Aspects of Holographic Complexity

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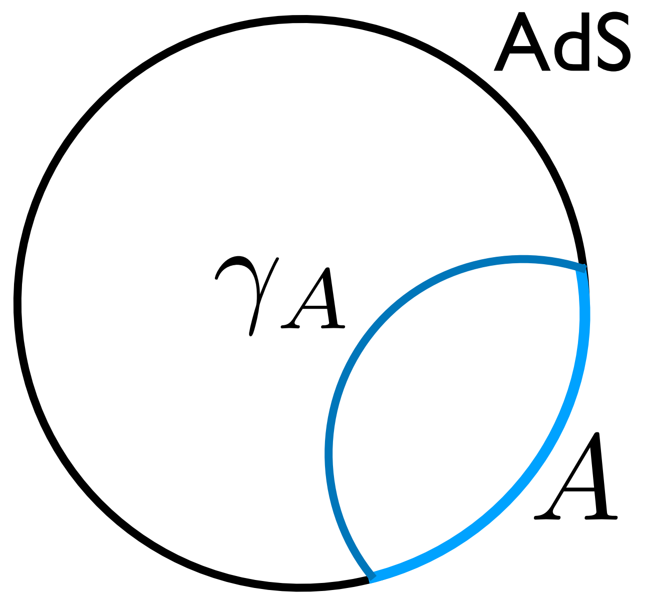
Based on work with A. Bernamonti, F. Bigazzi, D. Billo, L. Faggi, J. Hernandez, R. Myers, S. Ruan, J. Simón
PRL 123 (2019) no.8 081601 & J. Phys. A: Math. Theor. 53 (2020) 294002 + arXiv: 2108.09281

Introduction

Quantum Information & AdS/CFT

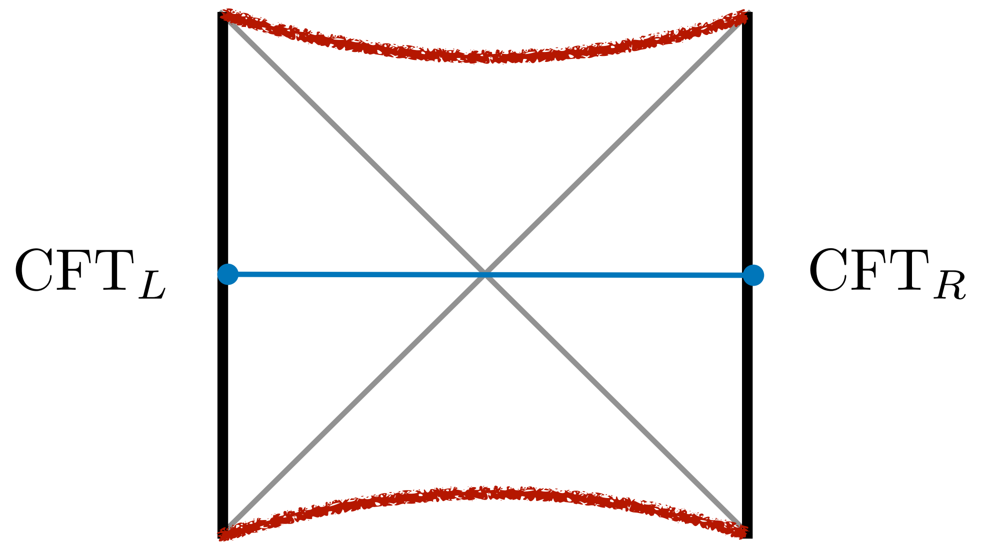
Interplay between quantum information and holography has led to a fruitful bulk-boundary dialogue

$$S_A = \mathrm{Min}\frac{\mathrm{Area}(\gamma_A)}{4G_N}$$



Spacetime geometry ~ entanglement

$$|\mathrm{TFD}\rangle = \sum_n e^{-\frac{\beta E_n}{2}}|n\rangle_L \otimes |n\rangle_R$$

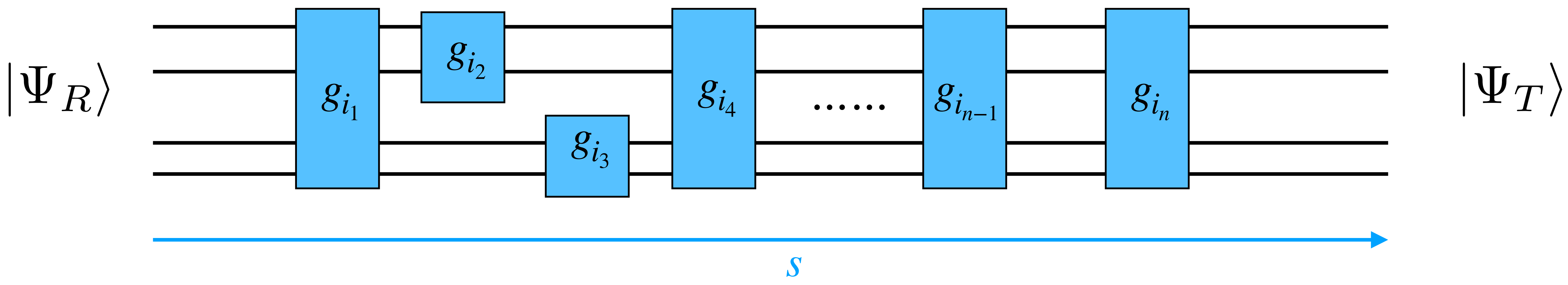


Computational complexity: how hard it is to implement a task?

Quantum Circuit Complexity

How difficult is it to prepare a particular target state?

Given a simple reference state (unentangled), how hard it is to prepare a target state acting with a unitary constructed from a set of generators of elementary gates



$$|\Psi_T\rangle = U_T|\Psi_R\rangle = g_{i_n}\dots g_{i_1}|\Psi_R\rangle$$

GATE

$$g_I = e^{i\epsilon \mathcal{O}_I}$$

GATE GENERATOR

Complexity quantifies the cost of the optimal circuit generating U_T

Nielsen's geometric approach

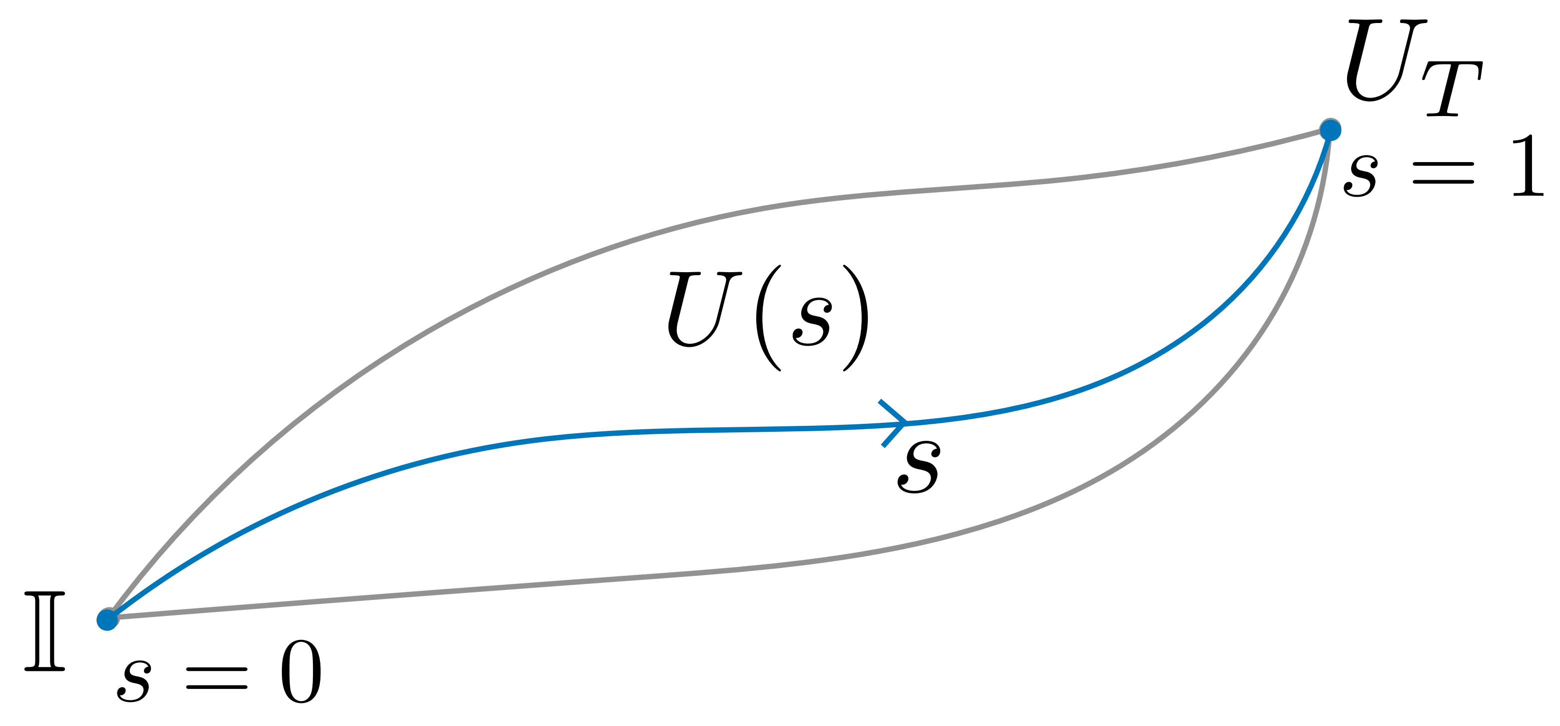
[Nielsen et al.]

Continuum representation of unitary transformations

$$U(\sigma) = \overleftarrow{\mathcal{P}}\exp\left[-i\int_0^{\sigma} ds H(s)\right]$$

$$H(s) = \sum Y^I(s)\mathcal{O}_I$$

CONTROL FUNCTION



Associates a cost to each trajectory in the space of unitaries

COST FUNCTION

$$\mathcal{C}\left(|\Psi_T\rangle\right) \equiv \text{Min}\int_0^1 ds\, F(U, Y^I)$$

Complexity: globally cost-minimizing trajectory in the space of unitaries

Cost Function

Classical mechanic problem with F playing the role of the Lagrangian

Form of F generally not fixed, but usually one requires

1. Smoothness
2. Positivity
3. Triangle inequality

Examples:

$$F_1(U, Y^I) = \sum_I |Y^I|$$ ~ COUNTING GATES

$$F_\kappa(U, Y^I) = \sum_I |Y^I|^\kappa$$

$\kappa = 2$ BASIS INDEPENDENT

SAME CIRCUIT AS F_2

$$F_2(U, Y^I) = \sqrt{\sum_I (Y^I)^2}$$ RIEMANNIAN GEOMETRY

QFT Complexity

Approach á la Nielsen adapted to free QFT: [Jefferson,Myers]

- UV regularization δ (lattice): family of coupled harmonic oscillators
- Gates built from a finite number N of simple generators $\mathcal{O}$ yielding a closed algebra
- Optimal circuit: geodesic problem in the resulting geometry

$|0\rangle$ vacuum [Jefferson,Myers]

$|\alpha\rangle$ coherent state [Guo,Hernandez,Myers,Ruan]

$|TFD\rangle$ [Chapman,Eisert,Hackl,Heller,Jefferson,Marrochio,Myers]

Other interesting approaches

[Chapman,Heller,Marrochio,Pastawski]
[Caputa,Kundu,Miyaji,Takayanagi,Watanabe]
[Caputa,Magan]
…

