(K)NOT MACHINE LEARNING

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21 September 2021
Workshop on Black Holes: BPS, BMS, and Integrability
Instituto Superior Técnico

Collaborators

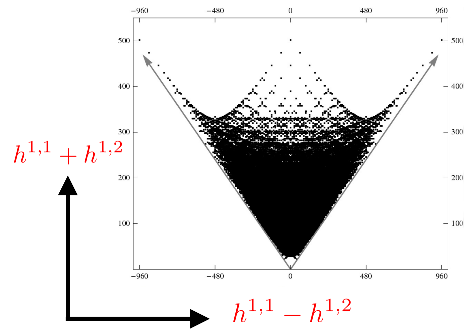
Jessica Craven

Arjun Kar

“Disentangling a Deep Learned Volume Formula”

arXiv:2012.03955

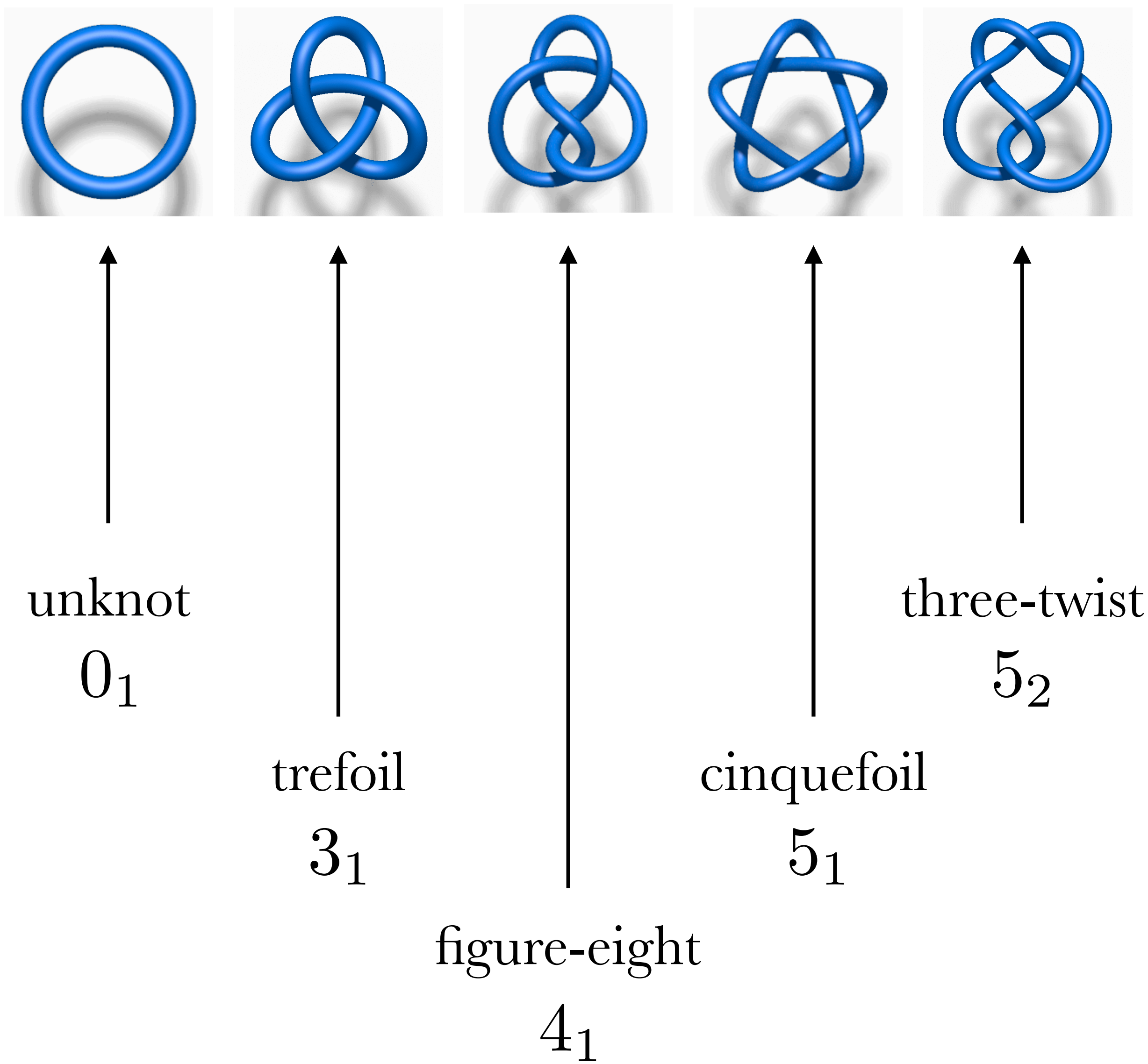
Mathematical Phenomenology



- Note patterns then look for an explanation
  - — Mirror symmetry is prototype example
  - — Knot theory provides another case study
- Use machine learning to train a computer to calculate in **hep-th**, **math**
  - — Black box gives **probably approximately correct** answers
  - — So far, we have mainly used ML to identify associations
  - — Want to bridge this success to new analytic results and methods
- Illustrate technology and then discuss this in a broader context

Dramatis Personae

Knot: $S^1 \subset S^3$; *e.g.*,



Dramatis Personae

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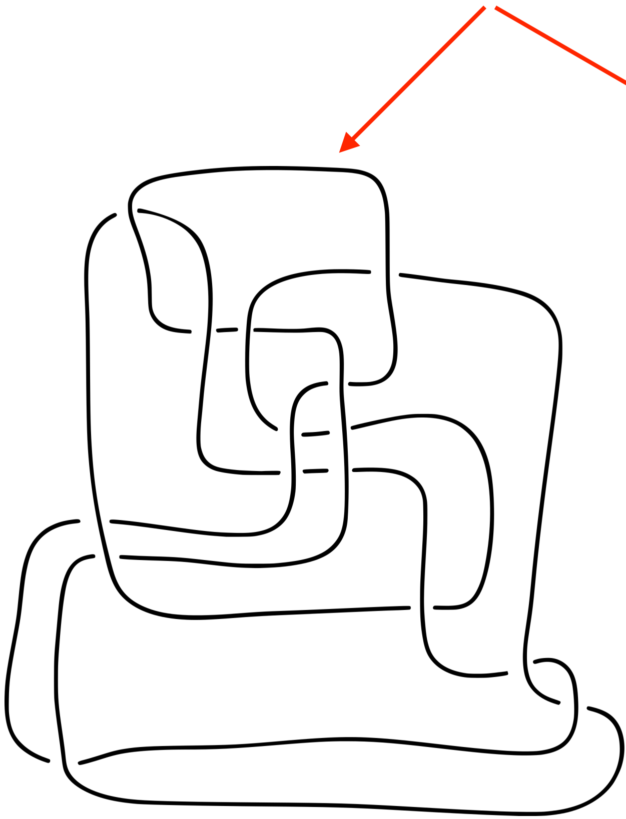
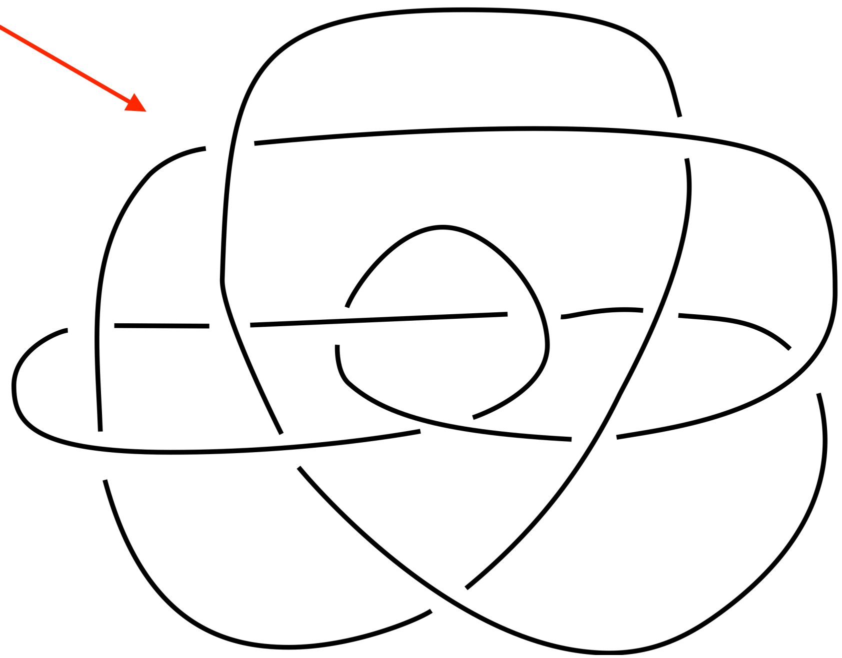


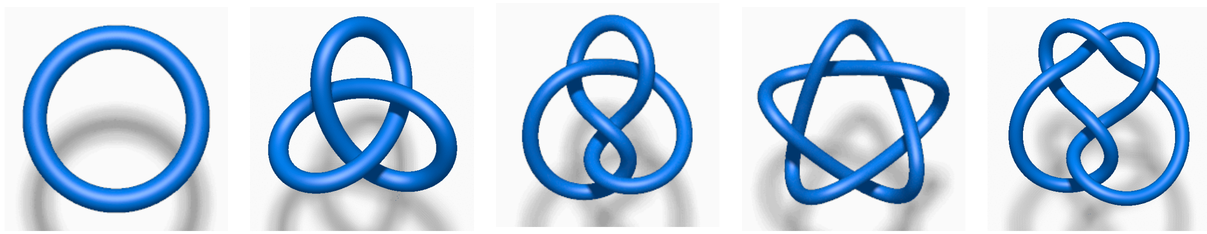
Thistlethwaite unknot



Ochiai unknot

Dramatis Personae

Knot: $S^1 \subset S^3$; *e.g.*,



Jones polynomial: $J(K;q) = (-q^{\frac{3}{4}})^{w(K)} \dfrac{\langle K \rangle}{\langle \bigcirc \rangle}$

$J(\bigcirc;q) = 1$

$\langle \times \rangle = q^{\frac{1}{4}} \langle \asymp \rangle + \dfrac{1}{q^{\frac{1}{4}}} \langle)(\rangle$

$w(K) =$ overhand − underhand

Jones (1985)

topological invariant: independent of how the knot is drawn

Question: how to calculate these?

Answer: quantum field theory!

Topological Invariants

- On a manifold $\mathcal{M}$ with metric $g_{\mu\nu}$, a topological invariant enjoys:

$$\frac{\delta}{\delta g_{\mu\nu}} \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = 0$$

- In Chern–Simons theory, the operators are Wilson loops

$$U_R(\gamma) = \mathrm{tr}_R \, \mathcal{P} \exp \left(i \oint_\gamma A \right)$$

- The colored Jones polynomial is a knot invariant:

$$J_n(K; q = e^{2\pi i/(k+2)}) = \frac{\int_{\mathcal{U}} [DA] \; U_n(K) e^{iS_{\mathrm{CS}}(A)}}{\int_{\mathcal{U}} [DA] \; U_n(0_1) e^{iS_{\mathrm{CS}}(A)}} = \frac{\langle U_n(K) \rangle}{\langle U_n(0_1) \rangle}$$

$$S_{\mathrm{CS}}(A) = \frac{k}{4\pi} \int_{\mathcal{M}} \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) , \quad Z(\mathcal{M}) = \int_{\mathcal{U}} [DA] \; e^{iS_{\mathrm{CS}}(A)}$$

Witten (1989)

Dramatis Personae

Knot: $S^1 \subset S^3$; e.g.,

Jones polynomial: $J(K;q) = (-q^{\frac{3}{4}})^{w(K)} \frac{\langle K \rangle}{\langle \bigcirc \rangle}$

$\langle \times \rangle = q^{\frac{1}{4}} \langle \asymp \rangle + \frac{1}{q^{\frac{1}{4}}} \langle)(\rangle$

$w(K) =$ overhand − underhand

vev of Wilson loop operator along K in $\square$ for $SU(2)$ Chern–Simons on S^3

Jones (1985)
Witten (1989)

$$J_2(4_1;q) = q^{-2} - q^{-1} + 1 - q + q^2 \ , \quad q = e^{\frac{2\pi i}{k+2}}$$

Hyperbolic volume: volume of $S^3 \backslash K$ is another knot invariant computed from tetrahedral decomposition of knot complement

Thurston (1978)
Mostow (1968)

Topological Invariants

- Volume appears as saddle point in $SL(2,\mathbb{C})$ Chern–Simons theory

$$\mathcal{Z}(\mathcal{M}) = \int_{\mathcal{U}_{\mathbb{C}}} [D\mathcal{A}][D\overline{\mathcal{A}}]\ \exp\left[\frac{it}{2}W(\mathcal{A}) + \frac{i\tilde{t}}{2}W(\overline{\mathcal{A}})\right]$$

$$W(A) = \frac{1}{4\pi}\int_{\mathcal{M}} \mathrm{tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right)$$

$$t = \ell + is\ , \quad \tilde{t} = \ell - is\ , \quad \ell \in \mathbb{Z}\ , \quad s \in \mathbb{C}$$

$$\mathcal{Z}(S^3 \setminus K) \supset \exp\left[\frac{is}{2\pi}\mathrm{Vol}(S^3 \setminus K) + i\ell\pi\,\mathrm{CS}(S^3 \setminus K)\right]$$

