Workshop on Black Holes, BPS and Quantum Information@Lisbon, Sep 21, 2021

Wormholes in coupled SYK/NAdS2 and their phase structures

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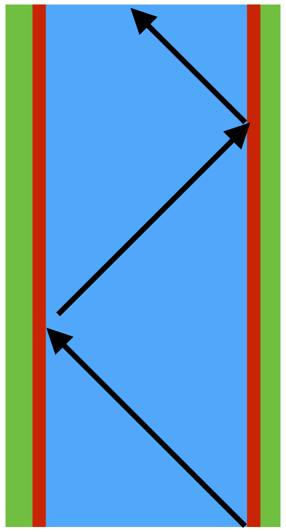
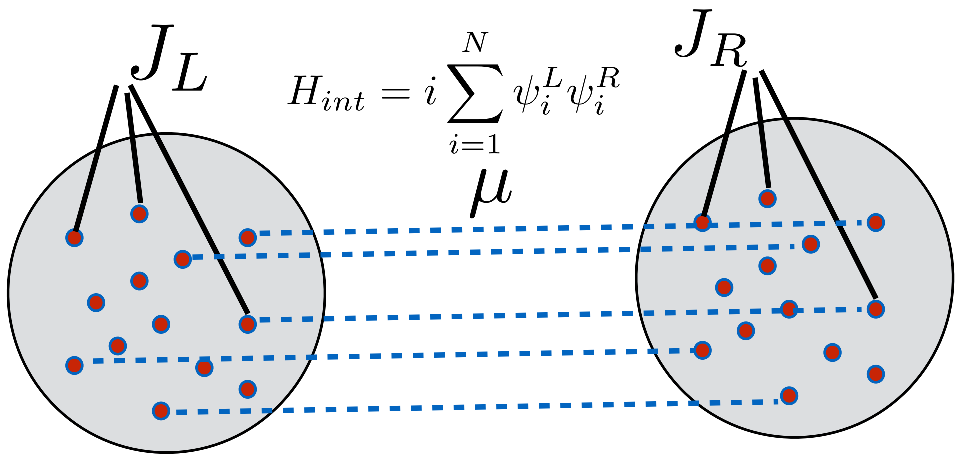
Based on arXiv:2011.12962 + work in progress

Today's focus:

- Traversable Wormholes in JT/ Two coupled SYK [Maldacena-Qi 18]

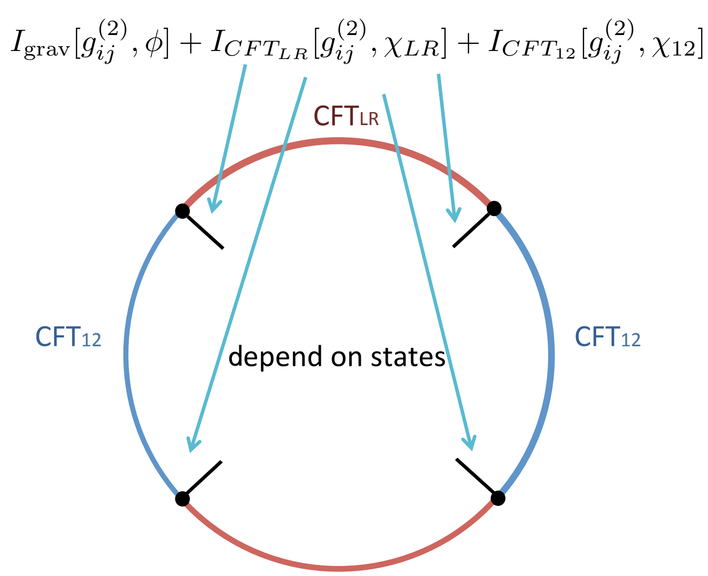


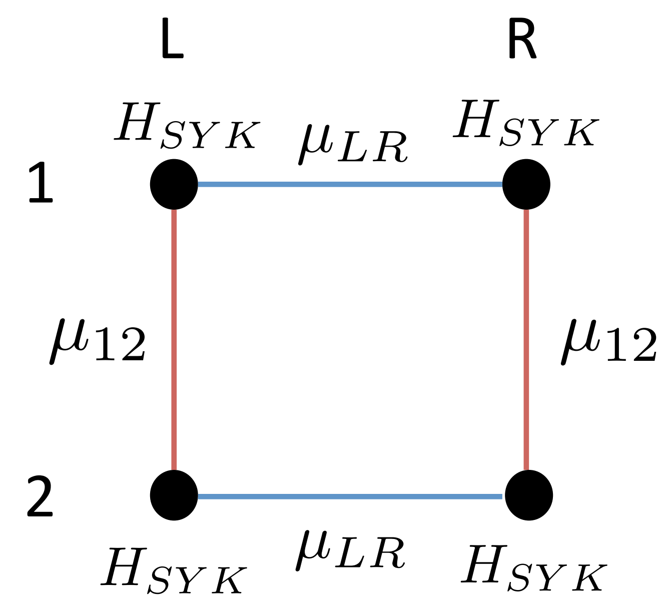
Traversable wormhole
=Global AdS2



two coupled SYK

- We generalize to JT with four boundaries/ Four coupled SYK





1. (Traversable) wormholes in 4d and in JT gravity

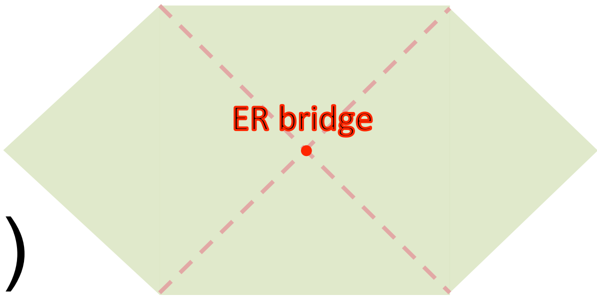
Wormholes: spacetime structure that connect distant regions.

・Spacial Wormholes: Closely related to quantum entanglement

[Israel, 76] [Maldacena,03] [Ryu-Takayanagi,06] [Raamsdonk,10] [Maldacena-Susskind,13]

Classically they are not traversable because of ANEC

(by quantum effect of matters we can break ANEC)



・(Euclidean) Spacetime Wormholes:

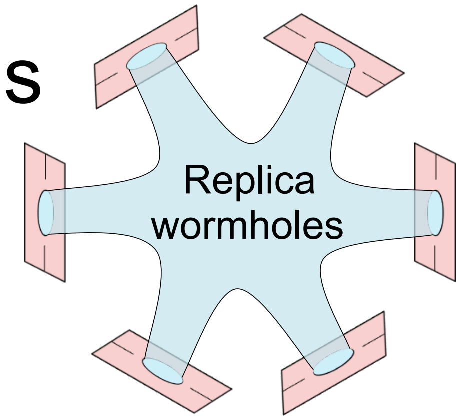
A kind of gravitational instanton.

Confusing object in AdS/CFT because they cause correlation of P.F.

[Coleman, 88] [Maldacena-Maoz, 04] [Arkani-hamed-Orega-Polchinski, 07]

Recently play an important role in BH information problems

[Saad-Shenker-Stanford, 19] [Penington-Shenker-Stanford-Yang, 19]
[Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini, 19]



Nearly AdS2 gravity

[Almeiri-Polchinski, 14] [Maldacena-Stanford-Yang, 16] [Jensen, 16] [Engelosoy-Mertens-Verlinde, 16]

Jackiw-Teitelboim (JT) gravity action

[Jackiw 85] [Teitelboim 83]

$$I[g_{\mu\nu},\phi] = \underbrace{-\frac{\phi_0}{16\pi G_N}\int\sqrt{g}R - \frac{\phi_0}{8\pi G_N}\int\sqrt{h}K}_{\text{topological}}$$
$$\underbrace{-\frac{1}{16\pi G_N}\int\phi\sqrt{g}(R+2) - \frac{1}{8\pi G_N}\int\sqrt{h}\phi_b K}_{\text{JT action}} \underbrace{+I_m[g,\chi]}_{\text{matter}}$$

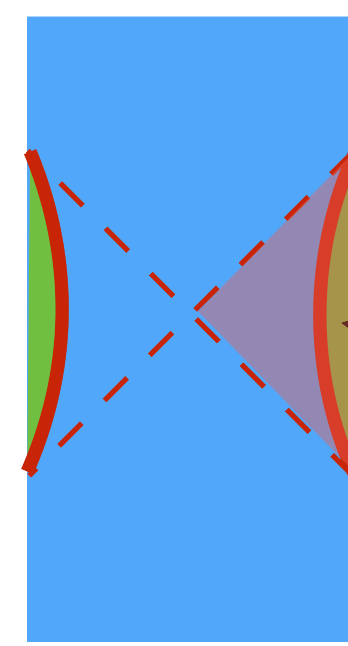
Boundary condition:

$$ds^2 = -\frac{du^2}{\epsilon^2},\ \ \phi|_{bdy} \equiv \phi_b(u) = \frac{\bar{\phi}_r}{\epsilon}$$

EOM:
$$\begin{cases} R+2=0 \longrightarrow \text{AdS}_2 \\ \nabla_\mu\nabla_\nu\phi - \nabla^2\phi + g_{\mu\nu}\phi = \frac{1}{8\pi G_N}\langle T^{mat}_{\mu\nu}\rangle \end{cases}$$

ex) BH solution:
$$\begin{cases} ds^2 = \left(\frac{2\pi}{\beta}\right)^2 \frac{-dt^2+dX^2}{\sinh^2\frac{2\pi}{\beta}X} \\ \phi(X) = \frac{2\pi\bar{\phi}_r}{\beta\tanh\frac{2\pi}{\beta}X} \\ \langle T^{mat}_{\mu\nu}\rangle = 0 \end{cases}$$

AdS$_2$



boundary $X=\epsilon$

Setup for traversable wormhole in JT gravity (1)

- Consider JT gravity w/two boundary + many matter fields

$$S = I_{JT} + I_m[g, \chi]$$

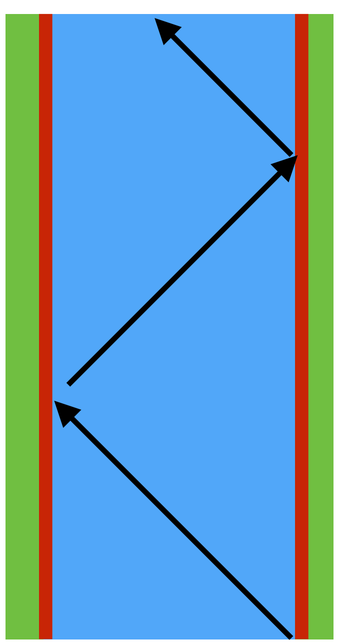
introduce double trace deformation for matters [Gao-Jafferis-Wall, 16] [Maldacena-Stanford-Yang, 17]

In dual description, we have

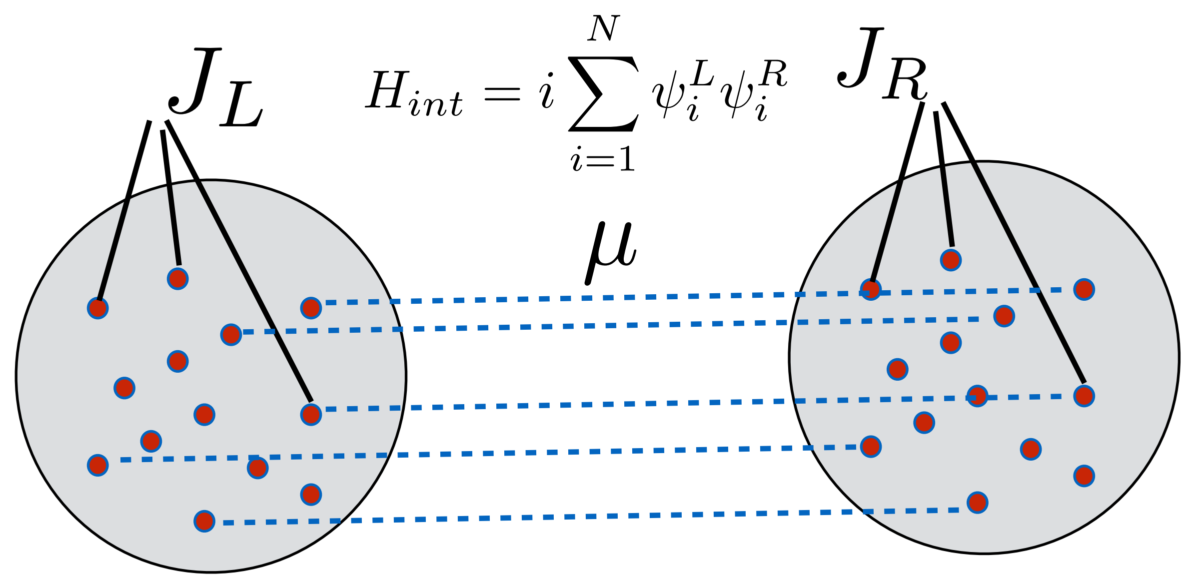
$$H = H_{QM_L} + H_{QM_R} + g\sum_{i=1}^{N} O_L^i(t) O_R^i(t)$$

In SYK case, $H_{QM} \to H_{SYK} = \sum J_{ijkl}\psi^i\psi^j\psi^k\psi^l \qquad O^i \to \psi^i$

- Both reduce to coupled Schwarzian theories



Traversable wormhole
=Global AdS2



two coupled SYK

Setup for traversable wormhole in JT gravity (2)

[Maldacena-Qi 18]
[Maldacena-Milekhin-Popov 18]

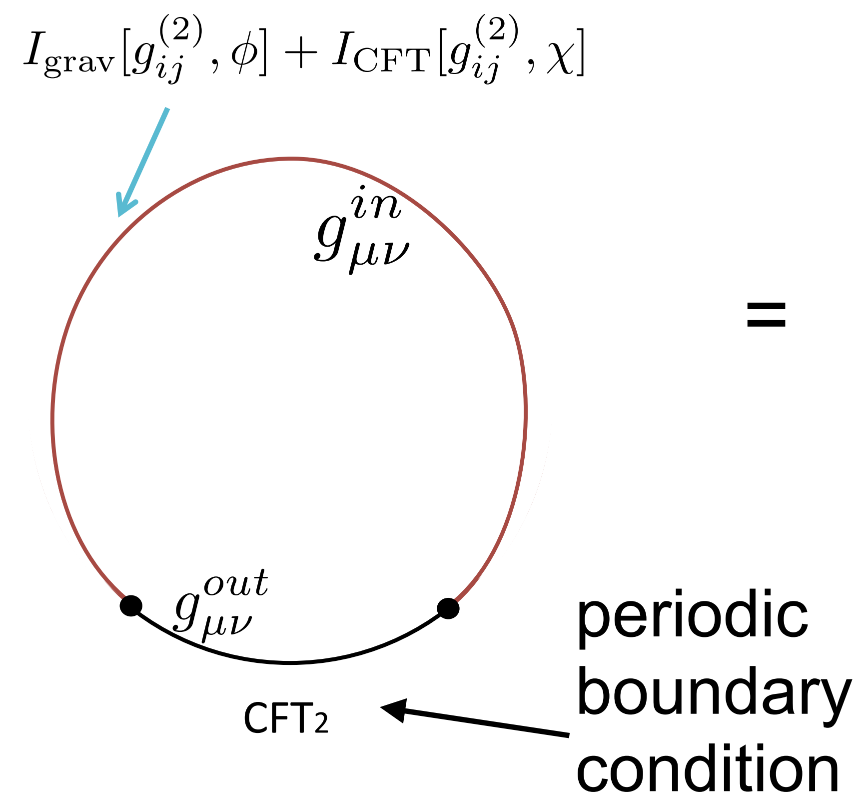
- Explicit traversable wormhole solution with conformal matters

$$\begin{cases} ds^2_{in} = ds^2_{\mathrm{AdS}_2} \quad \text{with} \quad ds^2_{in}|_{\mathrm{bdy}} = -\dfrac{dt^2}{\epsilon^2} \\ ds^2_{out} = \dfrac{-dt^2+dx^2}{\epsilon^2} \end{cases}$$

CFT: living on both of AdS_2 (in) and flat space (out) region

boundary condition *outside* is important to make traversable wormholes

2d Gravity

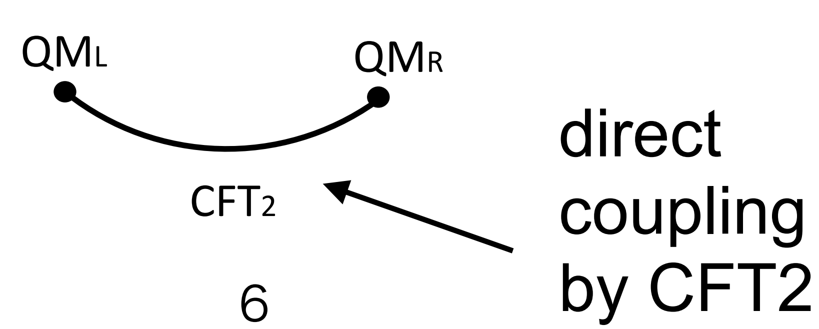


=

QM
(assume holography)

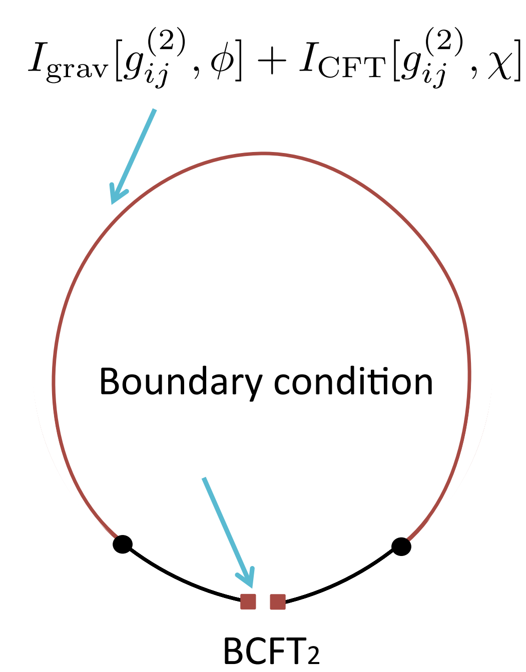
Ground state in
coupled system

$|G\rangle$



cf): boundary conditions

2d Gravity



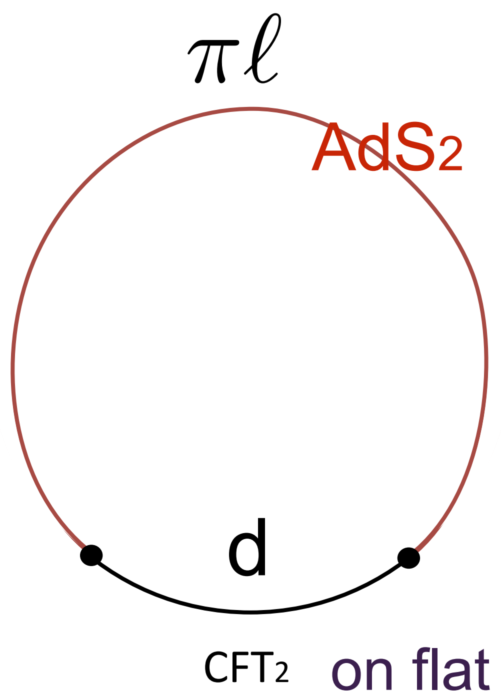
Traversable wormhole solution:

$$ds^2 = \frac{-dt^2 + d\sigma^2}{\ell^2 \sin^2 \frac{\sigma}{\ell}}\,, \quad \phi(\sigma) = \frac{2\bar{\phi}_r}{\pi\ell}\left[\frac{\frac{\pi}{2} - \sigma}{\tan\frac{\sigma}{\ell}} + 1\right]$$

$$\langle T^{mat}_{++}\rangle = \frac{c}{48\pi\ell^2} - \frac{\pi^2 c}{12(\pi\ell + d)^2}$$

Casimir energy: negative

($\frac{\pi^2 c}{48(\pi\ell + d)^2}$ for BCFT)



ℓ : "wormhole length", dynamically determined by EOM

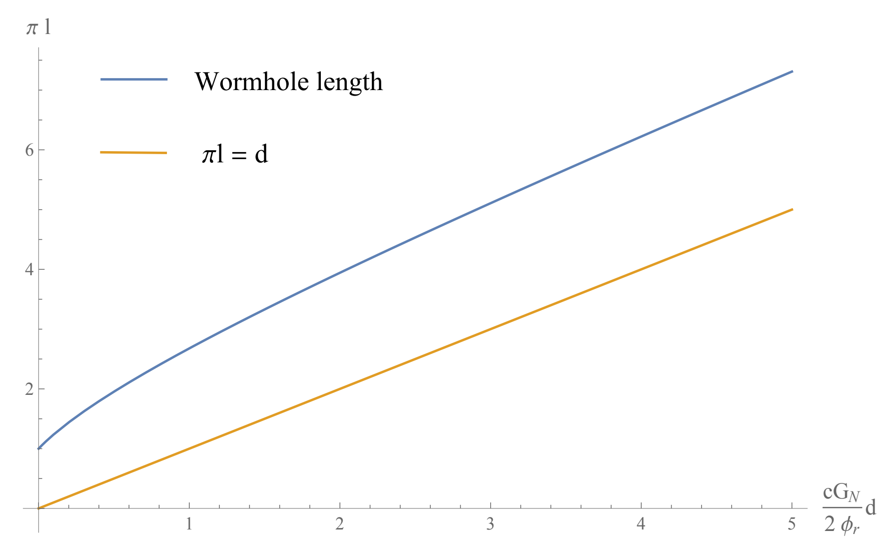
$$\ell = \ell(\bar{\phi}_r, c, d)$$

Alternatively: use variational method (approximate by TFD)

$$E(\ell) = 2 \times \frac{\pi\bar{\phi}_r}{4G_N} T_H^2 + \frac{c}{24\ell} - \frac{c\pi}{6(\pi\ell + d)}\,, \quad \ell = \frac{1}{2\pi T_H}$$