Holographic Complexity

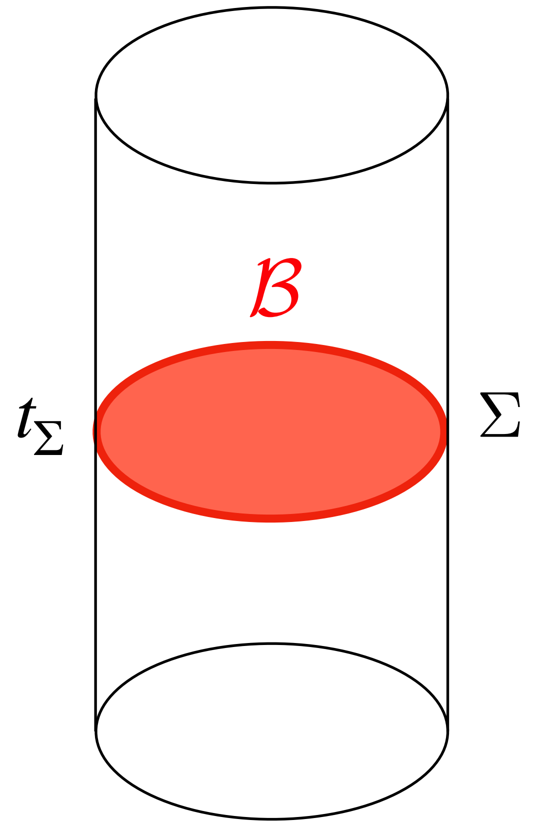
Holographic dual of the complexity of a target state $|\Psi_T\rangle$ on a boundary Cauchy slice Σ

Complexity = Volume of the extremal bulk hypersurface $\mathcal{B}$ anchored at the boundary on Σ

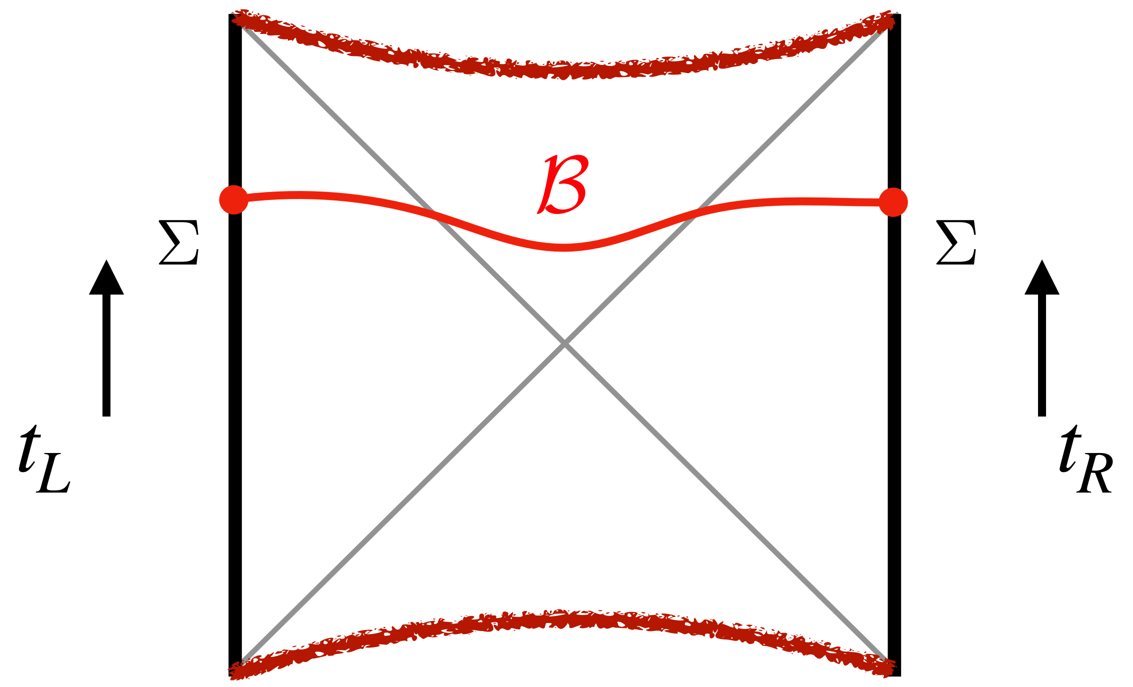
[Susskind, Stanford]

$$\mathcal{C}_{\rm V} = \max_{\partial\mathcal{B}=\Sigma} \frac{V(\mathcal{B})}{G_N \ell_{\rm bulk}}$$

ARBITRARY BULK SCALE



Global AdS

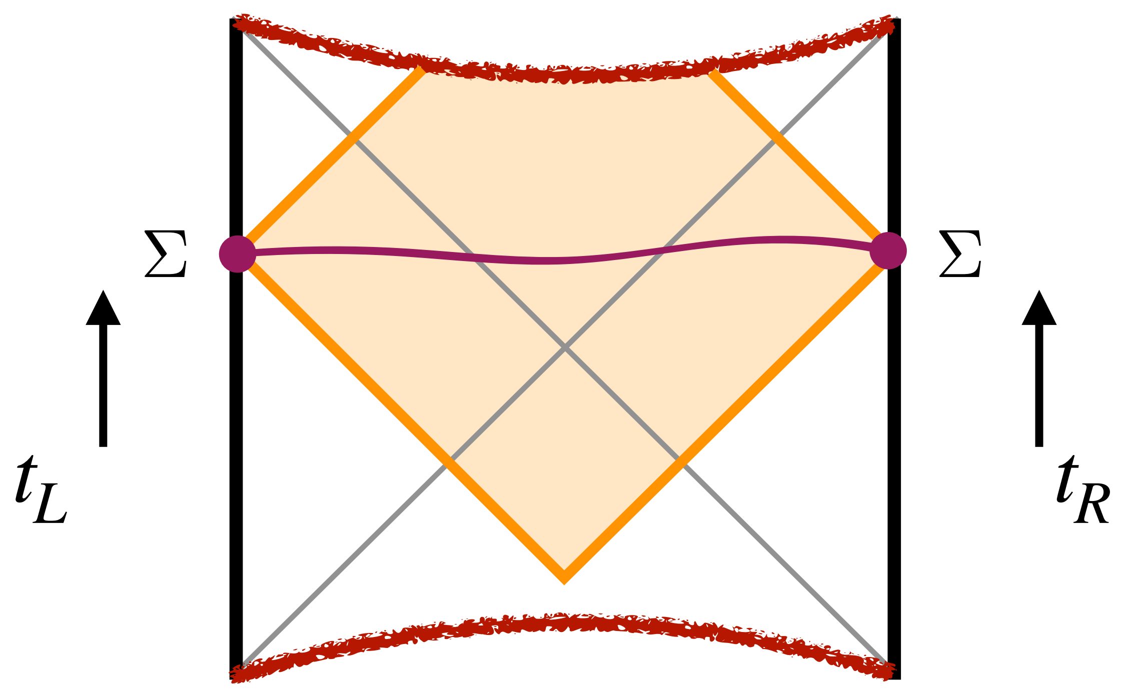


BTZ

Holographic Complexity

Complexity = Action evaluated on the Wheeler-DeWitt patch [Brown,Roberts,Susskind,Swingle,Zhao]

$$\mathcal{C}_A(\Sigma) = \frac{I_{\rm WDW}}{\pi}$$



WDW patch: domain of dependence of a bulk slice anchored to Σ

I_{WDW} imposing variational principle for Dirichlet BCs on WDW patch [Lehner, Myers, Poisson, Sorkin]

$$I_{\rm WDW} = I_{Bulk} + I_{GHY} + I_{\kappa} + I_{CT} + I_{Joints}$$

$$I_{CT} = \frac{1}{8\pi G_N}\int_{\partial\rm WDW} ds\, d^2\Omega\, \sqrt{\gamma}\,\Theta\, \log(\ell_{\rm ct}\Theta)$$

EXPANSION SCALAR (Θ)

ARBITRARY BULK SCALE ($\ell_{\rm ct}$)

ensures invariance under reparametrization of null directions

Holographic Complexity

Holographic complexity proposals reproduce a number of desirable properties

- probe the interior of black holes
- reproduce the expected late time complexity linear growth
- switchback effect

Interesting similarities and points of contact with (free) QFT results, for instance in the structure of leading UV divergences

$$\mathcal{C}_{A/V} \sim \mathcal{C}_{\kappa=2} \sim \frac{\mathrm{Vol}(\Sigma)}{\delta^{d-1}}$$

Holographic Complexity

Still have a very partial understanding

- General QFT formulation still lacking
- Understanding of holographic observables far from being exhaustive
- Incomplete map between the two sides

$$|\Psi_T\rangle \longleftrightarrow (g, \{\phi\}) \quad \text{classical bulk fields}$$

$$|\Psi_R\rangle \quad g_I \quad F(U, Y^I) \longleftrightarrow \text{?}$$

First Law of Holographic Complexity

A. Bernamonti, FG, J. Hernandez, R. Myers, S. Ruan and J. Simón

PRL 123 (2019) no.8 081601 & J. Phys. A: Math. Theor. 53 (2020) 294002

Complexity Variations

How complexity varies under a small change of the target state

$$\delta\mathcal{C} \equiv \mathcal{C}(|\Psi_T + \delta\Psi\rangle) - \mathcal{C}(|\Psi_T\rangle)$$

Why?

- Focus on the dependence of complexity on the target state
- May remove some ambiguities and yield sharper implications for holographic complexity
- Properties of new observables
- Variations often better observables (finite, physical implications…)

First Law of Complexity

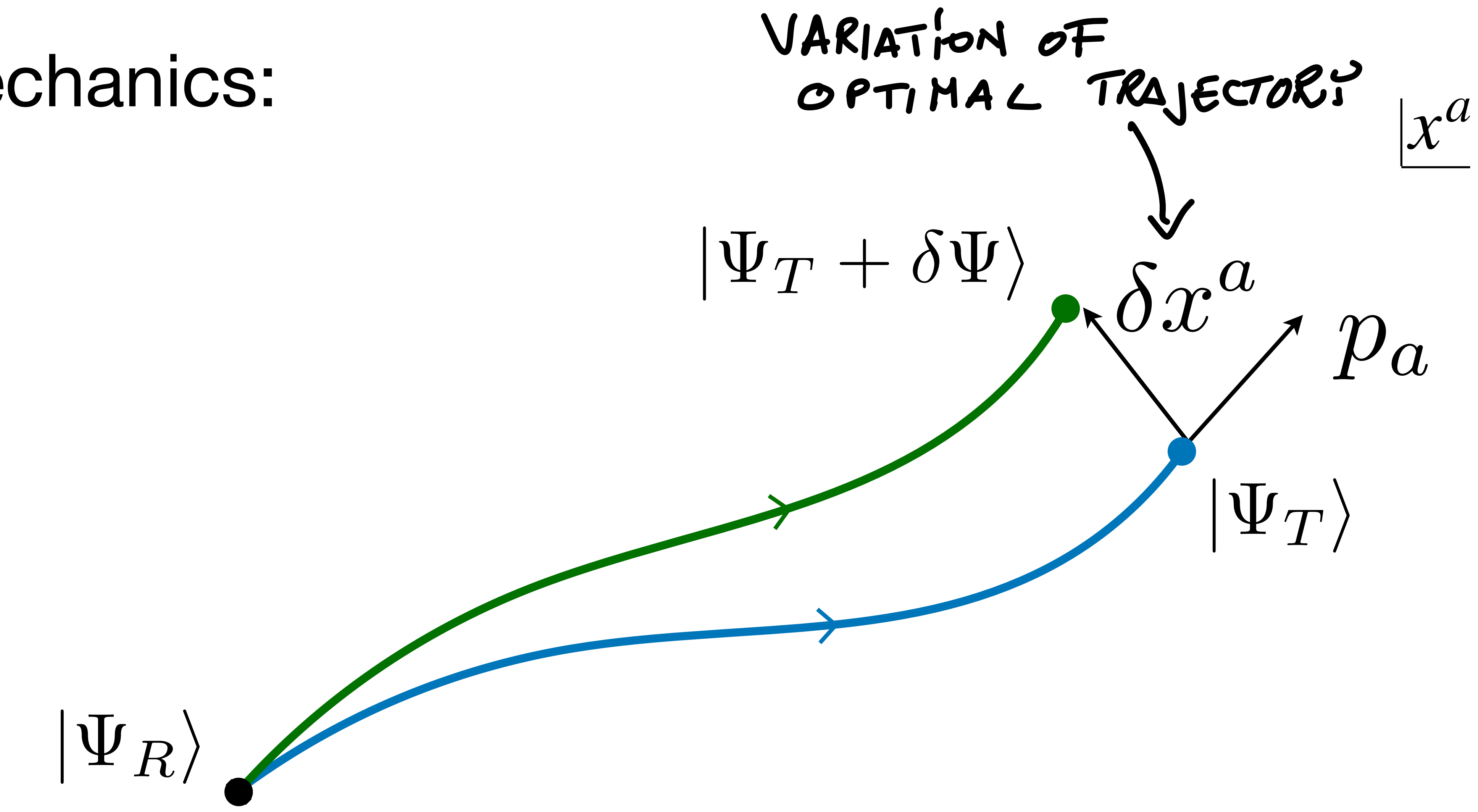
Using the analogy of Nielsen's approach to classical mechanics:

- 1st order variation

$$\delta\mathcal{C} = p_a \delta x^a \Big|_{s=1} \qquad \text{with} \qquad p_a = \frac{\partial F}{\partial \dot{x}^a}$$

- 2nd order variation

$$\delta\mathcal{C} = \frac{1}{2}\delta p_a \delta x^a \Big|_{s=1} \qquad \text{with} \qquad \delta p_a = \delta x^b \frac{\partial^2 F}{\partial x^b \partial \dot{x}^a} + \delta \dot{x}^b \frac{\partial^2 F}{\partial \dot{x}^b \partial \dot{x}^a}$$



Only contributions from the endpoint

Caveat: assumes perturbed optimal circuit stays close to the original optimal circuit

Holographic Framework

$$|\Psi_T\rangle = |0\rangle$$

Vacuum

$$|\Psi_T + \delta\Psi\rangle = |\varepsilon\alpha_j\rangle$$

Coherent state

Given an AdS bulk scalar $\phi(y^\mu) = \sum_n \left(u_n(y^\mu)a_n + u_n^*(y^\mu)a_n^\dagger\right)$

$$|\varepsilon\alpha_j\rangle = \mathrm{e}^{\varepsilon \sum D(\alpha_j)}|0\rangle \qquad D(\alpha_j) = \alpha_j a_j^\dagger - \alpha_j^* a_j$$

displacement op.

where a few modes $\{j\}$ are given classical expectation value with amplitude α_j

$$\langle\varepsilon\alpha_j|\phi|\varepsilon\alpha_j\rangle = \varepsilon \sum \left(\alpha_j u_j + \alpha_j^* u_j^*\right) \equiv \varepsilon\phi_{cl}$$

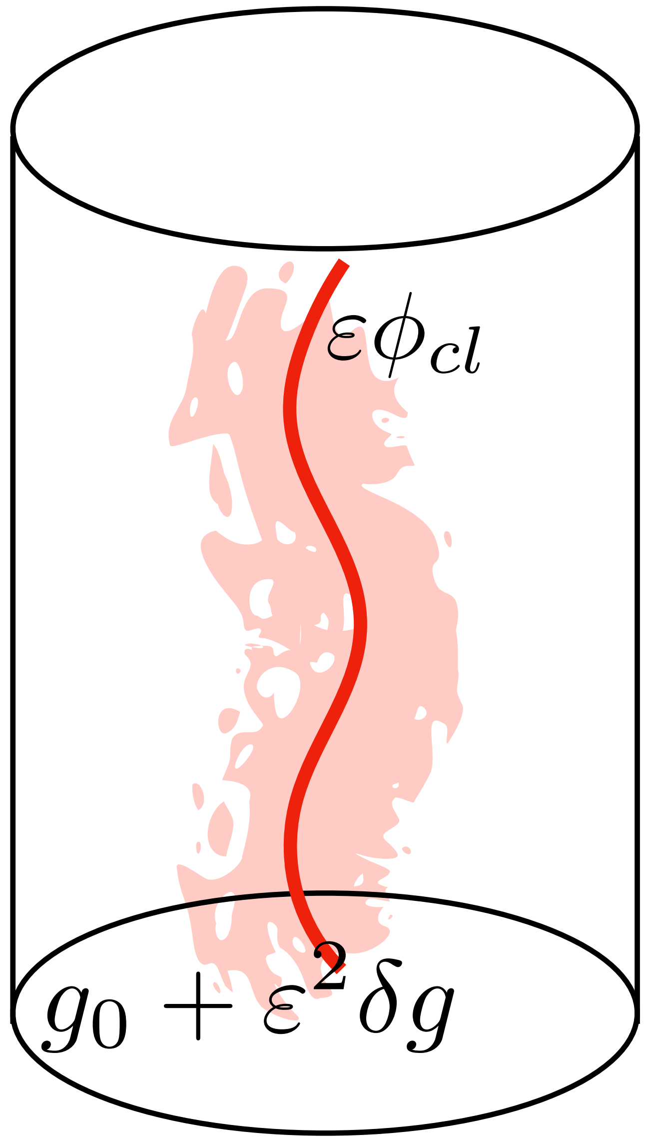
Holographic Framework

$|\varepsilon\alpha_j\rangle$ are also coherent states in the boundary CFT corresponding to excitations of the vacuum by the dual generalized free field operator $\mathcal{O}_\Delta$ and its descendants

- Quantum circuit technology in QFT [Jefferson,Myers] applied to coherent states [Guo,Hernandez,Myers,Ruan] can be equivalently applied in the bulk

- Simple classical gravity description suitable to evaluate holographic complexity

$\|\Psi_T\rangle = \|0\rangle$	$\|\Psi_T + \delta\Psi\rangle = \|\varepsilon\alpha_j\rangle$
Global AdS g_0	$(g_0 + \varepsilon^2 \delta g, \varepsilon\phi_{cl})$



Spherically symmetric perturbations and perturbatively in the amplitude ε of the coherent state

CA Variation

$$\delta I = I[g_0 + \varepsilon^2 \delta g, \varepsilon \phi_{cl}] - I[g_0] \stackrel{\mathcal{O}(\varepsilon^2)}{=} \delta I_{\text{WDW}} + I_{\delta\text{WDW}}$$

background fields on deformed WDW

fields variations on undeformed WDW

The terms contributing a finite piece

$$\delta I = \delta I_{EH} + \delta I_{\kappa} + \delta I_{CT} + \delta I_{\phi}$$

$$\delta I_{\text{gravity}} = 0$$

- Counterterm is essential for cancellation
- Not a general feature (non spherical setups, excited target state, …)

[Hashemi,Jafari,Naseh]

CA Variation

$$\delta\mathcal{C}_A = \frac{\delta I_\phi}{\pi} = -\frac{\varepsilon^2}{64\pi G_N}\int_{\partial\mathrm{WDW}} d\lambda\, d^{d-1}\Omega\,\sqrt{\gamma}\,\partial_\lambda\phi_{cl}^2$$

- Variation completely determined by the matter
- Purely $O(\varepsilon^2)$ contribution
- Scale independent: UV finite and independent of ℓ_{CT}
- Irrespectively of the cancellations, variation of each contribution to the action is localized on the boundary of undeformed WDW patch

CV Variation

$$\delta\mathcal{V} = \mathcal{V}_{\max}[g_0 + \varepsilon^2\delta g] - \mathcal{V}_{\max}[g_0] \overset{\mathcal{O}(\varepsilon^2)}{=} \delta\mathcal{V}_{\mathcal{B}_0} + \mathcal{V}_{\delta X^\mu}$$

0 ~ E.O.M.

δg on undeformed surface at t_Σ

deformation of the extremal surface

$$\delta\mathcal{C}_V = \frac{\delta\mathcal{V}}{G_N\ell} = \frac{\varepsilon^2}{(d-1)G_N\ell}\int_{t_\Sigma}\sqrt{h}\cos\rho^2 T_{tt}^{bulk}$$

- Purely $O(\varepsilon^2)$ contribution
- Scale independent and UV finite
- Expressed in terms of the bulk stress tensor

See also

[Belin,Lewkowycz,Sarosi]
[Jacobson,Visser]

CA-CV Comparison

Multi-mode coherent state $\phi_{cl} = \sum_{\{j\}} 2\,\alpha_j \cos(\omega_j t)\, e_j(\rho)$ $\omega_j = 2j + \Delta$

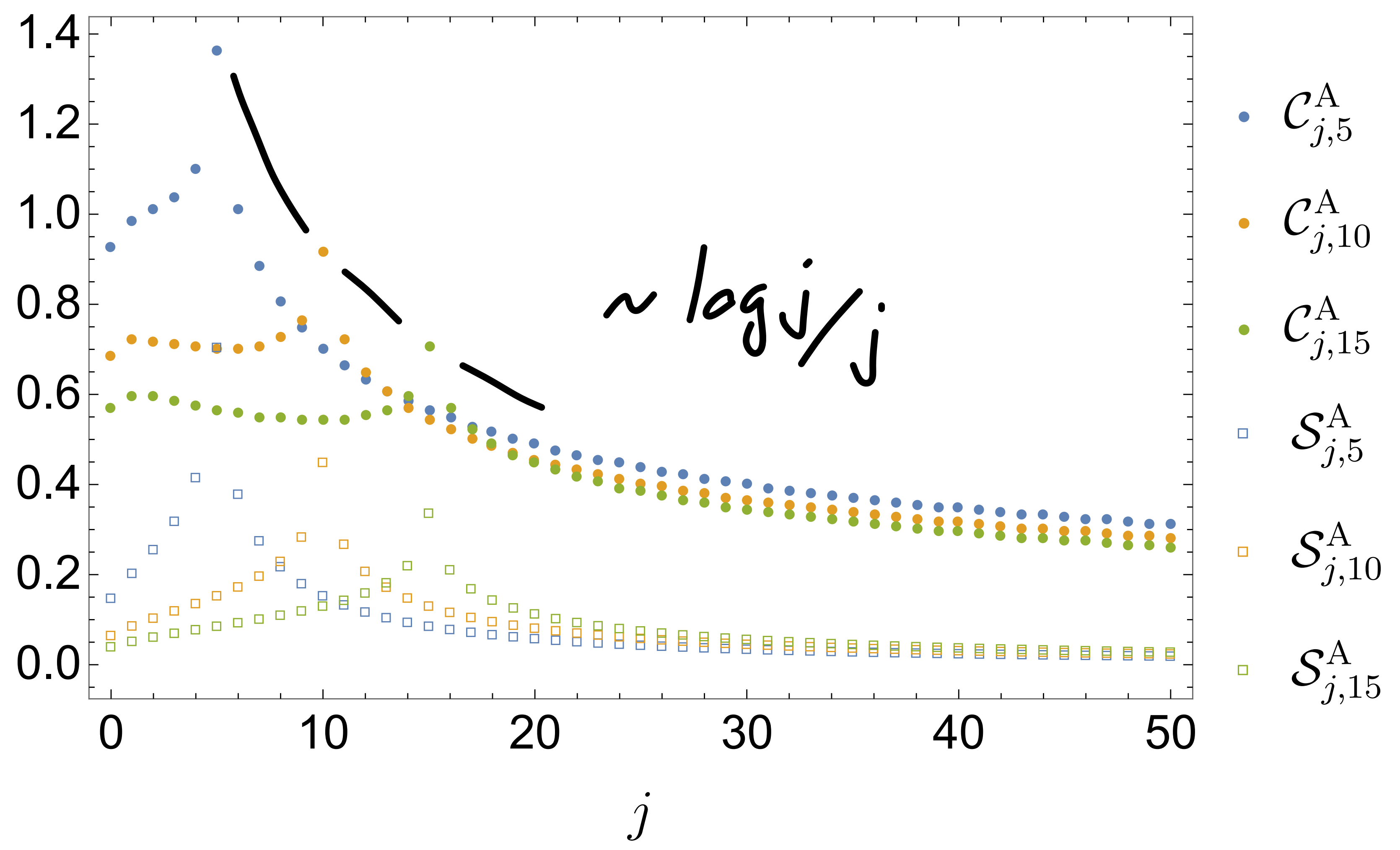
$$\delta\mathcal{C}_A = \varepsilon^2 \sum_{\{j,k\}} \alpha_j \alpha_k \left[\cos\omega_j t\ \cos\omega_k t\, C^A_{jk} + \sin\omega_j t\ \sin\omega_k t\, S^A_{jk}\right]$$