- The critical point responsible for this contribution is a flat $SL(2,\mathbb{C})$ valued connection $\mathcal{A}_+$, the geometric conjugate connection:

$$W(\mathcal{A}_+) = -\frac{i}{2\pi}\mathrm{Vol}(S^3 \setminus K) + \pi\,\mathrm{CS}(S^3 \setminus K)$$

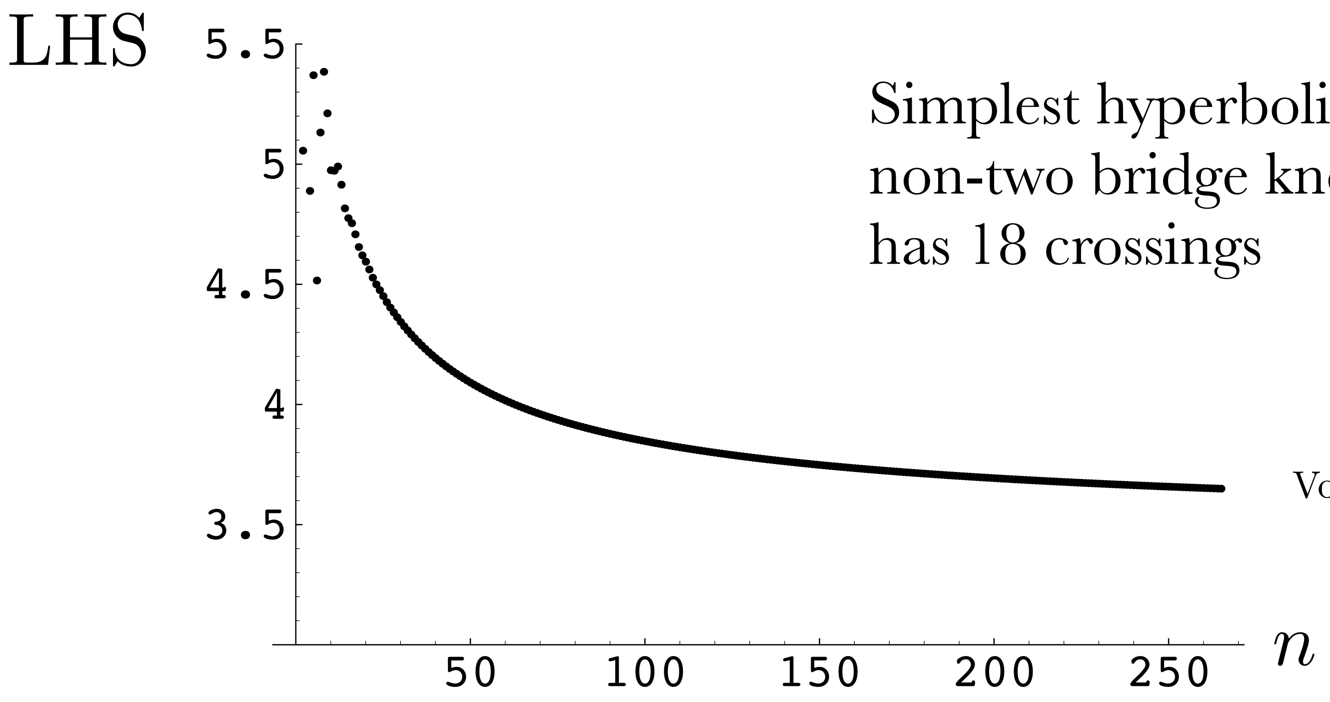
Dramatis Personae

Volume conjecture: $$\lim_{n\to\infty} \frac{2\pi \log |J_n(K;\omega_n)|}{n} = \mathrm{Vol}(S^3 \setminus K)$$

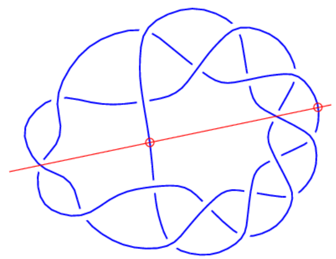
Kashaev (1997)
Murakami x 2 (2001)
Gukov (2005)

$\omega_n = e^{\frac{2\pi i}{n}}$

In fact, we take $n, k \to \infty$



Simplest hyperbolic non-two bridge knot, has 18 crossings



$\sigma_1^2(\sigma_1\sigma_2)^8$

Behavior is not monotonic!

Garoufalidis, Lan (2004)

Dramatis Personae

<u>Volume conjecture:</u>

$$\lim_{n\to\infty} \frac{2\pi \log |J_n(K;\omega_n)|}{n} = \mathrm{Vol}(S^3 \setminus K)$$

$$\omega_n = e^{\frac{2\pi i}{n}}$$

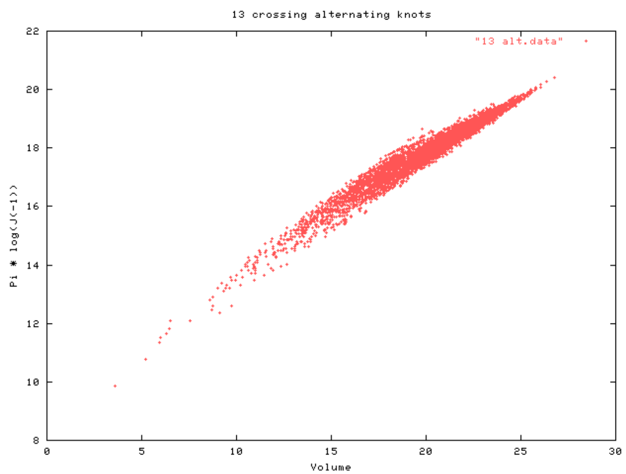
Kashaev (1997)
Murakami x 2 (2001)
Gukov (2005)

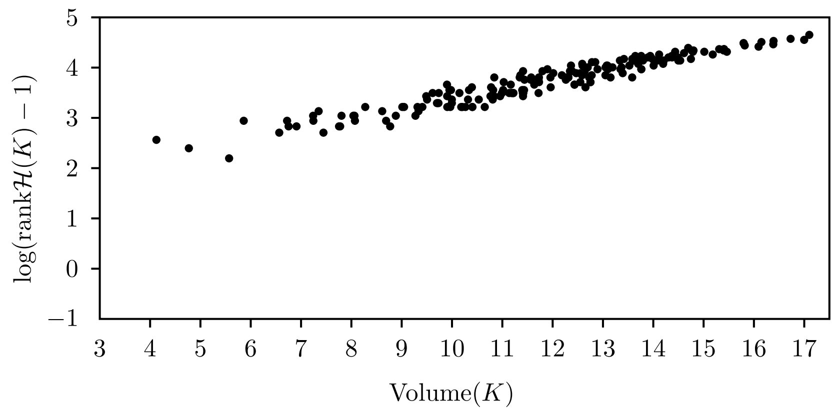
<u>Khovanov homology:</u> a homology theory $\mathcal{H}_K$ whose graded Euler characteristic is $J_2(K;q)$; explains why coefficients are integers

Khovanov (2000)
Bar-Natan (2002)

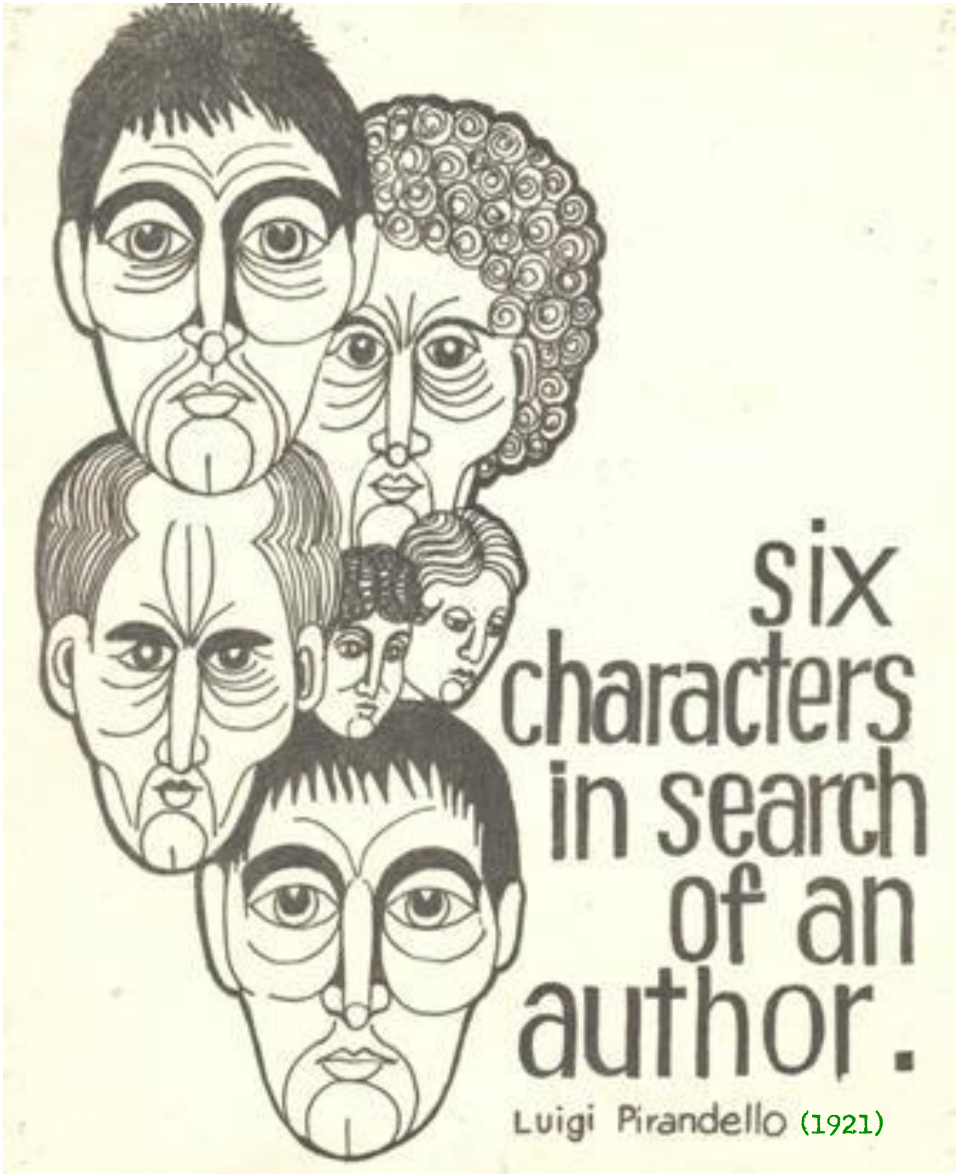
$$\log|J_2(K;-1)|\ ,\ \ \log(\mathrm{rank}(\mathcal{H}_K)-1) \propto \mathrm{Vol}(S^3\setminus K)$$

Dunfield (2000)
Khovanov (2002)