BH mass Weyl anomaly Casimir energy

minimize variational energy gives the same ℓ



$\pi\ell > d$: consequence of achronal ANEC

[Graham Olum 07]

Relations to 4d Black Holes

[Milekhin-Maldacena-Popov, 18]

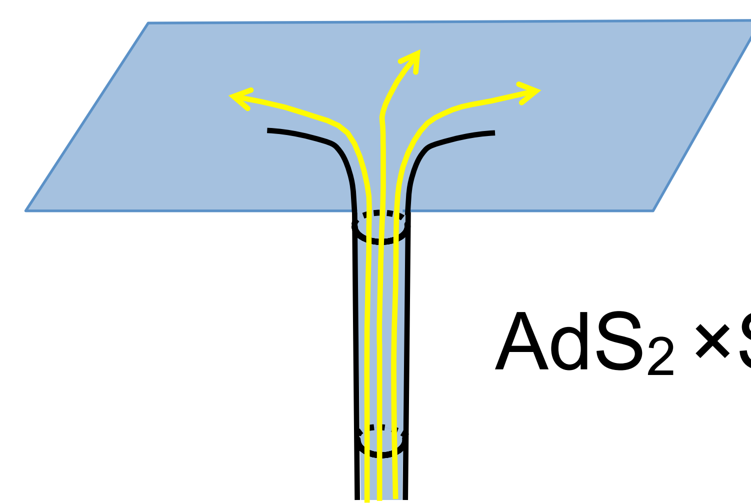
Theory: 4d gravity + Maxwell + massless Fermions

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G_N}R + \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi\right]$$

near extremal charged BH
→$AdS_2 \times S^2$ near horizon geometry
→appearance of nearly AdS_2 gravity

$$\begin{cases} ds^2 = -f(r)dt^2 + \dfrac{dr^2}{f(r)} + r^2 d\Omega_2^2 \\ f(r) = \left(1 - \dfrac{r_e}{r}\right)^2 \\ F = \dfrac{Q}{2}\sin\theta d\theta \wedge d\varphi \end{cases}$$

fermion under magnetic field
→ Landau degeneracy
→(1+1) d fermions on each magnetic line



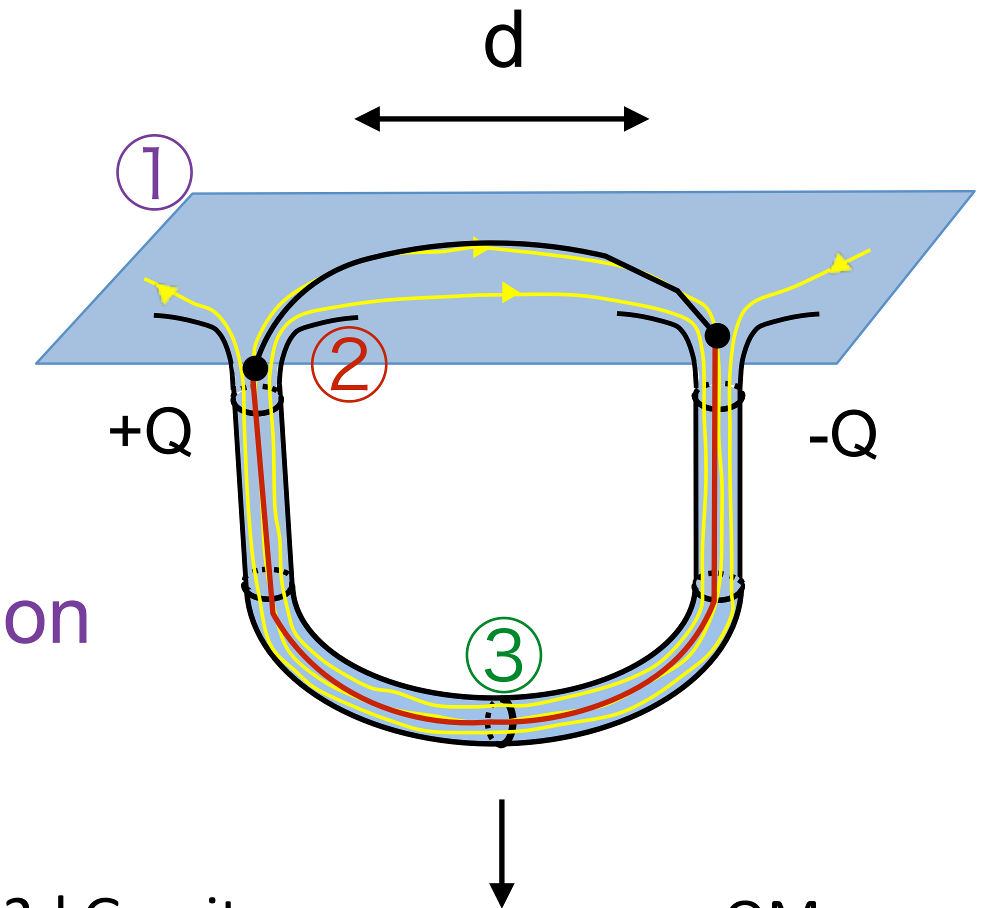
near horizon dynamics is described by
Jackiw-Teitelboim gravity + (1+1)d CFT

[Almheiri-Engerhardt-Marolf-Maxfield, 19] [Maldacena, 20]

Two oppositely magnetically charged BH

connection outside gives the direct interaction
→make wormholes traversable

[Gao-Jafferis-Wall, 16]



- ① :monopoles + anti-monopoles in flat region
- ② :magnetically charged BH
- ③ :Wormhole in JT + CFT

2d Gravity

QM
(assume holography)

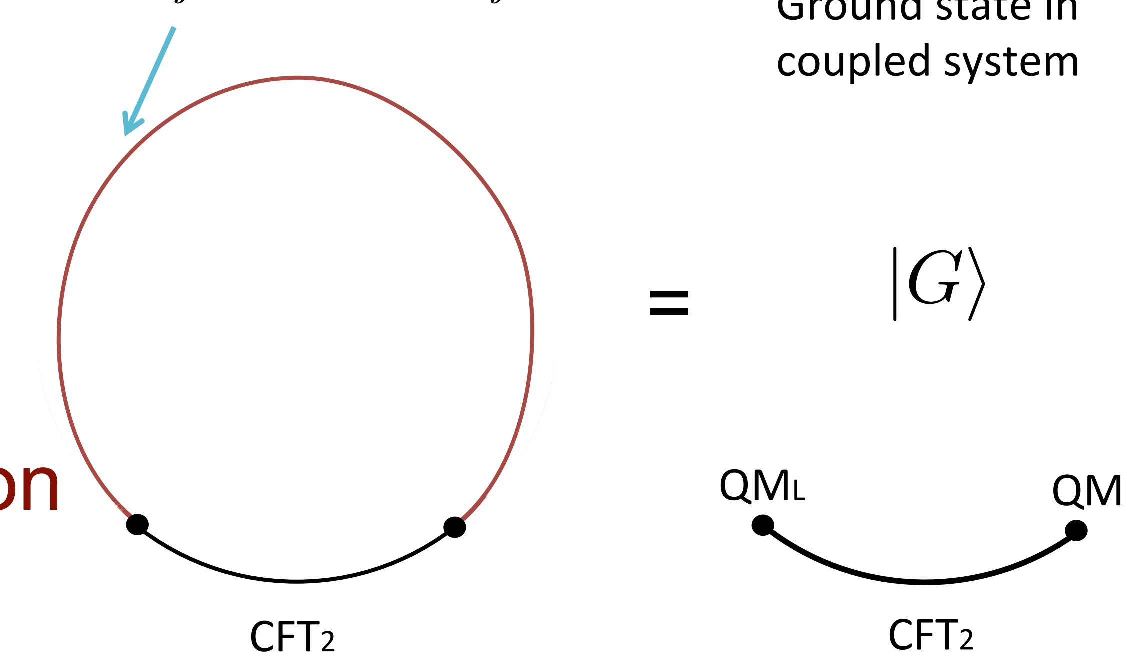
$I_{\text{grav}}[g_{ij}^{(2)}, \phi] + I_{\text{CFT}}[g_{ij}^{(2)}, \chi]$

Ground state in coupled system

Effectively described by…

(1+1)d CFT living on a circle

dynamical JT gravity turned on red region



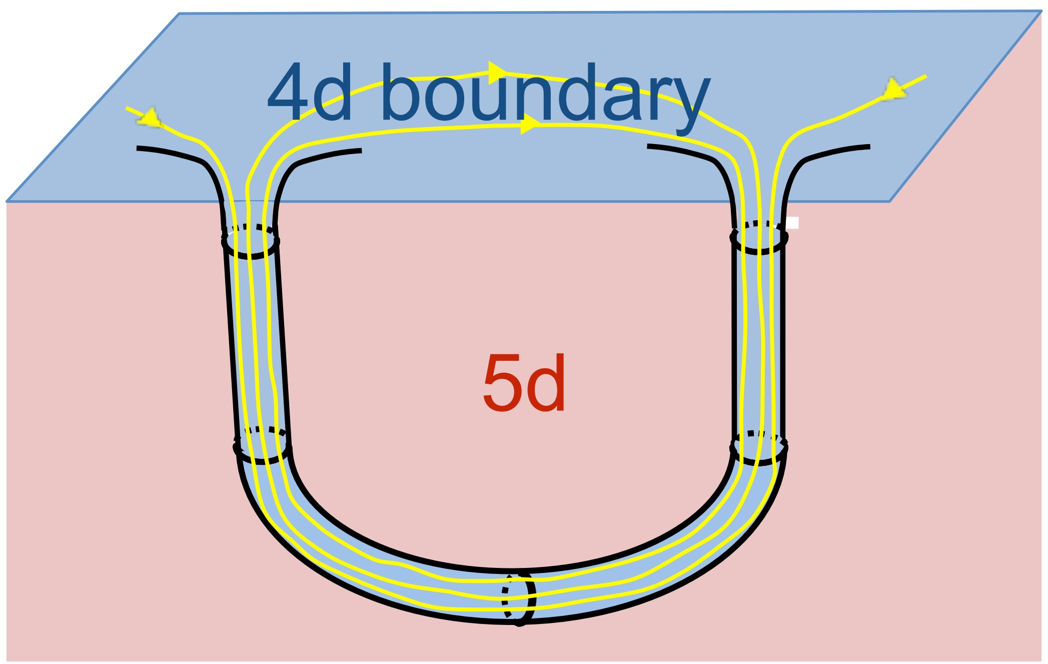
4d gravity + Holographic 4d CFT + Maxwell

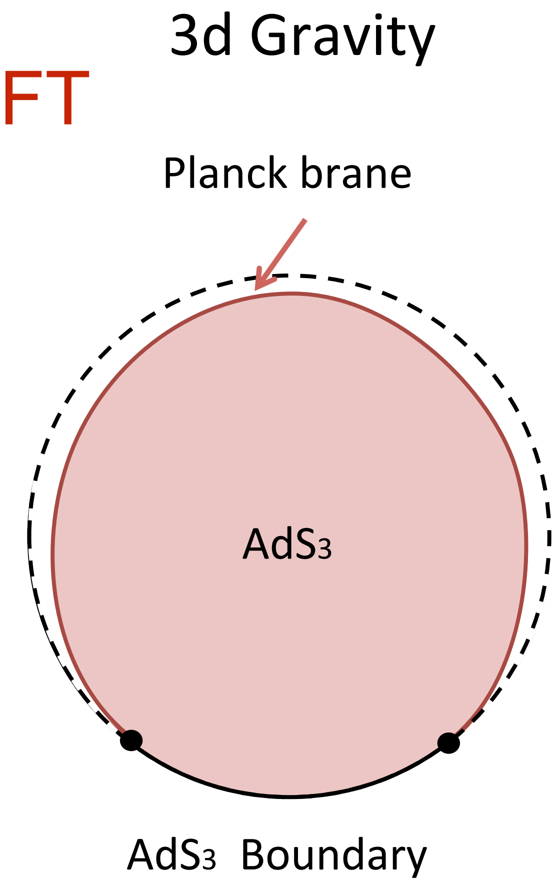
[Milekhin-Maldacena, 20]

= Randall-Sundrum II

magnetic field: $AdS_5 \to AdS_3 \times R^2$ [D'Hoker-Kraus, 08]

→ Jackiw-Teitelboim gravity + (1+1)d holographic CFT





[Almheiri-Mahajan-Maldacena-Zhao, 19]

Filling outside the wormholes, related to topological censorship

[cf: Galloway-Schleich-Witt-Woolgar, 99]

Holographic matter can be thought of geometrization of entanglement

1. Traversable wormholes in 4d and in JT gravity (Review)

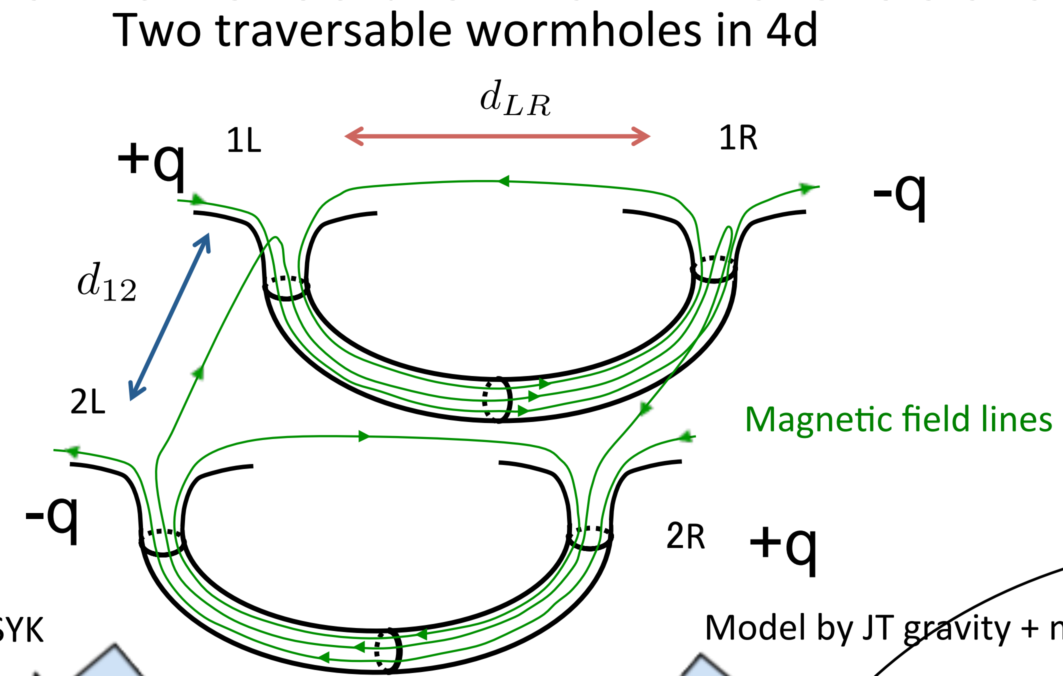
2. Two traversable wormholes by 4 coupled JT gravity

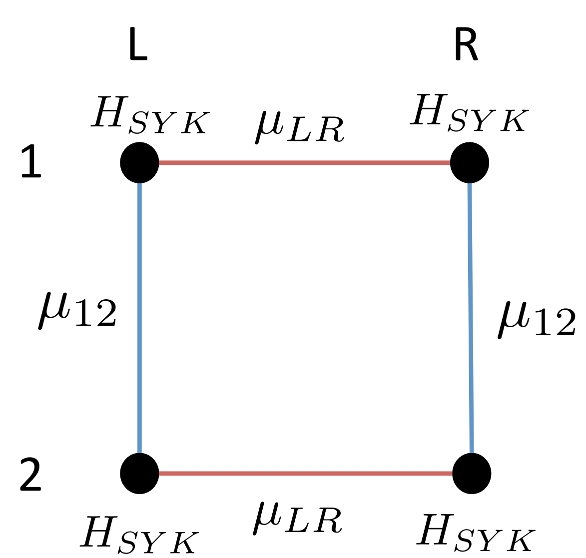
3. Relation to Euclidean wormhole: Bra-ket wormhole

4. Conclusion/ Future works

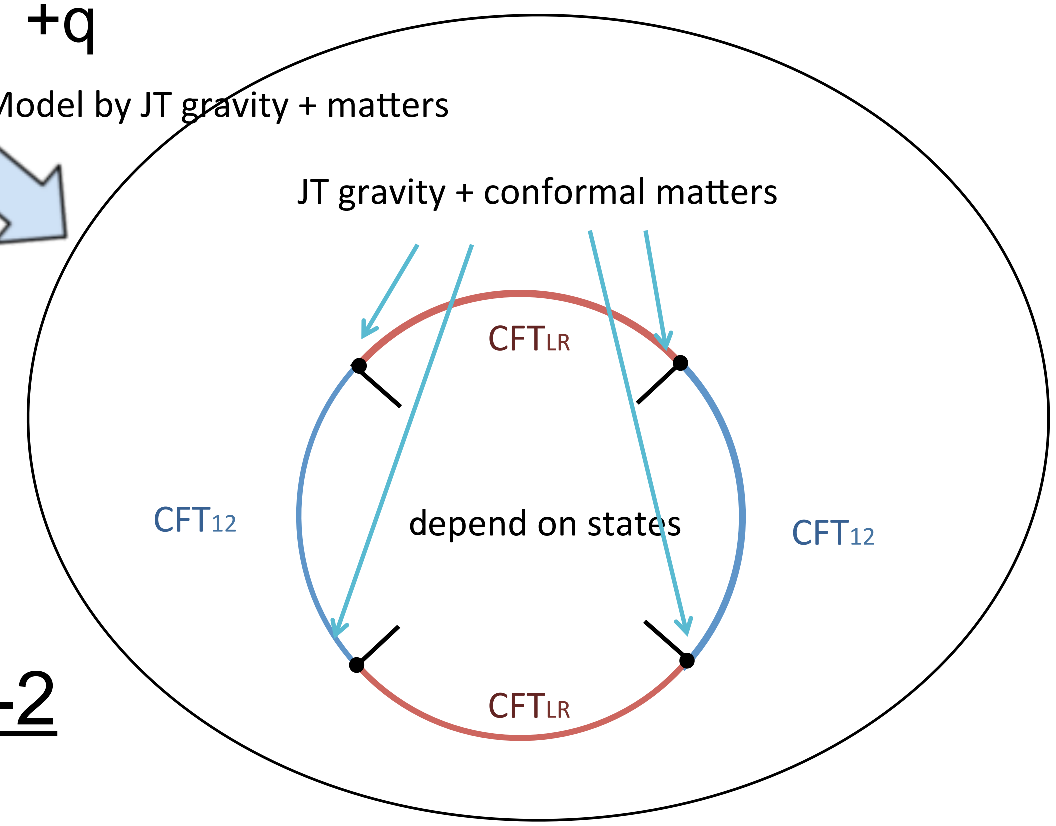
2. Four coupled JT gravities

Motivation:study of two traversable wormhole sectors in 4d:



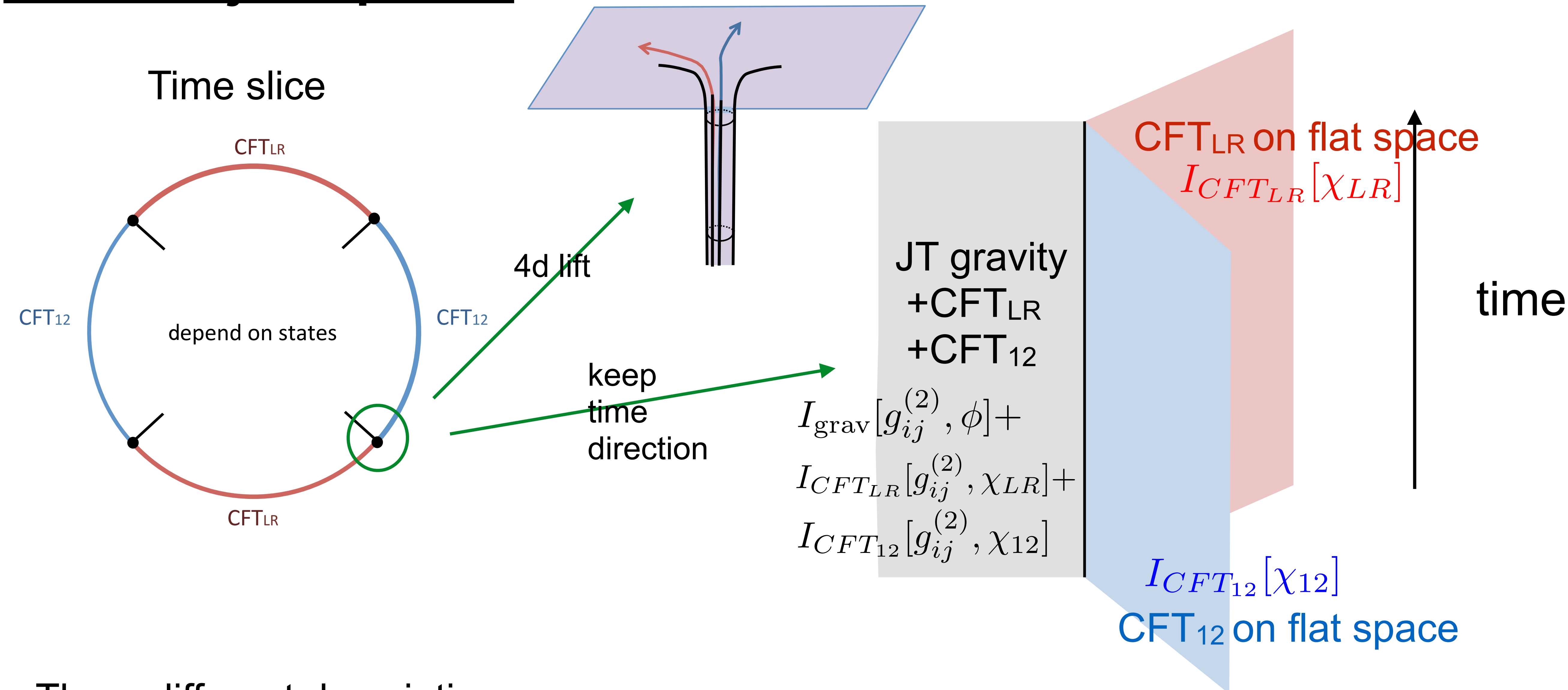


Z$_2$ symmetry:
exchange L-R and 1-2



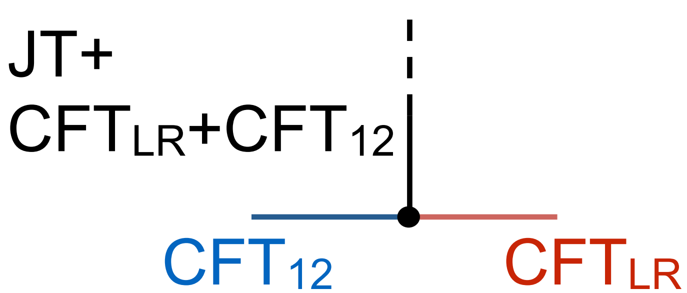
- Model by JT/SYK, study solutions, physical quantities, phase structures etc…
  - study the role of Z2 symmetry
  - the effect of boundary conditions outside the wormholes

Near the joint points

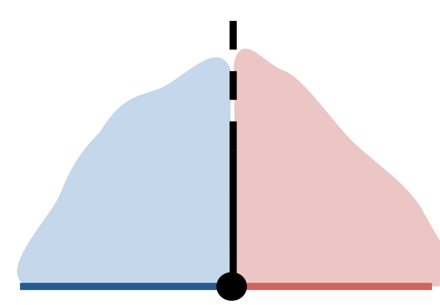


Three different descriptions

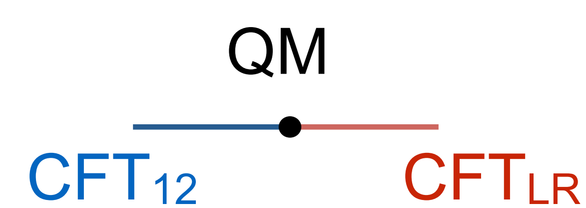
2d gravity



3d gravity

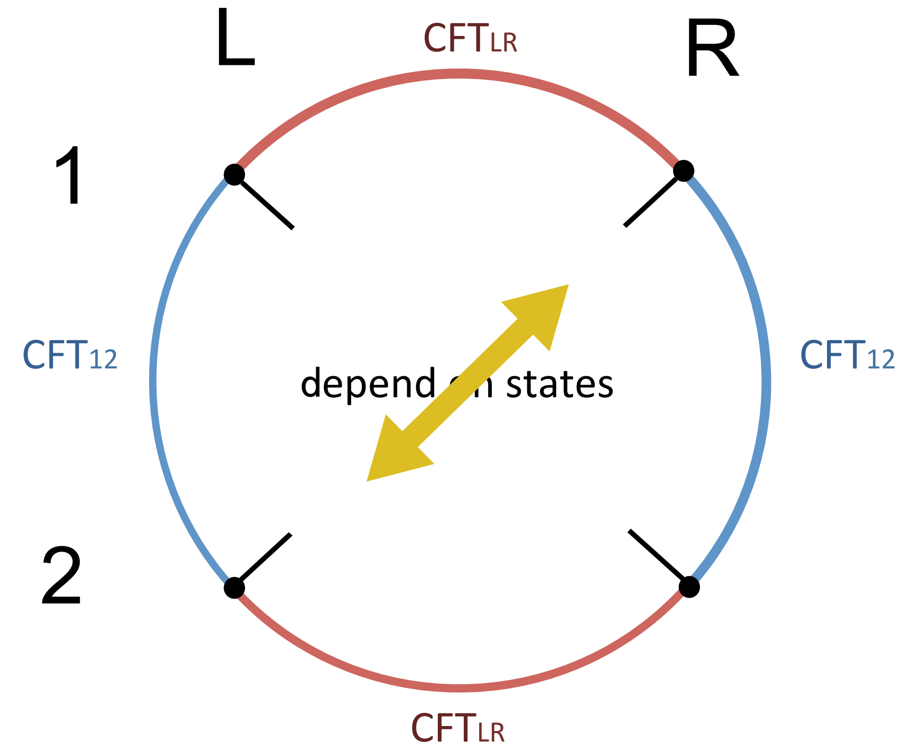


QM



three solutions:

There are several ways to connect :

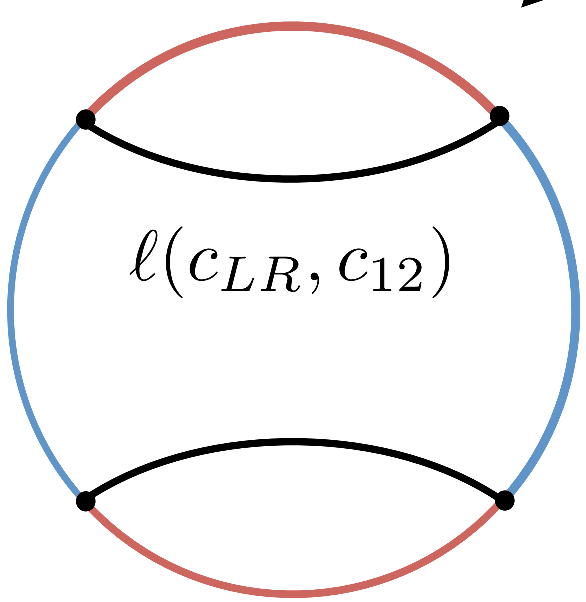


Wormhole solution:

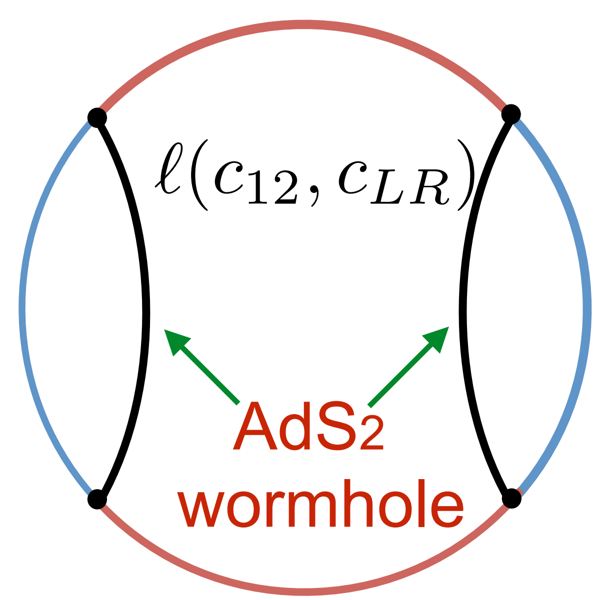
$$\begin{cases} ds^2 = \dfrac{-dt^2 + d\sigma^2}{\ell^2 \sin^2 \frac{\sigma}{\ell}} \\ \phi(\sigma) = \dfrac{2\bar{\phi}_r}{\pi \ell}\left[\dfrac{\frac{\pi}{2} - \sigma}{\tan \frac{\sigma}{\ell}} + 1\right] \end{cases}$$

symmetric solution:

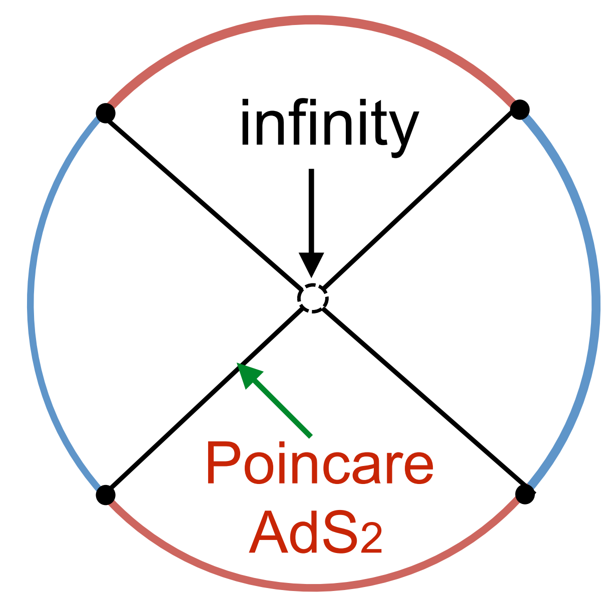
$$\begin{cases} ds^2 = \dfrac{-dt^2 + d\sigma^2}{\sigma^2} \\ \phi(\sigma) = \dfrac{\bar{\phi}_r}{\sigma} \end{cases}$$



L-R wormhole



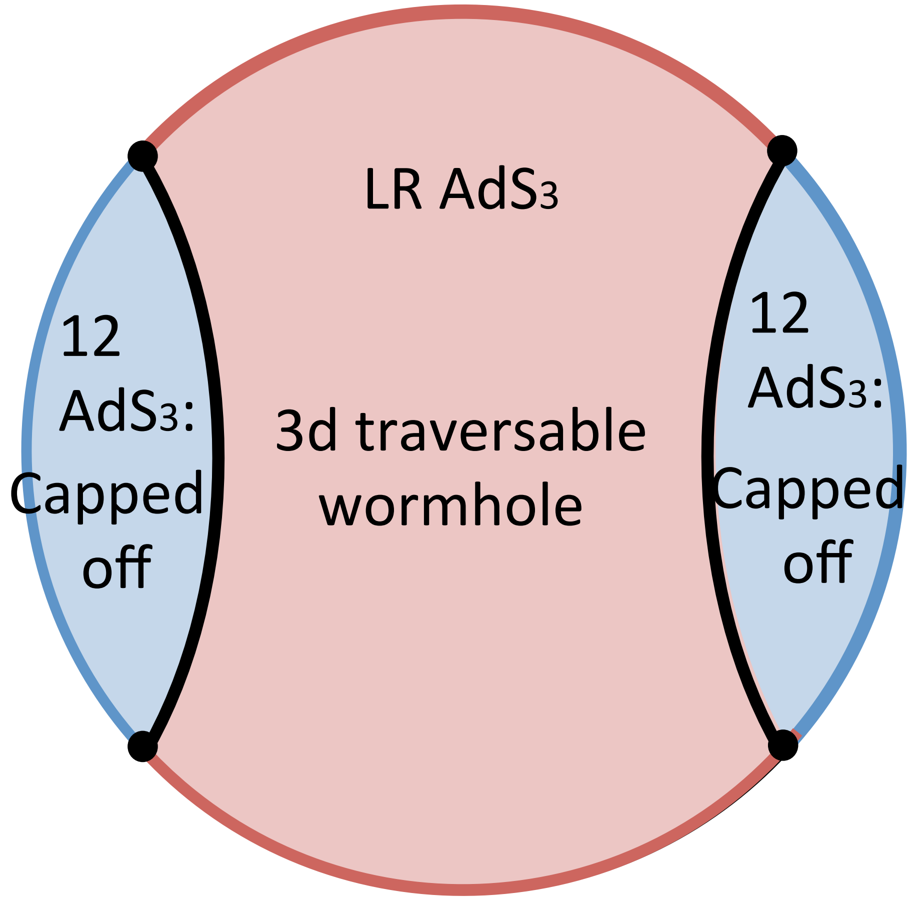
1-2 wormhole



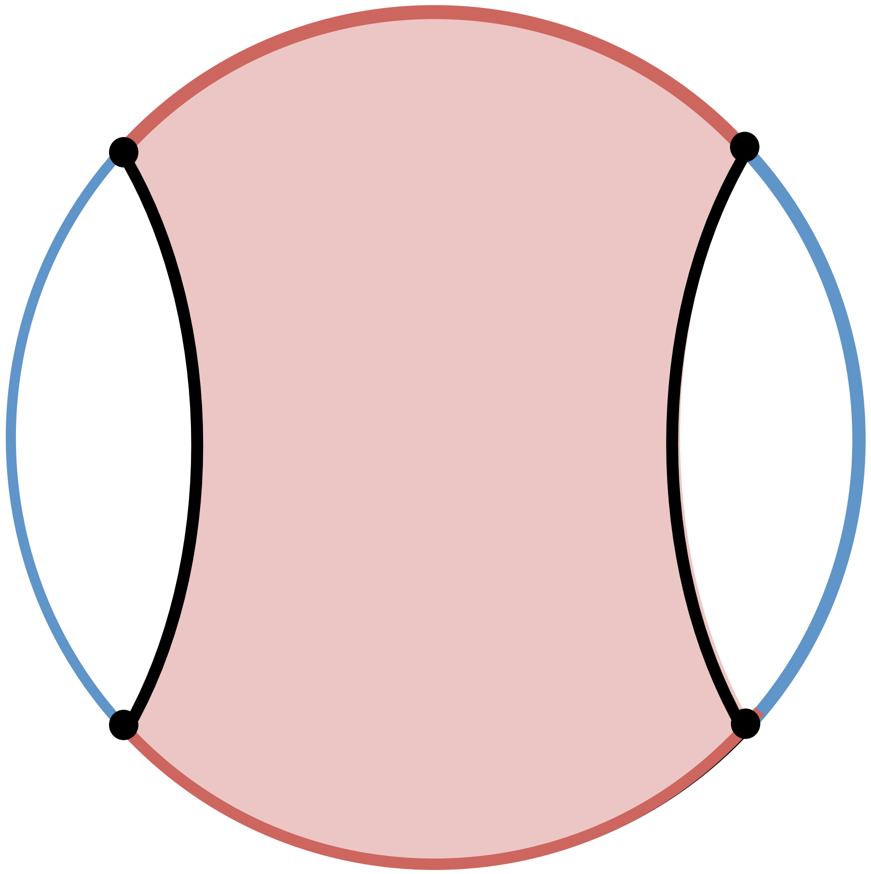
symmetric solution
(no wormhole,
only matter entanglement)

Wormhole solution with Holographic matters:

Both of CFT_{LR} and CFT_{12} are holographic



Only CFT_{LR} is holographic

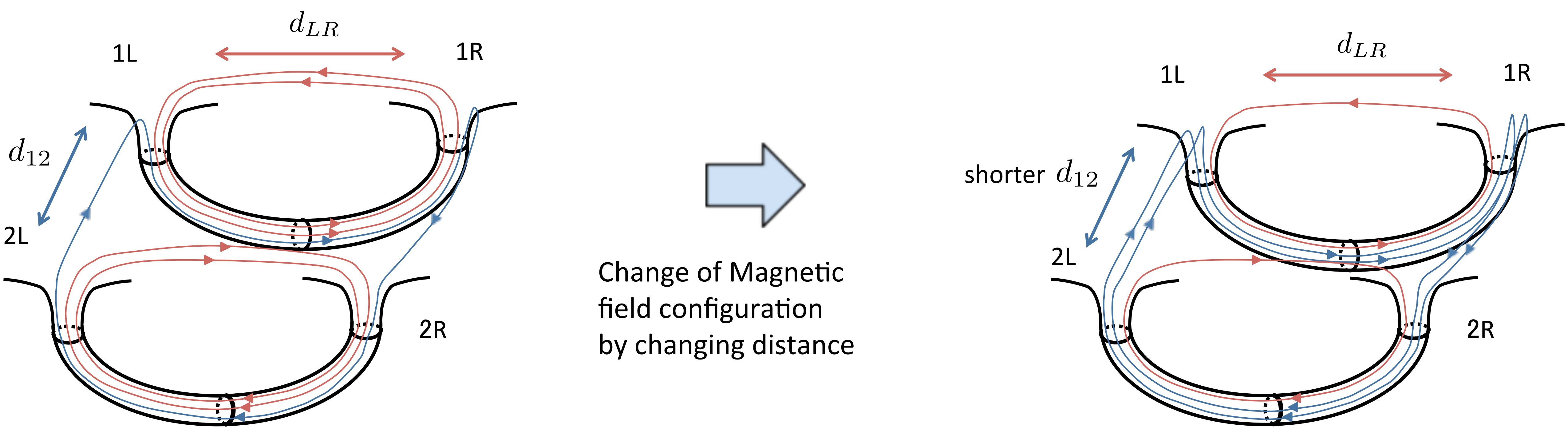


$$\langle T^{12}_{++}\rangle = \frac{c_{12}}{48\pi\ell^2} - \frac{\pi^2 c_{12}}{12(\pi\ell + d_{12})^2}$$

$$\langle T^{LR}_{++}\rangle = \frac{c_{LR}}{48\pi\ell^2} - \frac{\pi^2 c_{LR}}{48(\pi\ell + d_{LR})^2}$$

changing boundary conditions and phase transition (1):

We can compute energy in each solution:



Parameter: central charges c_{LR}, c_{12}

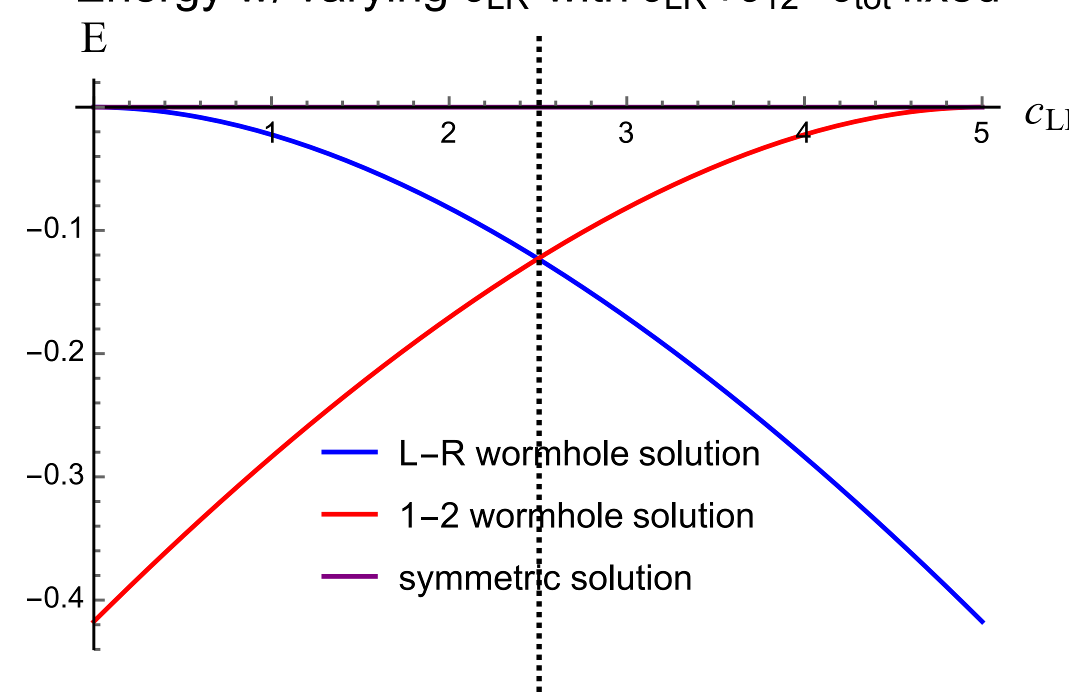
$c_{LR} + c_{12} = c_{tot}$:kept, change c_{LR}

change ℓ, then change energy E

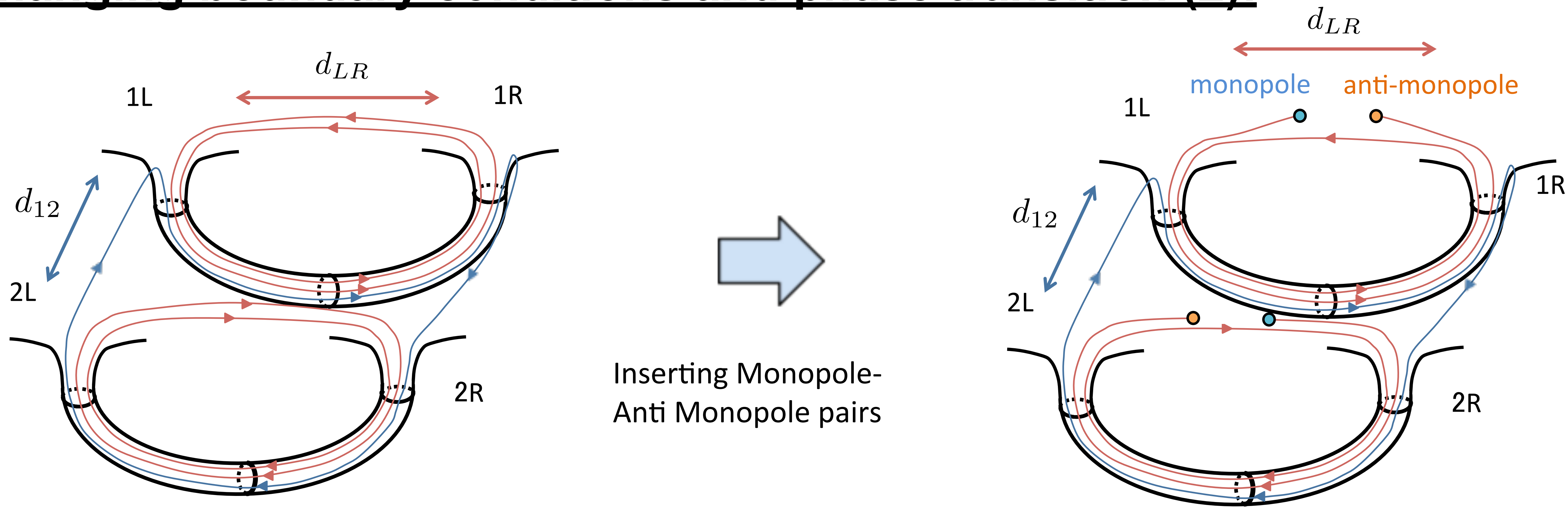
Theory has Z_2 symmetry at $c_{LR} = c_{12}$