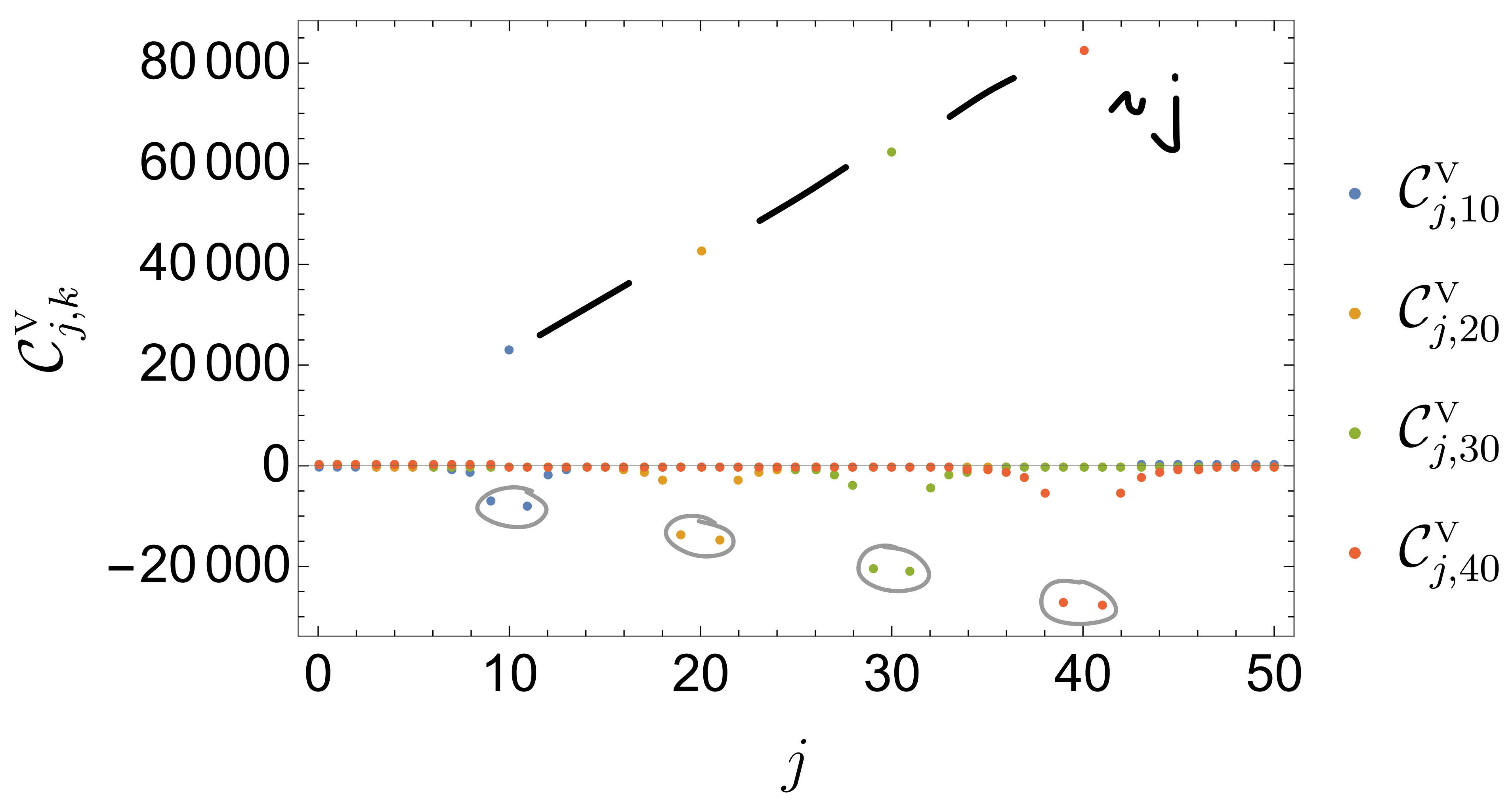
$$\delta\mathcal{C}_V = \varepsilon^2 \sum_{\{j,k\}} \alpha_j \alpha_k \left[\cos\omega_j t\ \cos\omega_k t\, C^V_{jk} + \sin\omega_j t\ \sin\omega_k t\, S^V_{jk}\right]$$

- Same functional oscillatory structure
- Mode mixing
- Scale independent coefficients C_{ij}, S_{ij}

CA-CV Comparison

- Different structure for the coefficients:





Different behaviour for CA and CV under perturbations

Similar conclusion to what found for "perturbations" given by defects

[Chapman,Ge,Policastro]
[Braccia,Cotrone,Tonni]

QFT Complexity Variation

Extension of the coherent states analysis of [Guo,Hernandez,Myers,Ruan] to general Gaussian states with non-vanishing first momenta

$$\delta\mathcal{C}_{\kappa=2} = \mathcal{C}_{Coh} - \mathcal{C}_{GS} = \varepsilon^2 \sum_{\{j\}} \alpha_j^2 [\cos^2 \omega_j t C_j^{\kappa=2} + \sin^2 \omega_j t S_j^{\kappa=2}]$$

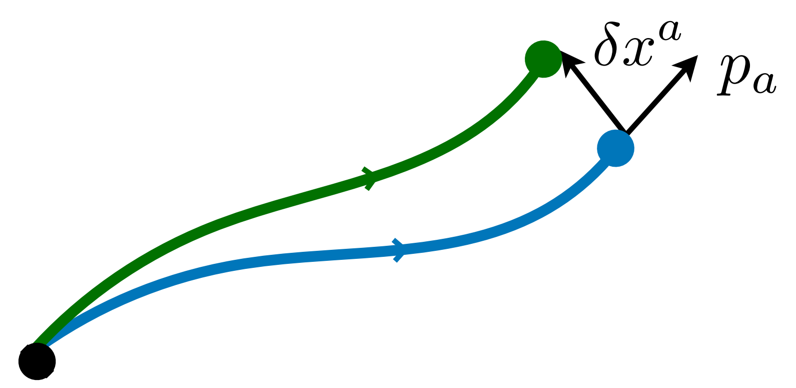
- Purely $O(\varepsilon^2)$ contribution
- Similar oscillatory structure as the holographic $\delta C_{A/V}$
- Diagonal: no mode mixing
- Dependence on various scales of the circuit models: memory of initial state and trajectory

$$C_j^{\kappa=2} = \frac{\log \frac{\omega_j}{\mu}}{\omega_j - \mu} f_j^2, \qquad S_j^{\kappa=2} = C_j^{\kappa=2} \frac{\omega_j^3}{\mu f_j^4}$$

REFERENCE STATE SCALE

GATE SCALE

Remarks



- $\delta\mathcal{C} \sim \varepsilon^2\alpha^2 \Rightarrow p_a\delta x^a|_{s=1} = 0$

  coherent state directions are orthogonal to the direction along the circuit preparing the CFT vacuum

- Holographic complexity variation: localized on the boundary of the WDW patch. $\partial\mathrm{WDW}$ = end of the quantum circuit?

- Diagonal vs mixed terms: holographic complexity may require more complicated cost functions?

Complexity with Rotation

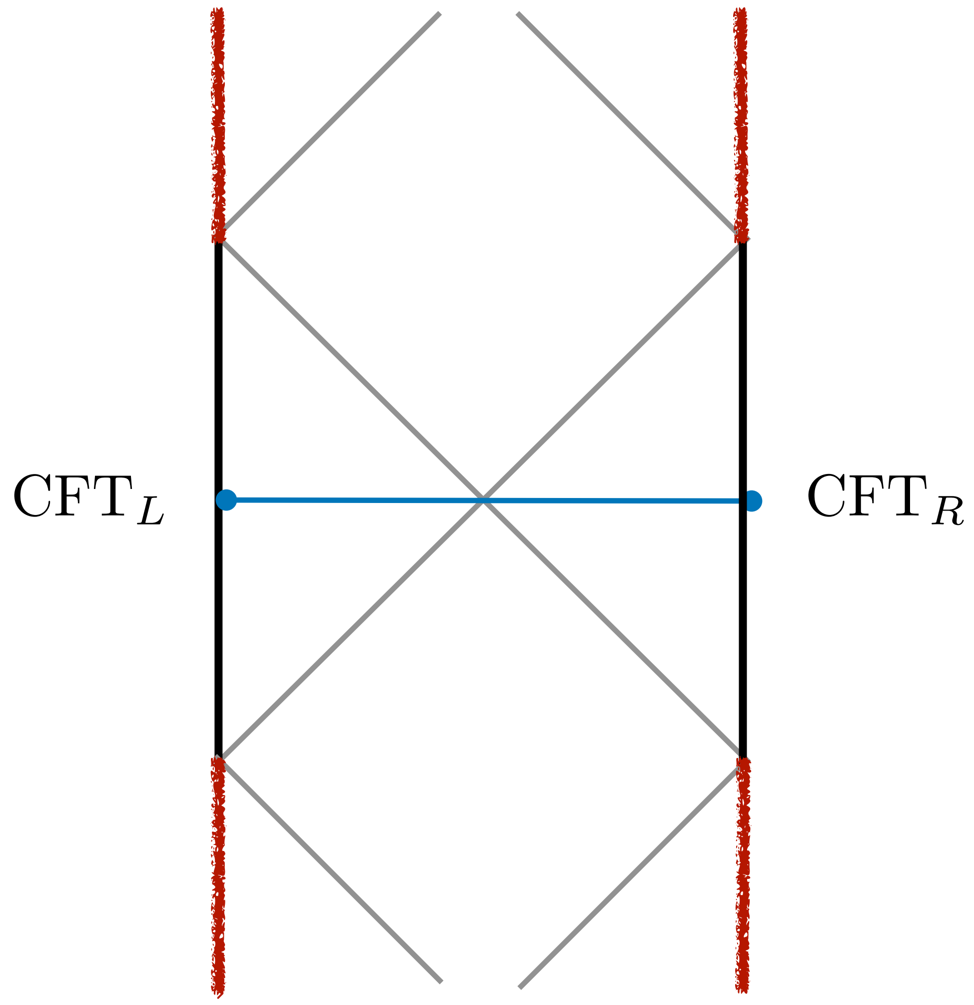
A. Bernamonti, F. Bigazzi, D. Billo, L. Faggi and FG
arXiv: 2108.09281

Motivations

- Mostly highly symmetric setups, mainly planar or spherically symmetric. Systems with rotation so far less understood
- Extra parameters and gravitational features to test complexity
- Interesting limits to probe, e.g., rotation with critical (speed of light) velocity

- Few holographic results: deserve to be studied further and extended (see later)
- QFT Nielsen's circuit complexity?

$$|rTFD\rangle = \frac{1}{\sqrt{Z(\beta,\Omega)}} \sum_n e^{-\frac{\beta}{2}(E_n+\Omega J_n)} |E_n, J_n\rangle_L |E_n, J_n\rangle_R$$



Rotating TFD

Simple model: two copies of a free boson on a circle

$$|rTFD\rangle = \frac{1}{\sqrt{Z(\beta,\Omega)}}\sum_n e^{-\frac{\beta}{2}(E_n+\Omega J_n)-it(E_n+\Omega J_n)}|E_n,J_n\rangle_L|E_n,J_n\rangle_R$$