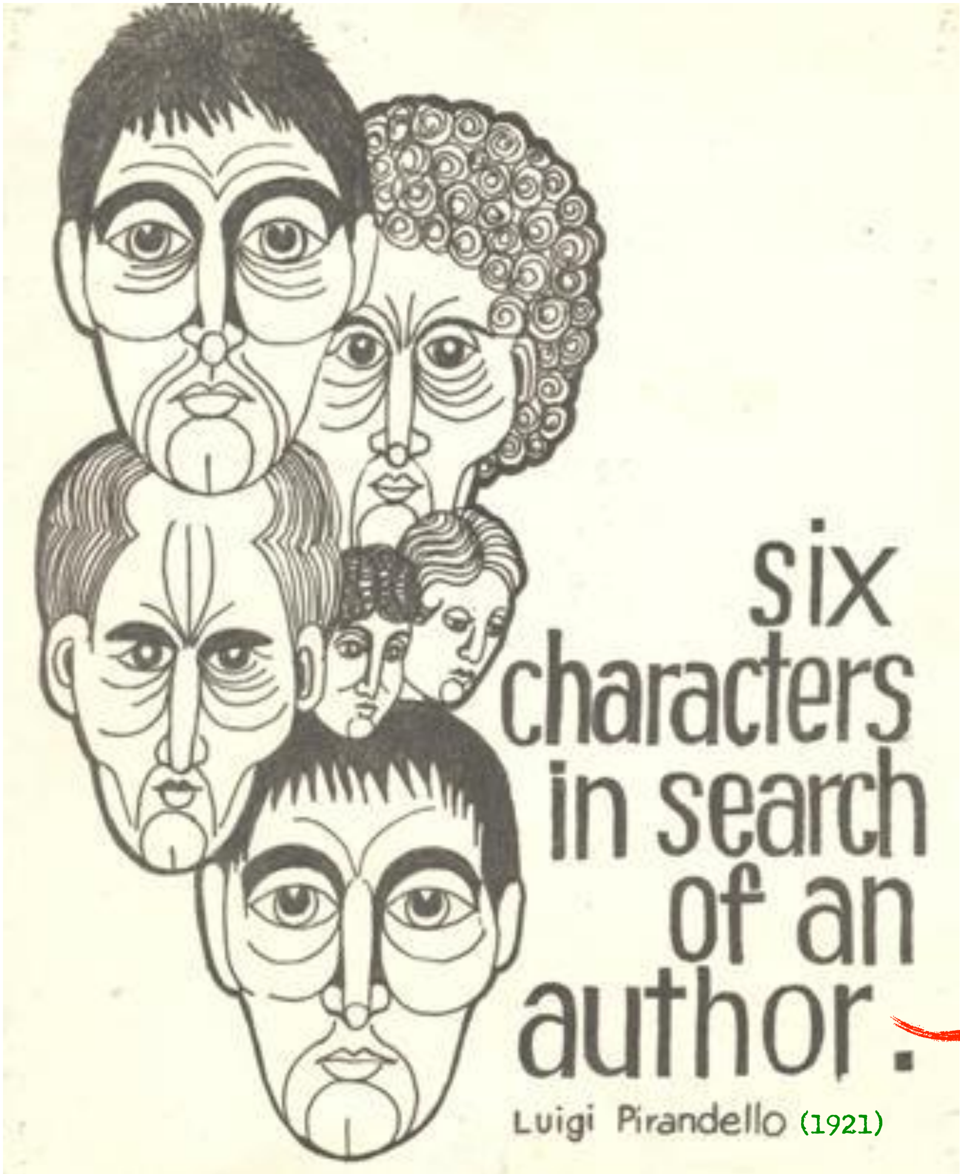




Dramatis Personae



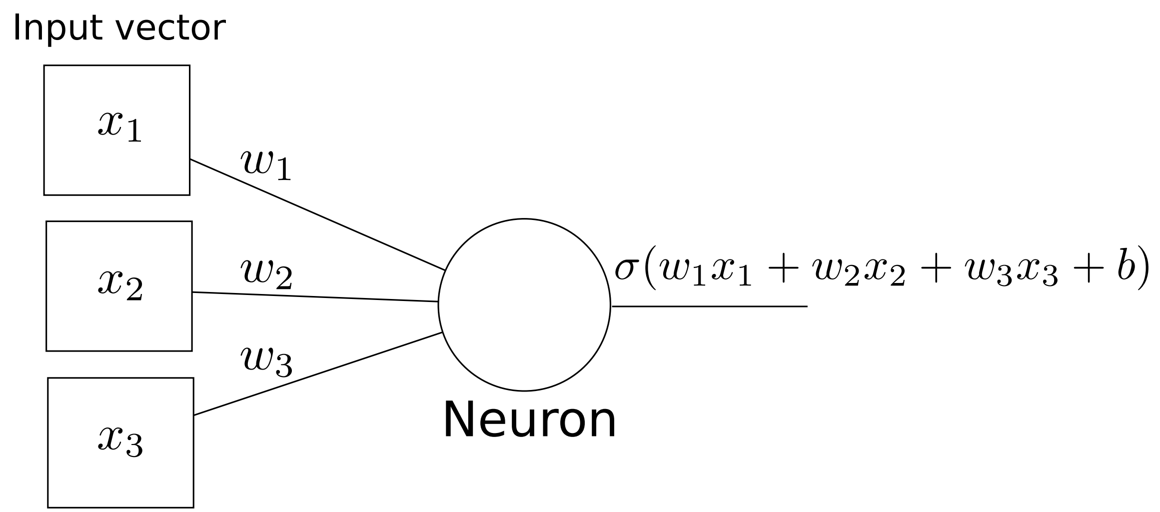
Dramatis Personae



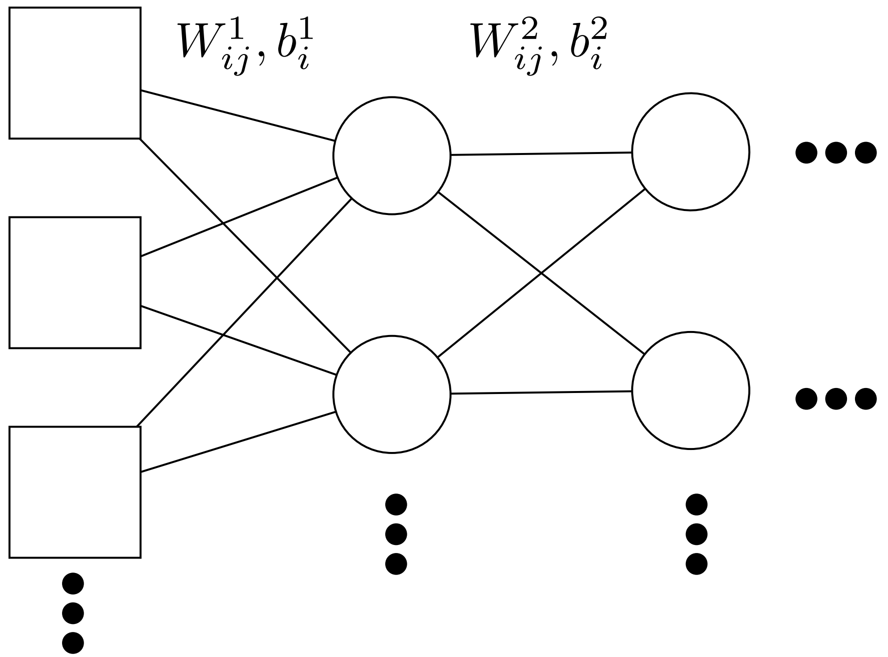
neural network

For other work on knots and machine learning, see Hughes (2016)
Levitt, Hajij, Sazdanovic (2019)
Gukov, Halverson, Ruehle, Sulkowski (2020)

Feedforward Neural Networks



Rosenblatt (1957)

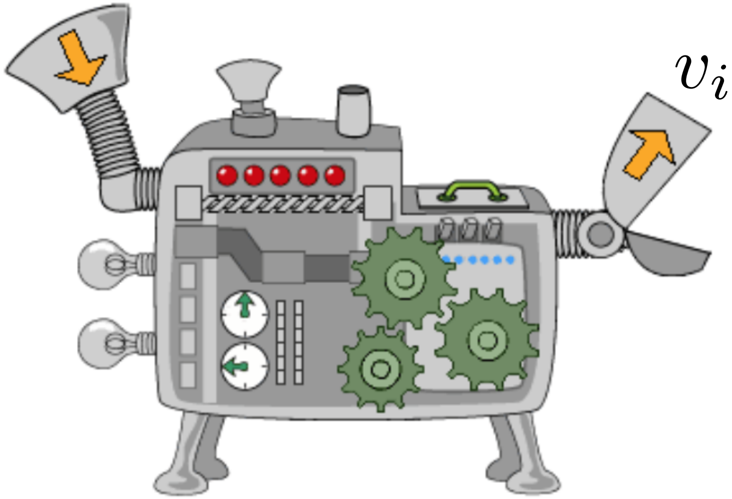


Mathematica 10+

Schematic representation of feedforward neural network. The top figure denotes the perceptron (a single neuron), the bottom, the multiple neurons and multiple layers of the neural network.

Neural Network

J_i



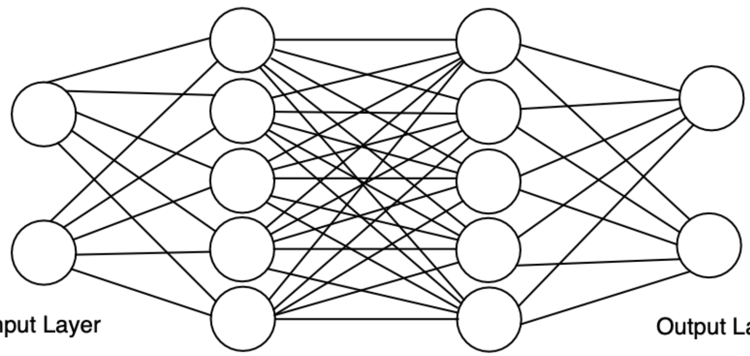
v_i

$$\{J_1, \ldots, J_n\} \longrightarrow \{v_1, \ldots, v_n\}$$

$$J_i \in T$$

$$\{J'_1, \ldots, J'_m\} \longrightarrow ???$$

$$J'_i \in T^c$$



$\vec{J}$ $\quad 100 \times 18 \quad 100 \times 100 \quad \sum_{a=1}^{100}$

12000 hyperparameters

Jones polynomials are represented as 18-vectors

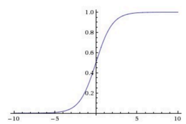
$$\vec{J}_K = (\min, \max, \text{coeffs}, 0, \ldots, 0)$$

Two layer neural network in Mathematica

$$f_\theta(\vec{J}_K) = \sum_a \sigma\left(W_\theta^2 \cdot \sigma(W_\theta^1 \cdot \vec{J}_K + \vec{b}_\theta^1) + \vec{b}_\theta^2\right)^a$$

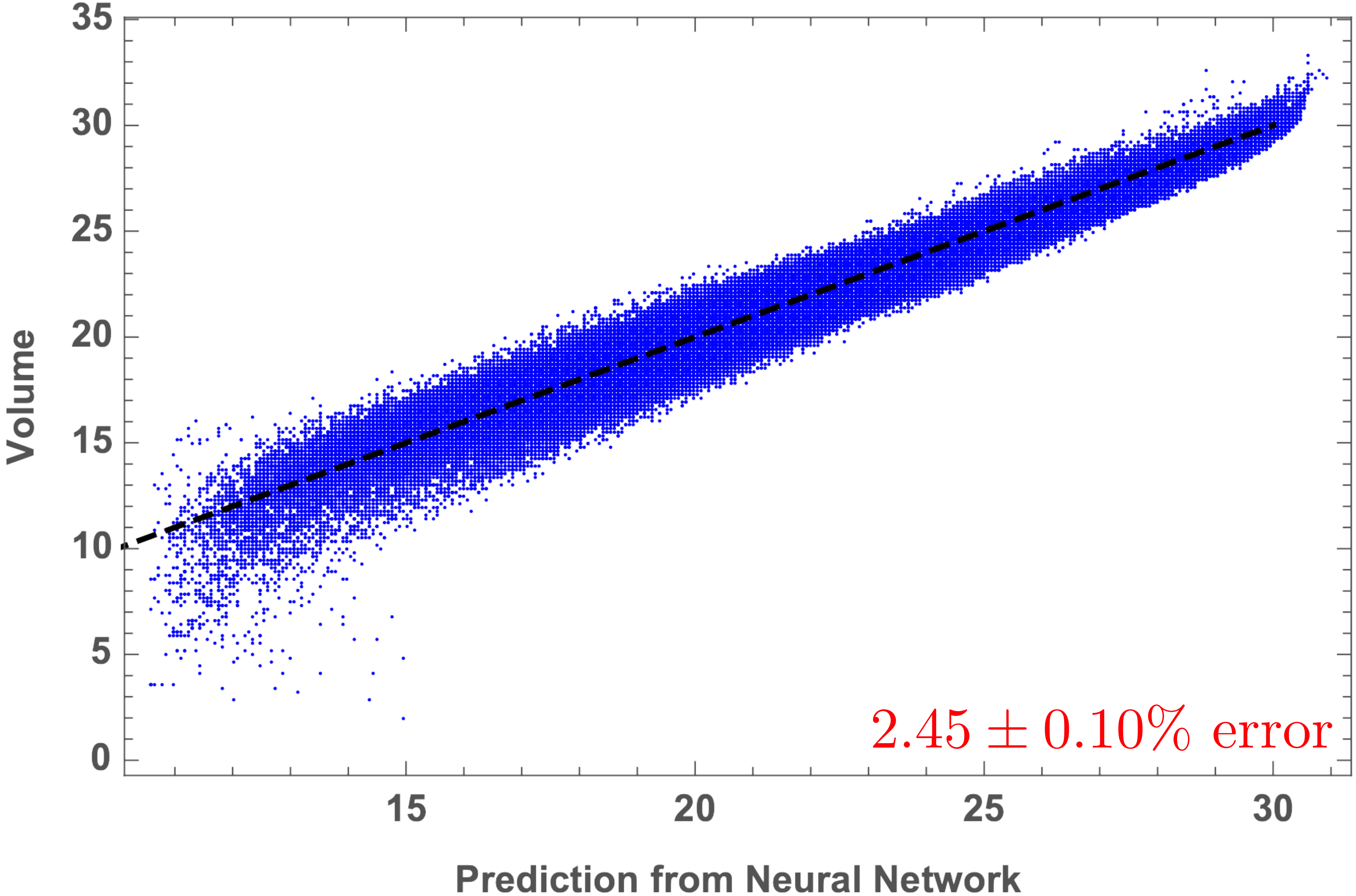
Logistic sigmoids for the hidden layers

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



VJ, Kar, Parrikar (2019)

Neural Network



trained on 10% of the 313,209 knots up to 15 crossings

VJ, Kar, Parrikar (2019)

Result

$$v_i = f(J_i) + \text{small corrections}$$

- J_i does not uniquely identify a knot

  e.g., 4_1 and K11n19 have same Jones polynomial, different volumes

- $174,619$ unique Jones polynomials

  2.83% average spread in volumes for a Jones polynomial
  intrinsic mitigation against overfitting

- Same applies to 1,701,913 hyperbolic knots up to 16 crossings

  (database compiled from **Knot Atlas** and SnapPy)

VJ, Kar, Parrikar (2019)

Result

$$v_i = f(J_i) + \text{small corrections}$$

- Neural network does better than more refined topological invariants

- Beyond the volume conjecture in Chern–Simons

  Jones polynomial (quantum) $\longleftrightarrow$ volume (classical)

  { weak coupling limit of $SL(2,\mathbb{C})$ Chern–Simons; strong coupling limit of $SU(2)$ }

- Failed experiments (*e.g.*, learning Chern–Simons invariant) also teach us something — maybe about the need for underlying homology theory

$$\lim_{n\to\infty} \frac{2\pi \, \log J_n(K; e^{2\pi i/n})}{n} = \mathrm{Vol}(S^3 \setminus K) + 2\pi^2 i \, \mathrm{CS}(S^3 \setminus K)$$

cf. Calabi–Yau Hodge numbers, line bundle cohomology, etc.

