invariant sector preserves $\mathcal{N} = 1$ SUSY: $(g_{\mu\nu},\, \psi^8_\mu)$, $(\phi_1,\, \sigma_1,\, \chi_1)$, $(\phi_2,\, \sigma_2,\, \chi_2)$.
- The superpotential of the $\mathcal{N} = 1$ theory is given by A_{88}, from which the scalar potential can be constructed.
- $$\xi_1 = \tanh\phi_1 e^{\mathrm{i}\sigma_1}, \quad \xi_2 = \tanh\phi_2 e^{\mathrm{i}\sigma_2},$$

  the $\mathcal{N} = 3$ point is given by

  $$\xi_1 = \frac{3}{5} - \frac{2\mathrm{i}}{5}, \quad \xi_2 = \frac{\mathrm{i}}{2},$$

which is a stationary point of the potential not the superpotential! The preserved three SUSYs lie outside the $\mathcal{N} = 1$ subsector.

- Using the uplift formulas [Guarino, Varela'15].
- In terms of the auxiliary coordinates on S^6

$$\begin{aligned}
\mu^1 &= \sin\xi\cos\theta_1\cos\chi_1, \quad \mu^2 = \sin\xi\cos\theta_1\sin\chi_1,\\
\mu^3 &= \sin\xi\sin\theta_1\cos\psi, \quad \mu^4 = \sin\xi\sin\theta_1\sin\psi,\\
\nu^1 &= \cos\xi\cos\theta_2, \quad \nu^2 = \cos\xi\sin\theta_2\cos\chi_2,\\
\nu^3 &= \cos\xi\sin\theta_2\sin\chi_2,
\end{aligned}$$

The 10D metric is given by

$$L^{-2}d\hat{s}_{10}^2 = \Delta^{-1}(\frac{3\sqrt{3}}{16}ds^2_{AdS_4}) + g_{mn}dy^m dy^n,$$

$$\Delta = 3^{\frac{9}{8}}2^{-\frac{3}{4}}(\cos 2\xi + 3)^{-\frac{1}{8}}\Xi^{-\frac{1}{4}}, \qquad \Xi = (24\cos 2\xi + 3\cos 4\xi + 37),$$

and the internal metric on the deformed S^6 is given as

$$g_{mn}dy^m dy^n = \frac{3\sqrt{3}}{4}(\Delta\Xi)^{-1}\Big[-\sin^2 2\xi d\xi^2 + 8(\cos 2\xi + 3)d\mu\cdot d\mu$$
$$+4(\cos 2\xi + 3)d\nu\cdot d\nu + 16\mu^A\eta^i_{AB}d\mu^B\epsilon^{ijk}\nu^j d\nu^k$$
$$-\frac{16}{\cos 2\xi + 3}(d\mu^A\eta^i_{AB}\mu^B\nu^i)^2\Big],$$

where $\eta^i s$ are the generators of $\mathrm{SO}(3)_L$,

$$\eta^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \eta^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \eta^3 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Denote $\mathcal{K}^i \equiv \mu^A\eta^i_{AB}d\mu^B$, various p-form fields can be expressed as

$$L^{-1}e^{\frac{3}{4}\phi_0}\hat{A}_{(1)} = \frac{2}{\cos 2\xi + 3}\nu^i\mathcal{K}^i,$$

$$L^{-2}e^{-\frac{1}{2}\phi_0}\hat{A}_{(2)} = -\Xi^{-1}\Big[-8d\nu^i \wedge \mathcal{K}^i + 6\sin 2\xi d\xi \wedge \nu^i\mathcal{K}^i$$
$$+2(3\cos 2\xi + 5)\nu^i\eta^i_{AB}d\mu^A \wedge d\mu^B$$
$$-(3\cos 2\xi + 5)\epsilon^{ijk}\nu^i d\nu^j \wedge d\nu^k\Big],$$

$$L^{-3}e^{\frac{1}{4}\phi_0}\hat{A}_{(3)} = \Xi^{-1}\Big[6\sin 2\xi\epsilon^{ijk}d\xi \wedge \mathcal{K}^i\nu^j \wedge d\nu^k$$
$$-2(3\cos 2\xi + 5)\epsilon^{ijk}\nu^i d\nu^j \wedge d\mu^A \wedge \eta^k_{AB}d\mu^B$$
$$-4\epsilon^{ijk}\mathcal{K}^i \wedge d\nu^j \wedge d\nu^k - \frac{8}{3}\csc^2\xi\epsilon^{ijk}\mathcal{K}^i \wedge \mathcal{K}^j \wedge \mathcal{K}^k\Big]$$
$$+\frac{3\sqrt{3}}{8}\Omega_{(3)}.$$

