D-instantons in Type II string theory on Calabi-Yau threefolds

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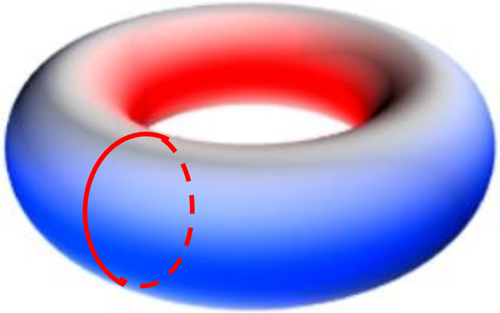
S.A., A.Sen, B.Stefanski arXiv:2108.04265
arXiv:2110.?????

Workshop on Black Holes, BPS and Quantum Information
September 22, 2021

Motivation



Instantons in string theory – *Euclidean branes* wrapped on non-trivial cycles of compactification manifold

Although exponentially suppressed in small g_s limit, they play important role for various reasons:

- crucial for non-perturbative dualities and for going beyond the perturbative formulation
- essential for moduli stabilization
- contain information on numerical invariants of compactification manifold

entropy of BPS black holes ←

But in contrast to gauge theories, until previous year, *no direct computation* of instanton effects in string theory was possible!!!

Breakthrough: understanding infrared and zero mode divergences through string field theory [A.Sen]

Goal: apply these ideas in the context of Calabi-Yau compactifications of type II string theory ⟶ *perfect match* with results based on dualities

The plan of the talk

1. Review of instanton corrections in CY compactifications

   → Hypermultiplet metric and D-instantons

2. D-instanton corrections to string amplitudes: problems and their resolution

3. Computation of (some) relevant contributions in type IIA

4. D-instantons in Type IIB

5. Conclusions

Instanton corrections in CY compactifications

The effective action of Type II string theory on a CY threefold is determined by the metric on the moduli space $\mathcal{M}_{\rm VM} \times \boxed{\mathcal{M}_{\rm HM}}$

special Kähler
no corrections in string coupling

quaternion-Kähler
corrections in string coupling

- **Classical metric (c-map)**

$$\begin{aligned} ds^2 \;&=\; d\phi^2 - e^{\phi}\,({\rm Im}\mathcal{N}^{-1})^{\Lambda\Sigma}\left(d\tilde\zeta_\Lambda - \mathcal{N}_{\Lambda\Lambda'}d\zeta^{\Lambda'}\right)\left(d\tilde\zeta_\Sigma - \bar{\mathcal{N}}_{\Sigma\Sigma'}d\zeta^{\Sigma'}\right) \\ &+\tfrac14\, e^{2\phi}\left(d\sigma + \tilde\zeta_\Lambda d\zeta^\Lambda - \zeta^\Lambda d\tilde\zeta_\Lambda\right)^2 + 4\mathcal{K}_{a\bar b}\,dz^a d\bar z^{\bar b} \end{aligned}$$

dilaton
$(e^{\phi} \sim g_s^2)$

NS-axion

RR-scalars

complex structure moduli (IIA)
complexified Kähler moduli (IIB)

$\mathcal{N}_{\Lambda\Sigma},\ \mathcal{K}$ – determined by holomorphic prepotential $F(z)$

- **1-loop correction** [Antoniadis,Minasian,Theisen,Vanhove '03
Robles-Llana,Saueressig,Vandoren '06, S.A. '07]

- **D-brane instantons**

$$e^{-2\pi|Z_\gamma|/g_s - 2\pi {\rm i}(q_\Lambda\zeta^\Lambda - p^\Lambda\tilde\zeta_\Lambda)}$$

$d_{\rm cycle}$	0	1	2	3	4	5	6
IIA :		×		D2		×	
IIB :	D(-1)		D1		D3		D5

- **NS5-brane instantons**

$$e^{-2\pi|k|\mathcal{V}/g_s^2 - {\rm i}\pi k\sigma}$$

Instanton corrections from dualities

Chain of dualities:

Robles-Llana,Saueressig,Theis,Vandoren ’07

IIA

IIB

$$\begin{tikzcd}
 & \{\alpha', \text{1-loop}\} \arrow[d, "\text{S-duality}"] \\
\{\text{A-D2}\} \arrow[d, "\text{e/m}"', "\text{duality}"] & \{\alpha', D(-1), D1\} \arrow[l, "\text{mirror}"'] \\
\{\text{all D2}\} \arrow[r, "\text{mirror}"] & \{\alpha', D(-1), D1, D3, D5\} \arrow[d, "\text{S-duality}"] \\
\{D2, NS5\} & \{\alpha', D(-1), D1, D3, D5, NS5\} \arrow[l, "\text{mirror}"']
\end{tikzcd}$$

Where we are:

Type IIA		Type IIB	
D2	to all orders	D1-D(-1)	to all orders
NS5	**?**	D3 D5-NS5	one-instanton approximation

D-instanton corrected metric

Instanton corrections are encoded into the *holomorphic contact structure* on the *twistor space* over $\mathcal{M}_{\rm HM}$

In practice: they can be extracted from *holomorphic Darboux coordinates* determined as solutions of integral equations

S.A.,Pioline,Saueressig,Vandoren '08

$$\mathcal{X}_\gamma(t) = \mathcal{X}^{\rm sf}_\gamma(t)\exp\left[\frac{1}{4\pi \mathrm{i}}\sum_{\gamma'}\Omega_{\gamma'}\langle\gamma,\gamma'\rangle\int_{\ell_\gamma}\frac{\mathrm{d}t'}{t'}\,\frac{t+t'}{t-t'}\,\log\left(1-\mathcal{X}_{\gamma'}(t')\right)\right]$$

$$\mathcal{X}^{\rm sf}_\gamma(t) = e^{-\frac{2\pi \mathrm{i}}{g_s}\left(t^{-1}Z_\gamma - t\bar{Z}_\gamma\right) - 2\pi \mathrm{i}\left(q_\Lambda\zeta^\Lambda - p^\Lambda\tilde{\zeta}_\Lambda\right)}$$

$\gamma = (p^\Lambda, q_\Lambda)$ – D-brane charge

$Z_\gamma = q_\Lambda z^\Lambda - p^\Lambda F_\Lambda$ – central charge

DT invariants of CY

the same as integral equation of Gaiotto-Moore-Neitzke for N=2 SYM / S^1

Given $\mathcal{X}_\gamma(t)$, there is a long and tedious procedure to extract the metric

It was realized in [S.A., Banerjee '14]

Small string coupling limit

We are interested only in the leading corrections in the small g_s limit in a given topological sector, i.e. for fixed axionic coupling $\Theta_\gamma \equiv q_\lambda \zeta^\Lambda - p^\Lambda \tilde{\zeta}_\Lambda$

All terms *non-linear* in DT invariants are subleading

(no need to solve integral equations)

$$\mathrm{d}s^2_{\mathrm{inst}} = \sum_\gamma \frac{\Omega_\gamma\, e^{(5\phi-\mathcal{K})/4}}{64\pi\sqrt{|Z_\gamma|}} \left(\sum_{k=1}^{\infty} k^{-1/2}\, e^{-k\mathcal{T}_\gamma} \right) \left(\mathcal{A}^2 + (\cdots)\mathrm{d}\mathcal{T}_\gamma \right)$$

such terms will be ignored

$\mathcal{T}_\gamma = 8\pi\, e^{(\mathcal{K}-\phi)/2}|Z_\gamma| + 2\pi\mathrm{i}\Theta_\gamma$ – instanton action

$$\mathcal{A} = |Z_\gamma| e^{(\mathcal{K}+\phi)/2} \left(\mathrm{d}\sigma + \tilde{\zeta}_\Lambda \mathrm{d}\zeta^\Lambda - \zeta^\Lambda \mathrm{d}\tilde{\zeta}_\Lambda + 8e^{-\phi}\mathrm{Im}\partial \log(e^{\mathcal{K}} Z_\gamma) \right) - 4\mathrm{i}\, \mathcal{C}_\gamma$$