VJ, Kar, Parrikar (2019)

Result

$$v_i = f(J_i) + \text{small corrections}$$

- Universal Approximation Theorem: feedforward neural network, sigmoid activation function, single hidden layer with finite number of neurons can approximate continuous functions on compact subsets of $\mathbb{R}^n$

Cybenko (1989)
Hornik (1991)

- Surprise here is simplicity of architecture that does the job

- We want a **not** machine learning knot result

Entr'acte

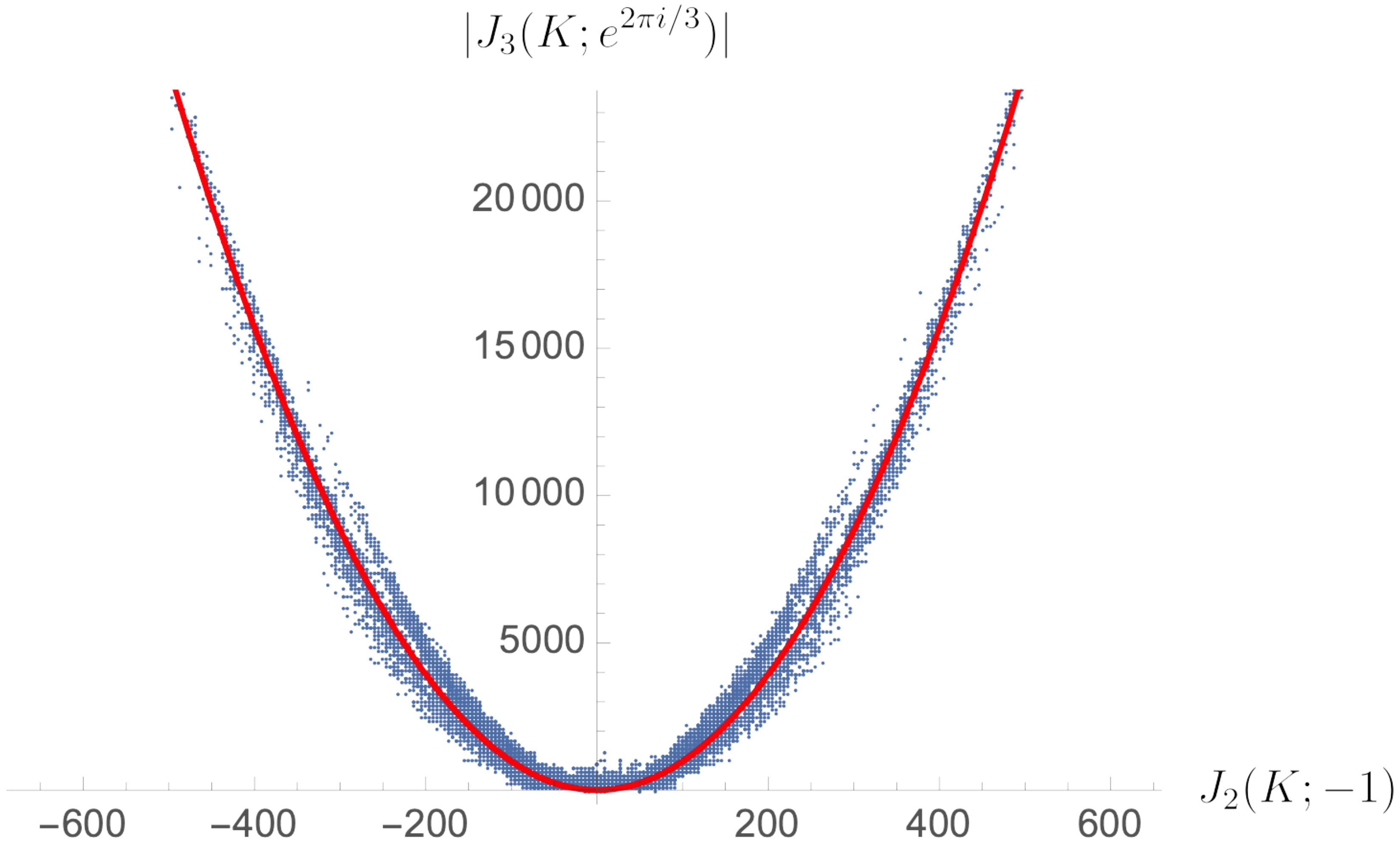
$$v_i = f(J_i) + \text{small corrections}$$

We seek to reverse engineer the neural network to obtain an analytic expression for the volume as a function of the Jones polynomial

To interpret the formula, we use machinery of analytically continued Chern–Simons theory

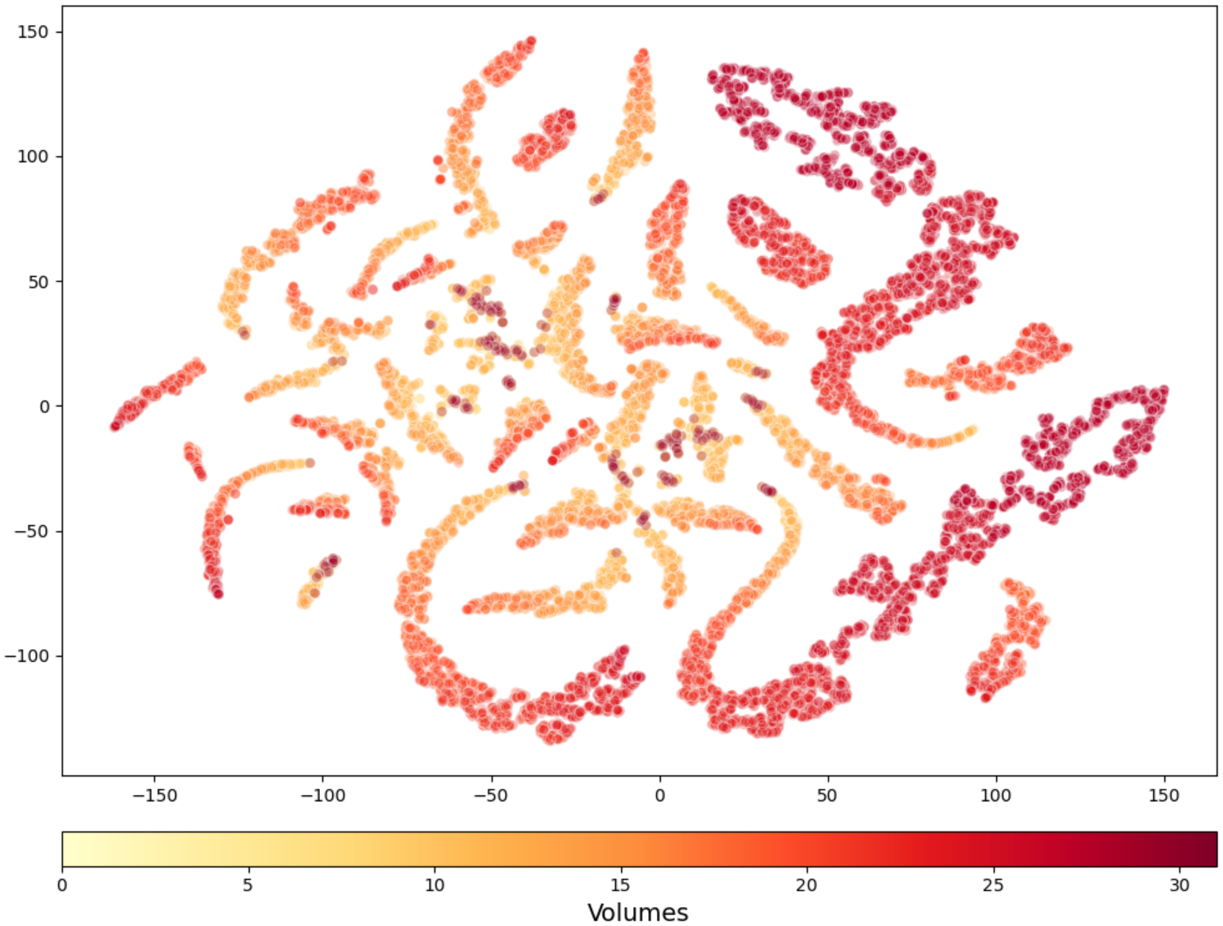
Towards the Volume Conjecture

- The volume conjecture: $\lim_{n\to\infty} \frac{2\pi \log |J_n(K;\omega_n)|}{n} = \mathrm{Vol}(S^3 \setminus K)$



- 11,921 colored Jones polynomials at $n = 3$

t-SNE

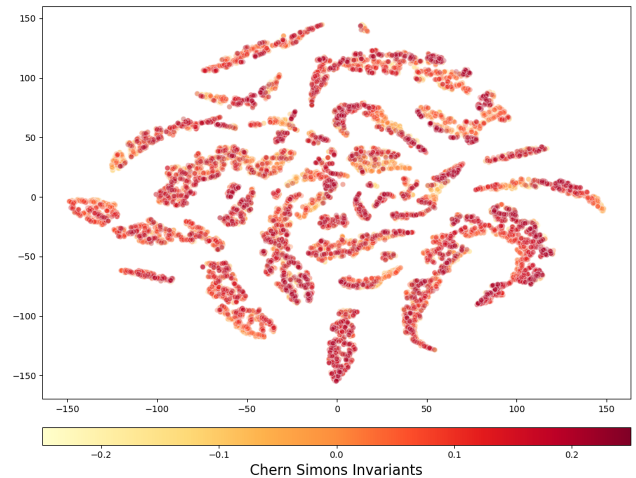


$$\lim_{n\to\infty} \frac{2\pi \log J_n(K;\omega_n)}{n}$$

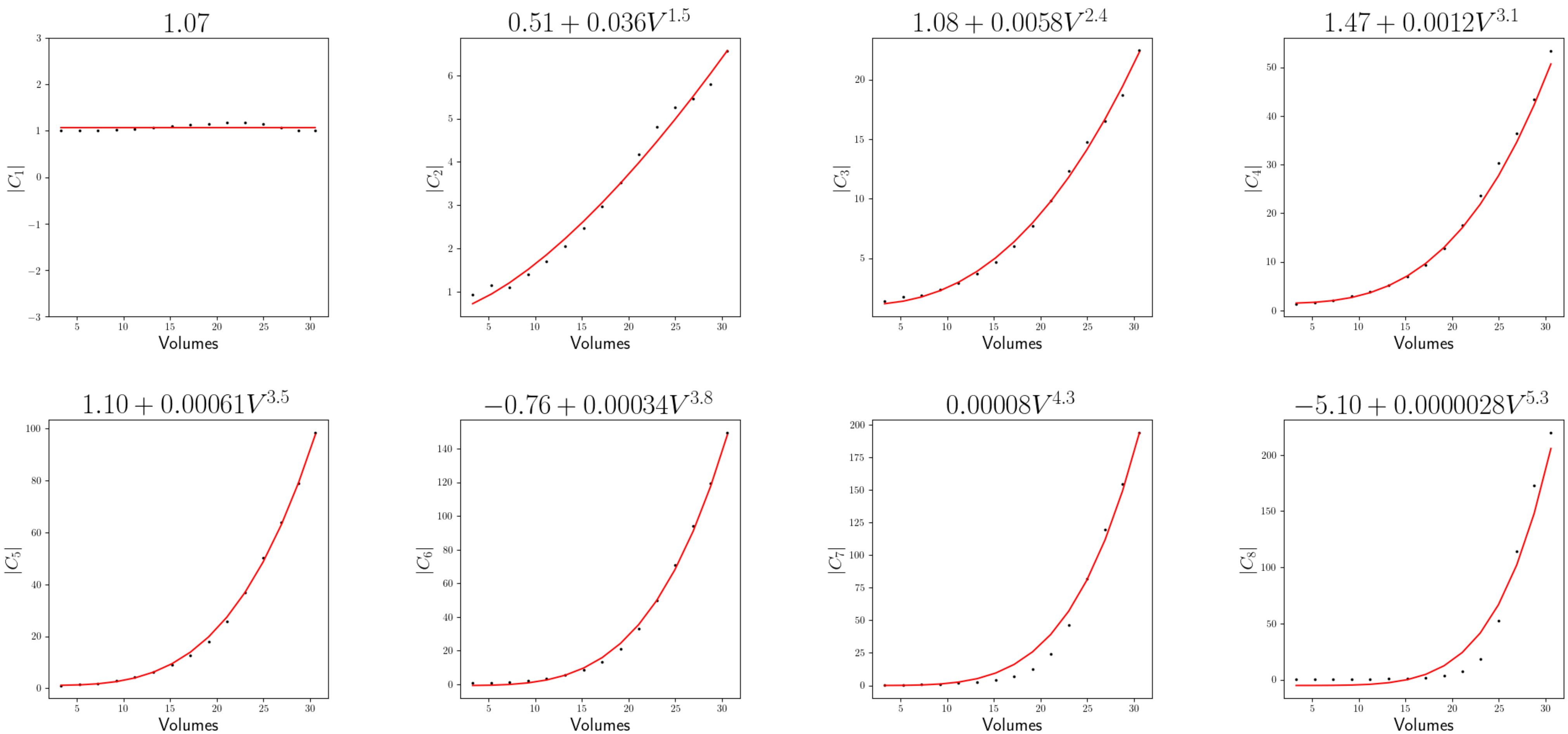
$$= \mathrm{Vol}(S^3 \setminus K)$$
$$+2\pi^2 i\, \mathrm{CS}(S^3 \setminus K)$$