Z_2 symmetry is broken at $c_{LR} = c_{12}$ point

Energy w/ varying c_{LR} with $c_{LR}+c_{12}=c_{tot}$ fixed



changing boundary conditions and phase transition (2):

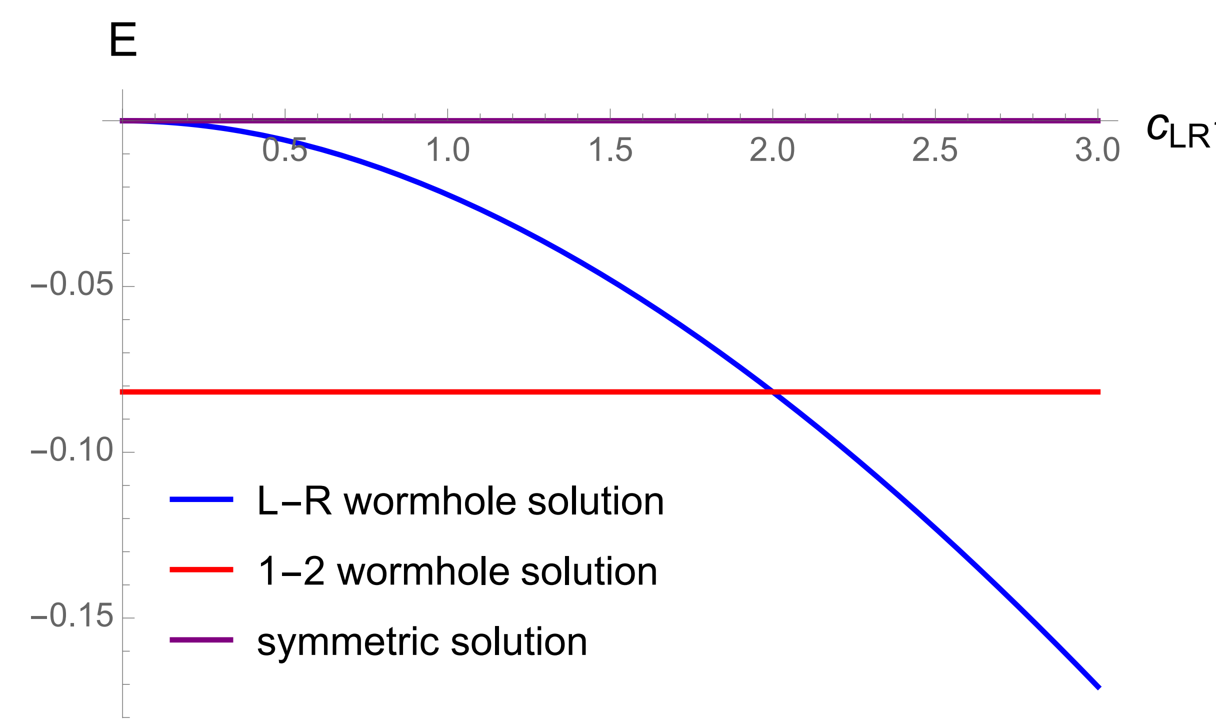


part of CFT_{LR} becomes BCFT [Callan, Rubakov…]

c_{12} :kept, $c_{LR}^{j} + c_{LR}^{s} = c_{LR}$:kept,
change c_{LR}^{j}

c_{LR}^{s} : central charge of BCFT

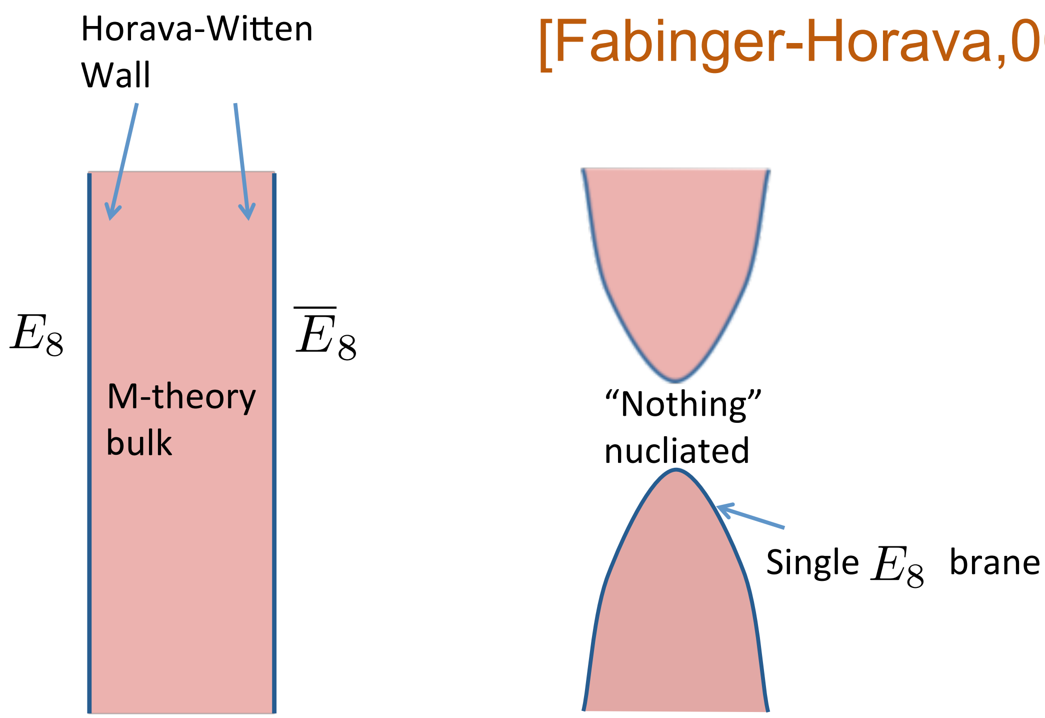
Phase transition at $c_{LR}^{j} = c_{12}$



Connection to Bubble of wormholes:

Bubble of wormhole

[Fabinger-Horava,00]

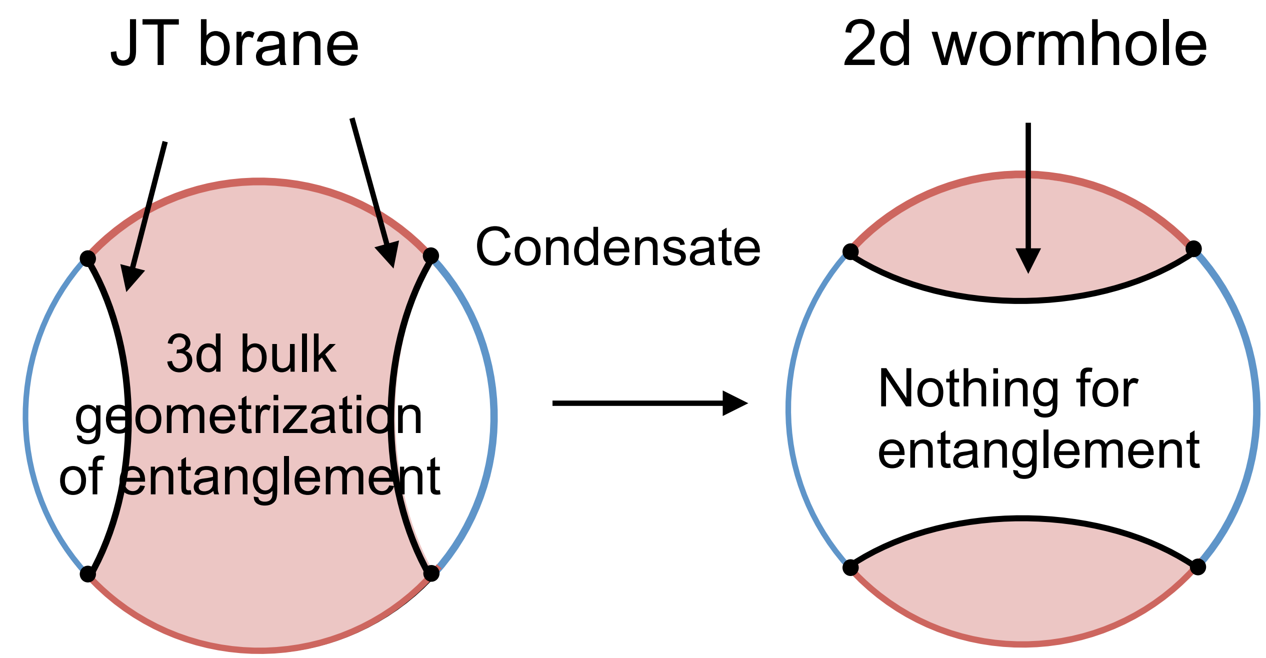


“Half of bubble of nothing [Witten,82] ”

Tachyonic E_8 string and its condensation were also constructed

[Horava-Keeler,07]

our case:



cf) wormhole from entangled tachyonic strings

[Jafferis-Schneider,21]

1. Traversable wormholes in 4d and in JT gravity (Review)

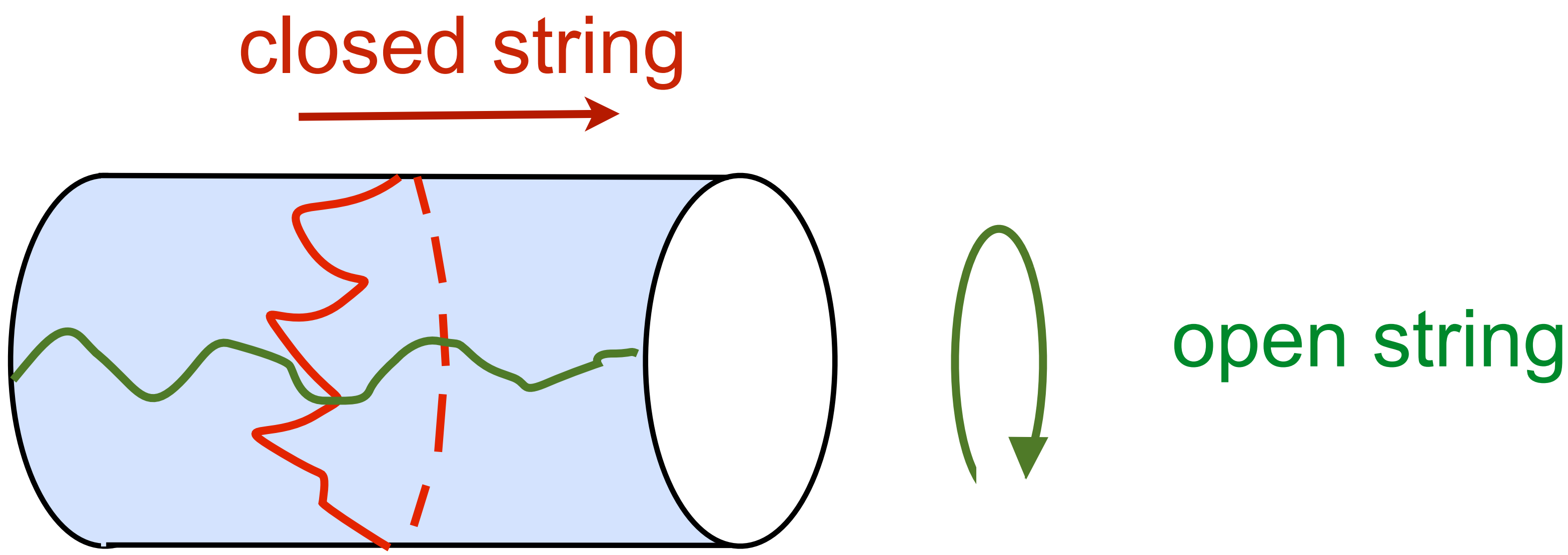
2. Two traversable wormholes by 4 coupled JT gravity

3. Relation to Euclidean wormhole: Bra-ket wormhole

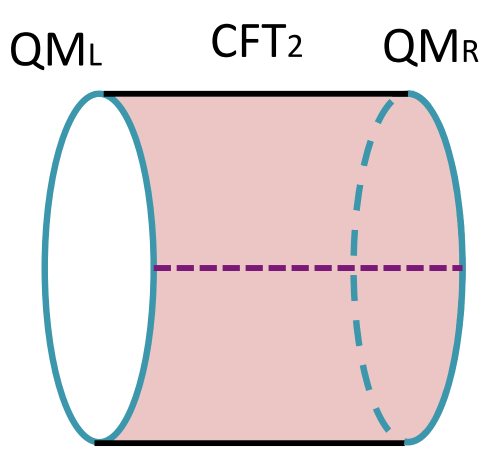
4. Conclusion/ Future works

3. Relation to Euclidean wormhole: Bra-ket wormhole interpretation

First remember the open closed duality in CFT



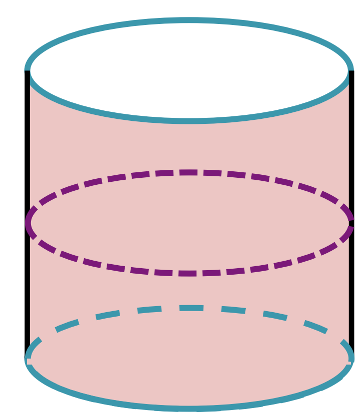
Similarly, in QFT coupled to holographic defect



"open channel"

= partition function of CFT coupled to QML and QMR

$$\mathcal{H} = \mathcal{H}^{\mathrm{line}}_{\mathrm{CFT}_2} \otimes \mathcal{H}_{\mathrm{QM}_L} \otimes \mathcal{H}_{\mathrm{QM}_R}$$



"closed channel"

= overlap of gravitationally prepared state

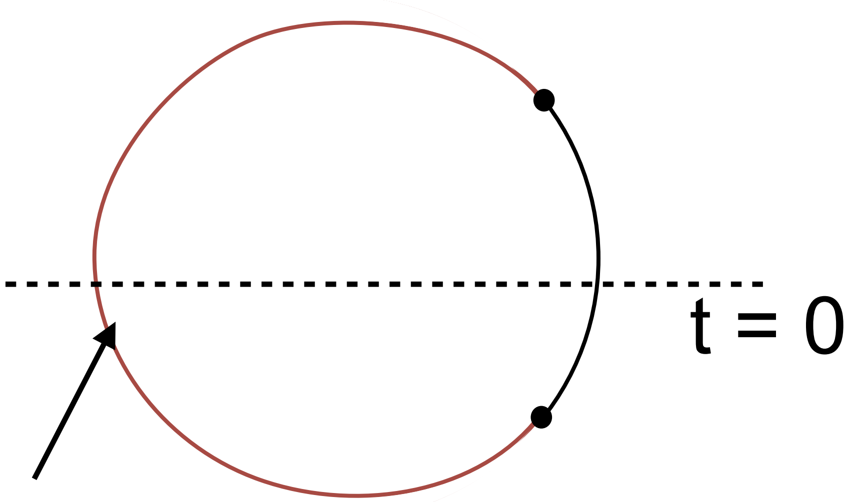
$$\mathcal{H} = \mathcal{H}^{\mathrm{cylinder}}_{\mathrm{CFT}_2}$$

[cf: Chen-Gorbenko-Maldacena, 20]

3. Bra-ket wormhole interpretation

start from traversable wormhole, exchange (Euclidean) time and space

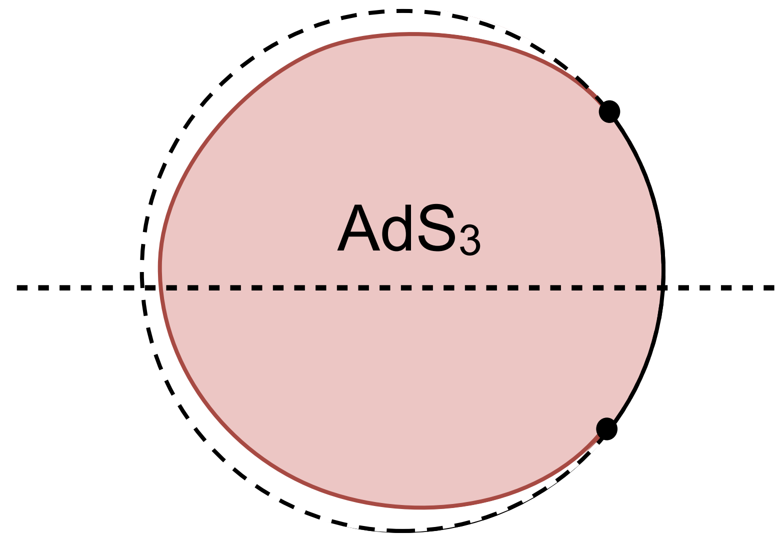
2d gravity



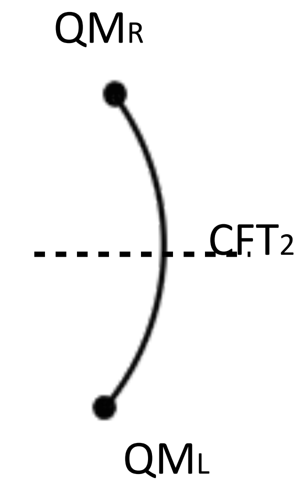
wormhole connects bra and ket

[cf: Page, 86]

3d gravity



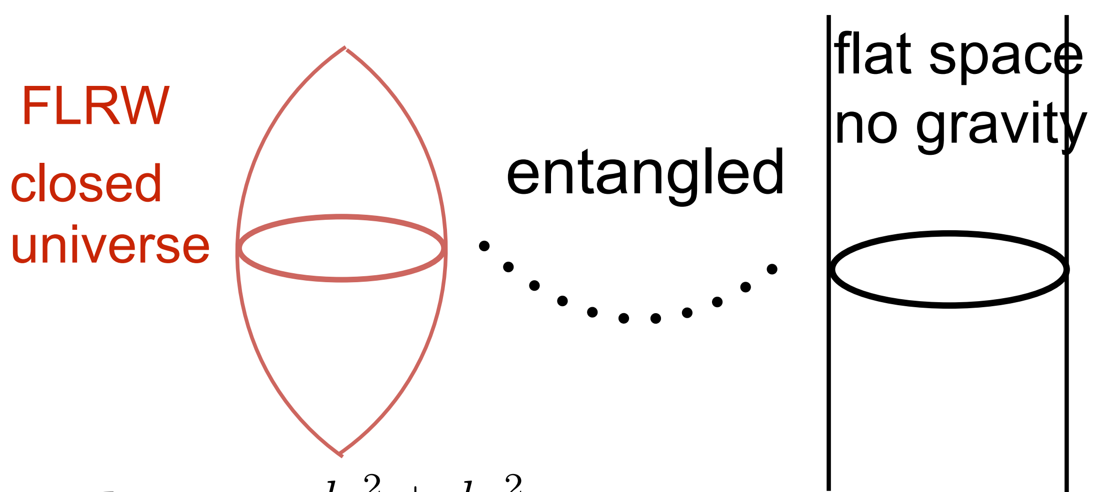
QM



$$|\Psi\rangle \in \mathcal{H}_{CFT}$$

$$|\Psi\rangle \sim \int \mathcal{D}J \; Z_{JT}[J(x)] e^{-\frac{\tau}{2} H_{CFT}} \, |J(x)\rangle_{CFT}$$

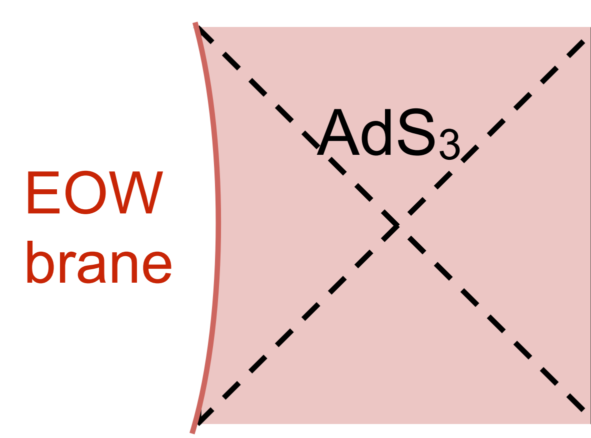
wick rotation $\tau = it$ in 2d gravity description



$$\begin{cases} ds^2 = \dfrac{-d\eta^2 + d\chi^2}{\cosh^2 \eta} \\ \phi(\eta) = \dfrac{c}{4}(1 - \eta \tanh \eta) \end{cases}$$