where $\mathcal{C}_\gamma = -\frac{1}{2}(\mathrm{Im}F)^{\Lambda\Sigma}\left(q_\Lambda - \mathrm{Re}F_{\Lambda\Xi}p^\Xi\right)\left(\mathrm{d}\tilde{\zeta}_\Sigma - \mathrm{Re}F_{\Sigma\Theta}\mathrm{d}\zeta^\Theta\right) - \frac{1}{2}\mathrm{Im}F_{\Lambda\Sigma}\, p^\Lambda\, \mathrm{d}\zeta^\Sigma$

Sen ’20

Instanton corrections to string amplitudes

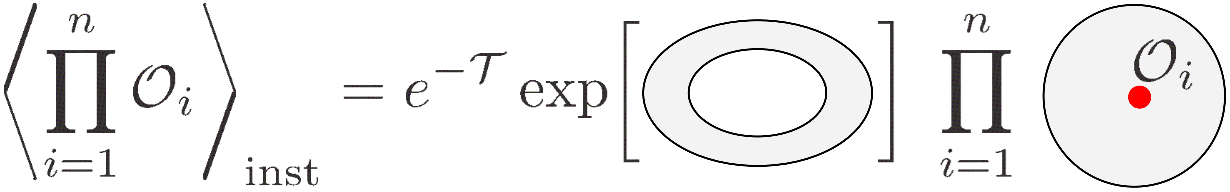
Amplitudes in the presence of instantons:

$$\langle \mathcal{O} \rangle = \frac{\int d\varphi_{\rm p} e^{S_{\rm p}} \mathcal{O} + e^{-\mathcal{T}} \int d\varphi_{\rm i} e^{S_{\rm in}} \mathcal{O}}{\int d\varphi_{\rm p} e^{S_{\rm p}} + e^{-\mathcal{T}} \int d\varphi_{\rm i} e^{S_{\rm in}}} \approx \langle \mathcal{O} \rangle_{\rm p} + e^{-\mathcal{T}} \left[\frac{\int d\varphi_{\rm i} e^{S_{\rm in}} \mathcal{O}}{\int d\varphi_{\rm p} e^{S_{\rm p}}} - \frac{\int d\varphi_{\rm p} e^{S_{\rm p}} \mathcal{O}}{\int d\varphi_{\rm p} e^{S_{\rm p}}} \, \frac{\int d\varphi_{\rm i} e^{S_{\rm in}}}{\int d\varphi_{\rm p} e^{S_{\rm p}}} \right]$$

includes *disconnected and bubble* diagrams such that

- each have either a vertex operator insertion or a boundary on D-instanton
- at least one diagram has both

The leading instanton contribution to n-point function:



$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i \right\rangle_{\rm inst} = e^{-\mathcal{T}} \exp\left[\quad \right] \prod_{i=1}^{n}$$

But there are problems:

- there are divergences due to zero modes
- the annulus amplitude formally vanishes

Example: in type IIB in 10d

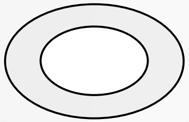
$$\int_0^\infty \frac{dt}{2t} \left[\frac{1}{2}\, \eta(\mathrm{i}t)^{-12} \left(\vartheta_3(0,\mathrm{i}t)^4 - \vartheta_4(0,\mathrm{i}t)^4 - \vartheta_2(0,\mathrm{i}t)^4 + \vartheta_1(0,\mathrm{i}t)^4 \right) \right] = 0$$

Annulus amplitude from string field theory

All divergences can be understood from *string field theory* [Sen ’20]

- the zero modes related to the collective coordinates of the D-instanton should be left unintegrated till the end of calculation
  - bosonic zero modes produce the momentum conserving delta-function
  - fermionic zero modes require insertion of zero mode vertex operators
- the divergence due to ghost zero modes arises due to the braekdown of the Seigel gauge $b_0|\Psi\rangle = 0$ used to get the worldsheet formulation, which is cured by working with a gauge invariant path integral