Volume is learnable from coefficients

Chern–Simons invariant probably is not



Coefficients



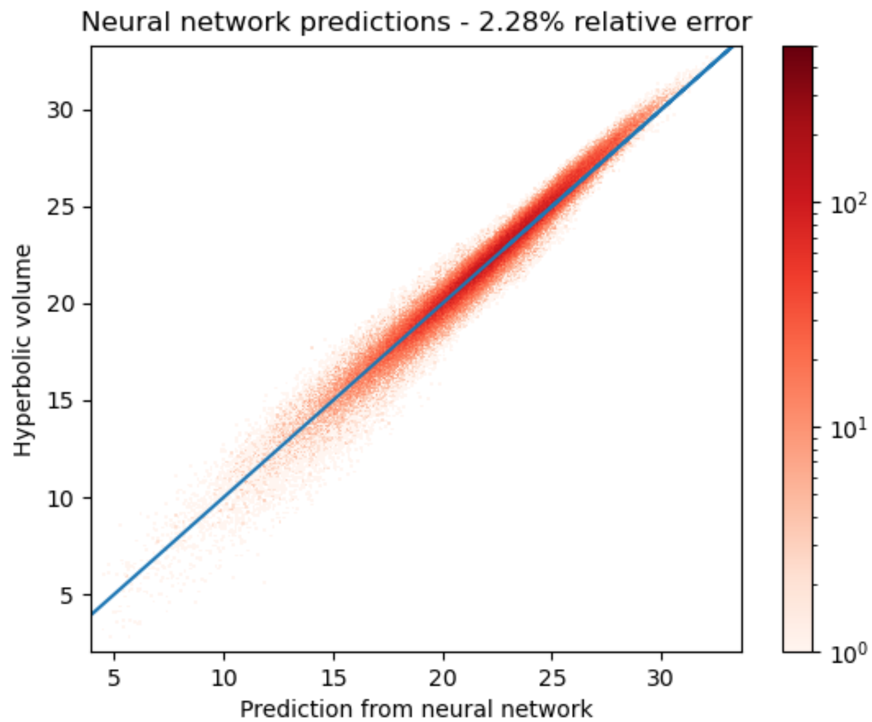
$$|C_n| \sim |C_{-n}|$$

Coefficients scale with volume as power law

No Degrees Needed

- Suppose we drop the degrees and provide only the coefficients; Jones polynomial is no longer recoverable from the input vector

- Results are unchanged!



<u>*N.B.*</u>: we have switched to `Python 3` using `GPU-Tensorflow` with `Keras` wrapper two hidden layers, 100 neurons/layer, ReLu activation, mean squared loss, `Adam` optimizer

Jones Evaluations

- Physics in Chern–Simons theory that leads to volume conjecture may also be responsible for information in $J_2(K;q)$

- Consider evaluations of Jones polynomial at roots of unity

- In particular, fix $r \in \mathbb{Z}$ and evaluate $j_p^r := J_2(K; e^{2\pi i p/(r+2)})$

- The input vector

$$\mathbf{v}_{\text{in}} = (\text{Re}(j_0^r), \text{Im}(j_0^r), \ldots, \text{Re}(j^r_{\lfloor (r+2)/2 \rfloor}), \text{Im}(j^r_{\lfloor (r+2)/2 \rfloor}))$$

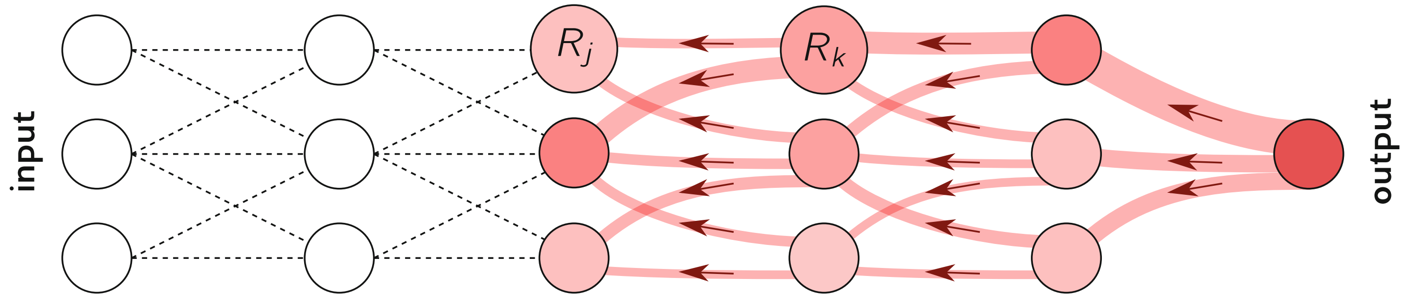
  does not degrade neural network performance

- In fact, we only need to feed in the magnitudes: $\mathbf{v}_{\text{in}} = (|j_0^r|, \ldots, |j_{\lfloor (r+2)/2 \rfloor}|)$

  Consistent with degrees not mattering

Layer-wise Relevance Propagation

- To determine which inputs carry the most weight, propagate backward starting from output layer employing a conservation property



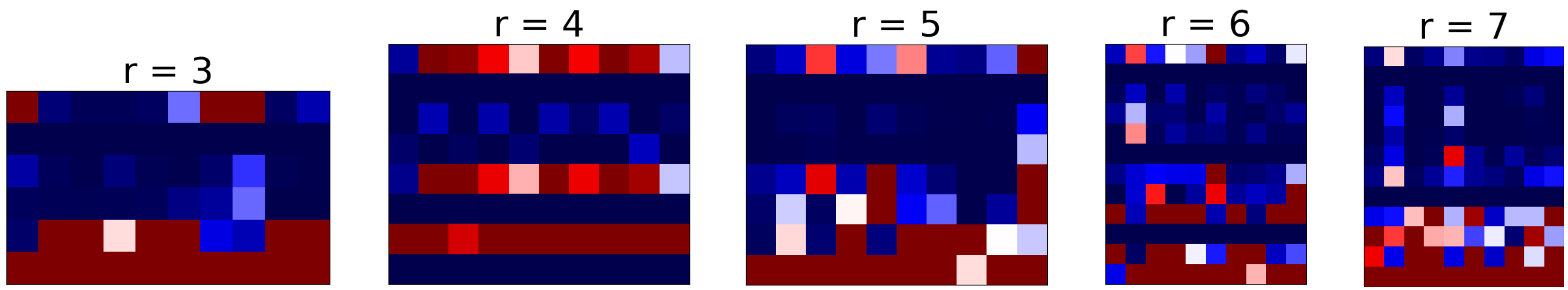
Montavon et al. (2019)

- Compute relevance score for a neuron using activations, weights, and biases

$$R_j^{m-1} = \sum_k \frac{a_j^{m-1} W_{jk}^m + N_{m-1}^{-1} b_k^m}{\sum_l a_l^{m-1} W_{lk}^m + b_k^m} R_k^m \quad , \quad \sum_k R_k^m = 1$$

j^{th} neuron in layer $m-1$

Layer-wise Relevance Propagation



- Each column is a single input corresponding to evaluations of the Jones polynomial at phases $e^{\frac{2\pi i p}{r+2}}$, $0 \leq 2p \leq r+2$, $p \in \mathbb{Z}$
- Ten different knots
- We show the relevances (red is most relevant) and notice that the same input features light up

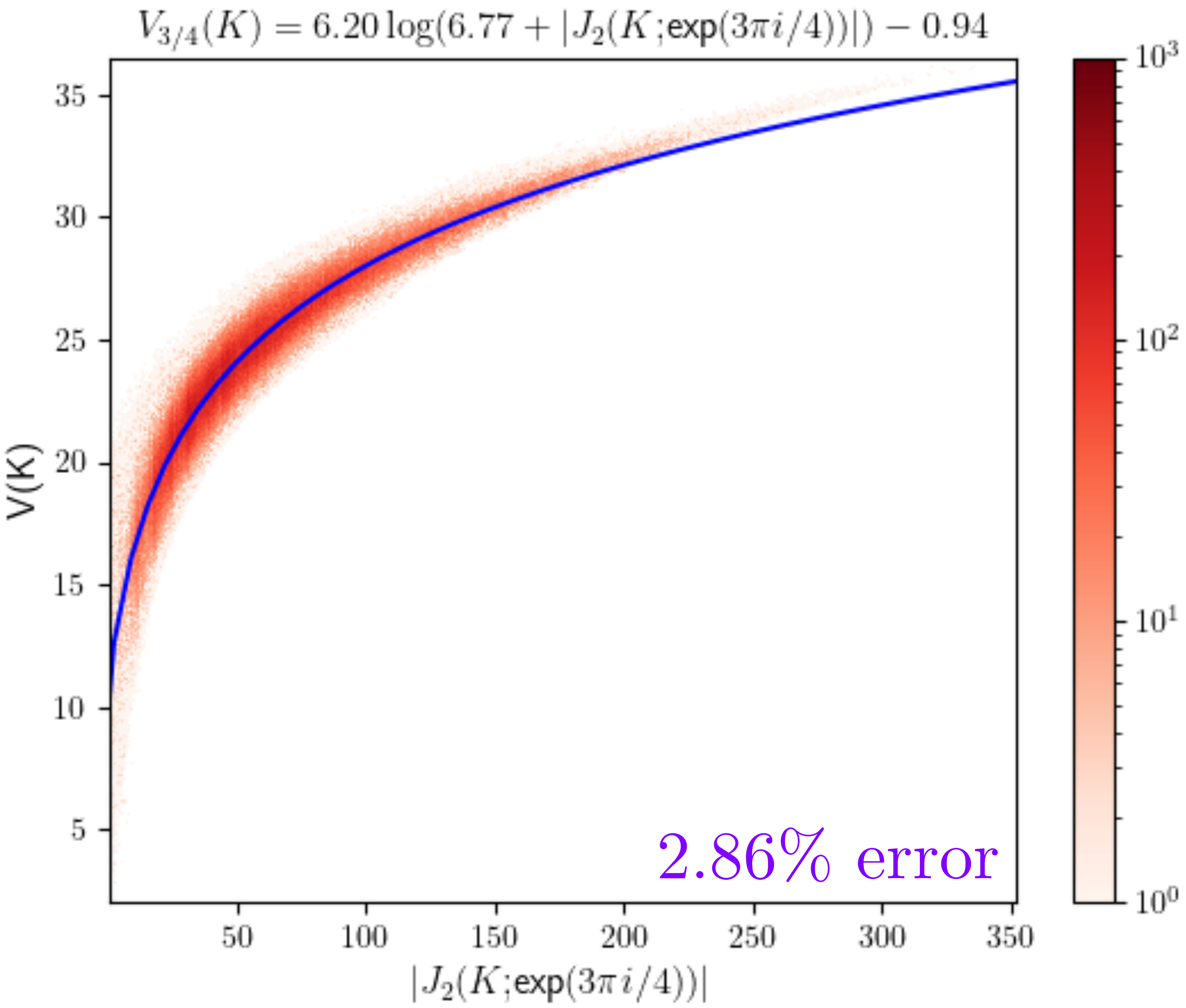
Relevant Phases

r	Error	Relevant roots	Fractional levels	Error (relevant roots)
3	3.48%	$e^{4\pi i/5}$	$\frac{1}{2}$	3.8%
4	6.66%	-1	0	6.78%
5	3.48%	$e^{6\pi i/7}$	$\frac{1}{3}$	3.38%
6	2.94%	$e^{3\pi i/4}, -1$	$\frac{2}{3}, 0$	3%
7	5.37%	$e^{8\pi i/9}$	$\frac{1}{4}$	5.32%
8	2.50%	$e^{3\pi i/5}, e^{4\pi i/5}, -1$	$\frac{4}{3}, \frac{1}{2}, 0$	2.5%
9	2.74%	$e^{8\pi i/11}, e^{10\pi i/11}$	$\frac{3}{4}, \frac{1}{5}$	2.85%
10	3.51%	$e^{2\pi i/3}, e^{5\pi i/6}, -1$	$1, \frac{2}{5}, 0$	4.39%
11	2.51%	$e^{8\pi i/13}, e^{10\pi i/13}, e^{12\pi i/13}$	$\frac{5}{4}, \frac{3}{5}, \frac{1}{6}$	2.44%
12	2.39%	$e^{5\pi i/7}, e^{6\pi i/7}, -1$	$\frac{4}{5}, \frac{1}{3}, 0$	2.75%
13	2.52%	$e^{2\pi i/3}, e^{4\pi i/5}, e^{14\pi i/15}$	$1, \frac{1}{2}, \frac{1}{7}$	2.43%
14	2.58%	$e^{3\pi i/4}, e^{7\pi i/8}, -1$	$\frac{2}{3}, \frac{2}{7}, 0$	2.55%
15	2.38%	$e^{12\pi i/17}, e^{14\pi i/17}, e^{16\pi i/17}$	$\frac{5}{6}, \frac{3}{7}, \frac{1}{8}$	2.4%
16	2.57%	$e^{2\pi i/3}, e^{7\pi i/9}, e^{8\pi i/9}, -1$	$1, \frac{4}{7}, \frac{1}{4}, 0$	2.45%
17	2.65%	$e^{14\pi i/19}, e^{16\pi i/19}, e^{18\pi i/19},$	$\frac{5}{7}, \frac{3}{8}, \frac{1}{9}$	2.46%
18	2.49%	$e^{4\pi i/5}, e^{9\pi i/10}, -1$	$\frac{1}{2}, \frac{2}{9}, 0$	2.52%
19	2.45%	$e^{2\pi i/3}, e^{16\pi i/21}, e^{6\pi i/7}, e^{20\pi i/21}$	$1, \frac{5}{8}, \frac{1}{3}, \frac{1}{10}$	2.43%
20	2.79%	$e^{8\pi i/11}, e^{9\pi i/11}, e^{10\pi i/11}, -1$	$\frac{3}{4}, \frac{4}{9}, \frac{1}{5}, 0$	2.4%