Particle number simultaneously labels Hamiltonian and momentum eigenstates

$$H = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\, \frac{\pi^2}{2} + \frac{m^2}{2}\phi^2 + \frac{1}{2}(\partial_x\phi)^2 = \sum_k \omega_k \left(a_k^\dagger a_k + \frac{1}{2}\right)$$

$$J = -\int_{-\frac{L}{2}}^{\frac{L}{2}} dx\; \pi\partial_x\phi = \sum_k p_k \left(a_k^\dagger a_k + \frac{1}{2}\right)$$

Rotating TFD

Gaussian state: factorizes into single-mode rotating TFD states

$$|rTFD\rangle = \otimes|rTFD\rangle_k \qquad |rTFD\rangle_k = \frac{1}{\sqrt{Z_k(\beta,\Omega)}}\sum_n e^{-\left(\frac{\beta}{2}+it\right)(\omega_k+\ \Omega p_k)(n+\frac{1}{2})}|n\rangle_{k,L}|n\rangle_{k,R}$$

can be written as TFD state with no rotation and mode-dependent effective temperature and time

$$|rTFD(\beta,t)\rangle_k = \frac{1}{\sqrt{Z_k(\beta,\Omega)}}\sum_n e^{-\left(\frac{\beta}{2}+it\right)(\omega_k+\ \Omega p_k)(n+\frac{1}{2})}|n\rangle_{k,L}|n\rangle_{k,R} \ = \frac{1}{\sqrt{Z_k(\beta_k)}}\sum_n e^{-\left(\frac{\beta_k}{2}+it_k\right)\omega_k(n+\frac{1}{2})}|n\rangle_{k,L}|n\rangle_{k,R} = |TFD(\beta_k,t_k)\rangle_k$$

with $\beta_k = \beta\left(1+\Omega\frac{p_k}{\omega_k}\right)$ $\qquad t_k = t\left(1+\Omega\frac{p_k}{\omega_k}\right)$ $\qquad \beta_k$ non-negative for $\Omega < 1$

Rotating TFD state can be given an effective description in terms of non-rotating TFD states

Rotating TFD Complexity

TFD circuit complexity: $F_{\kappa=2}$ optimal circuit does not mix modes and complexity obtained as the sum of complexities evaluated for each mode separately

$$\mathcal{C}_{\kappa=2} = \frac{1}{4}\sum_k \log^2\left(f_k^{(+)} + \sqrt{\left(f_k^{(+)}\right)^2 - 1}\right) + \log^2\left(f_k^{(-)} + \sqrt{\left(f_k^{(-)}\right)^2 - 1}\right)$$

$$f_k^{(\pm)} = \frac{1}{2}\left(\frac{\mu}{\omega_k} + \frac{\omega_k}{\mu}\right)\cosh 2\alpha_k \pm \frac{1}{2}\left(\frac{\mu}{\omega_k} - \frac{\omega_k}{\mu}\right)\sinh 2\alpha_k \cos\omega_k t \qquad \alpha_k = \frac{1}{2}\log\left(\frac{1+e^{-\beta\omega_k/2}}{1-e^{-\beta\omega_k/2}}\right)$$

↑ REFERENCE STATE SCALE

[Chapman,Eisert,Hackl,Heller,Jefferson,Marrochio,Myers]

Consequence: we can estimate the complexity of the rotating TFD state as the sum of complexity of single mode TFD states each with inverse temperature β_k and time t_k

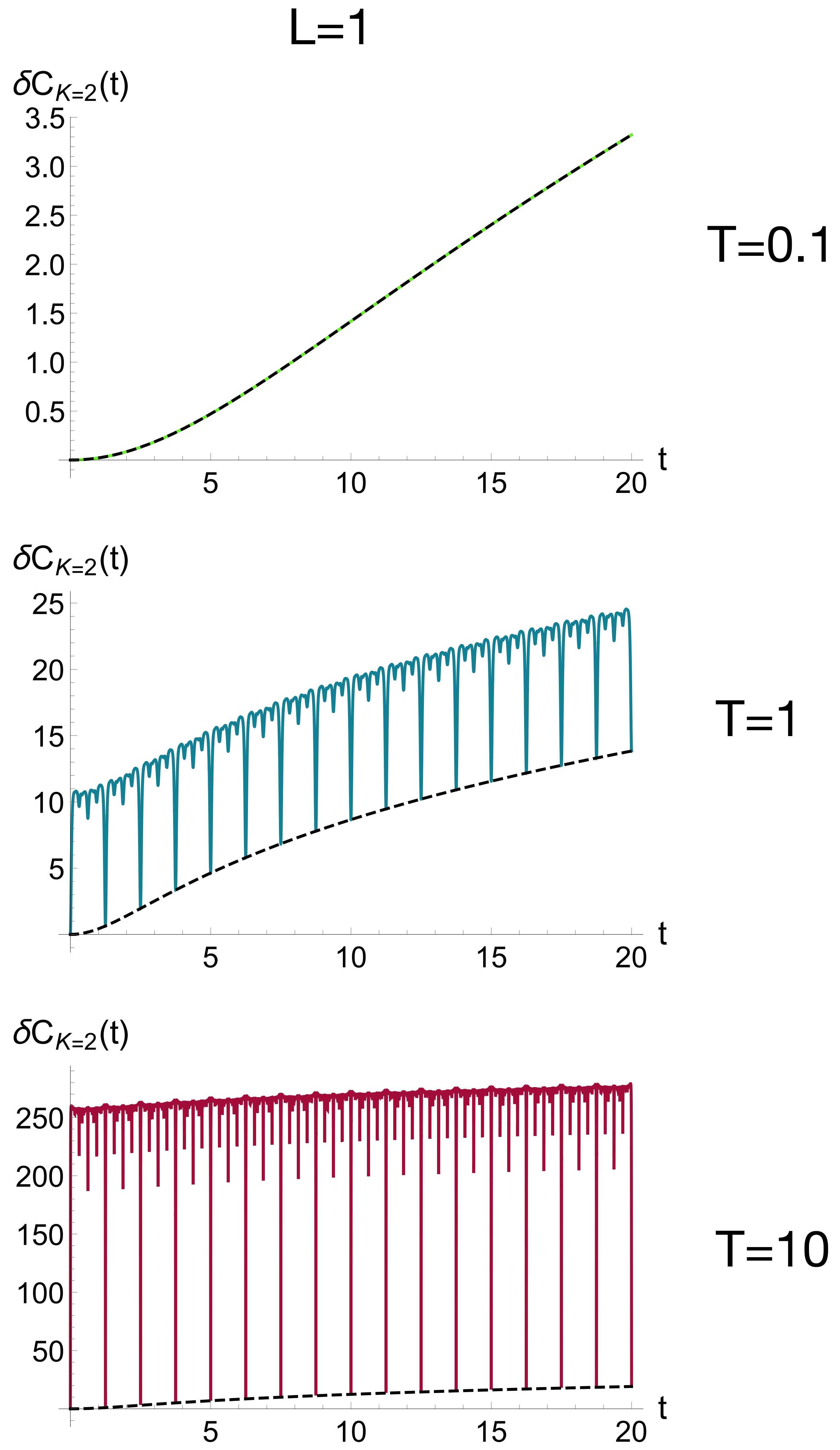
Time Dependence

Time variation $\mathcal{C}_{\kappa=2}(t) - \mathcal{C}_{\kappa=2}(0)$

- Low temperature: zero mode dominated
- Oscillations vs holographic linear increase: L-R moving d.o.f. propagating freely on the circle. Periodicity:

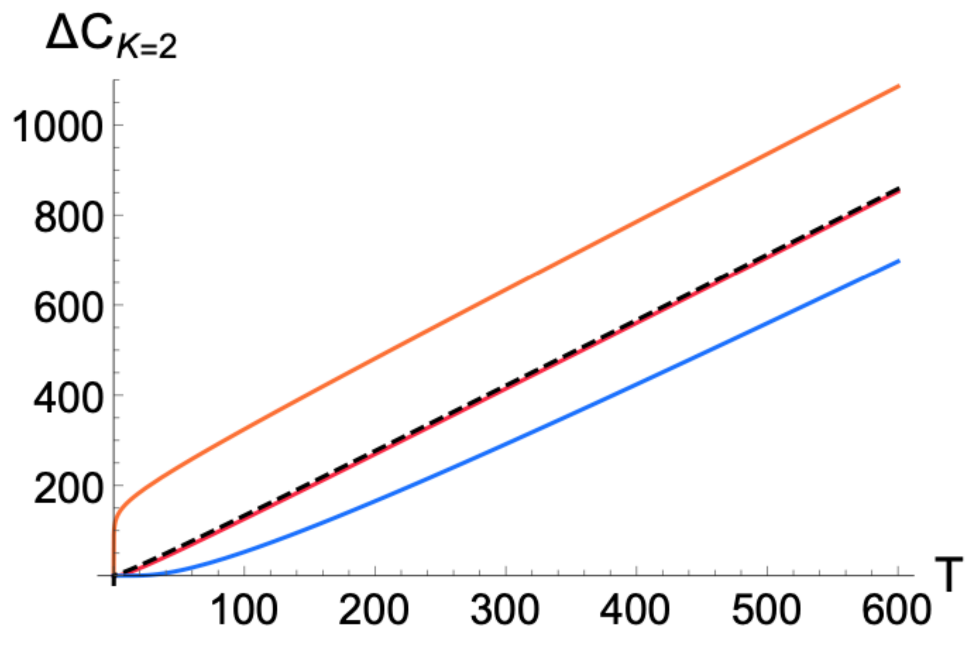
$$t \sim t + \frac{L}{2|k|(1 \pm \Omega)}$$

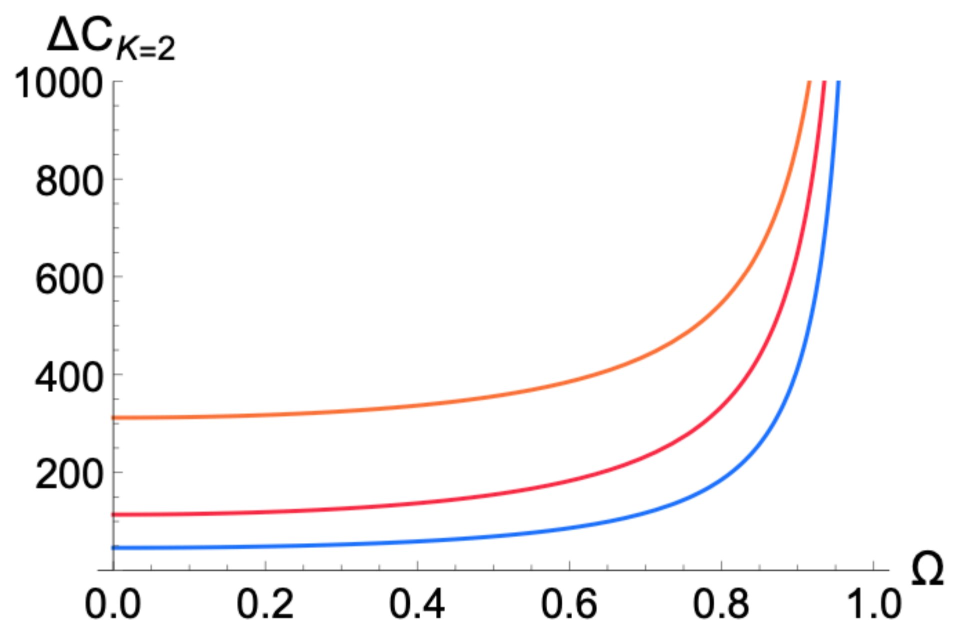
- High T: amplitude of different modes become comparable



Complexity of Formation

$$\Delta\mathcal{C}_{\kappa=2} = \mathcal{C}\left(|rTFD\rangle\right) - \mathcal{C}\left(|0\rangle_L|0\rangle_R\right) = 2\sum \text{arctanh}^2 e^{-\beta_k\omega_k/2}$$





- Scale independent
- Positive defined

High temperature and critical angular velocity $\Omega \to 1$ divergences:

$$\Delta\mathcal{C}_{\kappa=2} \sim \frac{7\zeta(3)}{2\pi}\frac{T}{1-\Omega^2} - \frac{1}{2}\log^2 T + \dots \qquad \text{as } T \to \infty$$

$$\Delta\mathcal{C}_{\kappa=2} \sim \frac{7\zeta(3)}{4\pi}\frac{T}{1-\Omega} - \frac{1}{2}\log^2 \frac{1}{1-\Omega} + \dots \qquad \text{as } \Omega \to 1$$

Holography: Review

The first estimates of late time holographic complexity growth rate -before complete understanding of action contributions of null boundaries- was given in [Brown,Roberts,Susskind,Swingle,Zhao][Cai,Ruan,Wang,Yang,Peng]

$$\lim_{t\to\infty} \frac{d\mathcal{C}}{dt} \sim (M - \Omega_+ J) - (M - \Omega_- J)$$

Lower dimensional case revisited in [Auzzi,Baiguera,Nardella et al.]: full time dependence of CV and CA growth rate

- Counterterm was not included: does it play any role?
- Complexity of formation i.e., cost of preparing the rotating state as compared to the cost of preparing vacuum?