The annulus amplitude becomes non-vanishing



$$\exp\Big[\qquad\Big] = \frac{\mathrm{i}\,C}{\sqrt{k}}\, g_o\, \Omega_\gamma \underbrace{\int \prod_\mu d\xi^\mu}_{\text{bosonic z.m.}} \underbrace{\int \prod_{\delta,\dot\delta=1}^{2} d\chi^\delta\, d\chi^{\dot\delta}}_{\text{fermionic z.m.}}$$

$C = 2^{-5}\,\pi^{-13/2}$

open string coupling

counts cycles

$$\mathrm{Re}\mathcal{T}_\gamma = \frac{1}{2\pi^2 g_o^2} \longrightarrow g_o^2 = \frac{2g_s}{V_\gamma} = \frac{e^{(\phi-\mathcal{K})/2}}{16\pi^3|Z_\gamma|}$$

bosonic z.m. → $(2\pi)^4\delta^{(4)}\left(\sum_i p_i\right)$

Metric vs. curvature

We are interested in the effective action for massless scalars. For such fields, 2- and 3-point amplitudes vanish ⟶ we need *4-point* function

The simplest 4-point function affected by the metric on $\mathcal{M}_{\rm HM}$ is generated by

$$\int d^4x\, \mathcal{R}_{ijkl} (\chi^i \bar{\chi}^j)(\chi^k \bar{\chi}^l)$$

Symmetric part of (the *Sp(n)* part of)
the curvature on $\mathcal{M}_{\rm HM}$
very complicated!

fermions from
hypermultiplets

Solution:
do not impose momentum conservation!

General structure I

The effective action: $$-\frac{1}{2}\int d^4x\, G_{ij}(\vec{\varphi})\,\partial_\mu\varphi^i\partial^\mu\varphi^j$$

$$G_{ij} = g_{ij} + \sum_\gamma e^{-\mathcal{T}_\gamma}\left(h_{ij}^{(\gamma)} + \cdots\right)$$

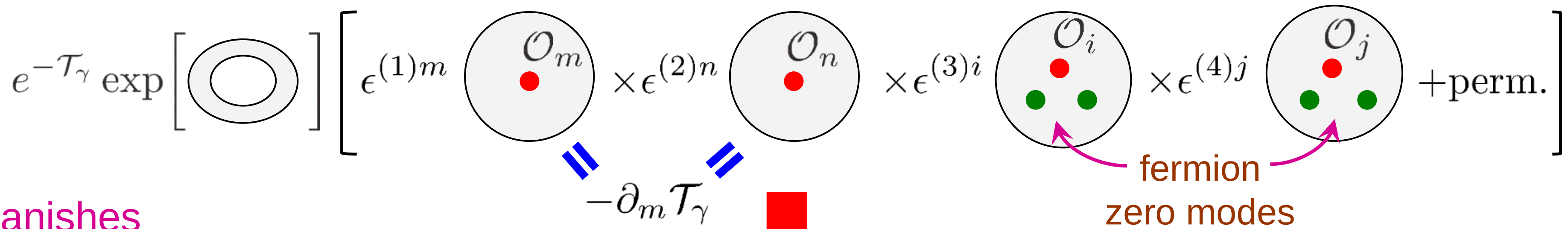
$$\varphi^i = \phi^i + \lambda^i$$

quantum fluctuations

The leading 4-λ term: $$-\frac{1}{4}\int d^4x\, e^{-\mathcal{T}_\gamma}\underbrace{\partial_m\mathcal{T}_\gamma\,\partial_n\mathcal{T}_\gamma}_{\text{because } \mathcal{T}_\gamma\sim 1/g_s} h_{ij}^{(\gamma)}(\vec{\phi})\,\lambda^m\,\lambda^n\,\partial_\mu\lambda^i\partial^\mu\lambda^j$$

$$\mathcal{A} = (2\pi)^4\delta^{(4)}\left(\sum_\alpha p^{(\alpha)}\right) e^{-\mathcal{T}_\gamma}\left[\epsilon^{(1)m}\,\epsilon^{(2)n}\,\epsilon^{(3)i}\,\epsilon^{(4)j}\,\partial_m\mathcal{T}_\gamma\,\partial_n\mathcal{T}_\gamma\,h_{ij}^{(\gamma)}(\vec{\phi})\,(p^{(3)}\cdot p^{(4)}) + \text{perm.}\right]$$