$$e^{ix} = e^{\frac{2\pi i}{k+2}}$$

Phenomenological Function

$\omega = e^{\frac{3\pi i}{4}}$ ⟷ $k = \frac{2}{3}$



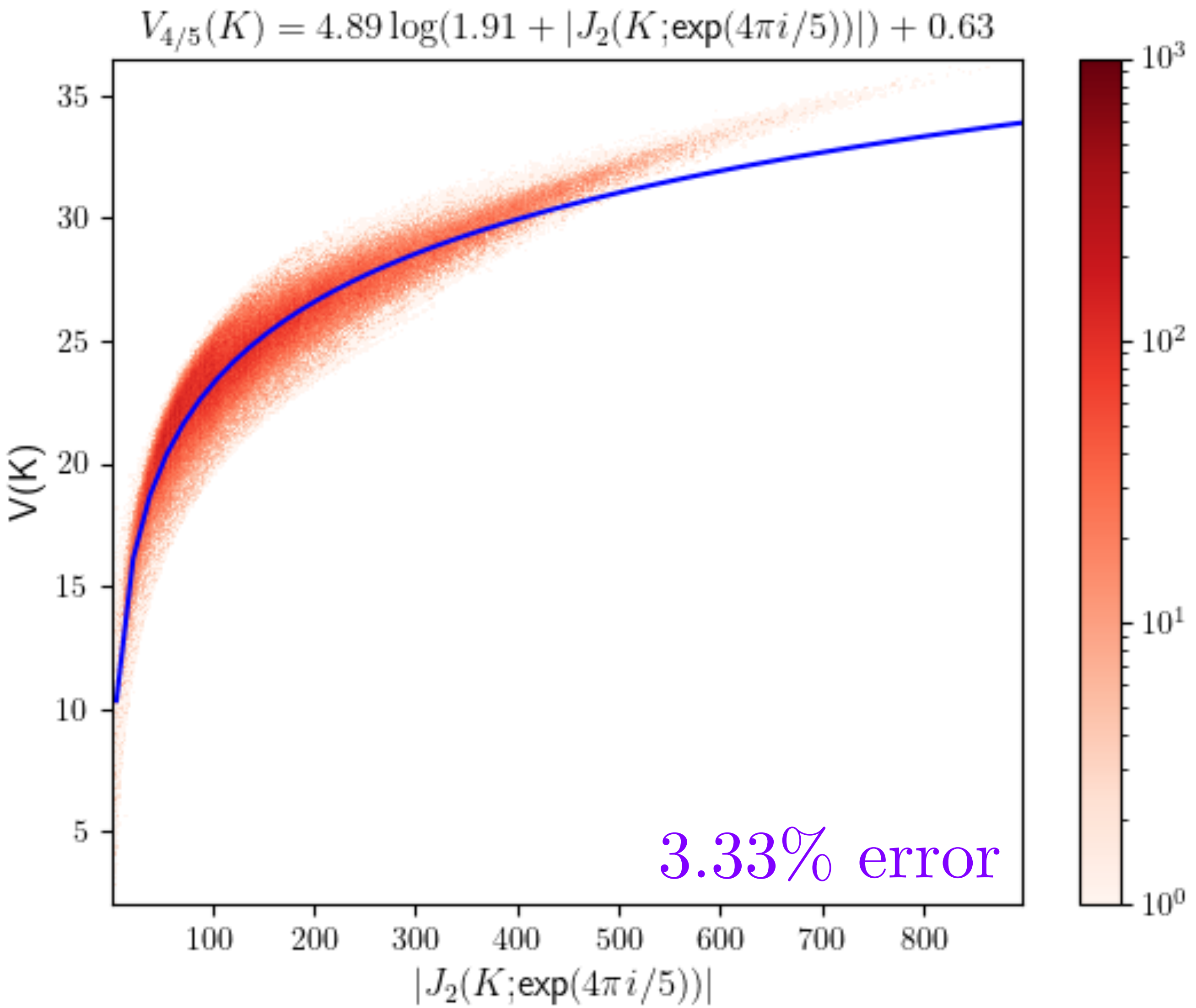
- Parameters fixed via curve fitting routines in Mathematica

Phenomenological Function

$\omega = e^{\frac{4\pi i}{5}}$

$\Updownarrow$

$k = \frac{1}{2}$



- Parameters fixed via curve fitting routines in Mathematica

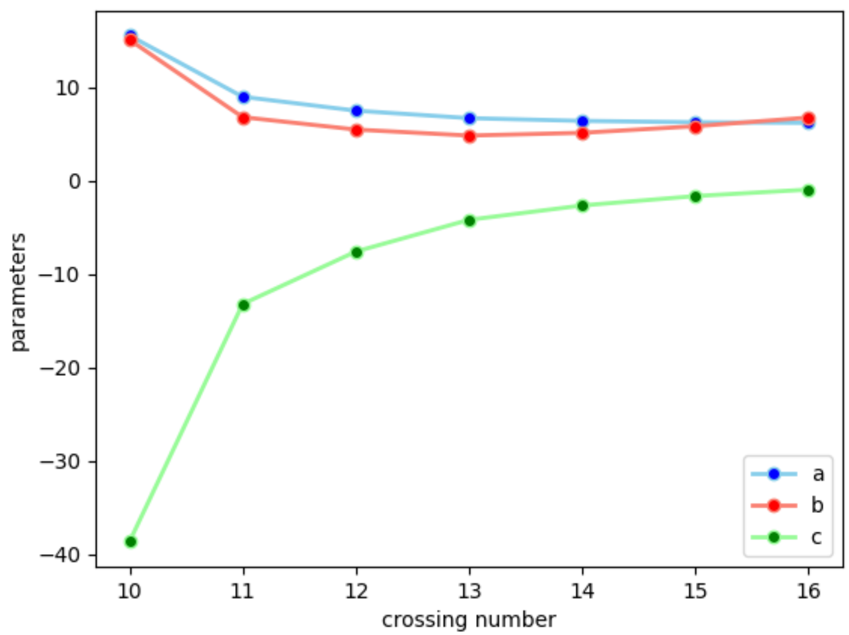
Phenomenological Function

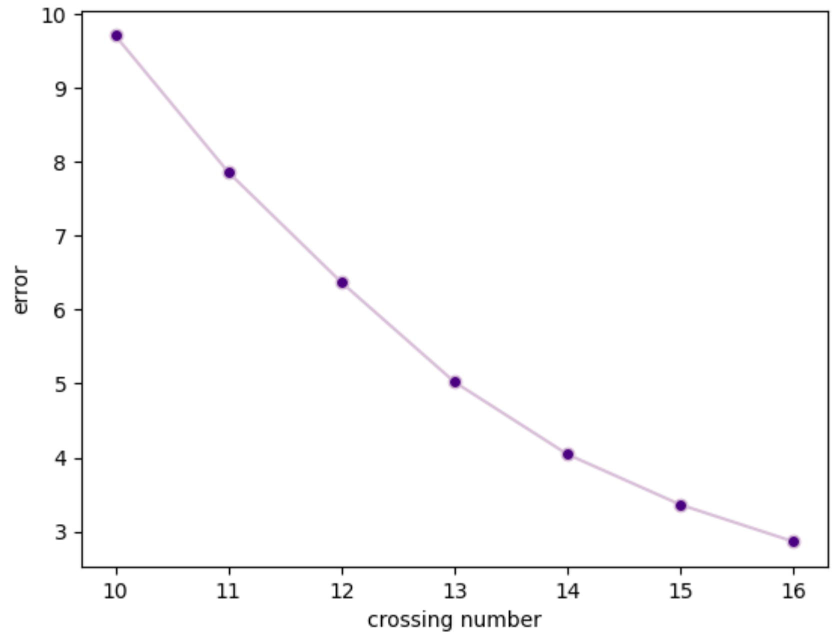
$$V_{3/4}(S^3 \setminus K) = 6.20 \log(|J_2(K; e^{\frac{3\pi i}{4}})| + 6.77) - 0.94$$

2.86% error compared to 2.28% error for neural network

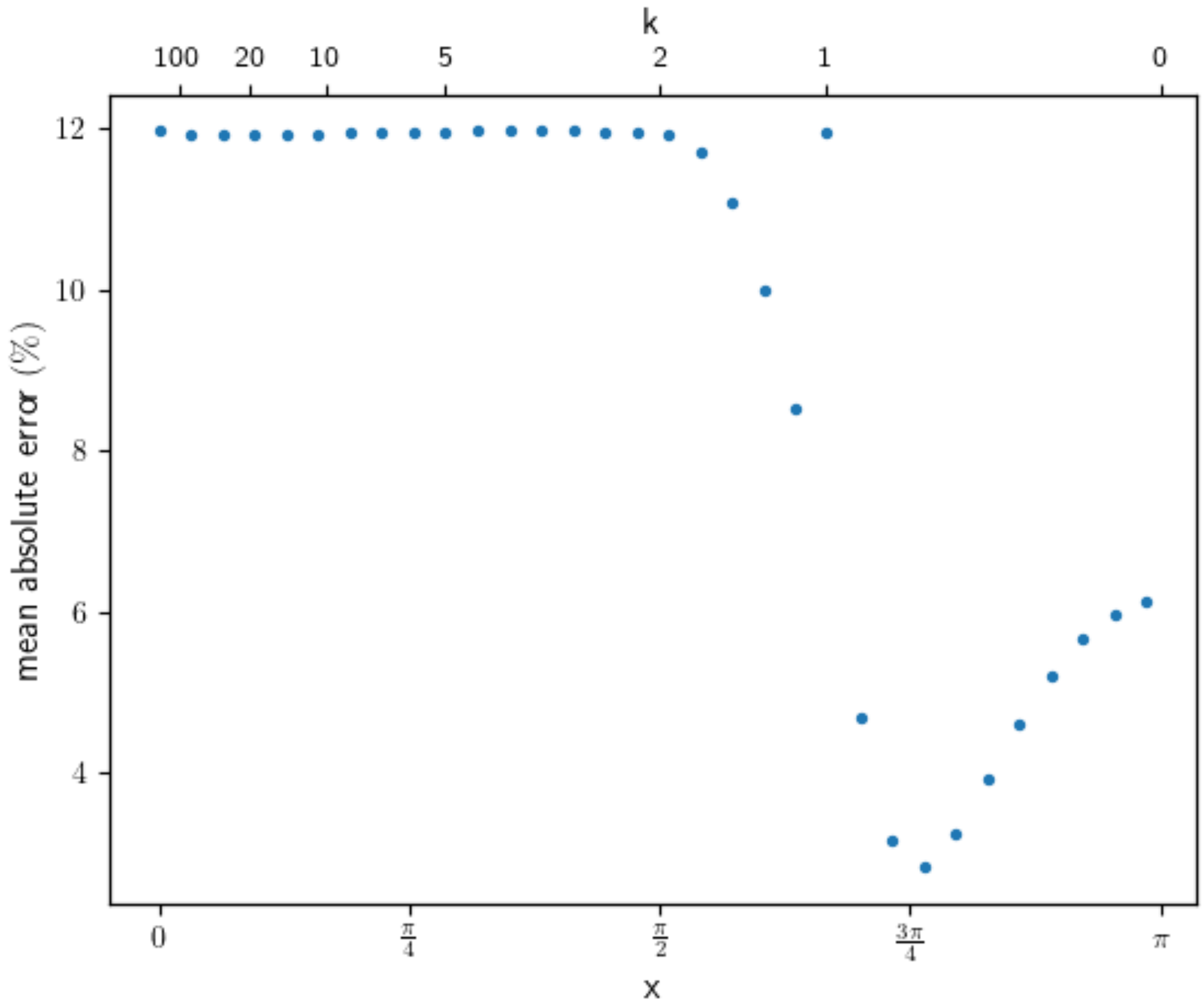
corresponds to Chern–Simons level $k = \frac{2}{3}$