[cf:Maldacena-Maoz,04]

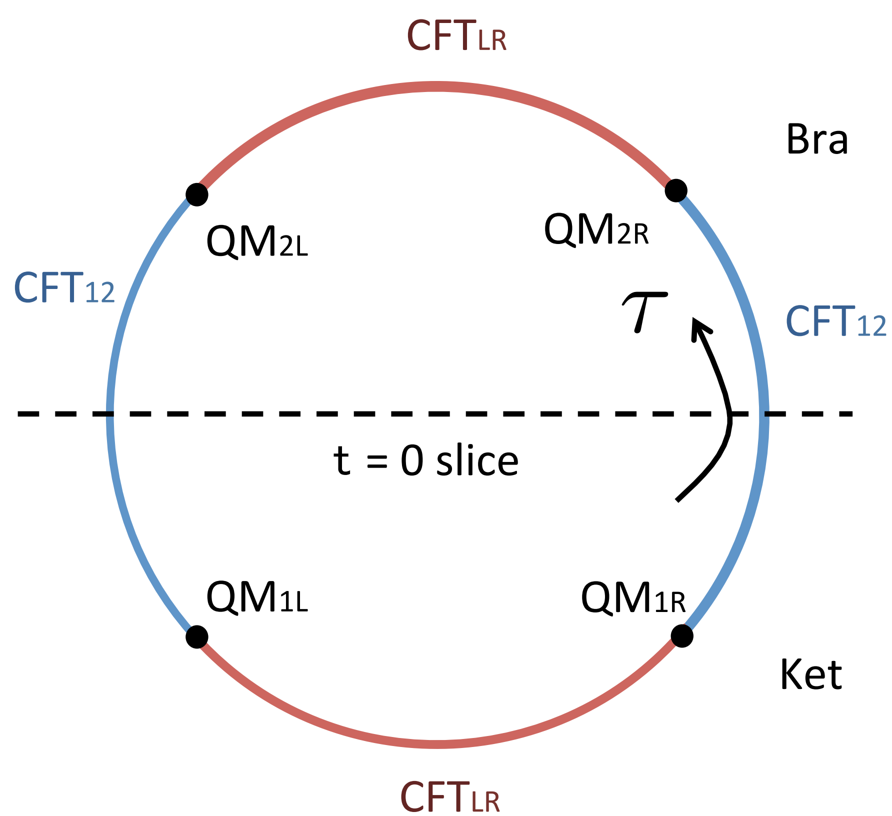
3d gravity description

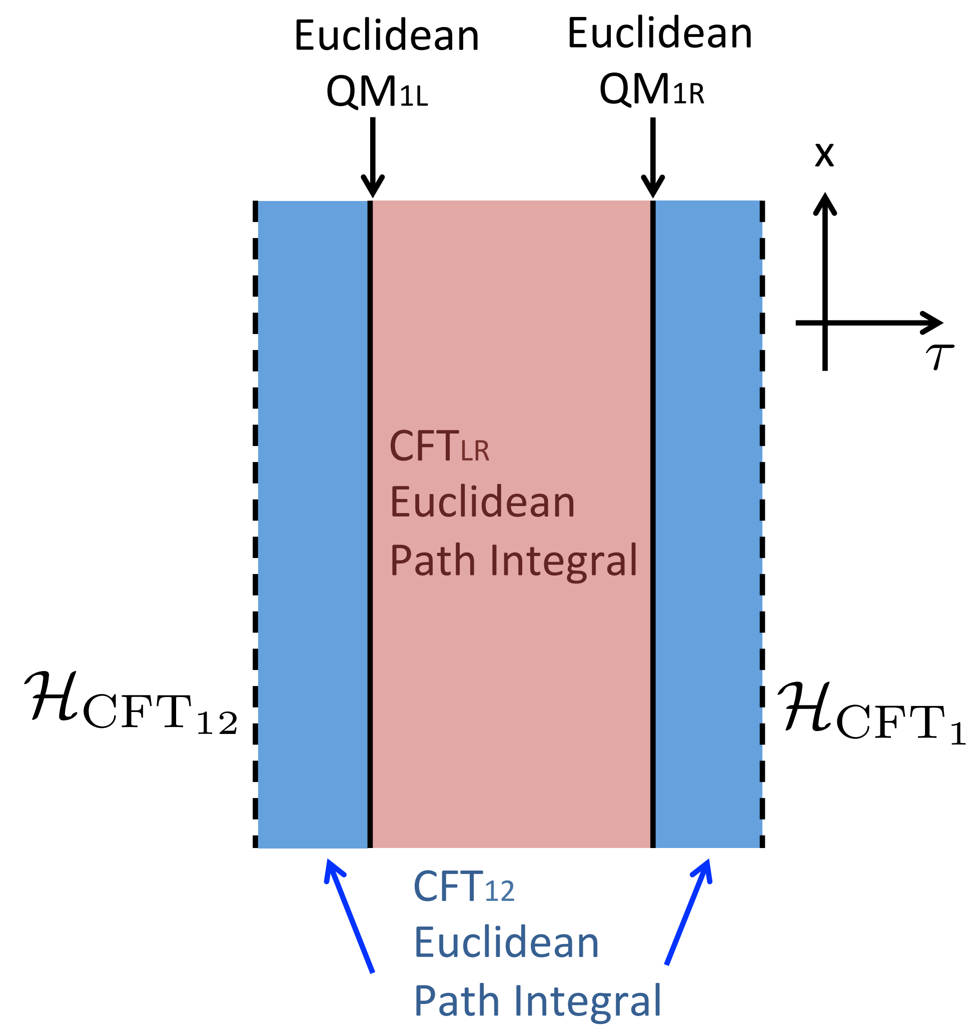


[cf:Cooper-Rozali-Swingle-Raamsdonk-Waddell-Wakeham, 18]

four coupled case:

QM description





prepare a state $|\Psi_{12}\rangle \in \mathcal{H}_{CFT_{12}} \otimes \mathcal{H}_{CFT_{12}}$

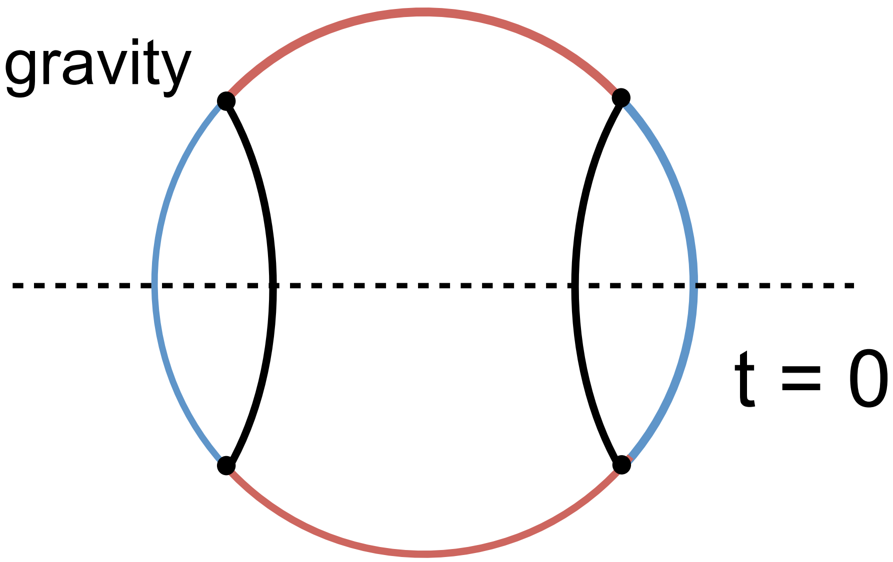
$\langle\Psi_{12}|\Psi_{12}\rangle = Z \sim e^{-LE}$ $\rightarrow$ smaller energy in traversable wormholes picture dominate

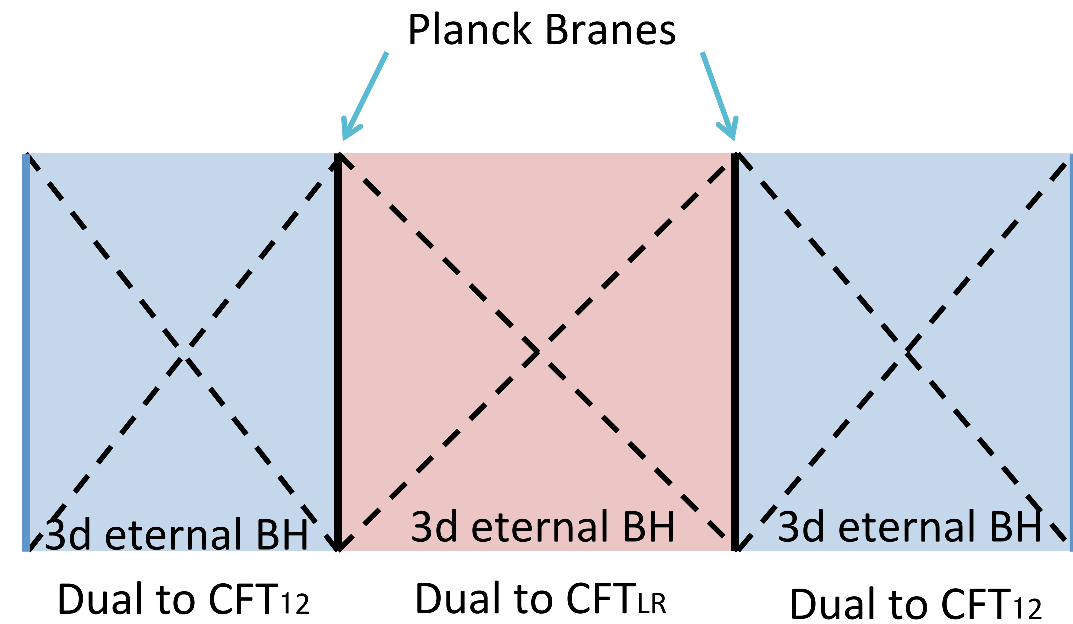
four coupled case:

3d Gravity

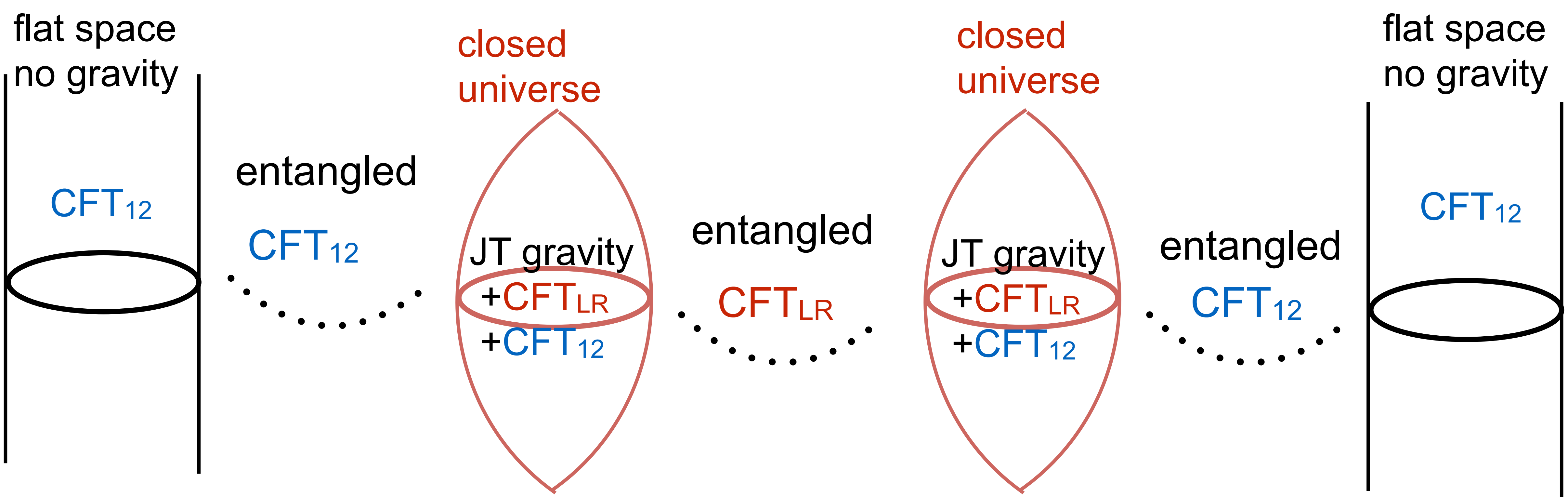
bra-ket wormhole state

2d gravity



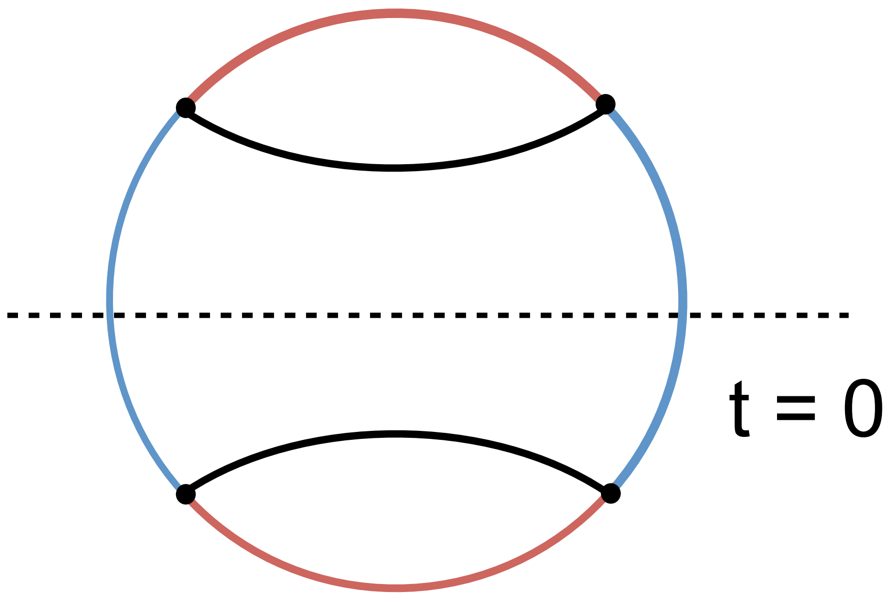


dominant for $c_{LR} < c_{12}$



four coupled case:

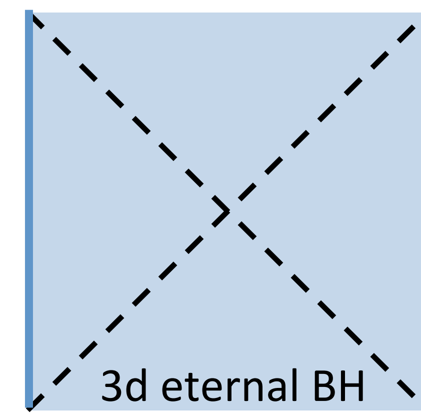
no bra-ket wormhole state



dominant for $c_{LR} > c_{12}$

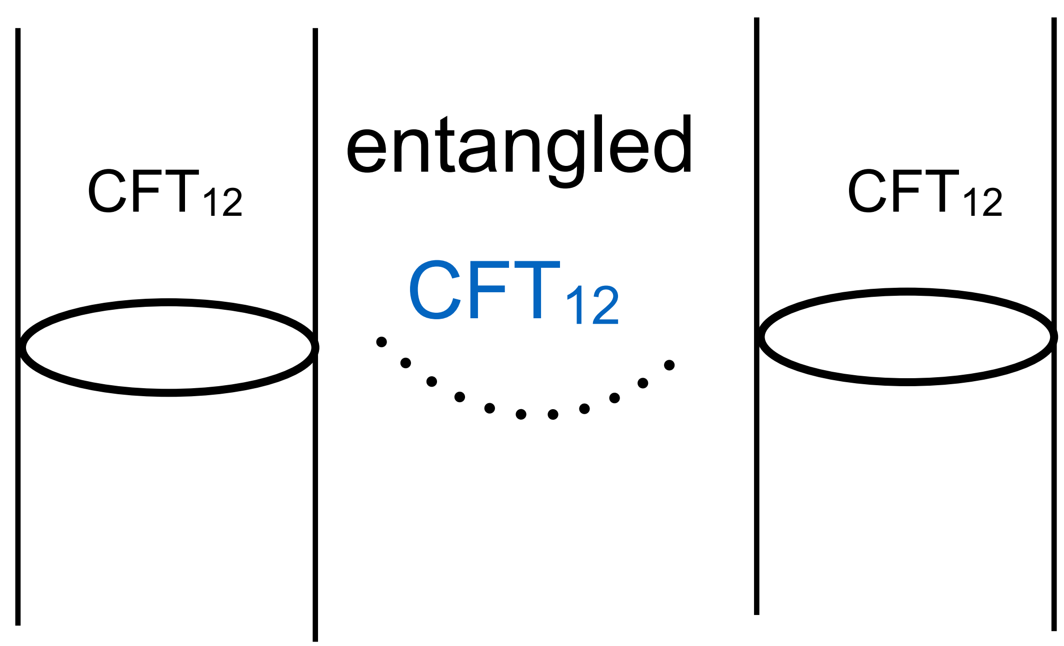
two closed universe annihilate
in Euclidean regime

3d Gravity



Dual to CFT_{12}

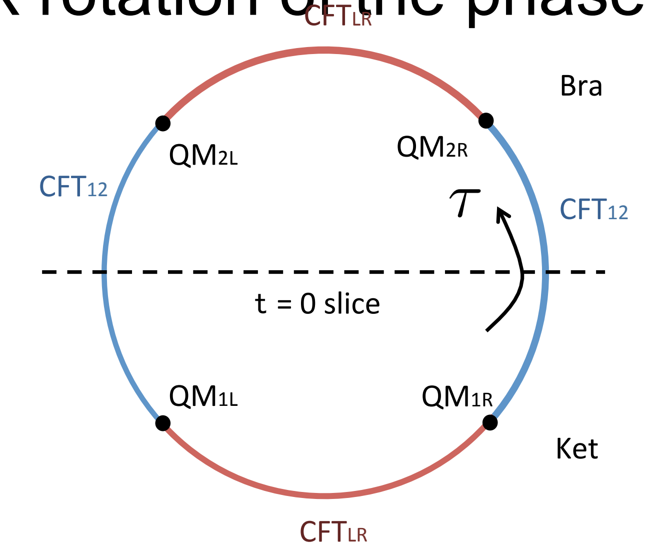
"2d gravity description"



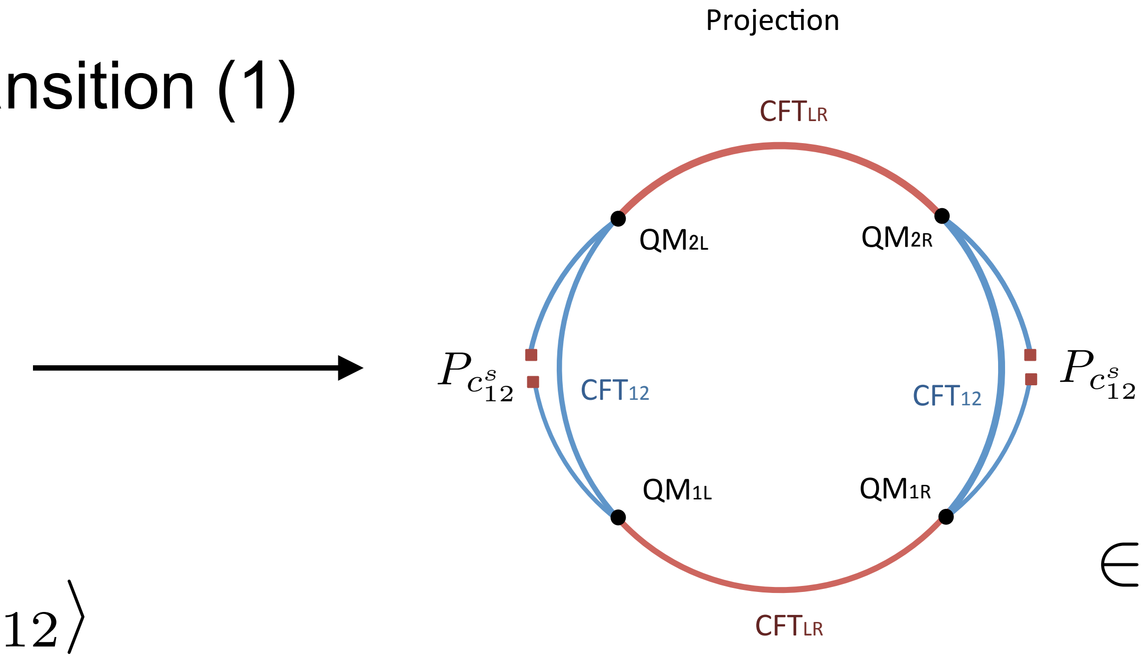
TFD state with $\beta = 2(\pi\ell + d_{12})$

Effect of projection by boundary states :

Wick rotation of the phase transition (1)