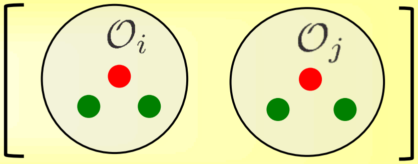
$$=$$



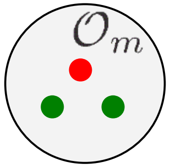
fermion zero modes

vanishes for $p = p'$

$$(p\cdot p')\,h_{ij}^{(\gamma)} = \sum_{k|\gamma}\frac{C}{\sqrt{k}}\,g_o\,\Omega_{\gamma/k}\int\prod_{\delta,\dot{\delta}=1}^{2} d\chi_a^\delta d\chi_a^{\dot{\delta}}\left[\quad\right]$$



General structure II



$$= \mathrm{i}\, a_m\, p_\mu\, \gamma^\mu_{\dot{\alpha}\alpha}\, \chi^\alpha \chi^{\dot{\alpha}}$$

↓

$$ds^2_{\text{inst}} = \sum_\gamma 2\pi e^\phi\, g_o\, \Omega_\gamma \left(\sum_{k=1}^{\infty} k^{-1/2}\, e^{-k\mathcal{T}_\gamma}\right) \left(\sum_m a_m\, d\lambda^m\right)^2 + \mathcal{O}(d\mathcal{T}_\gamma)$$

Caveat: this procedure is insensitive to the field redefinitions

$$\varphi^m \to \varphi^m + e^{-\mathcal{T}_\gamma} \xi^m(\vec{\varphi})$$

leading ↓ order

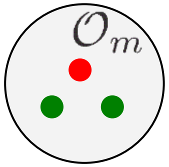
$$d\varphi^m \to d\varphi^m - e^{-\mathcal{T}_\gamma} \xi^m(\vec{\varphi})\, d\mathcal{T}_\gamma$$

↓

Terms $\sim d\mathcal{T}_\gamma$ cannot be compared

Example 1: NS-axion contribution

It remains to compute



$$= \mathrm{i}\, a_m\, p_\mu\, \gamma^\mu_{\dot\alpha\alpha}\, \chi^\alpha \chi^{\dot\alpha}$$

for $m = \sigma,\ (\zeta^\Lambda, \tilde\zeta_\Lambda),\ z^a,\ \phi$

NS-axion is the scalar dual to the *B* field. Thus, we have to analyze:

$$\mathrm{i}\pi\kappa\, T_2\, \underbrace{\mathcal{X}^\alpha \hat{\mathcal{X}}^\beta} \int_{-\infty}^{\infty} dz \left\langle V_B(\mathrm{i})\ \underbrace{c\, e^{-\phi/2} S_\alpha(0)}\ \underbrace{e^{-\phi/2} S_\beta(z)} \right\rangle$$

grav. coupling.

brane tension.

10d ferm. z.m.

B-field vertex operator

unintegrated and integrated vertex op. of open str. fermions

$$V_B = 2\, b_{\mu\nu}\, c\,\bar c\, \left(\partial X^\mu + \mathrm{i}\, p_\rho\, \psi^\rho \psi^\mu\right) e^{\mathrm{i}p\cdot X} e^{-\bar\phi} \bar\psi^\nu + \cdots$$

- doubling trick
- OPE
- contour integration
- $\mathcal{X}^\alpha = \eta \otimes \chi^\alpha$ and $\hat{\mathcal{X}}^\alpha = \bar\eta \otimes \chi^{\dot\alpha}$

covariantly constant spinor on CY

$$\frac{1}{2}\, \pi^2 \kappa\, T_2\, p_\rho\, b_{\mu\nu}\, \chi^\alpha \chi^{\dot\alpha}\, (\gamma^{\rho\mu\nu})_{\dot\alpha\alpha}\, \underbrace{(2\pi)^3 \delta^{(3)}(0)}$$

artefact of the flat space

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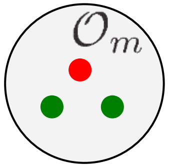