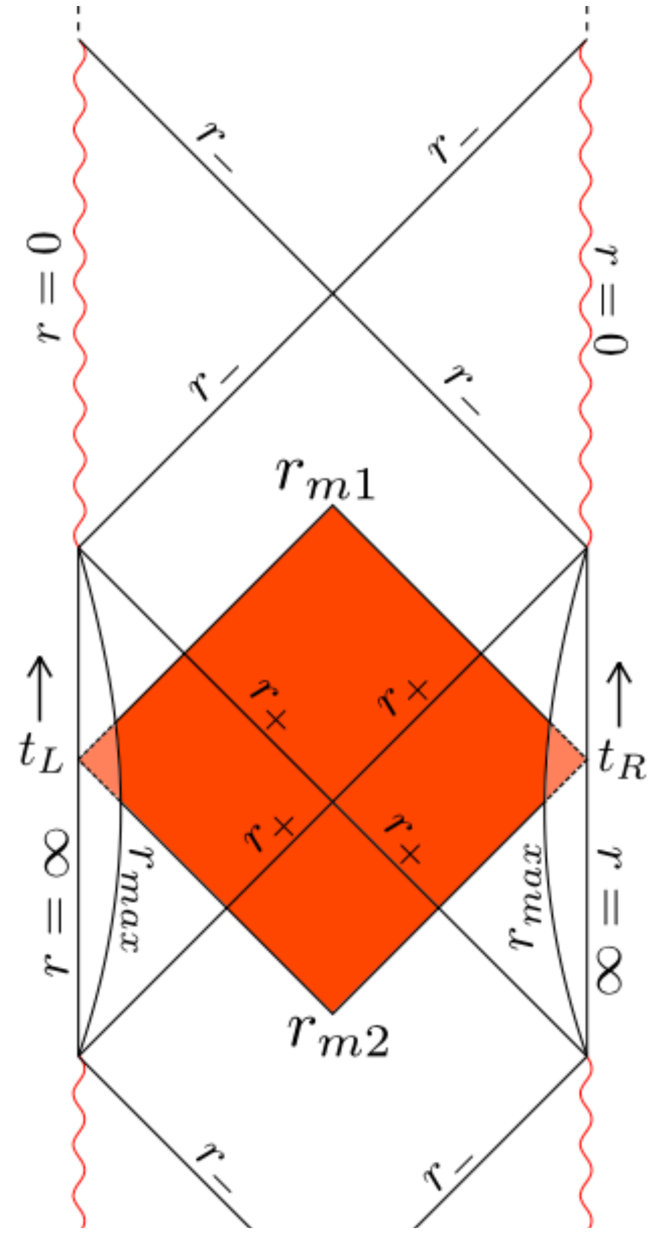
Rotating BTZ

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\varphi - \omega(r)\,dt\right)^2$$

$$f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} \qquad M = \frac{r_+^2 + r_-^2}{8} \qquad J = \frac{r_+ r_-}{4}$$

$$\omega(r) = \frac{r_+ r_-}{r^2} \qquad T = \frac{r_+^2 - r_-^2}{2\pi r_+} \qquad \Omega = \frac{r_-}{r_+}$$

$J < M$

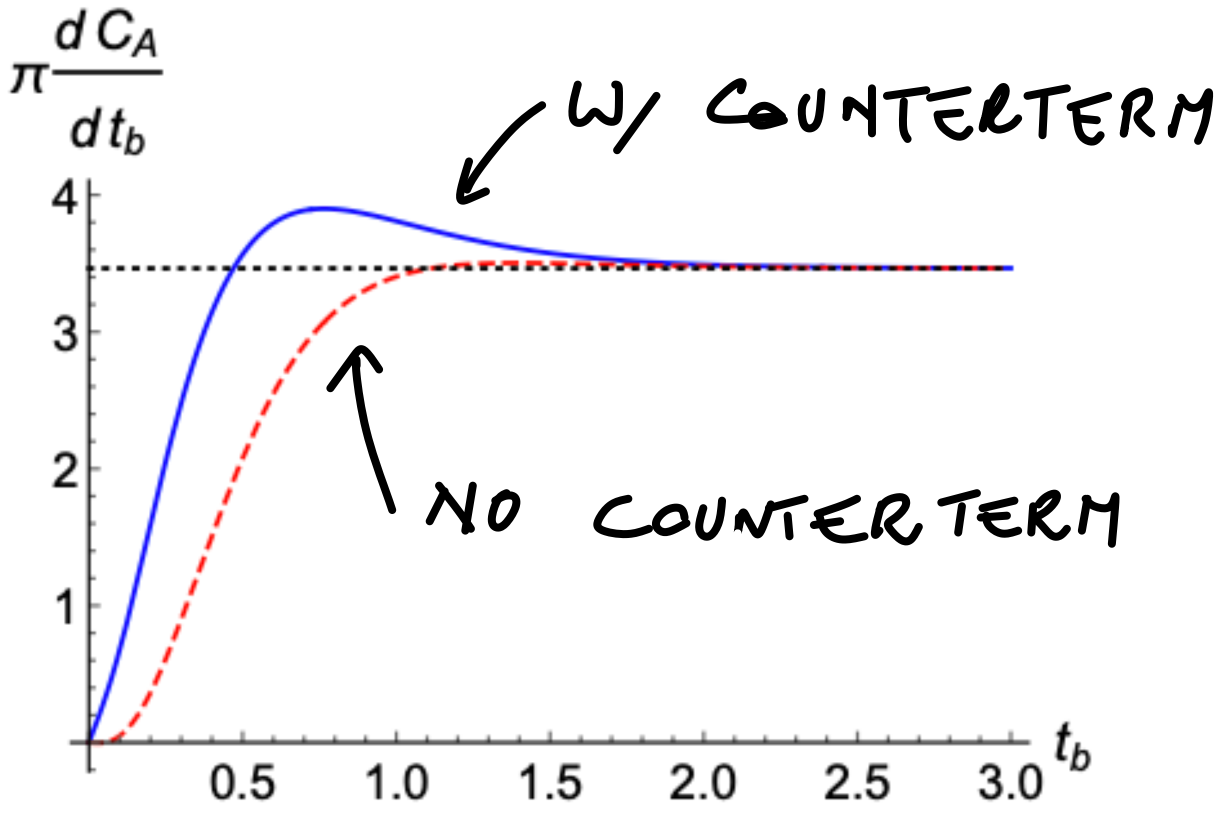


$t_L = t_R = \frac{t}{2}$

Revisited time dependence of holographic complexity taking into account all terms in the WDW action

- Inclusion of counterterm does not alter qualitative behaviour
- Expected late time linear growth

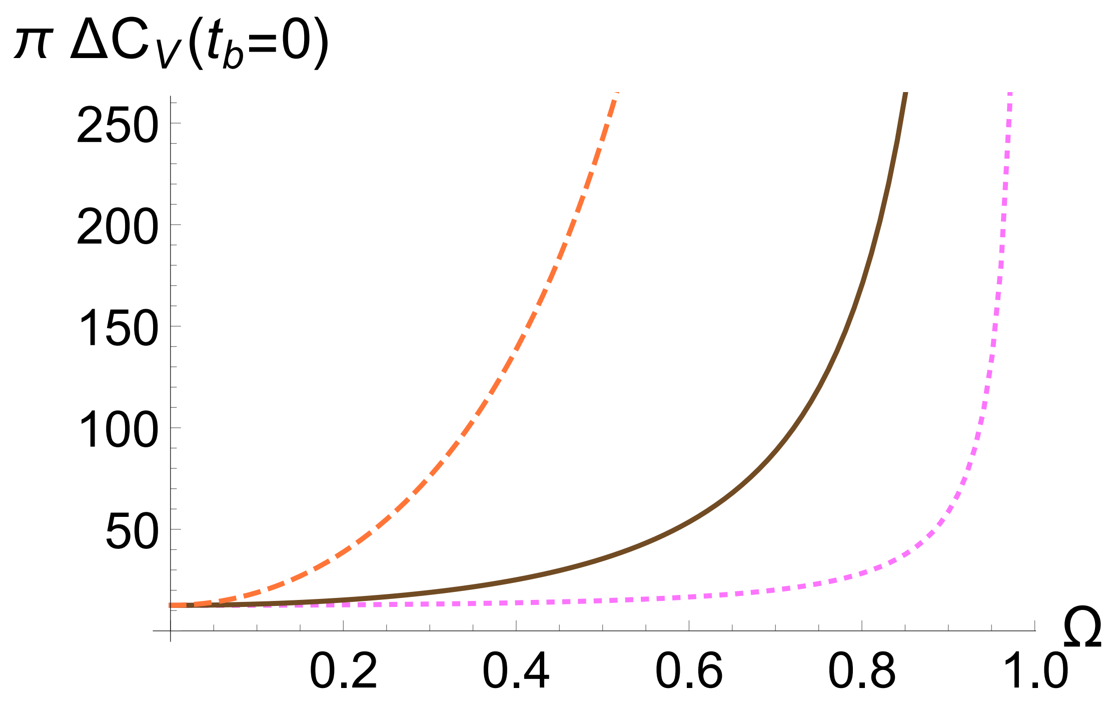
$$\lim_{t\to\infty} \frac{dC}{dt} \sim M - \Omega J$$