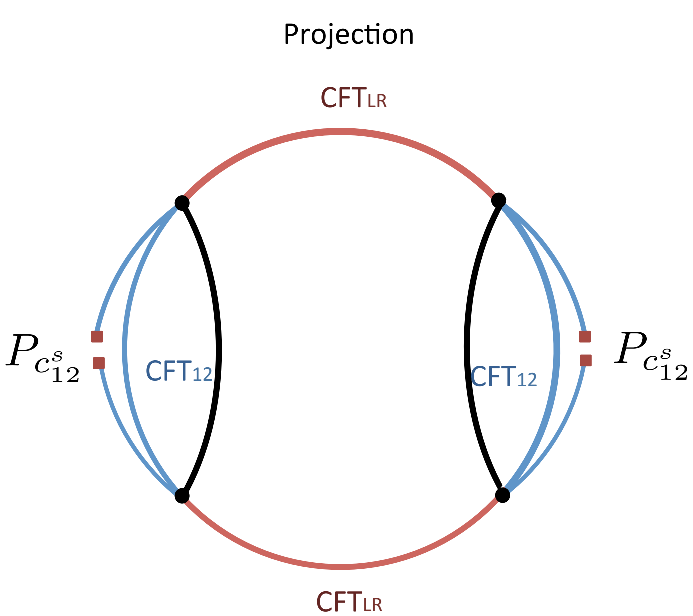


$|\Psi_{12}\rangle$

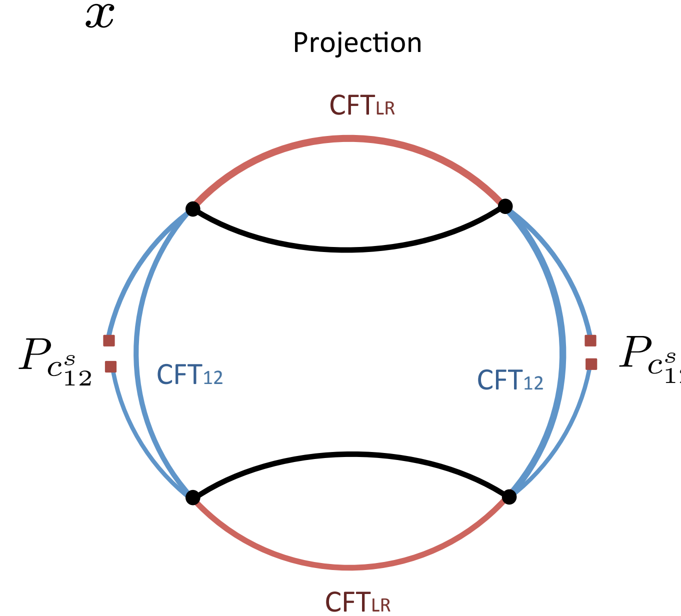


$$\langle B^L_{c_{12^s}}, B^R_{c_{12^s}} |\Psi_{12}\rangle$$
$$\in \mathcal{H}_{CFT^j_{12}} \otimes \mathcal{H}_{CFT^j_{12}}$$

Partial projection onto boundary states $|B\rangle \sim \prod_x |\psi_x\rangle$ for CFT$_{12^s}$



$c_{LR} < c_{12^j}$



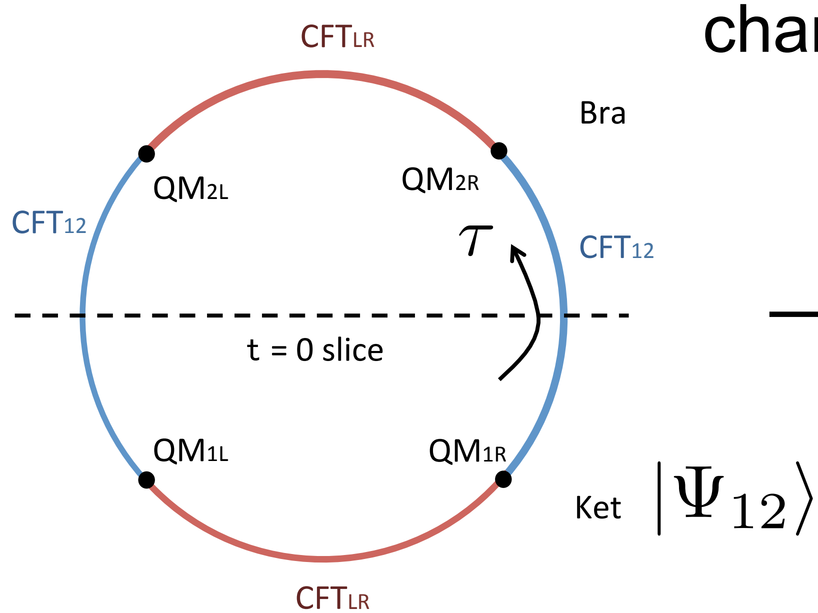
$c_{LR} > c_{12^j}$

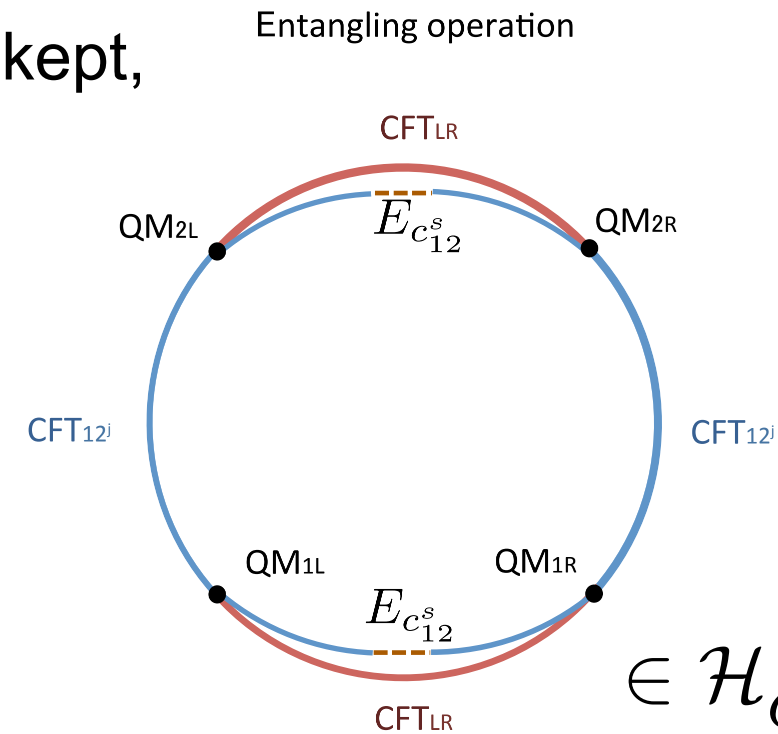
- Maybe sum over final state ~ average, wormhole appears
- It will be interesting to consider projection on Hawking radiations [cf: Marolf-Maxfield,20] [Qi ,21]

Effects of entangling operations:

Wick rotation of the phase transition (2)

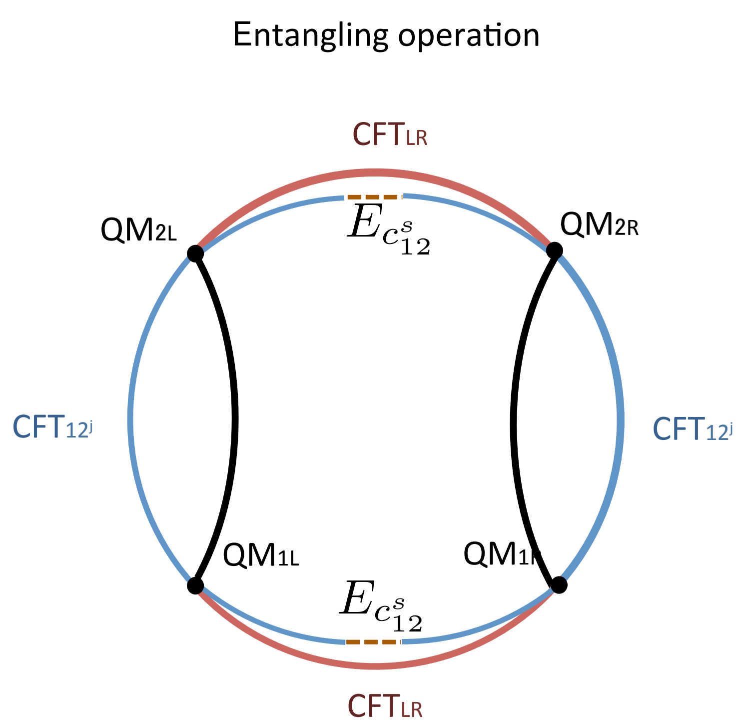
$c_{LR} + c_{12} = c_{tot}$:kept,
change c_{LR}



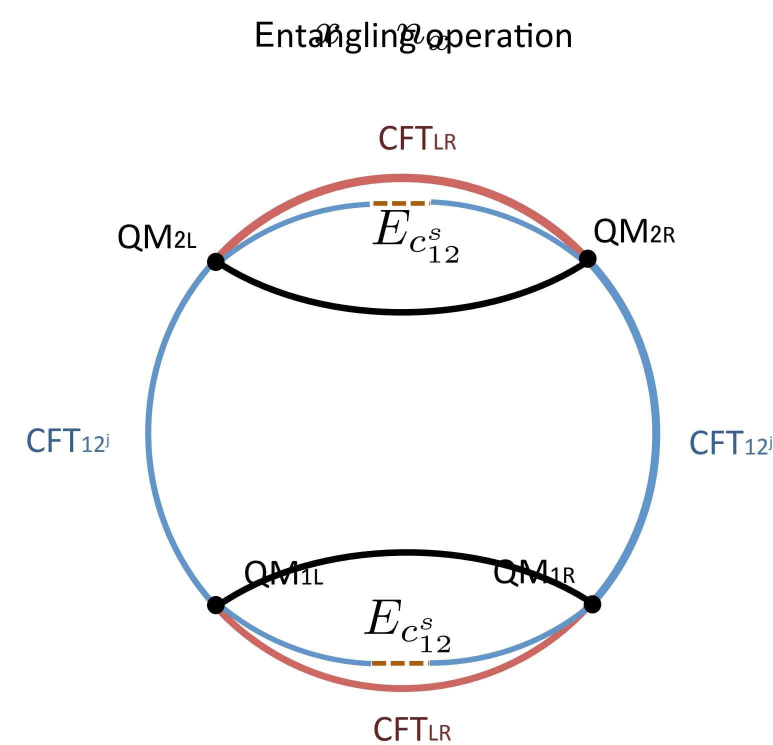


$$\langle I^{LR}_{c^s_{12}} | \Psi_{12} \rangle$$
$$\in \mathcal{H}_{CFT^j_{12}} \otimes \mathcal{H}_{CFT^j_{12}}$$

Partially entangling two side = projection onto $|I\rangle = \prod_x \sum_{n_x} |n_x\rangle |n_x\rangle$ for $CFT_{12}{}^s$



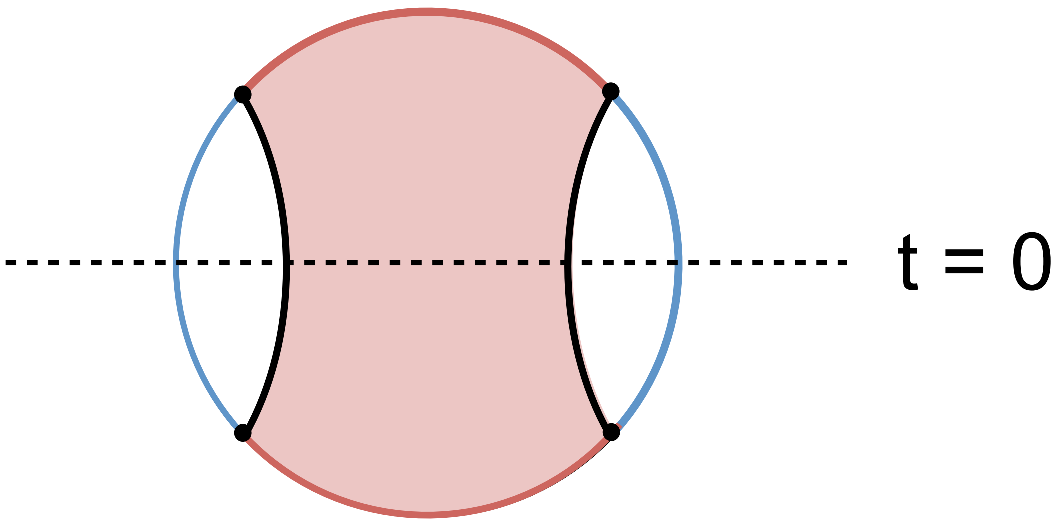
$c_{LR} < c_{12}$

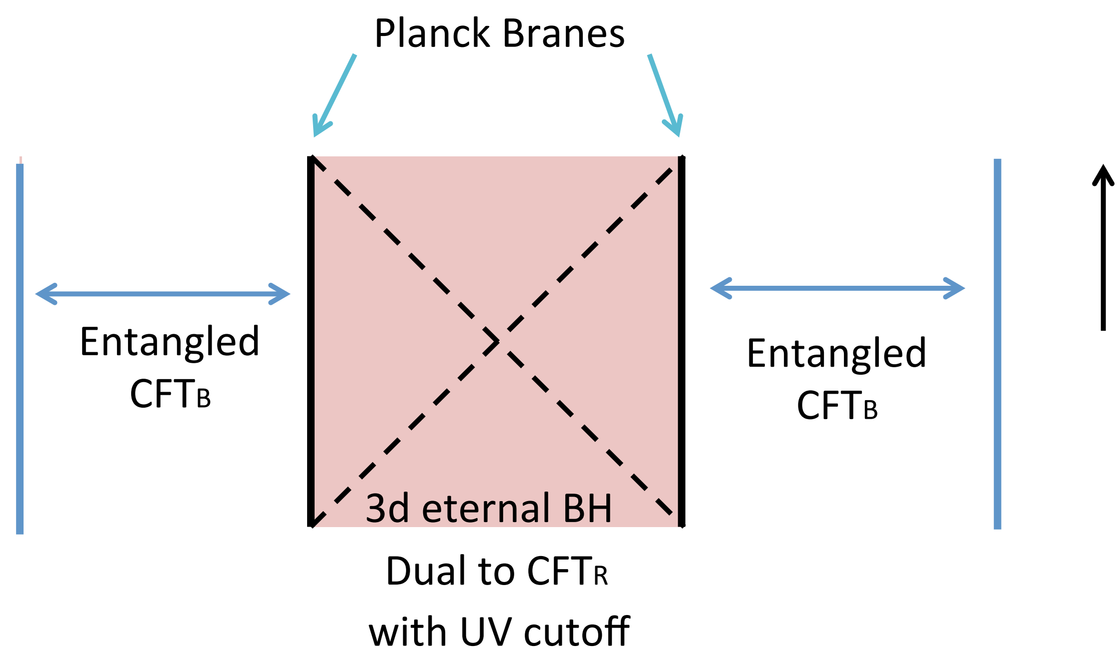


$c_{LR} > c_{12}$

Embed holographic states to free field Hilbert spaces:

"partially doubly holographic"





[cf:"Holo-ween" Simidzija-Raamsdonk, 20]

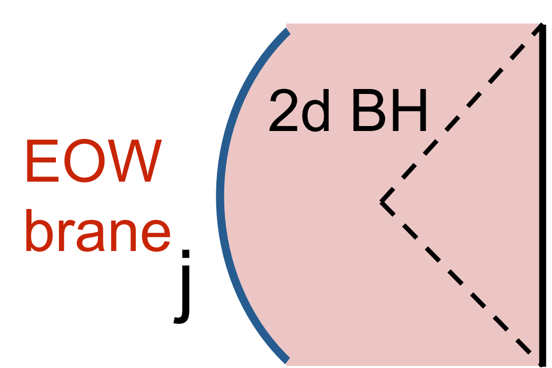
states in $\mathcal{H}_{CFT_B} \otimes \mathcal{H}_{CFT_B}$

- Take CFT_B to be bunch of free fields (or Ising CFTs)

  $\rightarrow$ realize holographic states in free fields Hilbert sp.

cf: JT gravity with auxiliary system

[Penington-Stanford-Shenker-Yang 19]

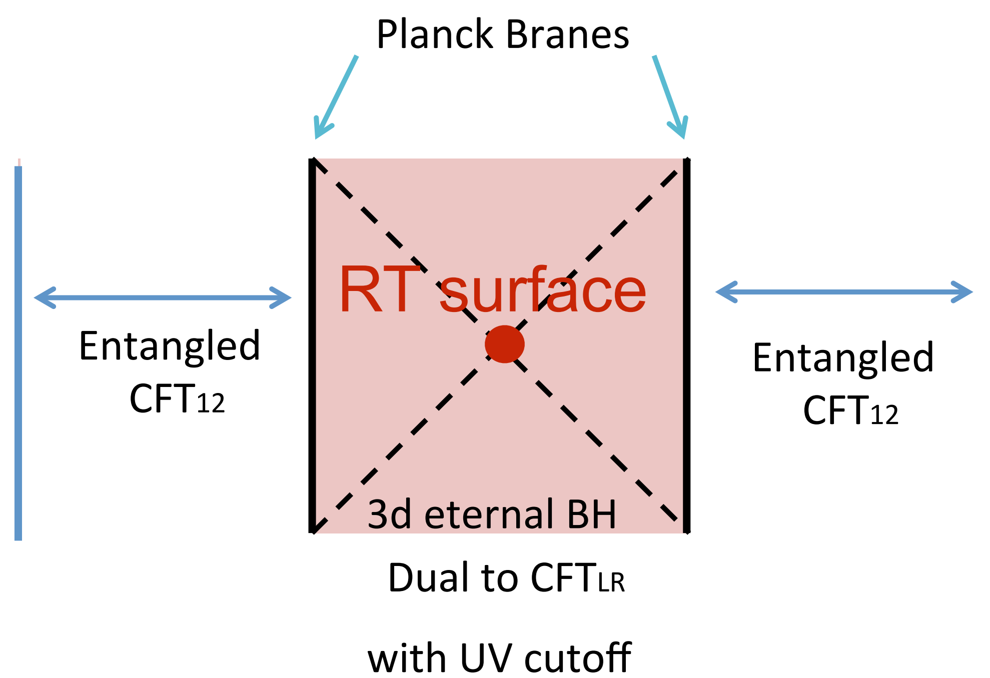


$= |\psi_j\rangle_B$

$$\rho_R \rightarrow |\rho_R\rangle = \sum \langle\psi_i|\psi_j\rangle_B |i\rangle_R \otimes |j\rangle_R \in \mathcal{H}_{aux} \otimes \mathcal{H}_{aux}$$

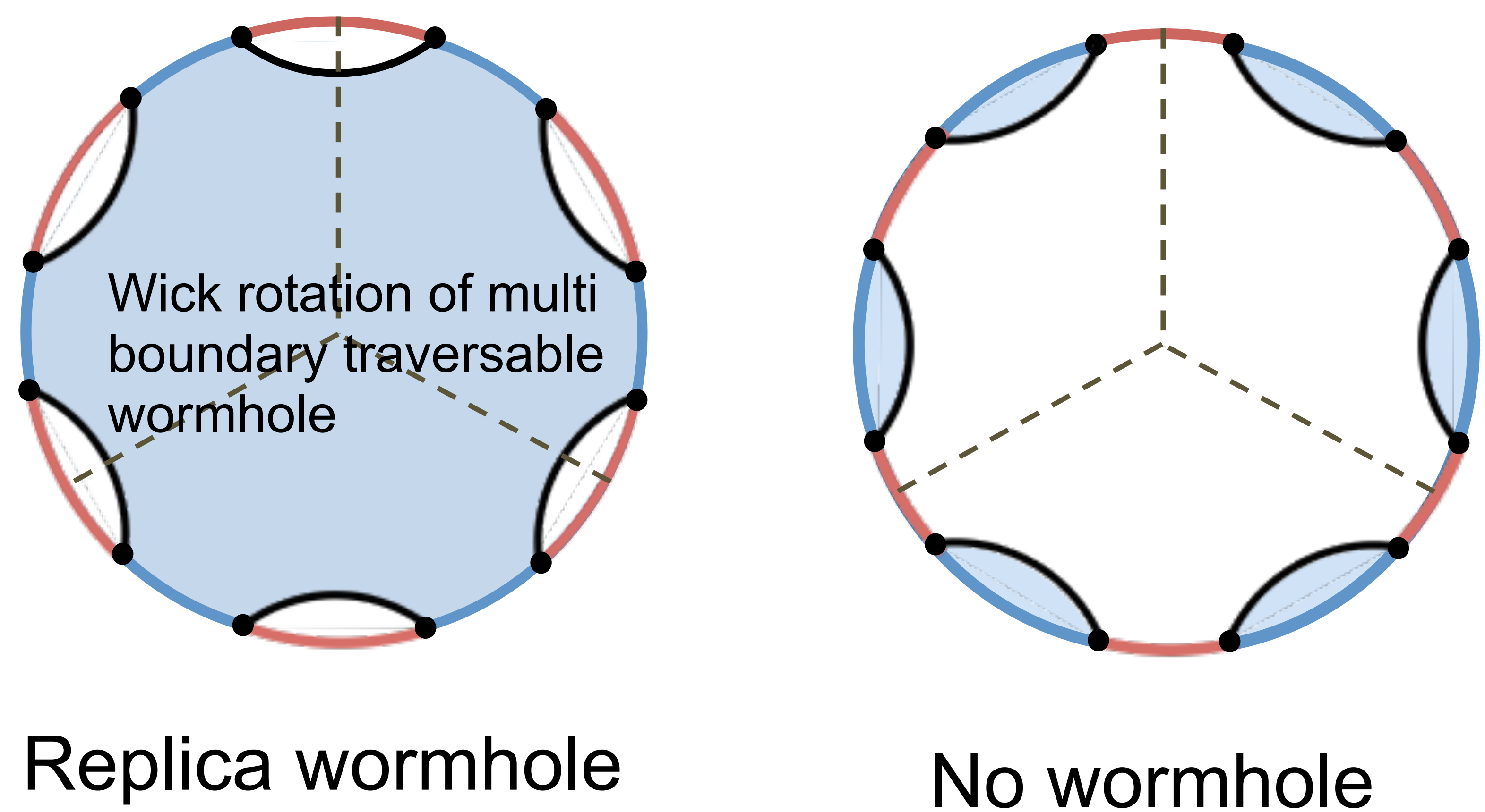
Entanglement entropy and Replica wormholes:

Entanglement entropy between two side is calculated using RT formula for states in non-holographic CFTs



Replica wormhole will justify this calculation

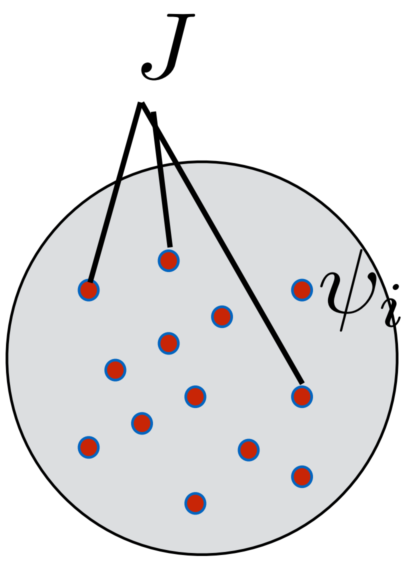
n=3 Renyi Entropy



SYK model: [Sachdev-Ye 93] [Kitaev 14,15]

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$ ($\dim\mathcal{H} = 2^{\frac{N}{2}}$)

Hamiltonian: $H_{SYK} = i^{\frac{q}{2}} \sum_{i_1<i_2<\cdots<i_q} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$



with $\langle J_{i_1 i_2 \cdots i_q} \rangle_J = 0$ and $\langle J^2_{i_1 i_2 \cdots i_q} \rangle_J = \dfrac{\mathcal{J}^2 (q-1)!}{q(2N)^{q-1}}$

Lagrangian: $L = \psi_i(\tau)\partial_\tau \psi_i(\tau) - i^{\frac{q}{2}} \sum_{i_1<i_2<\cdots<i_q} J_{i_1 \cdots i_q} \psi_{i_1} \cdots \psi_{i_q}$

- Solvable in the large N limit
- Exact diagonalization at fine N
- Have the same low energy action with 2d dilaton gravity

[Maldacena-Stanford, 16] [Maldacena-Stanford-Yang, 16]

Conformal Symmetry and its breaking in SYK:

After disorder average and Hubbard-Stratonovich type transformation, we obtain

$$S[G,\Sigma] = N\left[\log \mathrm{Pf}(\partial_\tau - \Sigma) - \int d\tau_1 \int d\tau_2 \Sigma(\tau_1,\tau_2) G(\tau_1,\tau_2) - \frac{\mathcal{J}^2}{q} G(\tau_1,\tau_2)^q\right]$$

At large N , (G,Σ) is classical. EOM is for $G(\tau_1,\tau_2) = \frac{1}{N}\langle\psi_i(\tau_1)\psi_i(\tau_2)\rangle$

$$\begin{cases} \cancel{\partial_{\tau_1} G(\tau_1,\tau_2)} - \int d\tau' \Sigma(\tau_1,\tau') G(\tau',\tau_2) = \delta(\tau_1 - \tau_2) \\ \Sigma(\tau_1,\tau_2) = \frac{\mathcal{J}^2}{q} G(\tau_1,\tau_2)^{q-1} \end{cases}$$

conformal sym:
$$\begin{cases} G(\tau_1,\tau_2) \to [f'(\tau_1) f'(\tau_2)]^{\Delta} G(f(\tau_1), f(\tau_2)) \\ \Sigma(\tau_1,\tau_2) \to [f'(\tau_1) f'(\tau_2)]^{1-\Delta} \Sigma(f(\tau_1), f(\tau_2)) \end{cases} , \quad \Delta = \frac{1}{q}$$

$\to$ solutions with power low: $G(\tau_1,\tau_2) = \frac{c_\Delta}{|\mathcal{J}(\tau_1 - \tau_2)|^{2\Delta}}$ spontaneously break conformal symmetry

- UV $\partial_{\tau_1} G(\tau_1,\tau_2)$ term breaks conformal (=reparametrization) symmetry

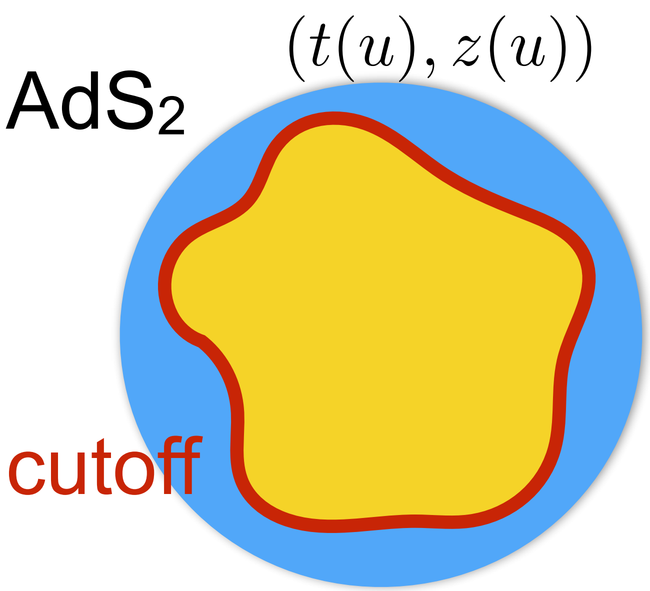
  leading explicit breaking effect is described by the action

$$S = -\frac{N\alpha_S}{\mathcal{J}} \int \{f(\tau),\tau\} , \qquad \{f(\tau),\tau\} = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2$$

Relation to JT gravity:

[Maldacena-Stanford-Yang,16]

dynamics are captured by AdS2 with finite cutoff $(t(u), z(u))$



- reparametrization is spontaneously broken by each geometry (cutoff)
- configuration space: $(g_{\mu\nu}, \phi)$

dilaton~ "entropion"

$$ds^2_{\rm bdy} = \frac{du^2}{\epsilon^2} \;, \quad \phi_{\rm bdy} = \phi_b = \frac{\phi_r}{\epsilon}$$

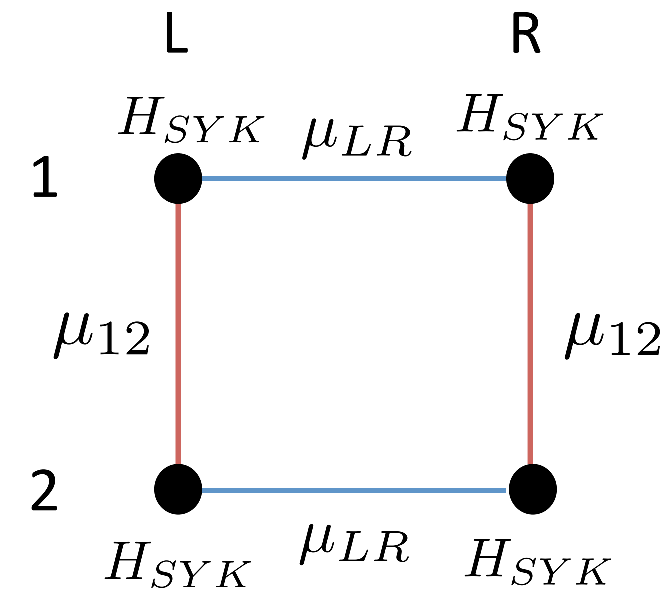
- JT gravity action breaks explicitly the reparametrization symmetry

$$\frac{\delta I_{\rm grav}}{\delta \phi} = 0 \;\; \& \;\; \frac{\delta I_{\rm grav}}{\delta g_{\mu\nu}} = 0 \quad \rightarrow \quad I = \frac{1}{\epsilon^2} - \phi_r \int \{t(u), u\}$$

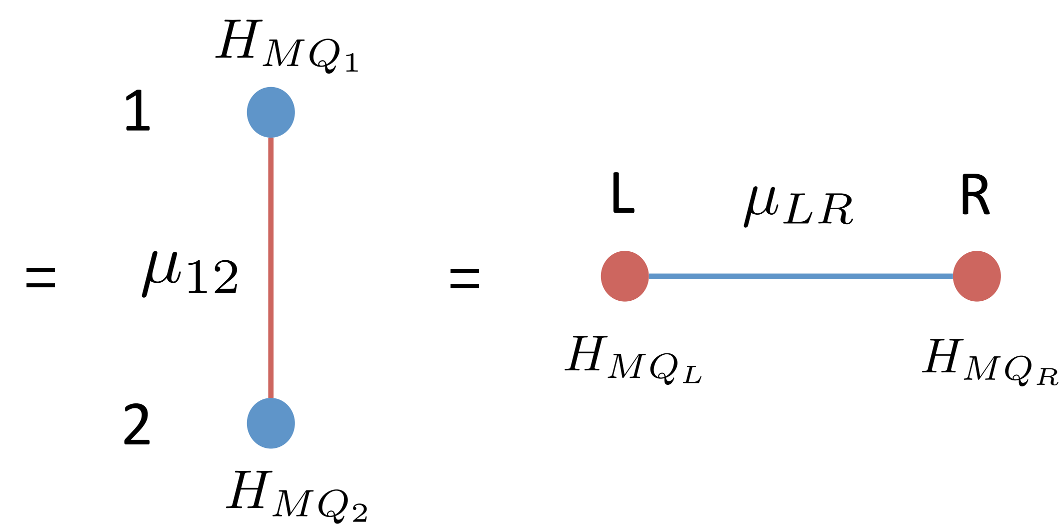
Four coupled SYK model: definition

The model:

$$H = H^{1L}_{SYK} + H^{1R}_{SYK} + H^{2L}_{SYK} + H^{2R}_{SYK}$$
$$+ i\mu_{LR}(\psi^{1L}\psi^{1R} + \psi^{2L}\psi^{2R})$$
$$+ i\mu_{12}(\psi^{1L}\psi^{2L} - \psi^{1R}\psi^{2R})$$



$$H_{SYK} = \sum J_{ijkl}\psi^i\psi^j\psi^k\psi^l$$



- Can be viewed as coupled Maldacena-Qi models
- Model two traversable wormholes in SYK
- "Duality" LR ↔ 12 , which is $\mathbb{Z}_2$ symmetry at $\mu_{12} = \mu_{LR}$
- Order parameter: $S^{dif} = \frac{1}{2}(S^{11}_{LR} + S^{22}_{LR}) + \frac{1}{2}(S^{12}_{LL} - S^{12}_{RR})$, $S^{\alpha\beta}_{AB} = -2i\psi^{\alpha}_{A}\psi^{\beta}_{B}$
- Introducing collective fields $G^{\alpha\beta}_{AB}(\tau_1,\tau_2) = \langle\psi^{\alpha}_{A}(\tau_1)\psi^{\beta}_{B}(\tau_2)\rangle$ and $\Sigma^{\alpha\beta}_{AB}(\tau_1,\tau_2)$

  Large N equations are (numerically) solvable

Solutions of the model:

Property of the hopping terms (per an SYK fermion on each site):

$$H_M = +i\mu_{LR}(\psi^{1L}\psi^{1R} + \psi^{2L}\psi^{2R}) + i\mu_{12}(\psi^{1L}\psi^{2L} - \psi^{1R}\psi^{2R})$$

Using the Jordan-Wigner transformation

$$\psi_{1L} = \sigma_x \otimes \mathbb{I} \ , \quad \psi_{1R} = \sigma_y \otimes \mathbb{I} \ , \quad \psi_{2R} = -\sigma_z \otimes \sigma_y \ , \quad \psi_{2R} = \sigma_z \otimes \sigma_x$$

mapping to two cite XY model

$$H_M = -\frac{1}{2}\mu_{LR}(\sigma_z \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_z) - \frac{1}{2}\mu_{12}(\sigma_y \otimes \sigma_y - \sigma_x \otimes \sigma_x)$$

Ground state: $|G(\mu_{LR}, \mu_{12})\rangle = \cos\frac{\theta}{2}|\uparrow\uparrow\rangle - \sin\frac{\theta}{2}|\downarrow\downarrow\rangle \ , \quad \tan\theta = \frac{\mu_{12}}{\mu_{LR}}$

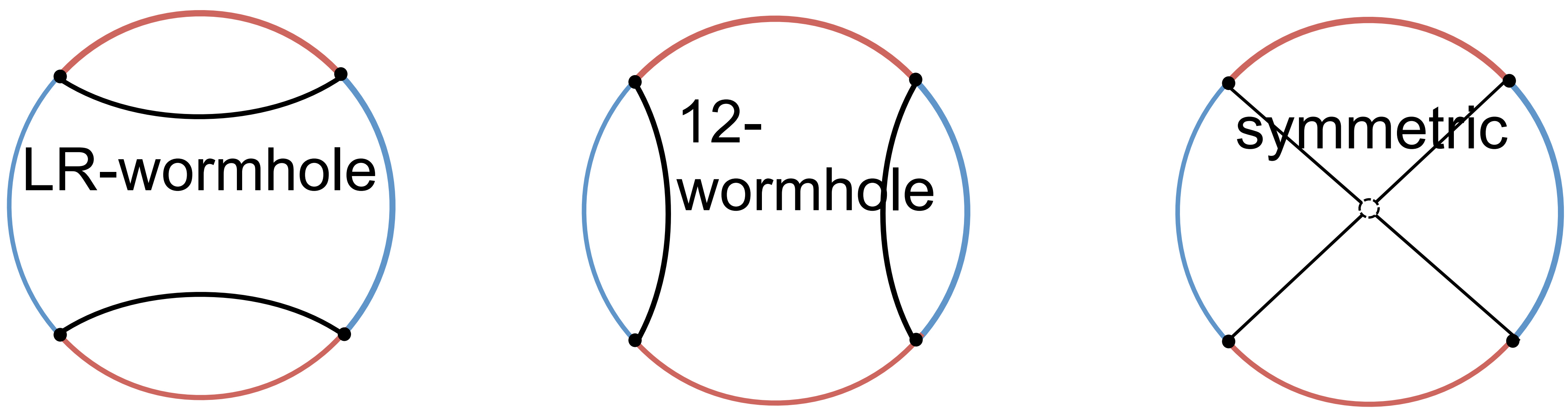
In particular, the ground state is unique.

Solutions of the model:

Including SYK terms   first focus on $\mu_{12} = \mu_{LR}$

small $\mu_{12} = \mu_{LR}$:

- They have 3 solutions: LR-wormhole, 12-wormhole and symmetric
- Actually $\mathbb{Z}_2$ symmetry is broken by wormhole solutions



(connection = entanglement = wormhole)

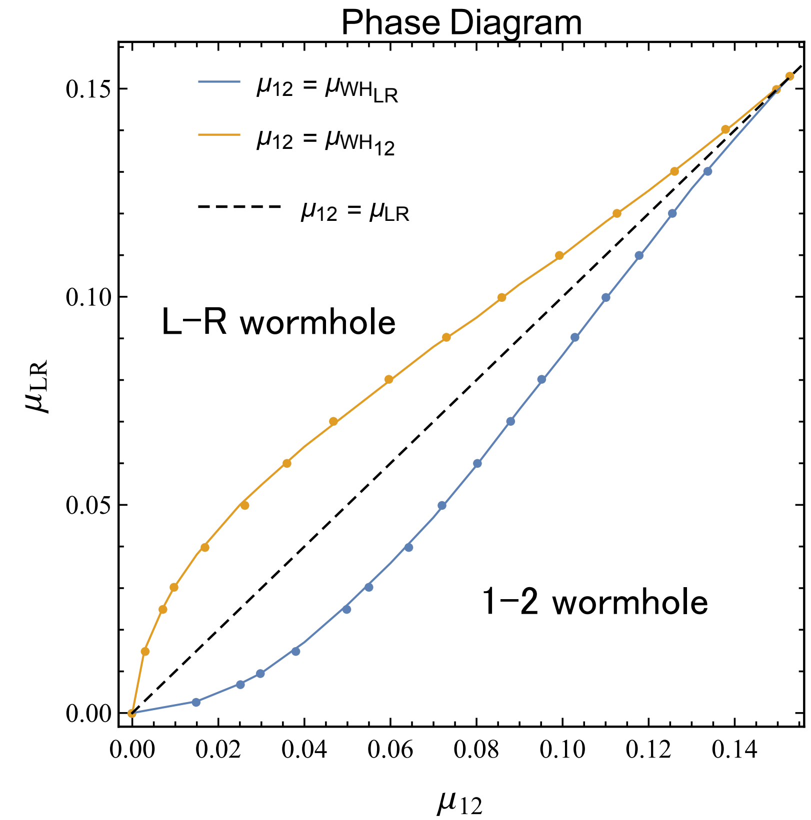
larger $\mu_{12} = \mu_{LR}$:

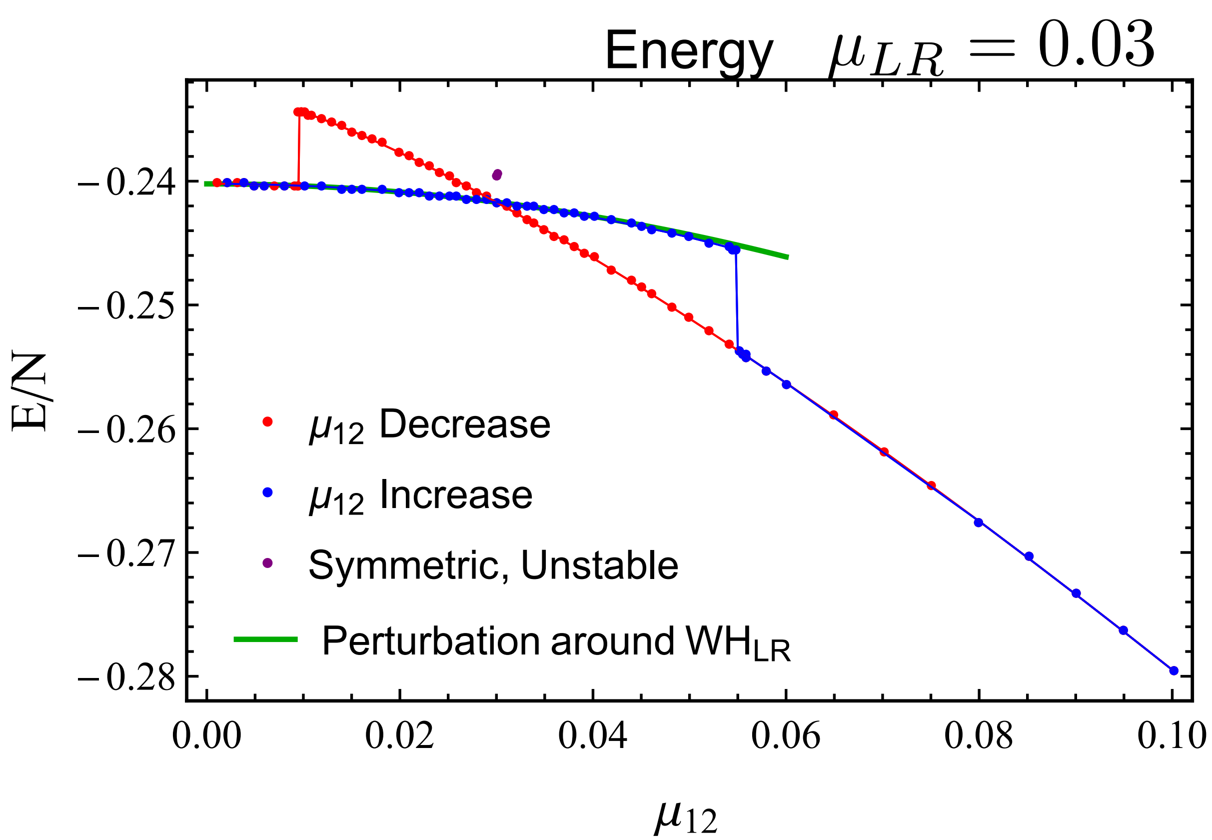
3 solutions coincide around $\mu_{LR} \approx 0.154$, symmetry is restored