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covariantly constant spinor on CY

$$\frac{1}{2}\, \pi^2 \kappa\, T_2\, p_\rho\, b_{\mu\nu}\, \chi^\alpha \chi^{\dot\alpha}\, (\gamma^{\rho\mu\nu})_{\dot\alpha\alpha}\ V_\gamma$$

volume of the wrapped cycle

Dualization

But we still have to go from the *B* field to the NS axion

- generalize the previous construction from scalars to 2-forms
- perform dualization in the presence of instanton corrections

$$\int d^4x \left(\frac{f}{12} H_{\mu\nu\rho}H^{\mu\nu\rho} - \underbrace{\frac{1}{6}\,\varepsilon^{\mu\nu\rho\tau} H_{\mu\nu\rho}A_\tau}_{\text{CS type of coupling}} \right) \quad \xleftrightarrow[\text{duality}]{} \quad \int \frac{d^4x}{2f} \left(\partial_\mu\sigma + A_\mu\right)\left(\partial^\mu\sigma + A^\mu\right)$$

$$H^{\mu\nu\rho} = f^{-1}\,\varepsilon^{\mu\nu\rho\tau}\left(\partial_\tau\sigma + A_\tau\right)$$

The dual actions are not equal, but their deformations due to $f \to f + \delta f$ are *equal* (to the first order) on the undeformed duality relation

$$\Downarrow$$

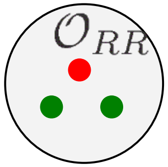
It is enough to substitute into the disk 1-point function

$$\partial_{[\rho}b_{\mu\nu]} = \frac{\kappa}{12\pi V}\,\varepsilon_{\rho\mu\nu}{}^{\tau}\left(\partial_\tau\sigma + \underbrace{\tilde\zeta_\Lambda\partial_\tau\zeta^\Lambda - \zeta^\Lambda\partial_\tau\tilde\zeta_\Lambda}_{A_\tau}\right)$$

$$T_2 V_\gamma = \mathrm{Re}\mathcal{T}_\gamma \quad \Downarrow$$

$$\boxed{a_\sigma \nabla\sigma = \frac{\pi\kappa^2}{4V}\,\mathrm{Re}\mathcal{T}_\gamma\left(d\sigma + \tilde\zeta_\Lambda\, d\zeta^\Lambda - \zeta^\Lambda\, d\tilde\zeta_\Lambda\right)}$$

Example 2: RR-field contribution



$$= \mathrm{i}\pi\kappa\, T_2\, \mathcal{X}^{\alpha}\hat{\mathcal{X}}^{\beta} \int_{-\infty}^{\infty} dz \left\langle V_{RR}(\mathrm{i})\ c e^{-\phi/2} S_{\alpha}(0)\ e^{-\phi/2} S_{\beta}(z) \right\rangle$$

New ingredient: for the doubling trick, we need to use the boundary condition for the spin field

$$\bar{S}^{\delta} = \frac{1}{3!}\, v_{\gamma,IJK}\, (\Gamma^{IJK})^{\delta\delta'} S_{\delta'}$$

volume form of the wrapped 3-cycle

$$-\mathrm{i}\,\frac{\pi^2}{36}\,\kappa\, T_2\, \chi^{\alpha}\chi^{\dot{\beta}}\, p_{\mu} \int_{L_{\gamma}} d^3x\ \tilde{C}_{\mathbf{pqr}}\, v_{\gamma,\mathbf{ijk}}\, (\bar{\eta}\Gamma^{M}\Gamma^{\mu\mathbf{pqr}}\Gamma^{\mathbf{ijk}}\Gamma_{M}\eta)_{\dot{\beta}\alpha}$$