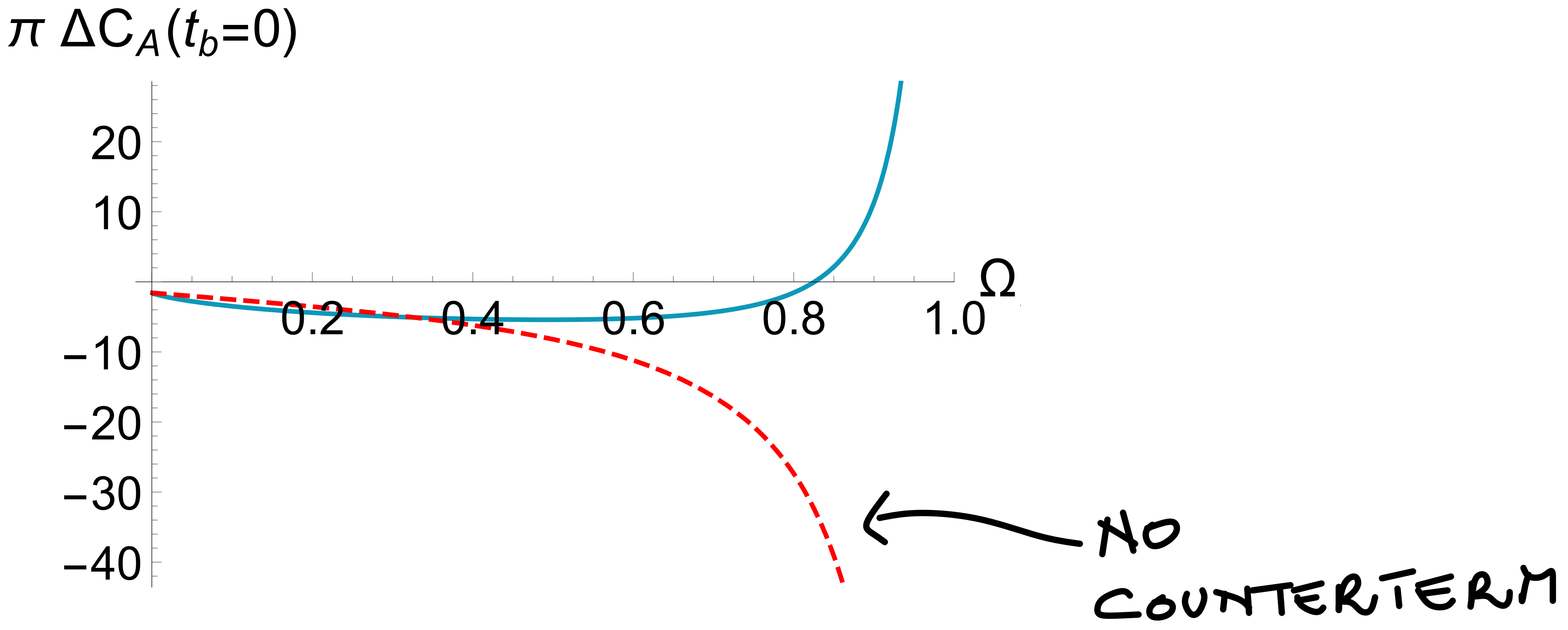


Rotating BTZ

To compare with the QFT results focus on complexity of formation $\Delta\mathcal{C} = \mathcal{C}(t=0) - 2\mathcal{C}_{\text{AdS}}$

Ω dependence:





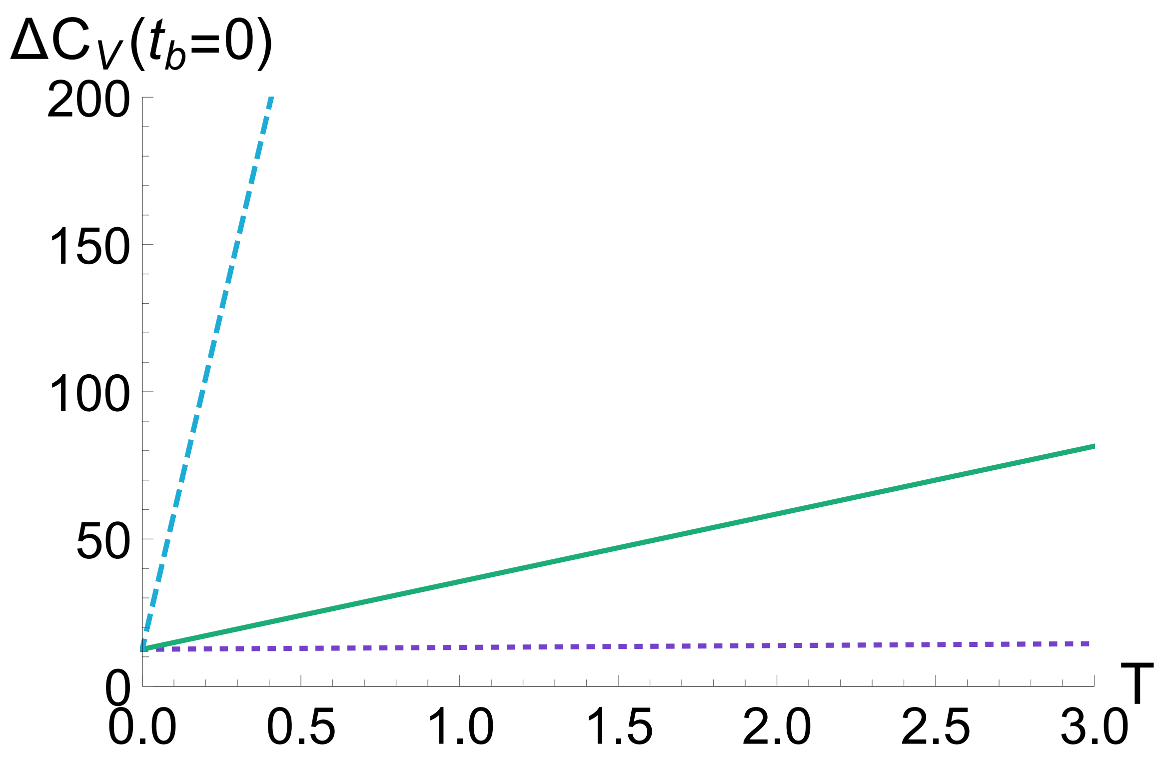
- CA and CV have the same $\Omega \to 1$ divergences when the counterterm is included

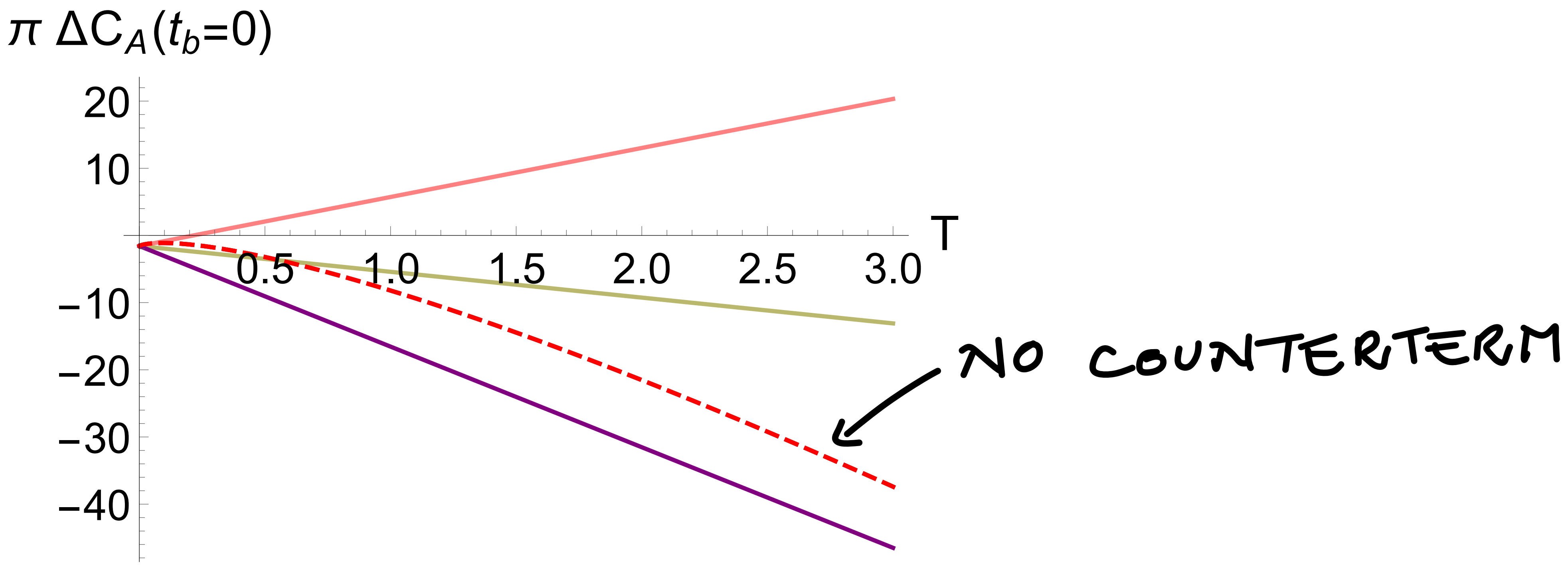
$$\Delta C \sim \frac{T}{1-\Omega} \log \frac{1}{1-\Omega} \, .$$

- Without countertem, same form but opposite sign
- Extra log factor as compared to QFT divergence

Rotating BTZ

T dependence:





- Matching linear behaviour for CV and CA only when counterterm is included
- Same linear divergence as for free QFT complexity at high T
- In tension with the hypothesis of a *third law of holographic complexity*:
  finite complexity of formation at zero T vs logarithmic divergence of higher dimensional charged and Myers-Perry black holes [CarmiChapmanMarrochioMyersSugishita]
  [AlBalushi,Hennigar,Kunduri,Mann]

Holography: Review

Higher dimensional rotating black holes: much harder technical task

Notable exception: odd-dimensional Myers-Perry AdS black holes with equal angular momenta in each orthogonal plane [AlBalushi,Hennigar,Kunduri,Mann]

Highlighted connection between holographic complexity and thermodynamic volume in the large black hole limit $r_+ \gg \ell$

- complexity of formation controlled by the thermodynamic volume

$$\Delta\mathcal{C} \sim S \log \frac{\Omega}{T} + \tilde{f}\, V_+^{\frac{D-2}{D-1}}$$

- late time growth rate

$$\lim_{t\to\infty} \frac{d\mathcal{C}}{dt} \sim P\,(V_+ - V_-)$$

[Couch,Fischler,Nguyen]

Kerr-AdS4

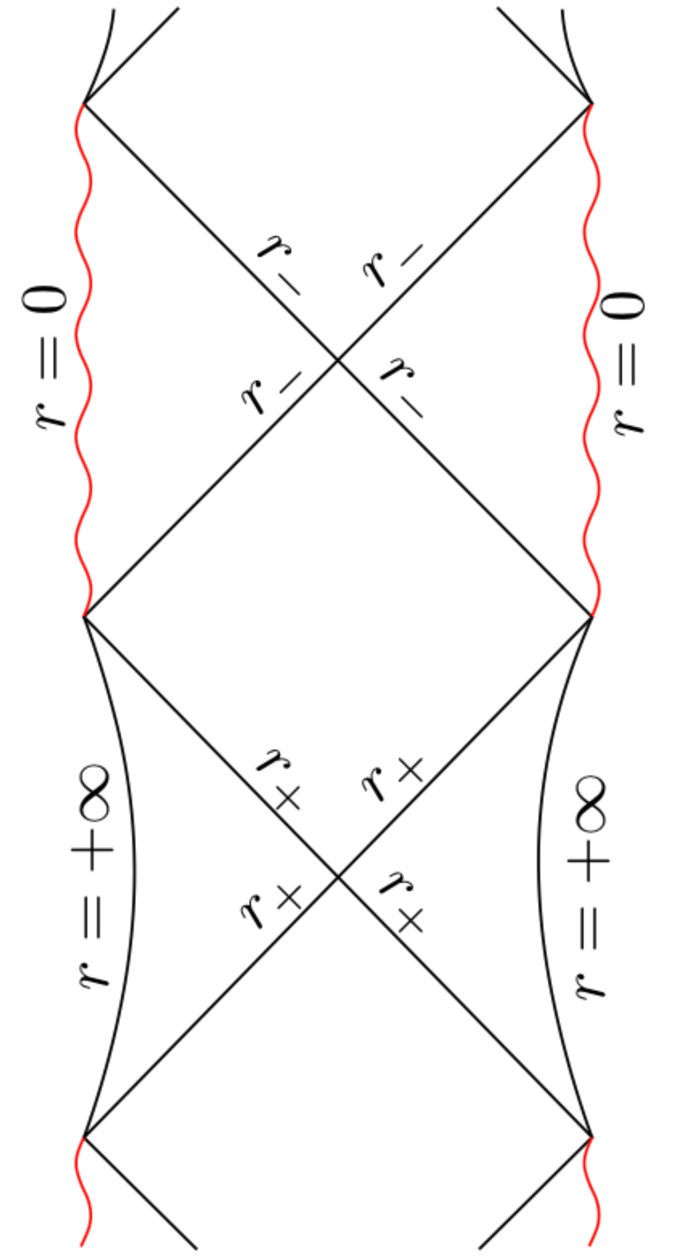
$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - \frac{a}{\Xi}\sin^2\theta d\varphi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\left(adt - \frac{r^2+a^2}{\Xi}d\varphi\right)^2$$

$$\Delta = (r^2+a^2)\left(1+\frac{r^2}{\ell^2}\right) - 2mr \qquad \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Xi = 1 - \frac{a^2}{\ell^2}$$

$$M = \frac{m}{G_N\,\Xi^2} \qquad J = aM$$

$$\Omega = \frac{a}{\ell}\frac{r_+^2+\ell^2}{r_+^2+a^2} \qquad T = \frac{r_+}{4\pi\left(r_+^2+a^2\right)}\left(1+\frac{a^2}{\ell^2}+3\frac{r_+^2}{\ell^2}-\frac{a^2}{r_+^2}\right)$$