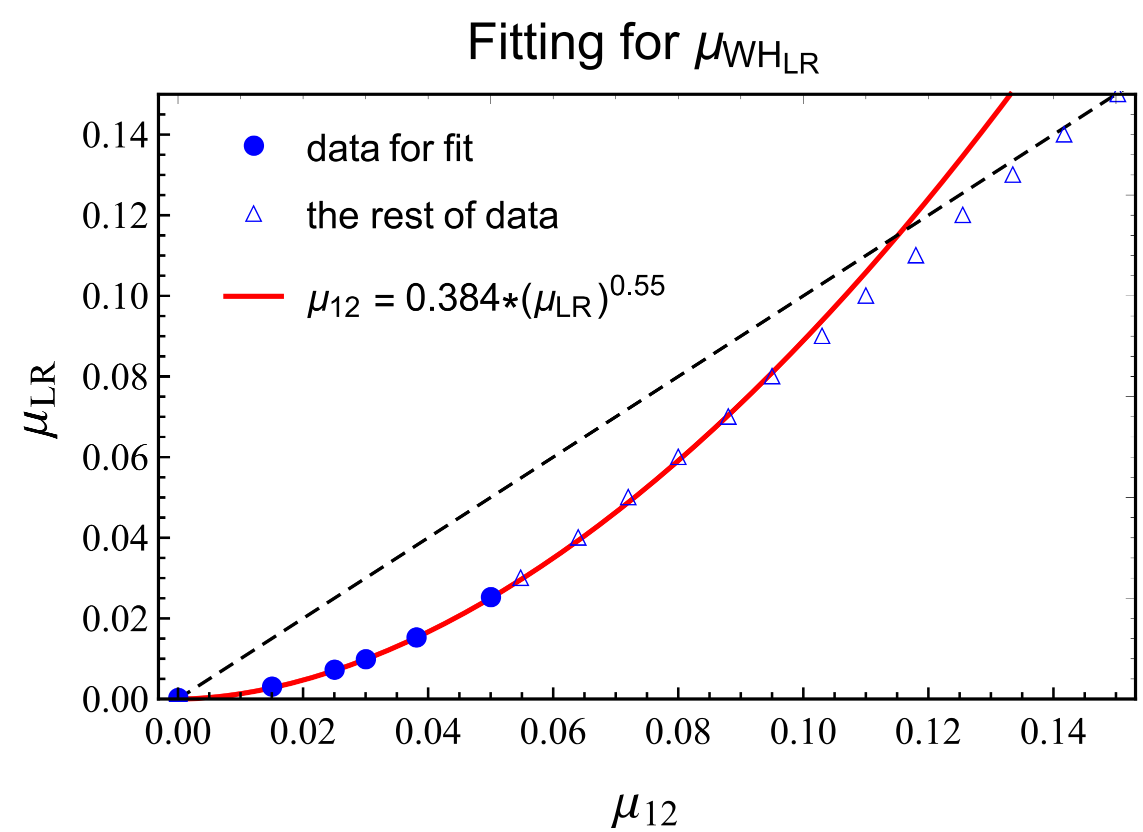
Symmetry breaking is strongly coupled phenomena

General parameters: fixed μ_{LR}

- we can study wormhole solutions for $\mu_{LR} \neq \mu_{12}$
- because of instability, we cannot reach the symmetric solutions
- for $\mu_{LR} \ll \mu_{12}$, L-R wormhole disappears
- True ground state + metastable entangled state



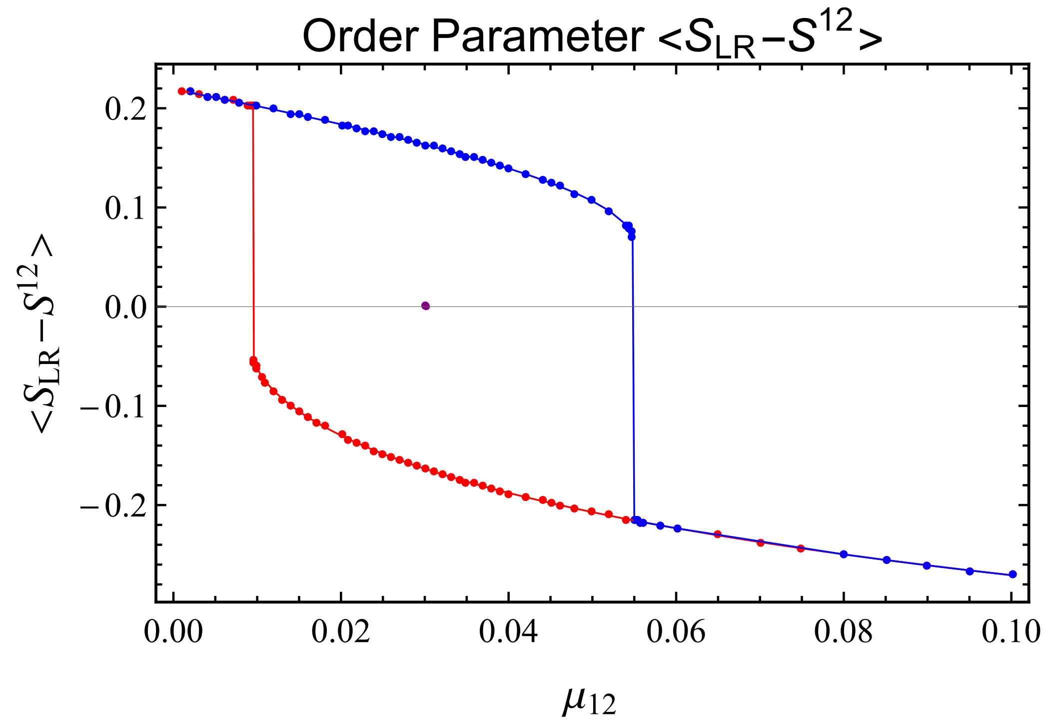


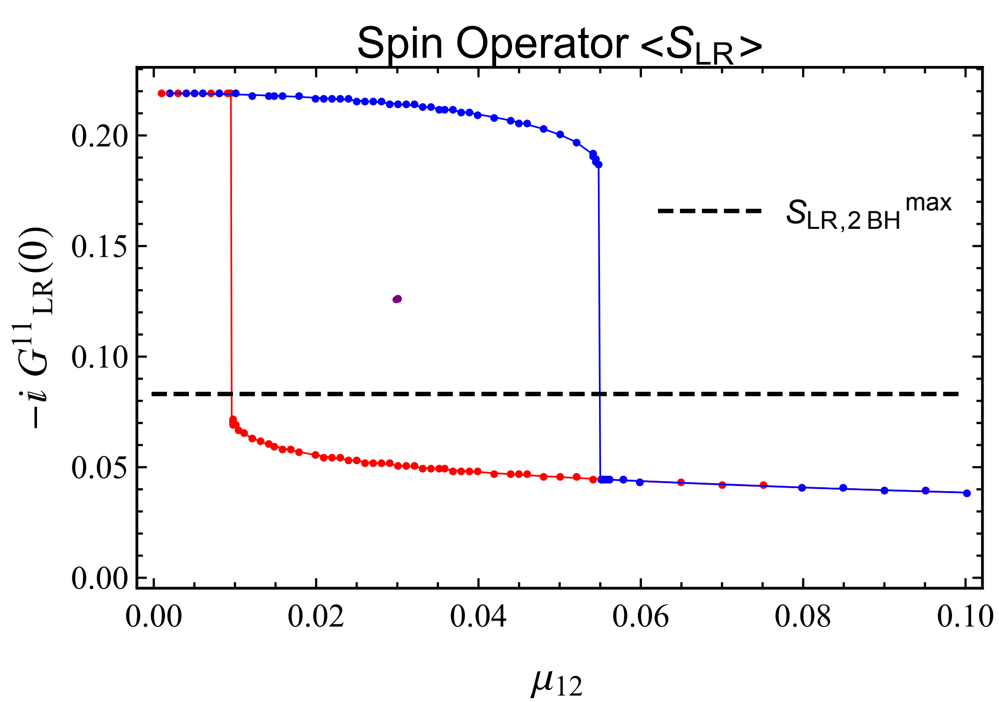


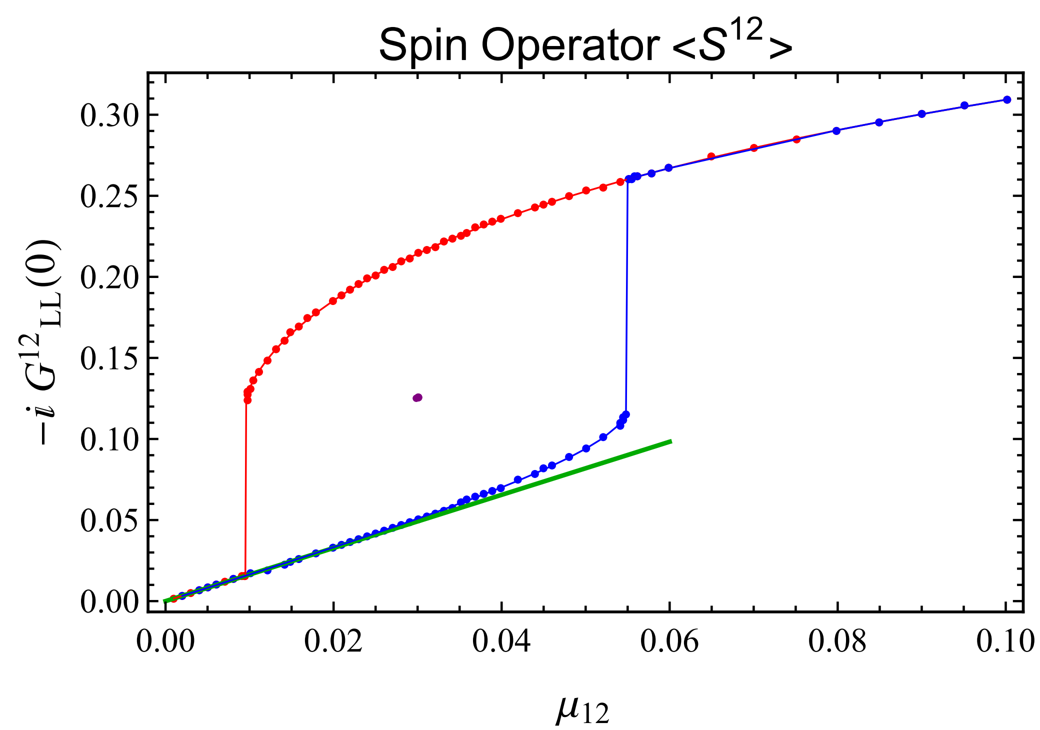
Order parameters:

- we can study the order parameter and check that they acquires a vev

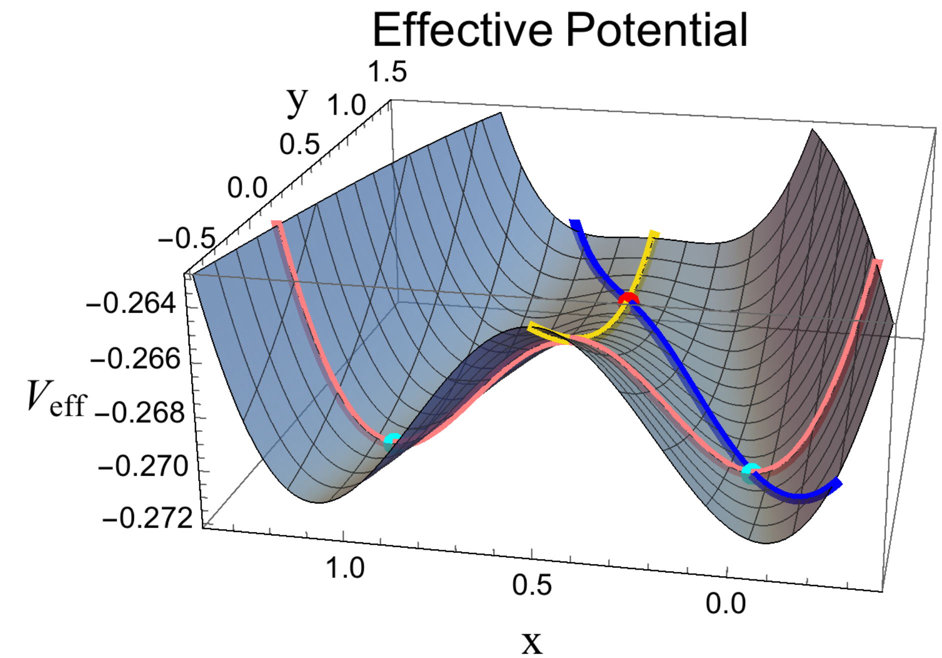
- seeing correlation
  ~ entanglement pattern







Effective potential:

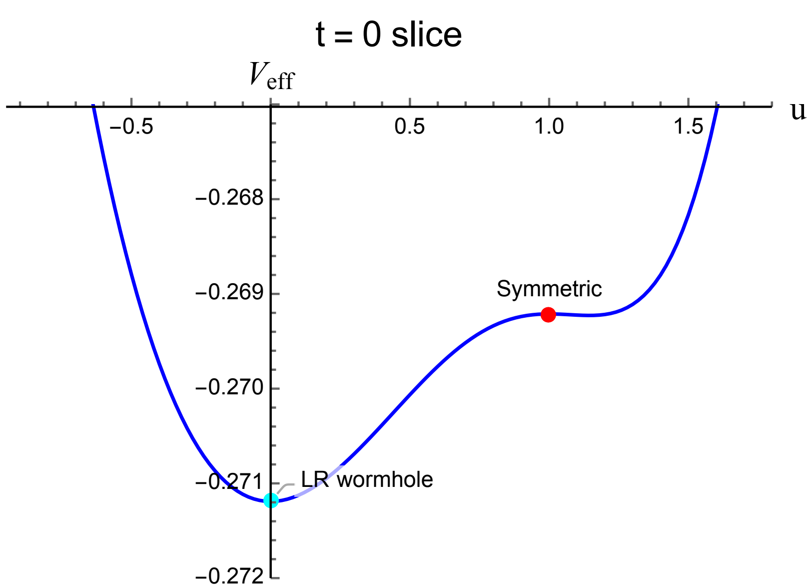


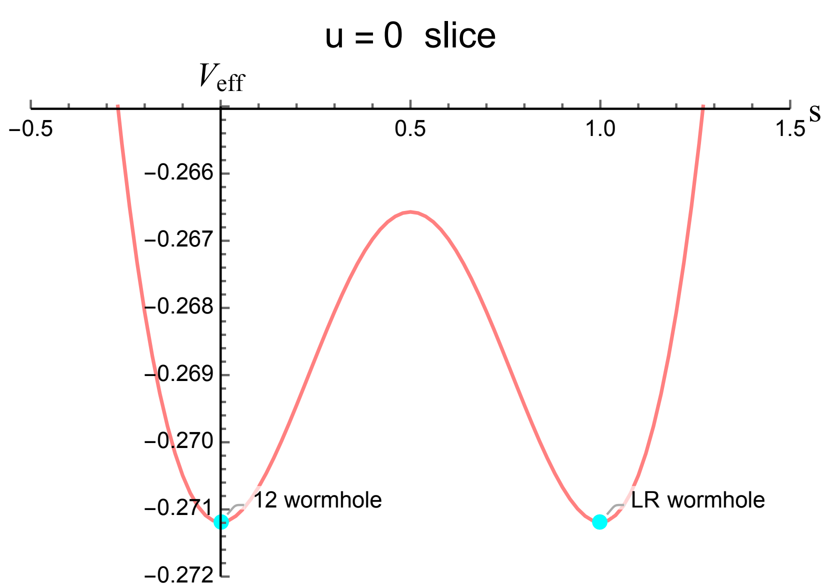
Evaluating the action $S_E/N \approx \beta V_{\rm eff}(G, \Sigma)$

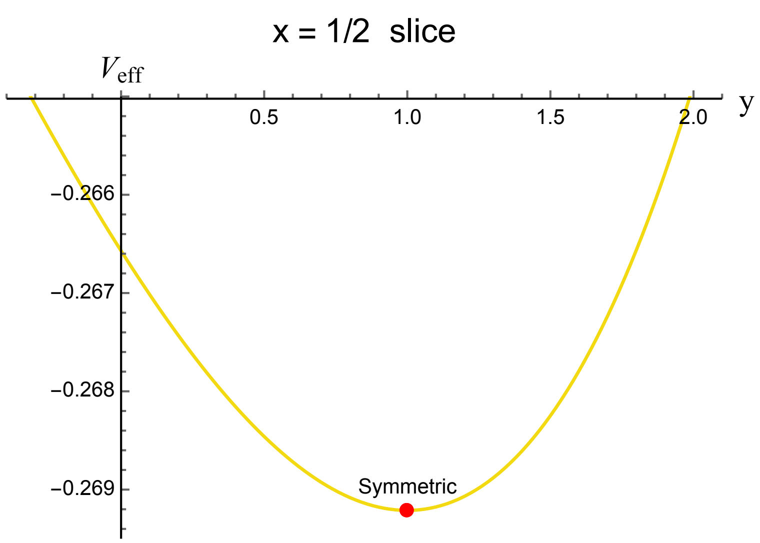
for *non-solutions* gives an effective potential

one slice:

$$\begin{cases} G(\tau) = sG_{WH_{LR}}(\tau) + tG_{WH_{12}}(\tau) + uG_{sym}(\tau) \ , \quad s+t+u=1 \\ \Sigma(\tau) = J^2 G(\tau)^{q-1} \end{cases}$$

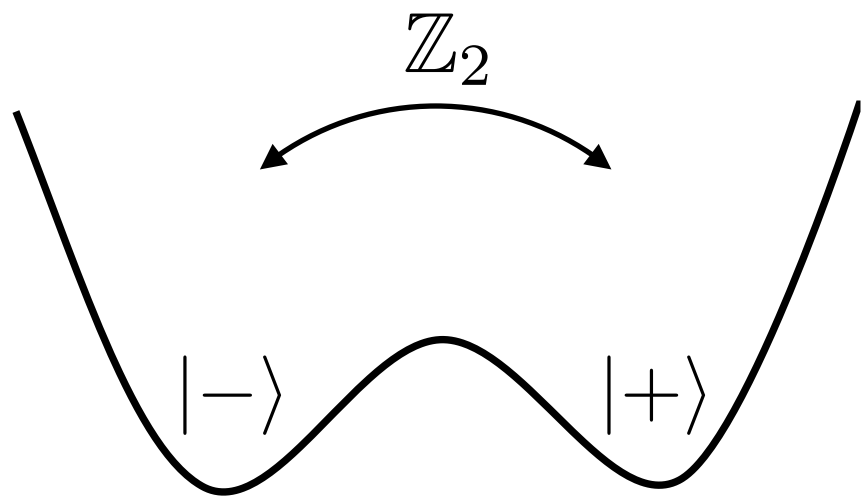






Instability = Tachyonic nature of the symmetric solution is manifest

Relation to double well potential :



- Two classical vacua (LR wormhole $\approx |+\rangle$, 12 -wormhole $\approx |-\rangle$)
- Tachyon saddle (symmetric saddle)
- At finite N (=Planck const), no symmetry breaking

$$\begin{cases} |0\rangle = |+\rangle + |-\rangle & \longleftarrow \ \mathbb{Z}_2 \ \text{even} \\ |1\rangle = |+\rangle - |-\rangle & \longleftarrow \ \mathbb{Z}_2 \ \text{odd} \end{cases}$$

Energy difference: $\Delta E \propto e^{-S_{\text{inst}}}$

Some Lessons from four coupled SYK/NAdS2:

- wormhole connection pattern = Z_2 symmetry breaking

  maybe related to the idea that graviton = NG boson? [Kraus-Tomboulis,02]

  (cf: Scalar in gravity multiplet = coset models)

- LR-wormhole phase: $H_{\mathrm{MQ}^1_{LR}} + H_{\mathrm{MQ}^2_{LR}} + \mu H^{12}_{int}$ is a good description

  (string theory : ∃ duality frame w/ (super) gravity , depend on solutions)

- Tachyonic solution: condensation makes wormholes

  imply condensation of entangled gas = wormholes

  [Jafferis-Schneider,21]

- Z_2 symmetry: gauged in the bulk and spectrum should be complete

  ($|LR\rangle + |12\rangle$, $|LR\rangle - |12\rangle$ $\rightarrow$ $|LR\rangle$ and $|12\rangle$ should be included)

  = even = odd

  sum over wormholes ↔ Z_2 charge completeness [Polchinski,03]

  (cf: some over SL(2,R) orbits for partition functions in 3d gravity)

  [Dijkraaf-Maldacena-Moore-Verlinde,00] [Maloney-Witten,07]

1. Traversable wormholes in 4d and in JT gravity (Review)

2. Two traversable wormholes by 4 coupled JT gravity

3. Bra-ket wormhole interpretation

4.Conclusion/ Future works

4.Conclusion

- We constructed four coupled SYK/ Jackiw-Teitelboim gravity (2k coupled of this type is straightforward)

- Both have two gapped ground state. Z_2 symmetry at special point is broken by wormhole configurations. Symmetric saddle is unstable in SYK.

- There is a first order phase transition, which exchanges wormhole configurations. In bra-ket wormhole interpretation, the transition is caused by Projection/Entangling operation.

- We basically studied the effect of cutting/connecting *outside* the wormholes

Future works

- We do not understand the role of Z_2 symmetry in bra-ket wormhole picture.
- Showing instability of symmetric saddles in JT gravity side.
- Bra-ket wormholes in SYK model side.
- Projection by boundary states causes phase transition and that helps the bra-ket factorization. Projection by energy eigenstates does not manifestly factorize. Should we include bra-ket wormholes in string/M-theory?
- Effect of measurements on Hawking radiations in evaporating BHs.
- Bra-ket wormhole can also be considered in de Sitter gravity.
  Its generalization to many de Sitter.