Kähler form

on CY $\bar{\eta}\Gamma^{\mathbf{ij}}\eta = \mathrm{i}\omega^{\mathbf{ij}} = -\mathrm{i}J^{\mathbf{i}}_{\mathbf{k}}g^{\mathbf{kj}}$

complex structure

$$-4\mathrm{i}\,\pi^2\kappa\, T_2\, (\gamma^{\mu})_{\dot{\beta}\alpha}\, \chi^{\alpha}\chi^{\dot{\beta}}\, p_{\mu} \int_{L_{\gamma}} \left(2\,\tilde{C} + \mathrm{i}\star\tilde{C} - 3\mathrm{i}\, J(\tilde{C})\right)$$

One must fix the relative normalization of $\tilde{C}$ and C giving rise to

$\zeta^{\Lambda},\ \tilde{\zeta}_{\Lambda}$

$$= -2\pi\mathrm{i} \int_{L_{\gamma}} C \quad \Rightarrow \quad \tilde{C} = \frac{\pi}{4\kappa\, T_2}\, C$$

Evaluation of the integrals

It remains to evaluate 3 integrals:

$$\int_{L_\gamma} C \;= q_\Lambda \zeta^\Lambda - p^\Lambda \tilde\zeta_\Lambda = \Theta_\gamma$$

$$\int_{L_\gamma} \star C \;= 2\mathscr{C}_\gamma - 4e^{\mathcal{K}}\mathrm{Re}\left[\bar Z_\gamma (z^\Lambda \tilde\zeta_\Lambda - F_\Lambda \zeta^\Lambda)\right]$$

$$\int_{L_\gamma} J(C) = -\frac{2}{3}\,\mathscr{C}_\gamma - \frac{4}{3K}\,\mathrm{Re}\left[\bar Z_\gamma (z^\Lambda \tilde\zeta_\Lambda - F_\Lambda \zeta^\Lambda)\right]$$

Basis: $\{A^\Lambda, B_\Lambda\} \in H_3(\mathfrak{Y})$

$\{\alpha_\Lambda, \beta^\Lambda\} \in H^3(\mathbb{Z}, \mathfrak{Y})$

$$\int_{A^\Lambda} \alpha_\Sigma = -\int_{B_\Sigma} \beta^\Lambda = \delta^\Lambda_\Sigma, \quad \int_{B_\Lambda} \alpha_\Sigma = \int_{A^\Lambda} \beta^\Sigma = 0$$

$$L_\gamma = q_\Lambda A^\Lambda - p^\Lambda B_\Lambda$$

$$C = \zeta^\Lambda \alpha_\Lambda - \tilde\zeta_\Lambda \beta^\Lambda$$

$\Omega = z^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda$ – hol. form of CY

where $\mathscr{C}_\gamma = -\frac{1}{2}(\mathrm{Im}F)^{\Lambda\Sigma}\left(q_\Lambda - \mathrm{Re}F_{\Lambda\Xi}p^\Xi\right)\left(\tilde\zeta_\Sigma - \mathrm{Re}F_{\Sigma\Theta}\zeta^\Theta\right) - \frac{1}{2}\,\mathrm{Im}F_{\Lambda\Sigma}\, p^\Lambda\, \zeta^\Sigma$

$$\star\alpha_\Lambda = \left(\mathrm{Re}\mathcal{N}(\mathrm{Im}\mathcal{N})^{-1}\right)_\Lambda^{\ \Sigma}\alpha_\Sigma - \left(\mathrm{Im}\mathcal{N} + \mathrm{Re}\mathcal{N}(\mathrm{Im}\mathcal{N})^{-1}\mathrm{Re}\mathcal{N}\right)_{\Lambda\Sigma}\beta^\Sigma$$

Suzuki ‘95

$$\star\beta^\Lambda = \left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma}\alpha_\Sigma - \left((\mathrm{Im}\mathcal{N})^{-1}\mathrm{Re}\mathcal{N}\right)^\Lambda_{\ \Sigma}\beta^\Sigma \qquad \mathcal{N}_{\Lambda\Sigma} = \bar F_{\Lambda\Sigma} + 2\mathrm{i}\,\frac{(\mathrm{Im}Fz)_\Lambda(\mathrm{Im}Fz)_\Sigma}{(z\mathrm{Im}F\bar z)}$$