$\theta = \pi/2$

Reduced symmetry but some partial progress

Kerr-AdS4

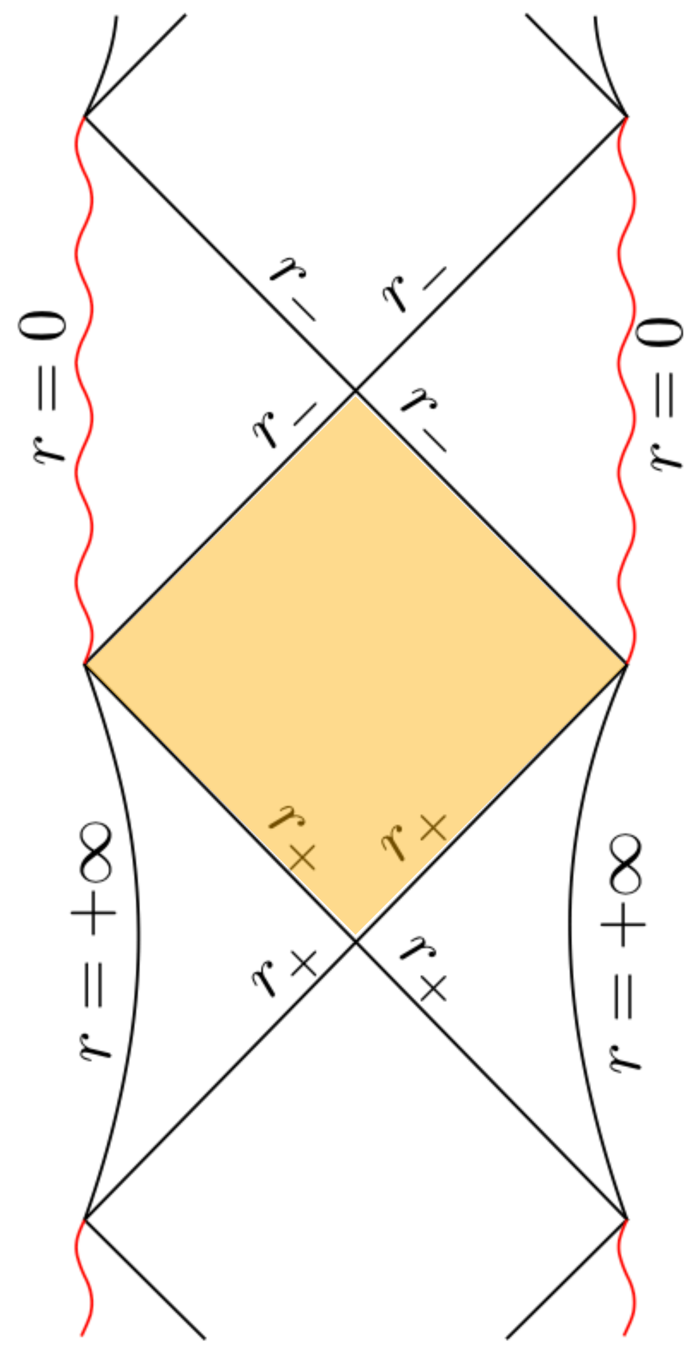
Recent analysis of null hypersurfaces [AlBalushi,Mann]: implicit description of the WDW patch

Precise treatment of the late time limit of the CA growth rate

- Only non-vanishing contributions: EH term and null-null joints at the past and future tips of the WDW patch
- Reproduces expected answer [Cai,Ruan,Wang,Yang,Peng]

$$\lim_{t\to\infty} \pi \frac{dC_{\rm A}}{dt} = \frac{r_+^3 - r_-^3 + \ell^2 \left(r_+ - r_-\right)}{2\, G_N \left(\ell^2 - a^2\right)} = (M - \Omega_+ J) - (M - \Omega_- J)$$

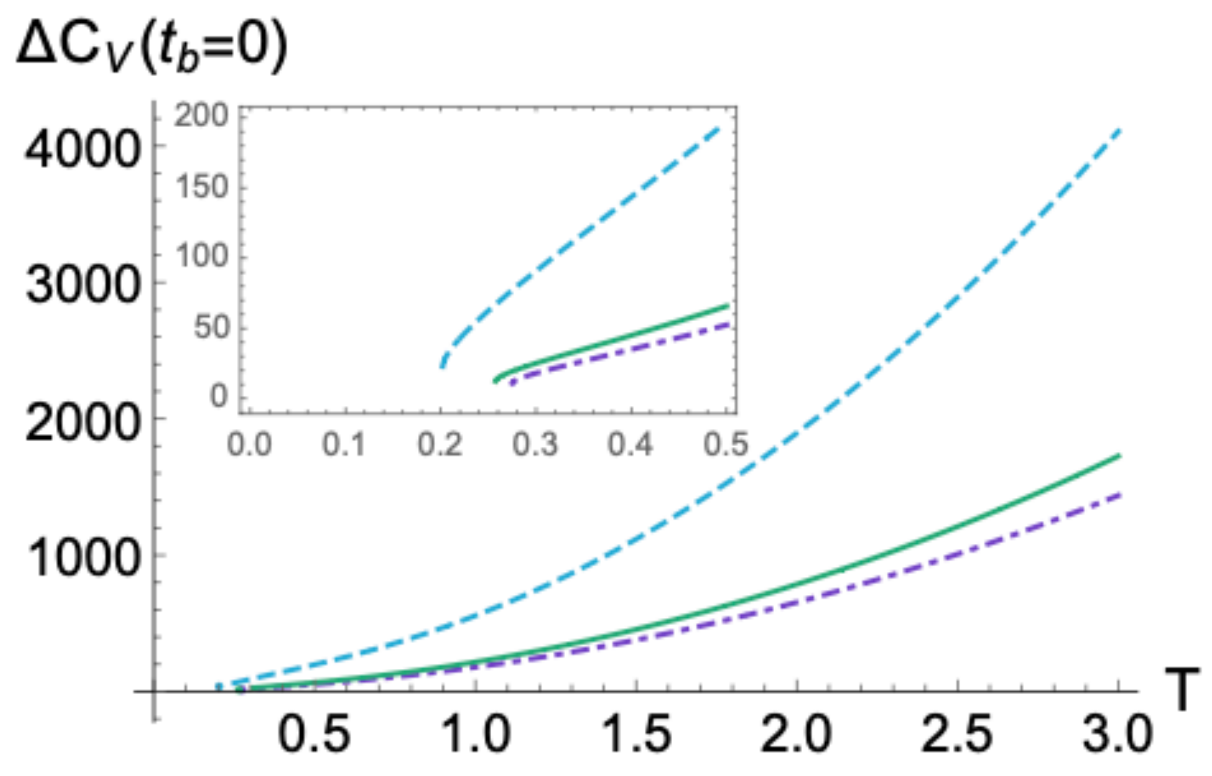
- Proportional to the difference of thermodynamic volumes in the large black hole limit $r_+ \gg \ell$

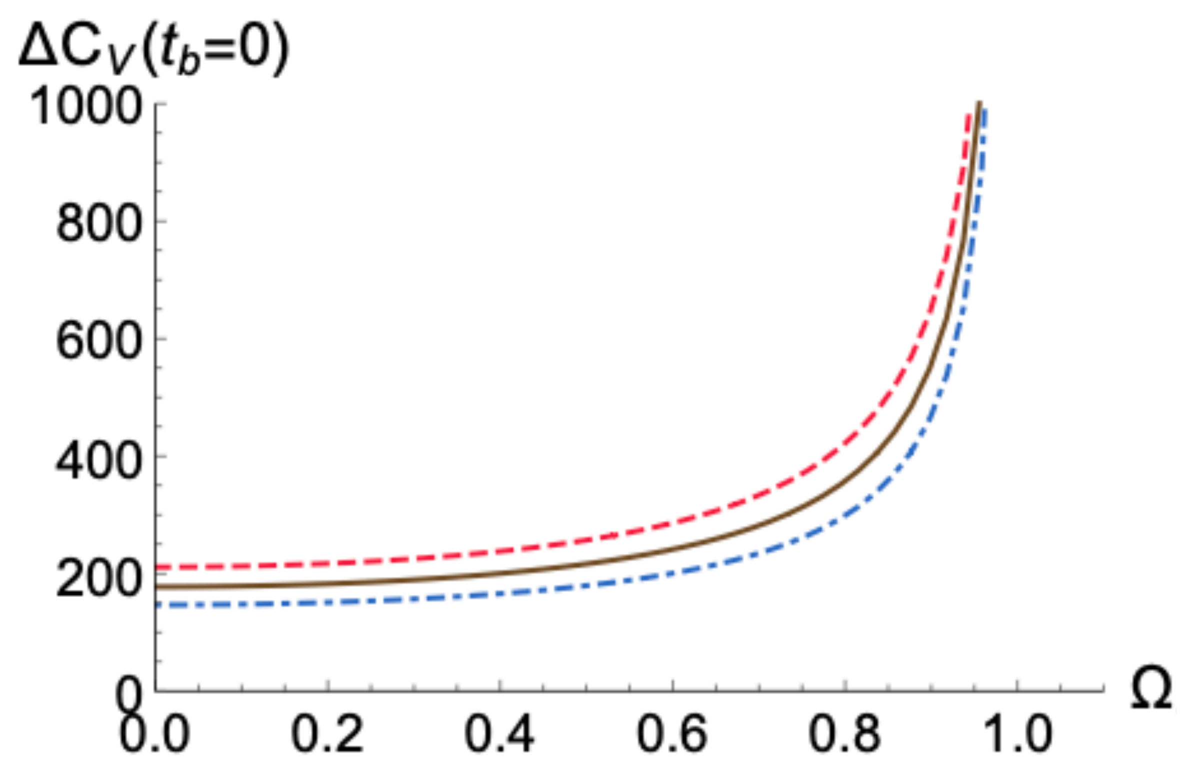


$\theta = \pi/2$

Kerr-AdS4

CV complexity of formation evaluated numerically





- Diverges at high T and for critical angular velocity $\Omega \to 1$
- Compatibility with the claim of [AlBalushi,Hennigar,Kunduri,Mann] on the scaling of the complexity of formation with the thermodynamic volume
- Unable to test it independently. Scaling of S and V fixed in the region of parameters corresponding to physical solutions (extremal solutions have superluminal rotation)

Remarks

- Map between rotating and non-rotating TFD in simple scalar toy model
- Holographic complexity: counterterm essential to match CA and CV behaviour for rotating BTZ black holes
- Qualitative agreement between QFT and holographic complexity results in the high temperature and critical angular velocity limit
- QFT complexity of formation scale independent while CA depends on the counterterm scale

Thank you!