- Parameters of fit robust as a function of crossing number



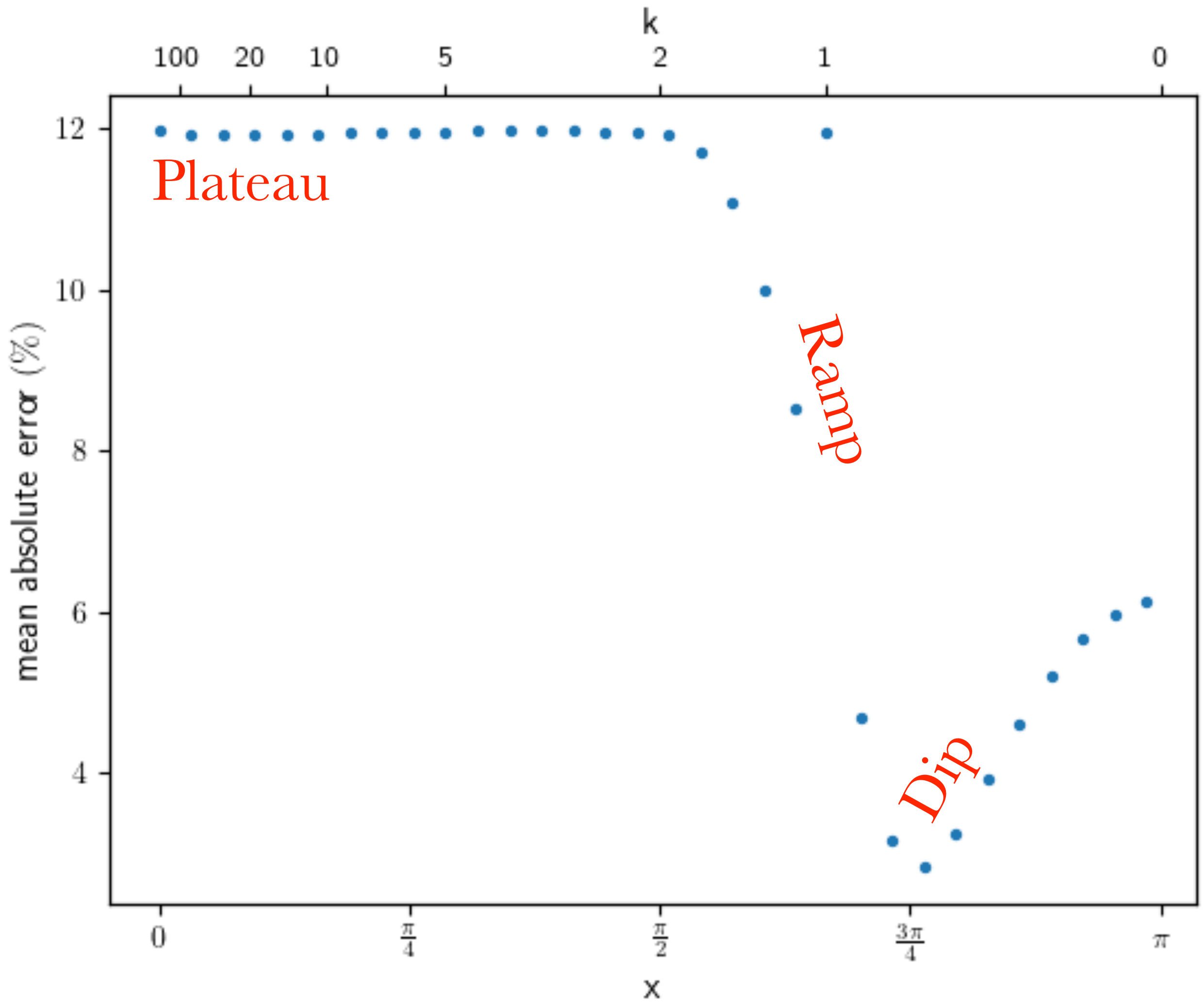


Best Phase



- Best fit to $V(x) = a\log(|J_2(K;e^{ix})| + b) + c$ at $x = 2.3$

Best Phase



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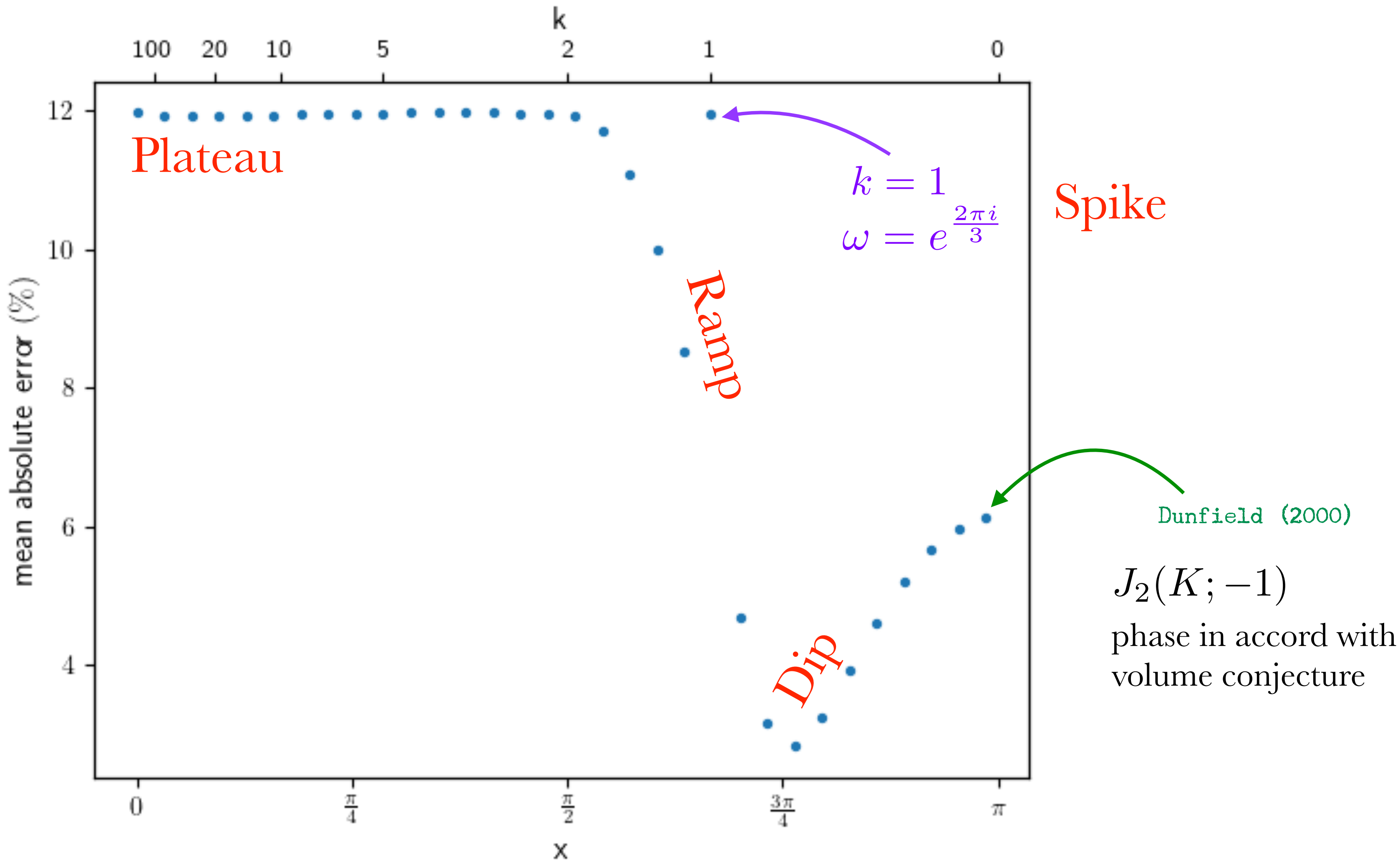


- Best fit to $V(x) = a \log(|J_2(K; e^{ix})| + b) + c$ at $x = 2.3$

Chern–Simons Theory

- Recall that $S_{\rm CS} = \frac{k}{4\pi}\int_{\mathcal{M}} {\rm tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$

- Under the gauge transformation $A_\mu \mapsto g^{-1}A_\mu g + g^{-1}\partial_\mu g$,

$$\Delta S_{\rm CS} = \frac{k}{4\pi}\int d^3x\ \epsilon^{\mu\nu\rho}\left(\partial_\mu {\rm tr}\left((\partial_\nu g)g^{-1}A_\rho\right) + \frac{1}{3}{\rm tr}\left(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g\right)\right)$$

- Associated to large gauge transformations, we recognize the *winding*

$$w(g) = \frac{1}{24\pi^2}\int d^3x\ \epsilon^{\mu\nu\rho}{\rm tr}\left(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g\right) \in \mathbb{Z}$$

- This implies that the level $k \in \mathbb{Z}$

- So what does fractional level mean?

- In Abelian Chern–Simons theory, can make sense of $k = \frac{1}{2}$

Moore (2019)

Analytic Continuation

- We can analytically continue the level

- Appeal to Morse theory; the integration cycle $\mathcal{C}$ used to compute path integral is decomposed in terms of Lefschetz thimbles

- The validity of the volume conjecture amounts to statement that geometric conjugate connection $\mathcal{A}_+$ contributes to $SU(2)$ path integral in neighborhood of $\gamma = \frac{n-1}{k} = 1$ at large k

  N.B.: for the Jones polynomial, $n = 2$, $\gamma = k^{-1}$

- As γ is varied, analytically continued integration cycle can pick up contributions from new critical points or lose current ones; these are Stokes phenomena that occur along Stokes lines in complex γ plane

Witten (2010)

Analytic Continuation

- For integer k, even if $\mathcal{A}_+$ saddle is present in integration cycle, it cancels with another saddle

- For general k, the leading contribution of the two saddles is $e^{iW(\mathcal{A}_+)}(1-e^{2\pi ik})$

- This survives in semi-classical limit $\gamma \to 1$, $k \to \infty$

  i.e., $e^{iW(\mathcal{A}_+)}(1-e^{2\pi ik})$ to path integral from pair of $SL(2,\mathbb{C})$ critical points after analytic continuation

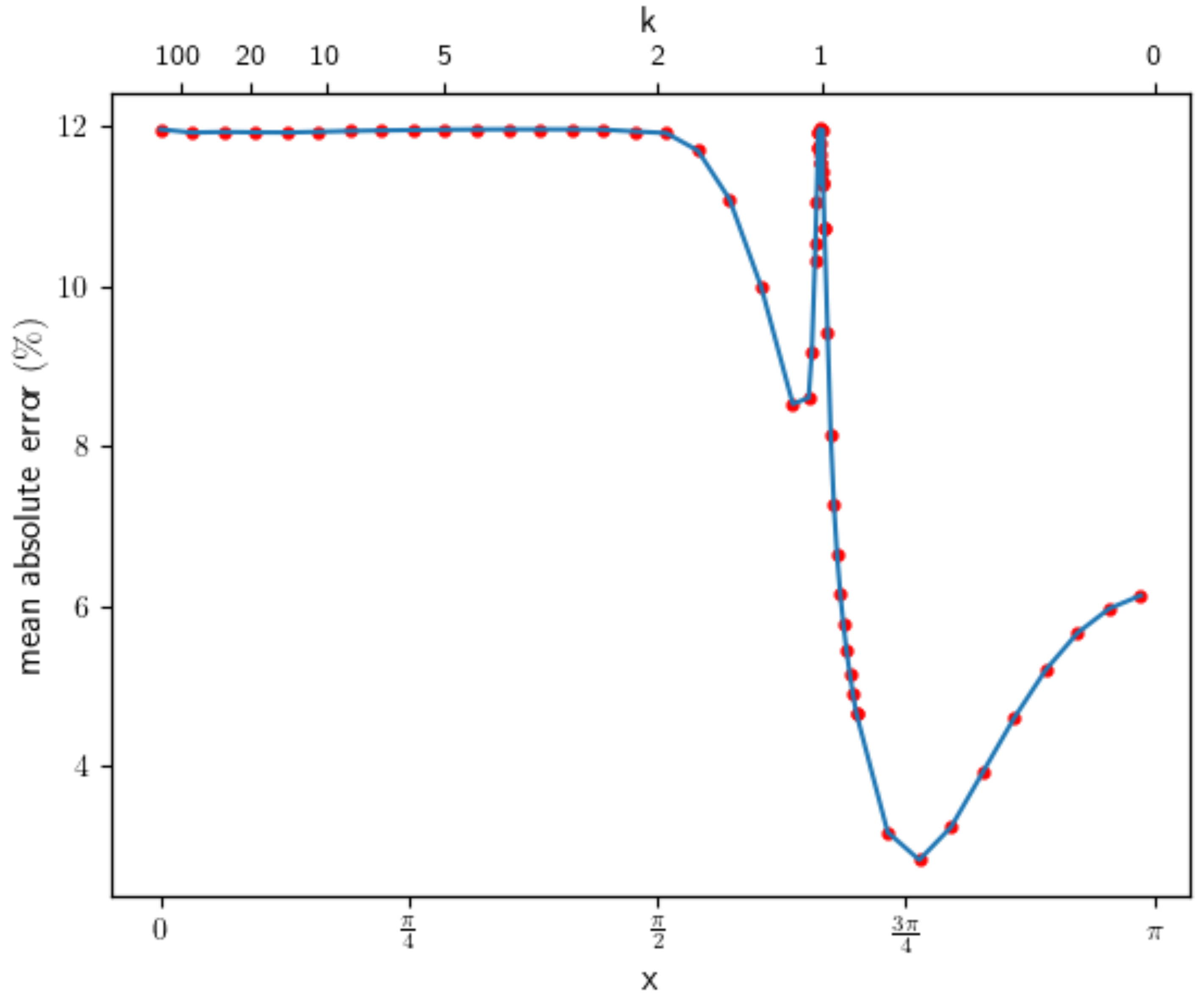
- Volume conjecture essentially states that this behavior occurs for geometric conjugate connection for every knot at $n=k+2$