Alternative basis: $\{\Omega, \chi_a, \bar\chi_a, \bar\Omega\} \in H^3 = H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$ $\qquad \chi_a(z) = \partial_{z^a}\Omega + \Omega\,\mathcal{K}_a$

$$C = \rho\,\Omega + \varrho^a \chi_a + \bar\varrho^a \bar\chi_a + \bar\rho\,\bar\Omega \quad \longrightarrow \quad J(C) = \mathrm{i}\left(\rho\,\Omega - \bar\rho\,\bar\Omega\right) + \frac{\mathrm{i}}{3}\left(\varrho^a\chi_a - \bar\varrho^a\bar\chi_a\right)$$

Relations: $\bar\rho = -\mathrm{i}e^{\mathcal{K}}\left(z^\Lambda\tilde\zeta_\Lambda - F_\Lambda\zeta^\Lambda\right) \qquad \mathrm{Im}\left(\varrho^a\partial_{z^a}Z_\gamma + (\rho + \varrho^a\mathcal{K}_a)Z_\gamma\right) = \mathscr{C}_\gamma$

Final result

$$a_\Lambda d\zeta^\Lambda + a^\Lambda d\tilde{\zeta}_\Lambda = -2\pi^3 \left[d\Theta_\gamma + 2\mathrm{i}\, \mathcal{C}_\gamma \right]$$

$$a_{z^a} dz^a + a_{\bar{z}^a} d\bar{z}^a = \mathrm{i}\pi^2 \mathrm{Re}\mathcal{T}_\gamma\, \bar{\partial} \log(e^{\mathcal{K}} \bar{Z}_\gamma)$$

$$a_\phi d\phi = \frac{\mathrm{i}\pi^2}{2} \mathrm{Re}\mathcal{T}_\gamma\, d\phi$$

$$ds^2_{\mathrm{inst}} = \sum_\gamma 2\pi e^\phi g_o\, \Omega_\gamma \left(\sum_{k=1}^\infty k^{-1/2} e^{-k\mathcal{T}_\gamma} \right) \left(\sum_m a_m\, d\lambda^m \right)^2 + \mathcal{O}(d\mathcal{T}_\gamma)$$

Perfectly coincides with the instanton corrected metric predicted by dualities!

Type IIB and mirror symmetry

Mirror symmetry

$\mathrm{IIA}/\mathfrak{Y} \;=\; \mathrm{IIB}/\tilde{\mathfrak{Y}}$

mirror CYs

$$\mathcal{M}^{\rm IIA}_{\rm HM}(\mathfrak{Y}) = \mathcal{M}^{\rm IIB}_{\rm HM}(\tilde{\mathfrak{Y}})$$

Two metrics are identical

But if we want to express the metric in terms of natural Type IIB fields (transforming simply under S-duality), we need *mirror map*

receives instanton corrections

D(-1), D1 – known; D3 – partially; D5 – make no sense without NS5

However, at leading order in g_s their effect is $\mathcal{O}(d\mathcal{T}_\gamma)$ → classical mirror map is sufficient

The resulting metric is exactly reproduced by a similar calculation of string amplitudes

New features:

- more branes to analyze
- existence of bound states of branes of different type
- Kähler geometry instead of complex structure moduli

Conclusions

- String field theory is able to fix all apparent divergences and ambiguities in instanton contributions to string amplitudes
- String amplitudes perfectly reproduce the results predicted by dualities, which provides a highly non-trivial test for both approaches
- There is a way to extract directly the *metric* on the moduli space and not only its *curvature*

Future directions:

- Compute instanton corrections where dualities do not help
  (phenomenologically relevant $\mathcal{N} = 1$ string compactifications)
- Get insights about *NS5-brane instantons* in CY compactifications which remain not fully understood
- Go beyond the leading order in the string coupling

Thank you!