where $d\Omega_{(3)} = \mathrm{vol}(AdS_4)$, $L^2 = 2^{-\frac{1}{12}}g^{-25/12}m^{1/12}$ and $e^{\phi_0} = 2^{\frac{5}{6}}g^{\frac{5}{6}}m^{-\frac{5}{6}}$.

Holographic checks of the duality between $\mathcal{N} = 3$ solution in massive IIA and $\mathcal{N} = 3$ Chern-Simons matter CFT

- Matching the symmetry
  bulk residual SUSY transform under SO(3)$_D \to$ CFT SO(3)$_r$ symmetry. Bulk SO(3)$_R \to$ CFT SO(3)$_f$.

- An infinite tower of Kaluza-Klein gravitons arises in AdS_4 upon compactifying massive IIA on M_6. The short gravitons should be dual to the BPS spin-2 operators in the CFT.

- We solved the spectrum of KK gravitons around the $\mathcal{N} = 3$, $AdS_4 \times M_6$ vacuum [Pang, Rong'15]. Some of them form short $\mathcal{N} = 3$ spin-2 multiplets (BPS graviton). Others form long $\mathcal{N} = 3$ spin-2 multiplets.

$$E_0 = j_r + 3, \quad j_r \in \mathbb{Z}^+ \cup \{0\}, \qquad j_f = 0.$$

- The fact that the short gravitons are all singlets of $SO(3)_f$ is not obvious, since $SO(3)_f$ commutes with $SO(3)_r$. Our results match with the CFT computation [Minwalla, P. Narayan, T. Sharma, V. Umesh and X. Yin '11].

- Matching the free energies on both sides.
- Tree level free energy of the $\mathcal{N}=3$ solution.

$$F_{\rm gravity} = \frac{16\pi^3\ell^2}{(2\pi\ell_s)^8 g^6}{\rm Vol}(S^6), \quad \ell^2 = \frac{3\sqrt{3}}{16}g^{-7/3}(m/2)^{1/3},$$

- Holographic dictionary

Page charge $\to$ rank of gauge group

$$-\frac{1}{(2\pi\ell_s)^5}\int_{M^6} e^{\frac{1}{2}\phi}\hat{*}F_{(4)}+A_{(2)}\wedge dA_{(3)}+\frac{1}{6}mA_{(2)}\wedge A_{(2)}\wedge A_{(2)} = N,$$

$$\mathcal{N}=3 \quad \text{solution}: \quad \frac{1}{(2\pi\ell_s)^5 g^5}\frac{16\pi^3}{3} = N.$$

Chern-Simons level k is related to the Romans mass [Gaiotto and Tomasiello '09].

$$m = F_{(0)} = \frac{k}{2\pi\ell_s},$$

- $$F_{\text{gravity}} = \frac{9\pi}{40} 3^{1/6} k^{1/3} N^{5/3}.$$

  .

- Free energy of SU(N) gauged Chern-Simons theory with 2 adjoint chiral multiplets on S^3 can be computed via localization. In the limit $N \gg k$

$$F_{\text{SCFT}}^{\mathcal{N}=3} = \frac{9\pi}{40} 3^{1/6} k^{1/3} N^{5/3} = F_{\text{gravity}} \,,$$

- Is supergravity computation valid in the regime $N \gg k$?

  In the string frame, curvature radius of bulk solution $R/\ell_s \sim (N/k)^{1/6} \gg 1$. String coupling $g_s \sim 1/(N^{\frac{1}{6}} k^{\frac{5}{6}})$. Yes, supergravity approximation is valid.

Conclusion

- We provided evidence to a new example of AdS_4/CFT_3, in which the bulk solution is the $\mathcal{N} = 3$ $AdS_4 \times M_6$ solution in massive IIA. The boundary CFT is the so far simplest one, with one SU(N) Chern-Simons gauge field and two adjoint chiral multiplets.
- More tests of this duality shall be carried out in future.

Thank You!