Witten (2010)

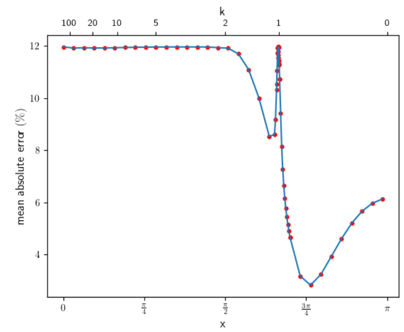
Hypothesis

The approximation formula works well for levels k for which $\mathcal{A}_+$ makes a contribution to the Chern–Simons path integral, and its accuracy increases with fraction of knots in dataset that receive such a contribution

The Shape of Things



The Shape of Things



Plateau: $k > 2$

$$\mathrm{Vol}(S^3 \setminus K) = \langle \mathrm{Vol}(S^3 \setminus K) \rangle$$

this gives 11.97% error for knots up to 16 crossings

corresponds to latent correlations in the dataset

Minimum: near $k = \frac{2}{3}$ or $\gamma = \frac{3}{2}$

Lefschetz thimbles contain geometric conjugate $SL(2, \mathbb{C})$ connection we expect in semiclassical limit for most knots

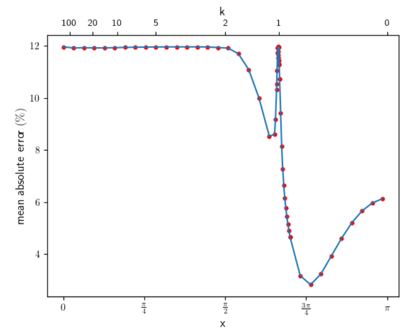
Dip: $0 < k < \frac{2}{3}$

knots retain geometric conjugate connection even as $k \ll 1$ or $\gamma \gg 1$

this is explanation for observation that $\log |J_2(K; -1)| \propto \mathrm{Vol}(S^3 \setminus K)$

Dunfield (2000)

The Shape of Things



<u>Ramp:</u> $\frac{2}{3} < k < 2$ interpolating regime

knots lose access to geometric conjugate connection

at $k = \frac{3}{2}$ or $\gamma = \frac{2}{3}$, the geometric conjugate connection of 4_1 enters

Witten (2010)

<u>Spike:</u> near $k = 1$

at integer values of level with $k + 1 \geq n$, the path integral receives contributions only from $SU(2)$ valued critical points

i.e., no analytic continuation is necessary

because we lose knowledge of the geometric conjugate connection, the error becomes high

<u>Conclusion:</u> geometric conjugate connection is crucial to success of approximation formula

A Better Formula

- Our reverse engineered function gave 2.86% error compared to 2.28% error for neural network; the latter is essentially intrinsic

- Can we do better with a formula? If so, how much better?

- Define a new error measure

$$\sigma = \frac{\text{variance of (actual volume} - \text{predicted volume)}}{\text{variance of volumes in dataset}}$$

[suggested to us in correspondence with `Fischbacher, Münkler`]

σ-measure is shift/rescaling invariant

- Can ask what fraction of variance is left unexplained

A Better Formula

$$\sigma = \frac{\text{variance of (actual volume} - \text{predicted volume)}}{\text{variance of volumes in dataset}}$$

- By this measure, the neural network gives $\sigma = 0.033$

  while our functional approximation gives $\sigma = 0.068$

- If we just assign the average volume to every knot in the dataset, $\sigma = 1$; this corresponds to plateau

- There is room for improvement, but it is remarkable that a function with only three fit parameters works so well

Other Experiments

- Different representations of Jones polynomial work just as well

- Khovanov polynomial predicts volume with 97.2% accuracy; HOMFLY-PT polynomial does less well with 93.9% accuracy

$$J_2(K;q) = \frac{Kh(K;-1,q)}{q+q^{-1}} = P(K;q^{-1},q^{1/2}-q^{-1/2})$$

- Chern–Simons invariant does not look to be learnable from Jones polynomial in various experiments

- Symbolic regression using PySR gives formulae with 96.6% accuracy, but not so interpretable

Cranmer, et al. (2020)

Prospectus

- Inequalities à la volume-ish theorem using analytically continued Chern–Simons theory; investigate this for higher colors as well
- Use numerics to study Stokes phenomena in Chern–Simons theory
- Monotonic version of the volume conjecture
- Relations between other topological invariants
- Better understanding of what problems are machine learnable in mathematics and physics — failed experiments may teach us something!
- Reverse engineer other machine learned results

Smooth Poincaré in 4d

- Conjecture: A four manifold with homotopy type S^4 is diffeomorphic to S^4

- Freedman proved it is homeomorphic

- Does S^4 admit exotic smooth structures?

- The Rasmussen s-invariant cannot be used to detect counterexamples

- Find topologically slice knots that are not slice

Freedman, Gompf, Morrison, Walker (2010)
Manolescu, Piccirillo (2021)

- Perhaps ML can assist in identifying further interesting knots

Craven, Hughes, VJ, Kar

A View to Holography

- Beautiful recent work by Hashimoto, *et al.*

- Train on lattice data for chiral condensate vev vs. quark mass

- Obtain metric from parameters in neural network and follow AdS/QCD dictionary despite not being in large-N limit

- Predict form of quark–antiquark potential

Hashimoto, Sugishita, Tanaka, Tomiya (2018)
Hashimoto (2019)
Akutagawa, Hashimoto, Sugimoto (2020)

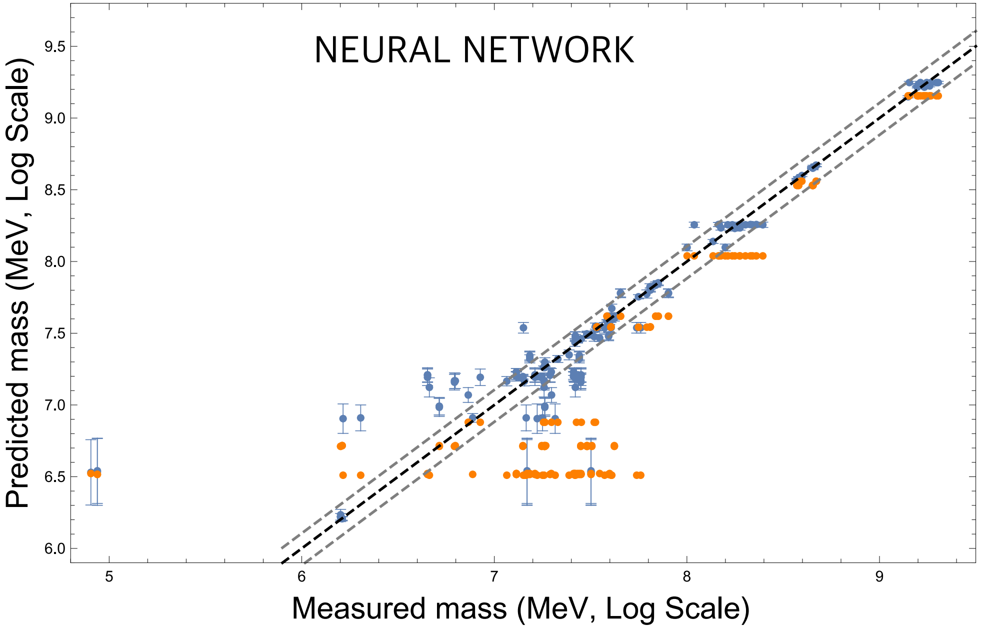
- Investigations in CFT

Chen, He, Lal, Zaz (2020)
Kuo, Seif, Lundgren, Whitsitt, Hafezi (2021)
Kántor, Niarchos, Papageorgakis (2021)

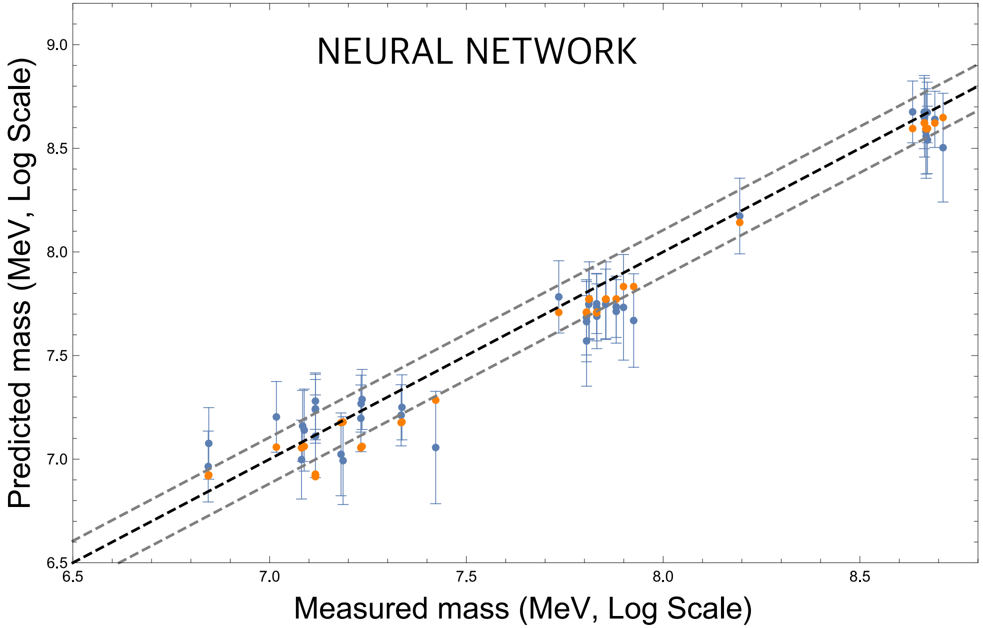
- Look at interesting black hole datasets

Baryons and Mesons

Mesons



Baryons



13.0% error

39.3% error

9.7% error

8.7% error

u : 336 MeV

d : 340 MeV

s : 486 MeV

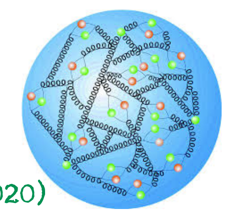
c : 1550 MeV

b : 4730 MeV

Constituent quark mass = QCD binding energy

= amount of energy to add to spontaneously emit meson containing given valence quark

The neural network is doing something else!



Gal, VJ, Mayorga Peña, Mishra (2020)

The Future

- Machine learning identifies associations
- Want to convert this to analytics — *i.e.*, how does a machine learn?
- What problems in physics and mathematics are machine learnable?
- Can a machine do interesting science?

hep-th

- Use machine learning to classify papers into **arXiv** categories
- 65% success at exact subject, 87% success at formal vs. phenomenology
- Mapping words to vectors contextually, we discover syntactic identities

$$\text{Paris} - \text{France} + \text{Italy} = \text{Rome}$$

$$\text{king} - \text{man} + \text{woman} = \text{queen}$$

He, VJ, Nelson (2018)

hep-th

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- An idea generating machine for **hep-th**:

$$\text{symmetry} + \text{black hole} = \text{Killing}$$
$$\text{symmetry} + \text{algebra} = \text{group}$$
$$\text{black hole} + \text{QCD} = \text{plasma}$$
$$\text{spacetime} + \text{inflation} = \text{cosmological constant}$$
$$\text{string theory} + \text{Calabi}-\text{Yau} = \text{M}-\text{theory} + G_2$$

He, VJ, Nelson (2018)

String Data

- string_data 2017 (Northeastern)
- string_data 2018 (LMU, Munich)
- Physics $\cap$ ML (Microsoft)
- string_data 2020 (CERN)
- NSF Institute for Artificial Intelligence and Fundamental Interactions, 2020–present (Harvard, MIT, Northeastern, Tufts)
- string_data 2021 (Wits)
- string_data 2022 (Turing Institute)

OBRIGADO!